Problem 1.

Given that RBF seed function is chosen to be Gaussian kernel, and the cost function J is square error,

we get $J = \frac{1}{2}\{y_d(n) - \sum_{k=1}^{N} w_k(n) \exp(-\frac{\|x(n) - c_k(n)\|^2}{2\sigma_k^2(n)})\}^2$

$$= \frac{1}{2}[y_d(n) - W^T(n) \cdot \psi(n)]^2$$

where $y_d(n)$ is the desired output at iteration (n)

$c_k(n)$ is the center vector for kth Gaussian kernel

$\sigma_k(n)$ is its spread parameter.

$w_k(n)$ is the weights between kth kernel node with output node

$x(n)$ is input.

$W(n) = [w_1(n), w_2(n), \cdots, w_N(n)]^T$.

$\psi(n) = [\phi\{x(n), c_1, \sigma_1\}, \phi\{x(n), c_2, \sigma_2\}, \cdots, \phi\{x(n), c_N, \sigma_N\}]$

ϕ is Gaussian kernel

$$w(n+1) = w(n) - \eta_w \frac{\partial}{\partial w} J \Big|_{W=W(n)}$$

$$= w(n) + \eta_w [y_d(n) - W^T(n) \cdot \psi(n)] \cdot \psi(n)$$

$$= w(n) + \eta_w e(n) \psi(n)$$

where $e(n) = y_d(n) - W^T(n) \cdot \psi(n)$

$$c_p(n+1) = c_p(n) - \eta_c \frac{\partial}{\partial c_p} J(n) \Big|_{c_p = c_p(n)}$$

$$= c_p(n) - \eta_c \frac{\partial}{\partial c_p(n)} \frac{1}{2} \left[y_d(n) - \sum_{p=1}^{N} w_p(n) \cdot \exp\left(-\frac{\|x(n) - c_p(n)\|^2}{2\sigma_p^2(n)}\right)\right]^2$$

$$\frac{\partial J(n)}{\partial c_p(n)} = \left[y_d - \sum_{p=1}^{N} w_p(n) \cdot \exp\left(-\frac{\|x(n) - c_p(n)\|^2}{2\sigma_p^2(n)}\right)\right] \cdot (-w_p(n)) \cdot$$

$$\exp\left(-\frac{\|x(n) - c_p(n)\|^2}{2\sigma_p^2(n)}\right) \cdot \left(-\frac{1}{2\sigma_p^2(n)}\right) \cdot \left[-2\,(x(n) - c_p(n))\right]$$

$$= e(n) \cdot (-w_p(n)) \cdot \left(\frac{1}{\sigma_p^2(n)}\right) \cdot \phi\left[x(n), c_p(n), \sigma_p\right] \cdot \left[x(n) - c_p(n)\right]$$

where $e(n) = y_d(n) - W^T(n) \cdot \varphi(n)$

$$\therefore c_p(n+1) = c_p(n) + \eta_c \frac{e(n)\, w_p(n)}{\sigma_p^2(n)} \cdot \phi\left[x(n), c_p(n), \sigma_p\right] \cdot \left[x(n) - c_p(n)\right]$$

$$\sigma_p(n+1) = \sigma_p(n) - \eta_\sigma \frac{\partial}{\partial \sigma_p} J(n) \Big|_{\sigma_p = \sigma_p(n)}$$

$$= \sigma_p(n) - \eta_\sigma \frac{\partial}{\partial \sigma_p(n)} \frac{1}{2} \left[y_d(n) - \sum_{p=1}^{N} w_p(n) \cdot \exp\left(-\frac{\|x(n) - c_p(n)\|^2}{2\sigma_p^2(n)}\right)\right]^2$$

$$\frac{\partial J(n)}{\partial c_p(n)} = \left[y_d(n) - \sum_{p=1}^{N} w_p(n) \cdot \exp\left(-\frac{\|x(n) - c_p(n)\|^2}{2\sigma_p^2(n)}\right)\right] \cdot (-w_p(n)) \cdot$$

$$\exp\left(-\frac{\|x(n) - c_p(n)\|^2}{2\sigma^2}\right) \cdot \left(-\frac{\|x(n) - c_p(n)\|^2}{2}\right) \cdot (-2) \cdot (\sigma_p^{-3}(n))$$

$$= \frac{e(n) \cdot (-w_p(n))}{\sigma_p^3(n)} \cdot \phi\left[x(n), c_p(n), \sigma_p\right] \cdot \|x(n) - c_p(n)\|^2$$

thus $\sigma_p(n+1) = \sigma_p(n) + \eta_\sigma \dfrac{e(n)\, w_p(n)}{\sigma_p^3(n)} \cdot \phi\left[x(n), c_p(n), \sigma_p\right] \|x(n) - c_p(n)\|^2$

where $e(n) = y_d(n) - W^T(n) \cdot \varphi(n)$