BURSA TECHNICAL UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES
DEPT. OF MECHANICAL ENGINERING

STUDENT NAME:	
COURSE TITLE:	MECH 603 Advanced Mechanical Vibrations
ASSIGMENT TITLE:	Vibration of MDoF system and continuous media
ASSIGMENT NUMBER	2

ISSUE DATE	**13 DEC 2012**	**DEADLINE DATE**	**30 DEC 2012**	**SCORE**	

LECTURER NAME: Assist. Prof. Dr. Hakan Gökdağ **SIGNED: …………………………**

DATE:…………………

Student declaration: (must be signed before submission)

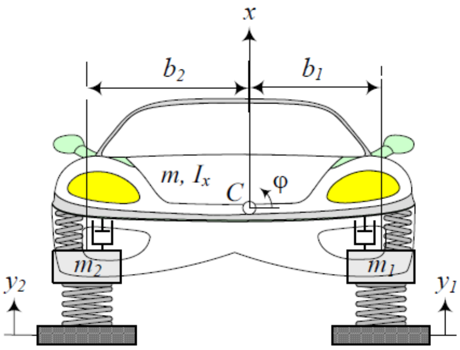
I declare that this assignment is my own work

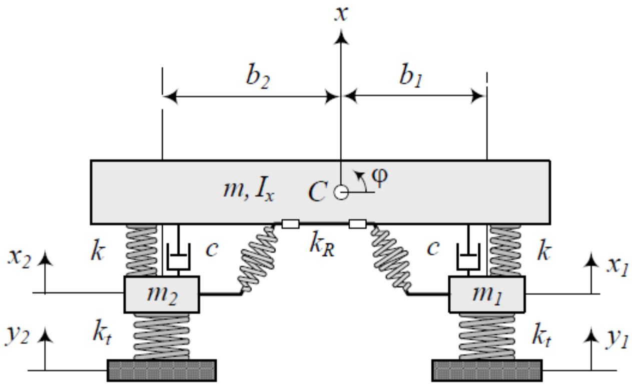
Student signature: …………………… Date : ………………..

Please write your report by pen or computer, not pencil!

Problem 1

Half-car model of a vehicle is given in the figure. The physical properties are as follows: m=420kg, m_1=m_2=53kg, I_x=820 kgm^2, b_1=0.7m, b_2=0.75m, k =10000Nm^{-1}, k_t=200000 Nm^{-1}, k_R=25000 Nm^{-1}, k_4= 1972900 Nm^{-1}, c=200 Nsm^{-1}. The vheicle is subject to base motions y_1 and y_2.





Equations of motion are:

$$[m]\,\ddot{\mathbf{x}} + [c]\,\dot{\mathbf{x}} + [k]\,\mathbf{x} = \mathbf{F}$$

where,

$$\mathbf{x} = \begin{bmatrix} x \\ \varphi \\ x_1 \\ x_2 \end{bmatrix}$$

$$[m] = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_x & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix}$$

$$[c] = \begin{bmatrix} 2c & cb_1 - cb_2 & -c & -c \\ cb_1 - cb_2 & cb_1^2 + cb_2^2 & -cb_1 & cb_2 \\ -c & -cb_1 & c & 0 \\ -c & cb_2 & 0 & c \end{bmatrix}$$

$$[k] = \begin{bmatrix} 2k & kb_1 - kb_2 & -k & -k \\ kb_1 - kb_2 & kb_1^2 + kb_2^2 + k_R & -kb_1 & kb_2 \\ -k & -kb_1 & k + k_t & 0 \\ -k & kb_2 & 0 & k + k_t \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 0 \\ 0 \\ y_1 k_t \\ y_2 k_t \end{bmatrix}.$$

a) Find the undamped natural frequencies and vibration modes by

(i) Solving characteristic polynomial

(ii) Matrix iteration method.

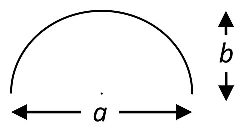
(Note: Each step is to be given in detail. Built-in commands such as "eig", "eigs" etc. in Matlab environment or any other functions of different software packages (Mathematica, Mapple, etc) are not to be used)

(iii) Plot the vibration modes, comment them and make conclusions.

(iv) Compute modal mass and stiffness values.

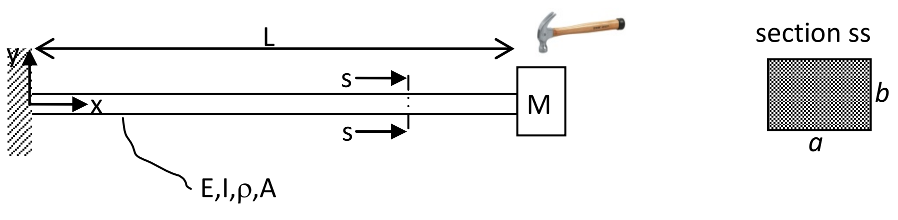
(v) Show that vibration modes are orthogonal to each other with respect to mass and stiffness matrices.

b) Suppose that while moving with a speed V = 60 km/h the right tire passes over a half sine wave-like bump on the road as shown below, where a=30cm and b=5cm. Assuming zero initial conditions plot the 5 seconds responses of the coordinates x_1 and x_2.



Problem 2

A beam with fixed one end is carrying a rigid tip mass M, and exhibits transverse vibrations. Physical properties are as follows: L=1m, E=210GPa, ρ=7800kgm^{-3}, sizes of the rectangular cross section a=2cm b=1cm, M is twice of the beam mass.



a) Derive the equation of motion along with the suitable boundary conditions.

b) Find the first five natural frequencies (Hz) and vibration modes of the system. Plot the vibration modes in the same figure.

c) Assume that M=0, and an impulse force with magnitude 10N is applied on the tip at t=0. Plot the shape of the beam after 1, 3, and 5 seconds.