ECE 602 A4

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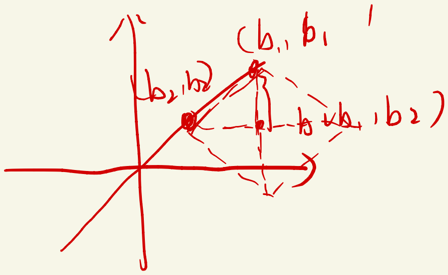
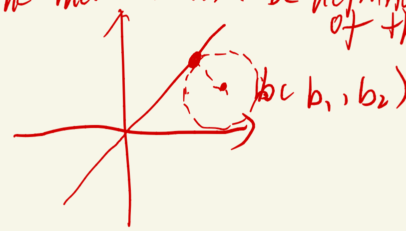
Exercise 1

① l_1-norm: the median of the coefficients of b.



we can see that in 2-D space any points between (b_2, b_2) and (b_1, b_1) will lead to a optimal solution, which means x can be in range $[b_1, b_2]$.
As the dimensen increase, we can see that the median of b will definitely lead to best result.
It could not be only one result, but the median will be definitely one of them.

② l_2-norm: the average $1^T b / m$.



m is the dimension of the space.

For example, in two dimension, $f(x) = x$ and minimize $\|1x - b\|_2$ means the shortest length from b to f(x)

Assume, $y = -x + h$ going through point $b(b_1, b_2)$ we can get $h = b_1 + b_2$. To get the solution x, we need to solve $\begin{cases} y = x \\ y = -x + b_1 + b_2 \end{cases} \Rightarrow x = y = \frac{b_1 + b_2}{2}$

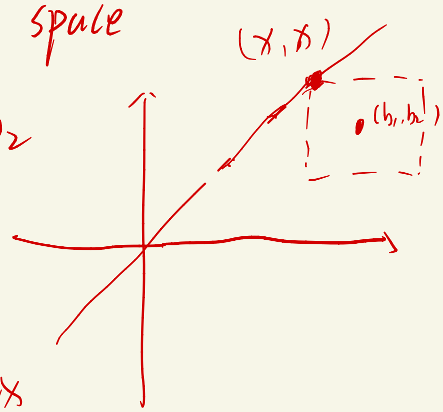
In higher dimension, use the same think method.

③ l_∞-norm: the midrange point $(\max b_i - \min b_i)/2$

for example, in 2-D space

we assume $b_1 > x > b_2$

In higher dimension, the optimal solution occur at the vertex



$b_1 - x = x - b_2$

$x = \frac{b_1 - b_2}{2}$

Exercise 3

(a) The robust least-squares problem, with interval matrix, is

minimize $\sup_{A \in \mathcal{A}} \|Ax-b\|_2$, which is equivalent

to minimize $\sup_{A \in \mathcal{A}} \|Ax-b\|_2^2$. The latter one can

be reformulated as minimize y^Ty

subject to $-y \le Ax-b \le y$, for all $A \in \mathcal{A}$

we also can easily get that $\sup_{A \in \mathcal{A}} (Ax-b)_i = \sum_{j=1}^{n} (\bar{A}_{ij} x_j + R_{ij}|x_j|) - b_i$

$\inf_{A \in \mathcal{A}} (Ax-b)_i = \sum_{j=1}^{n} (\bar{A}_{ij} x_j - R_{ij}|x_j|) - b_i$

Therefore, the original problem can be written as

$$\begin{aligned} &\text{minimize} \quad y^Ty \\ &\text{subject to} \quad \bar{A}x + R|x| - b \le y \\ &\qquad\qquad\quad \bar{A}x - R|x| - b \ge -y \end{aligned}$$

In QP form:

$$\begin{aligned} &\text{minimize} \quad y^Ty \\ &\text{subject to} \quad \bar{A}x + Rz - b \le y \\ &\qquad\qquad\quad \bar{A}x - Rz - b \ge -y \\ &\qquad\qquad\quad -z \le x \le z \end{aligned}$$

The variables are $x \in R^n$, $y \in R^m$, $z \in R^n$

(b) results shown in pdf file