Statistical Methods in Fast Fracture of Ceramics

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30.12.2013 / Fracture Mechanics Class Fall 2013

Outline

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Indeterminism of the fracture of ceramic materials

- Ceramic is brittle.
- Identical specimens of a ceramic material may exhibit large variations in fracture stress.
- Strength of brittle material depends on the volume.

Indeterminism of the fracture of brittle materials

Fracture mechanics view:

- Many flaws
- Random flaw size and orientation.

Outline

Key contributions

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- Walladi Weibull 1939,
  "A Statistical Theory of The Strength of Materials."
- Batdorf 1974,
  "A Statistical Theory for the Fracture of Brittle Structures Subjected to Nonuniform Polyaxial Stresses"

Summary

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- Weibull Theory
  - Weakest link theory.
  - Considers only the uniaxial stress.
  - Pure statistical
- Batdorf
  - Multiaxial stress addition to Weibull Theory.
  - Linear Elastic Fracture Mechanics

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Derivation of Weibull’s Relation

Derivation of Weibull’s Equation

Probability of failure of an individual volume element:

$$(1 - P) = (1 - P_0)^V$$

$$ln(1 - P) = Vln(1 - P_0) \Longrightarrow \text{risk of rupture } B$$

Derivation of Weibull's Relation

Derivation of Weibull's Equation

Probability of failure = 1 $- e^{B}$=1 $- e^{\int_V N(\sigma) dV}$

$$\int_V N(\sigma)\, dV = -\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m$$

Derivation of Weibull’s Relation

Derivation of Weibull’s Equation

Probability of failure

$$P_f = 1 - e^B \quad = \quad 1 - exp\left[\int_V N(\sigma)\, dV\right]$$

Weibull’s relation

$$\int_V N(\sigma)\, dV = -\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m$$

Derivation of Weibull's Relation

Derivation of Weibull's Equation

Weibull's probability of failure equation

$$P_{f,V} = 1 - exp\left[\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

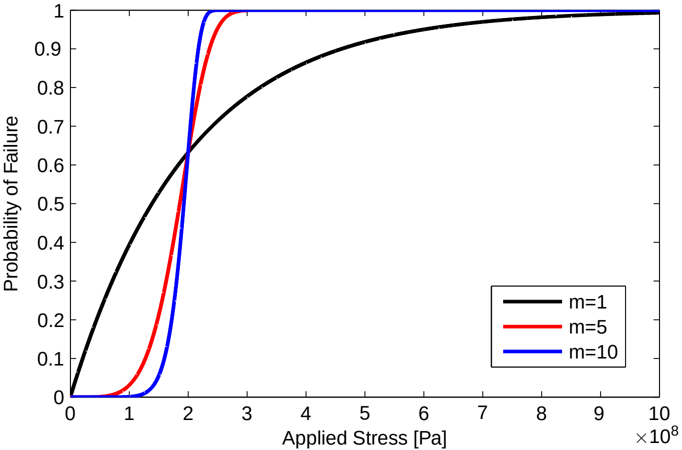
σ : applied stress

σ_0: characteristic strength for a volumeV_0

m: Weibull modulus

Plot of Weibull’s Equation using different modulus

Plot of Weibull’s Equation



Weibull's Equation Properties

Weibull's Equation Properties

- considers size effects
- considers only the uniaxial stress
- computational simplicity
- Phenomenological $\Longrightarrow$ not based on Fracture Mechanics Theory.

Batdorf Approach

Batdorf Approach

$$P_f = 1 - exp\left[-\int_{\Omega}\int_{V}\int_{0}^{\sigma} dV\, d\Omega\, d\sigma_c \frac{dN(\sigma_c)}{d\sigma_c}\left(\frac{H(\sigma_e, \sigma_c)}{4\pi}\right)\right]$$

$H(\sigma_e, \sigma_c)$: equals to 1 if $\sigma_e > \sigma_c$

$N(\sigma_c)$: Number of cracks that has critical stress higher than σ_c

σ_c: Critical stress

Ω: Area of unit sphere where $\sigma_{equavilent} > \sigma_c$

Batdorf Approach

Batdorf Approach

Properties

- assumes random flaw orientation
- consistent crack geometry

Relation Between Statistics and Experiments

Relation Between Statistics and Experiments

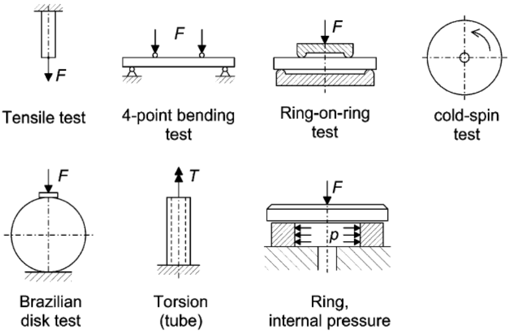
Statistical models should base on real data from the experiments. Therefore, experiments should be done.

Experiments

Experiment Properties

- There are various type of experiments for the characterization of brittle material.
- Experiments can be done both numerically and in laboratory conditions.

Experiment types



"The influence of failure criteria on strength prediction of ceramic components", Patrick Scheunemann

Data collection and fit procedure for statistical representation

- Collect σ , probability of fracture data from specimens.
- Determine σ_0 and m in Weibull equation.

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Experiment types

There is a computer code that is avaliable for statistical analysis of experiments.

CARES

An Interactive Code For Experiments and Curve Fitting fo Ceramics

- developed at NASA
- currently a commercial program
- Abilities
  - Weibull parameter estimates
  - Fatigue parameter estimates
  - Multiaxial stress states
  - Volume flaw surface flaw analysis
  - PIA, Weibull NSA, and Batdorf models
  - Fast fracture reliability analysis
  - Time-dependent reliability analysis (power law, Paris law, Walker law)
  - Proof testing (PIA and Batdorf theories)

Applications

Applications

Aerospace Applications:

- Mars Microprobe Aeroshell
- Small expendable turbojet
- APU Turbine wheel and nozzle

Automotive Applications:

- Valves and engine components
- Turbine engine and components (scroll, combustor, rotor, insulation)
- Turbocharger wheel

Propulsion and Power Applications:

- Turbine blade

Bioengineering Applications:

- Dental Crowns
- Hip joint

Summary

- Ceramic materials are brittle and they exhibit large variations on fracture strength.
- Brittleness makes use of statistical fracture is a must for the prediction of failure.
- Currently we have some tools that are usefull for use of statistical fracture is a must for the prediction of failure.