

CHE 318 Lecture 8

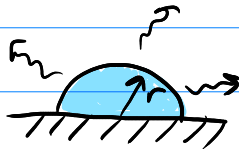
Jan 21 - 2026

Continue examples for P.S.S in spheres

1) Compare if (solving $N_A(t) = \text{const.} \Rightarrow$
 solving $N_A(t) = \text{time dependent}$)

2) Compare sphere & slab geometries

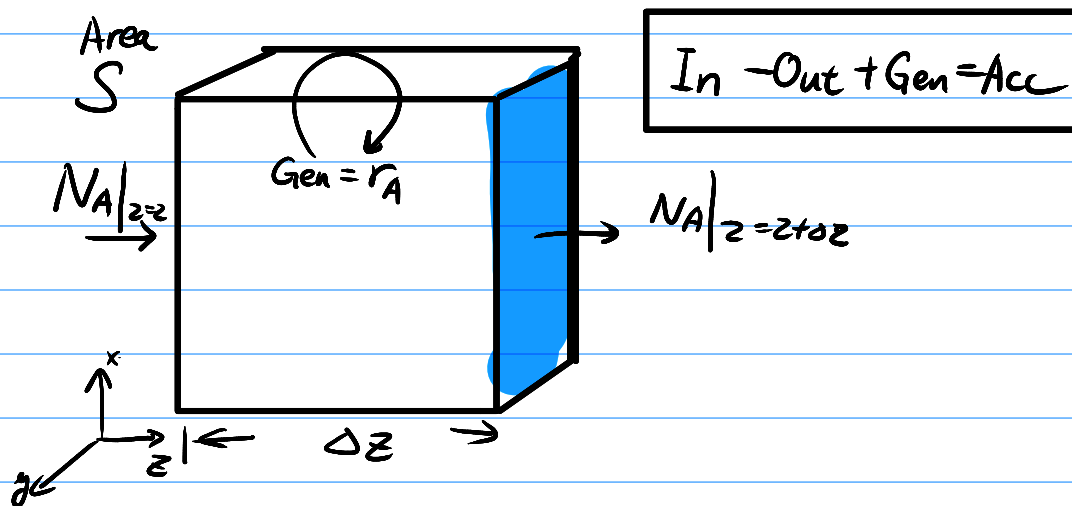
What if geometry is hemisphere? (evaporation of droplet on surface)



$$\frac{\bar{N}_A}{2\pi} \left(\frac{1}{r_i} - 0 \right) = \frac{D_{AB} P_T}{RT} \cdot \frac{1}{P_{Bm}} (P_{A_i} - 0)$$

Same principle but different coeff (2π vs 4π)

U.S.S mass transport

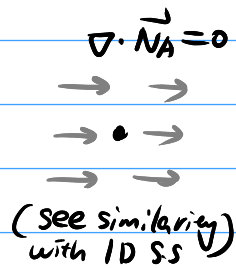
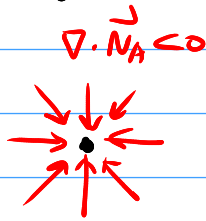
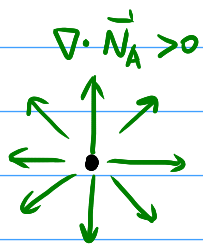


Derivation of govern eqn

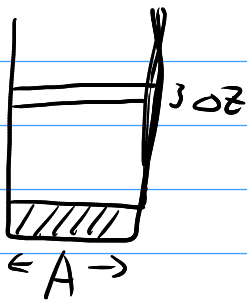
$$\begin{aligned} N_A|_{z=z} - N_A|_{z=z+\Delta z} &= - \left(\frac{N_A|_{z+\Delta z} - N_A|_z}{\Delta z} \right) \cdot \Delta z \\ &\quad \downarrow \text{Def. of derivative} \\ &= - \frac{\partial N_A}{\partial z} \cdot \Delta z \end{aligned}$$

Divergence operator

$$\nabla \cdot \vec{N}_A = \frac{\partial \vec{N}_A}{\partial x} + \frac{\partial \vec{N}_A}{\partial y} + \frac{\partial \vec{N}_A}{\partial z}$$



Solving U.S.S for stagnant B



Governing Eq (M.B)

$$\frac{\partial C_A}{\partial t} = - \frac{\partial N_A}{\partial z}$$

$$N_A = - C_T D_{AB} \frac{\partial x_A}{\partial z} + x_A (N_A + N_B)$$

↑ Cannot directly say $N_B = 0$!

Constraints between N_A & N_B

$$\frac{\partial C_T}{\partial t} = - \frac{\partial N_T}{\partial z} = - \frac{\partial N_A}{\partial z} - \frac{\partial N_B}{\partial z} = 0$$

$$\frac{\partial N_A}{\partial z} = - \frac{\partial N_B}{\partial z}$$

B.C.s

$$\begin{cases} x_A(z=0, t) = x_{A0} & x_A(L, t) = 0 \\ N_B(z=0, t) = 0 \end{cases} \quad \begin{array}{c} \uparrow \\ \text{No flux BC} \end{array}$$

$$\rightarrow N_A(0, t) = - \frac{C_T D_{AB}}{1 - x_{A0}} \frac{\partial x_A}{\partial z} \bigg|_{z=0}$$

$$\begin{aligned} \frac{\partial C_A}{\partial t} &= C_T \frac{\partial x_A}{\partial t} = - \frac{\partial \left(- \frac{C_T D_{AB}}{1 - x_{A0}} \frac{\partial x_A}{\partial z} \right)}{\partial z} = \frac{\partial [x_A \cdot (N_A + N_B)]}{\partial z} \\ &= - C_T D_{AB} \frac{\partial^2 x_A}{\partial z^2} = \frac{\partial x_A}{\partial z} \cdot (N_A + N_B) \end{aligned}$$

↖ 2 parts...

What is $N_T = N_A + N_B$?

$$N_T(z \neq 0, t) = N_T(z=0, t) = N_A(0, t) + N_B(0, t)$$

$$= - \frac{C_T D_{AB}}{1 - x_{A0}} \frac{\partial x_A}{\partial z} \bigg|_{z=0}$$

NOT a constant over t !

Final expression for Govern. Equ.

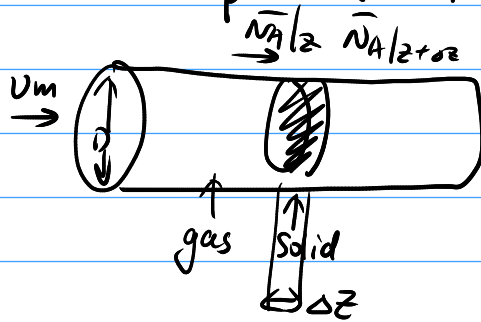
$$\frac{\partial x_A}{\partial t} = + D_{AB} \frac{\partial^2 x_A}{\partial z^2} + \left[\frac{D_{AB}}{1 - x_{A0}} \frac{\partial x_A}{\partial z} \right]_{z=0} \frac{\partial x_A}{\partial z}$$

And initial conditions

initial gradient!

$$x_A(z \neq 0, 0) = 0 \quad \text{Analytical solution exists}$$

Example 2 Reaction through catalyst wall



Reaction & M.T.

In the catalyst region Rate of reaction
 $r = k(C_{As} - C_A)$

$$\text{In} - \text{Out} + \boxed{\text{Gen}}^{\neq 0} = \text{Acc}$$

$$\frac{\pi D^2}{4} (N_A|_z - N_A|_{z+\Delta z}) + \pi D \cdot \Delta z \cdot k(C_{As} - C_A)$$

$$= \frac{\pi D^2}{4} \cdot \frac{\partial C_A}{\partial t}$$

$$\rightarrow - \frac{\partial N_A}{\partial z} + \left(\frac{4k}{D}\right)(C_{As} - C_A) = \frac{\partial C_A}{\partial t}$$

Use flux eq with v_m (easier!)

$$N_A = -D_{AB} \frac{\partial C_A}{\partial z} + C_A \cdot v_m$$

Constant v_m case

$$\frac{\partial N_A}{\partial z} = -D_{AB} \frac{\partial^2 C_A}{\partial z^2} + \frac{\partial C_A}{\partial z} v_m$$



$$\frac{\partial C_A}{\partial t} = - \left(-D_{AB} \frac{\partial^2 C_A}{\partial z^2} + \frac{\partial C_A}{\partial z} v_m \right) + \frac{4k}{D} (C_{As} - C_A)$$

- 1) Need initial & BC
- 2) Numerical integration