1) $\int \frac{x^4+x^2-6x}{x^3}dx = \int \left(x+\frac{1}{x}-\frac{6}{x^2}\right)dx = \frac{x^2}{2} + \ln|x| + \frac{6}{x} + C$

2) $\int \cos 2x \, dx = \frac{1}{2} \sin 2x + C$

3) $\int \frac{dx}{(3x+2)^4} = \frac{1}{3} \cdot \frac{1}{-3} \cdot \frac{1}{(3x+2)^3} + C = -\frac{1}{9} \cdot \frac{1}{(3x+2)^3} + C$

4) $\int \frac{5x-1}{\sqrt{4-x^2}}dx = \int \frac{5x\,dx}{\sqrt{4-x^2}} - \int \frac{dx}{\sqrt{4-x^2}} = \frac{1}{2}\int \frac{5dt}{\sqrt{4-t}} - \arcsin\frac{x}{2} =$

$= -1 \cdot 2 \cdot \frac{1}{2} \cdot 5\sqrt{4-x^2} - \arcsin\frac{x}{2} + C = -5\sqrt{4-x^2} - \arcsin\frac{x}{2} + C$

5) $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = 2\int \frac{d\sqrt{x}}{(1+\sqrt{x})} = 2\int \frac{dt}{1+t} = 2\ln(1+\sqrt{x}) + C$

6) $\int x^2 \cos x \, dx = \left| \begin{matrix} u = x^2 \\ dv = d\sin x \\ v = \sin x \end{matrix} \right| = x^2 \sin x - \int 2x \sin x \, dx =$

$= \left| \begin{matrix} u = x \\ v = -\cos x \end{matrix} \right| = x^2 \sin x - 2\left(-x\cos x + \int \cos x \, dx\right) =$

$= x^2 \sin x + 2x\cos x - 2\sin x + C = (x^2-2)\sin x + 2x\cos x + C$

7) $\int \arctan x \, dx = \left| \begin{matrix} u = \arctan x \\ dv = dx \end{matrix} \right| = x \operatorname{arctg} x - \int \frac{x}{1+x^2}dx =$

$= x \cdot \operatorname{arctg} x - \frac{1}{2}\int \frac{dt}{1+t} = x \operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + C$

8) $\int \frac{dx}{(x-1)^5} = -\frac{1}{4(x-1)^4} + C$

9) $\int \frac{(x+6)}{x^2-2x+17}dx = \int \frac{(x+6)dx}{(x-1)^2+16} = \int \frac{(x-1)\,dx}{(x-1)^2+16} + 7\int \frac{dx}{(x-1)^2+16} =$

$= \frac{1}{2}\int \frac{d(x-1)^2}{(x-1)^2+16} + 7\int \frac{d(x-1)}{(x-1)^2+16} = \frac{1}{2}\ln\left(16+(x-1)^2\right) + \frac{7}{4}\operatorname{arctg}\frac{x-1}{4} + C$

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10) $\int \frac{x\,dx}{(x^2-1)(x^2+1)} = \frac{1}{2}\int \frac{dx^2}{(x^2-1)(x^2+1)} = \frac{1}{2}\int \frac{dt}{t^2-1} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| + C =$

$= -\frac{1}{4}\ln\left|\frac{1+x^2}{1-x^2}\right| + C$

11) $\int \frac{\sqrt[3]{x}\,dx}{\sqrt[3]{x^2}-\sqrt{x}} = \int \frac{x^{\frac{1}{3}}\,dx}{x^{\frac{2}{3}}-x^{\frac{1}{2}}} = \int \frac{dx}{x^{\frac{1}{3}}-x^{\frac{1}{6}}} = \int \frac{dx}{x^{\frac{1}{6}}(x^{\frac{1}{6}}-1)} =$

$= \left| \begin{array}{l} t = x^{\frac{1}{6}} \\ dt = \frac{1}{6}\cdot\frac{1}{x^{5/6}}\,dx \\ dx = 6x^{5/6}\,dt = 6t^5\,dt \end{array} \right| = 6\int \frac{t^5\,dt}{t(t-1)} = 6\int \frac{t^4\,dt}{t-1} = 6\int (t^3+t^2+t+1)\,dt +$

$+ 6\int \frac{dt}{t-1} = 6\left(\frac{t^4}{4}+\frac{t^3}{3}+\frac{t^2}{2}+t+\ln|t-1|\right) + C =$

$= \frac{3}{2}t^4 + 2t^3 + 3t^2 + 6t + 6\ln|t-1| + C =$

$= \frac{3}{2}\sqrt[3]{x^2} + 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|1-\sqrt[6]{x}| + C$

12) $\int \sqrt[3]{x}\cdot\sqrt[3]{1+3\sqrt[3]{x^2}}\,dx = \left| \begin{array}{l} \sqrt[3]{x^2} = t \\ dt = \frac{2}{3}\cdot\frac{1}{\sqrt[3]{x}}\,dx \\ dx = \frac{3}{2}\sqrt[3]{x}\,dt \end{array} \right| = \frac{3}{2}\int \sqrt[3]{x^2}\,\sqrt[3]{1+3\sqrt[3]{x^2}}\,dt =$

$= \frac{3}{2}\int t\,\sqrt[3]{1+3t}\,dt = \left| \begin{array}{l} 1+3t = y^3 \\ t = \frac{1}{3}(y^3-1) \\ dt = y^2\,dy \end{array} \right| = \frac{3}{2}\int \frac{1}{3}(y^3-1)\,y\,y^2\,dy =$

$= \frac{1}{2}\int (y^6-y^3)\,dy = \frac{1}{2}\left(\frac{y^7}{7}-\frac{y^4}{4}\right) + C = \frac{(1+3\sqrt[3]{x^2})^{7/3}}{14} - \frac{(1+3\sqrt[3]{x^2})^{4/3}}{8} + C$

13) $\int \frac{dx}{\sin x \sin 2x} = \int \frac{\cos x\,dx}{2\sin^2 x\cos^2 x} = \left| \begin{array}{l} t = \sin x \\ dt = \cos x\,dx \end{array} \right| = \frac{1}{2}\int \frac{dt}{t^2(1-t^2)} =$

$= \frac{1}{2}\int \frac{dt}{t^2} + \frac{1}{2}\int \frac{dt}{1-t^2} = -\frac{1}{2}\cdot\frac{1}{t} + \frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| + C =$

$= -\frac{1}{2}\cdot\frac{1}{\sin x} + \frac{1}{4}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$

14) $\int \cos 5x \cdot \cos 3x \, dx = \frac{1}{2}\int (\cos 2x + \cos 8x)\, dx =$

$= \frac{1}{2}\left(\frac{1}{2}\sin 2x + \frac{1}{8}\sin 8x\right) + C = \frac{1}{4}\sin 2x + \frac{1}{16}\sin 8x + C$

15) $\int_0^{\pi} (2\pi + \sin 2x)\, dx = x^2\Big|_0^{\pi} - \frac{1}{2}\cos 2x\Big|_0^{\pi} = \pi^2 - 0 = \pi^2$

16) $\int_{1/2}^{1} \sqrt{4x-2}\, dx = \frac{1}{4}\cdot\frac{2}{3}\cdot(4x-2)^{3/2}\Big|_{1/2}^{1} = \frac{1}{6}\left(2^{3/2} - 0\right) = \frac{\sqrt{2}}{3}$

17) $\int_0^{+\infty} e^{-4x}\, dx = -\frac{1}{4}e^{-4x}\Big|_0^{+\infty} = \frac{1}{4}(-0+1) = \frac{1}{4}$

18) $\int_0^1 \ln x \, dx = \left|\begin{matrix} u = \ln x \\ v = x \end{matrix}\right| = x\ln x\Big|_0^1 - \int_0^1 x\frac{dx}{x} = -1.$