

CHE 318 Lecture Q8

Jan 21 - 2026

Continue examples for P.S.S in spheres

1) Compare if solving $N_A(t) = \text{const.} \Rightarrow$

(solving $N_A(t) = \text{time dependent}$)

2) Compare sphere & slab geometries

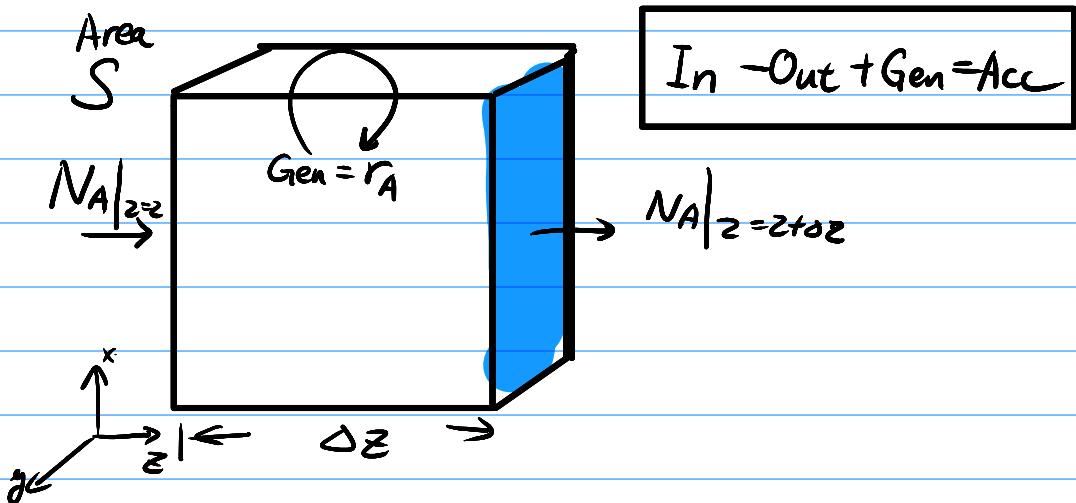
What if geometry is hemisphere? (evaporation of droplet on surface)



$$\frac{\bar{N}_A}{2\pi} \left(\frac{1}{r_i} - 0 \right) \cdot \frac{D_{AB} P_T}{RT} \cdot \frac{1}{P_{Bm}} (P_{A1} - 0)$$

Same principle but different coeff (2π vs 4π)

U.S.S mass transport



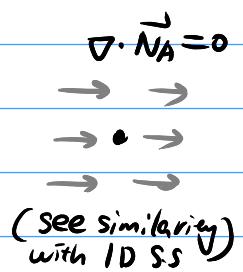
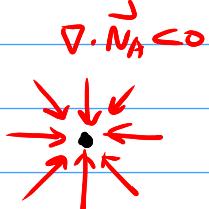
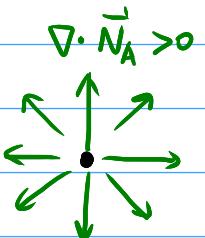
Derivation of govern eqn

$$\begin{aligned} N_A|_{z=0} - N_A|_{z=\Delta z} &= - \left(\frac{N_A|_{z=\Delta z} - N_A|_0}{\Delta z} \right) \cdot \Delta z \\ &= - \frac{\partial N_A}{\partial z} \cdot \Delta z \end{aligned}$$

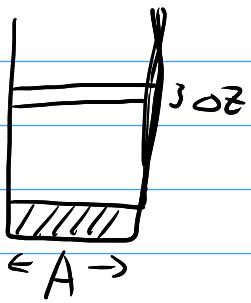
$\Downarrow \text{Def. of derivation}$

Divergence operator

$$\nabla \cdot \vec{N}_A = \frac{\partial \vec{N}_A}{\partial x} + \frac{\partial \vec{N}_A}{\partial y} + \frac{\partial \vec{N}_A}{\partial z}$$



Solving U.S.S for stagnant B



Governing Eq (M, B)

$$\frac{\partial C_A}{\partial t} = - \frac{\partial N_A}{\partial z}$$

$$N_A = - C_T D_{AB} \frac{\partial x_A}{\partial z} + x_A (N_A + N_B)$$

Constraints between N_A & N_B

Cannot directly say $N_B = 0$!

$$\frac{\partial C_T}{\partial t} = - \frac{\partial N_T}{\partial z} = - \frac{\partial N_A}{\partial z} - \frac{\partial N_B}{\partial z} = 0$$

$$\frac{\partial N_A}{\partial z} = - \frac{\partial N_B}{\partial z}$$

B.C.s

$$\begin{cases} x_A(z=0, t) = x_{A0} & x_A(z=L, t) = 0 \\ N_B(z=0, t) = 0 & \end{cases} \rightarrow N_A(z=0, t) = - \frac{C_T D_{AB}}{1-x_{A0}} \frac{\partial x_A}{\partial z} \Big|_{z=0}$$

No flux BC

$$\frac{\partial C_A}{\partial t} = C_T \frac{\partial x_A}{\partial t} = - \frac{\partial (-C_T D_{AB} \frac{\partial x_A}{\partial z})}{\partial z} - \frac{\partial (x_A \cdot (N_A + N_B))}{\partial z}$$

2 parts

$$= -C_T D_{AB} \frac{\partial^2 x_A}{\partial z^2} = \frac{\partial x_A}{\partial z} \cdot (N_A + N_B)$$

What is $N_T = N_A + N_B$?

$$N_T(z \neq 0, t) = N_T(z=0, t) = N_A(0, t) + \cancel{N_B(0, t)}$$

$$= x_A \frac{\partial (N_A + N_B)}{\partial z}$$

$$= - \frac{C_T D_{AB}}{1-x_{A0}} \frac{\partial x_A}{\partial z} \Big|_{z=0}$$

Not a constant over t !

Final expression for Govern. Eqa.

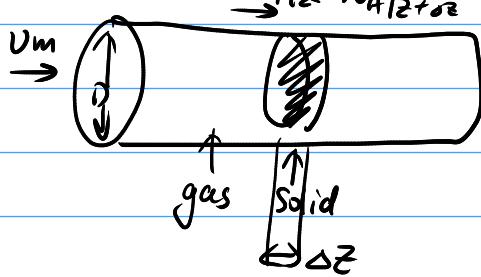
$$\frac{\partial x_A}{\partial t} = + D_{AB} \frac{\partial^2 x_A}{\partial z^2} + \left[\frac{D_{AB}}{1 - x_{A0}} \frac{\partial x_A}{\partial z} \Big|_{z=0} \right] \frac{\partial x_A}{\partial z}$$

And initial conditions

initial gradient!

$$x_A(z=0, 0) = 0 \quad \text{Analytical solution exists}$$

Example 2 Reaction through catalyst wall



Reaction & M.T.

In the Catalyst region Rate of reaction

$$r = k(C_{AS} - C_A)$$

$$\text{In} - \text{Out} + \boxed{\text{Gen}} \neq 0 = \text{Acc}$$

$$\frac{\pi D^2}{4} (N_A/z - N_A/z + \Delta N_A) + \pi D \cdot \Delta z \cdot k(C_{AS} - C_A)$$

$$= \frac{\pi D^2}{4} \cdot \frac{\partial C_A}{\partial z}$$

$$\hookrightarrow - \frac{\partial N_A}{\partial z} + \left(\frac{4k}{D} \right) (C_{AS} - C_A) = \frac{\partial C_A}{\partial z}$$

Use flux eq with v_m (easier!)

$$N_A = - D_{AB} \frac{\partial C_A}{\partial z} + C_A \cdot v_m$$

Constant v_m case

$$\frac{\partial N_A}{\partial z} = - D_{AB} \frac{\partial^2 C_A}{\partial z^2} + \frac{\partial C_A}{\partial z} v_m$$



$$\frac{\partial C_A}{\partial z} = - (- D_{AB} \frac{\partial^2 C_A}{\partial z^2} + \frac{\partial C_A}{\partial z} v_m) + \frac{4k}{D} (C_{AS} - C_A)$$

- 1) Need initial & BC
- 2) Numerical integration