

Recap of Lecture 8

- Introduction to machine learning
 - Unsupervised, supervised and reinforcement learning
- Clustering
 - K-means clustering
 - Hierarchical clustering
- Dimensionality reduction, visualization and 'structure' analysis
 - Principal Component Analysis (PCA)

A long time ago in a galaxy far, far away....





Every linear algebra class

Me: What are eigenvectors

Teacher: You can think of them as an ndimensional kernel subspace

Me: No I can't

3:08 PM - 28 Mar 2016

702 Retweets **1,384** Likes















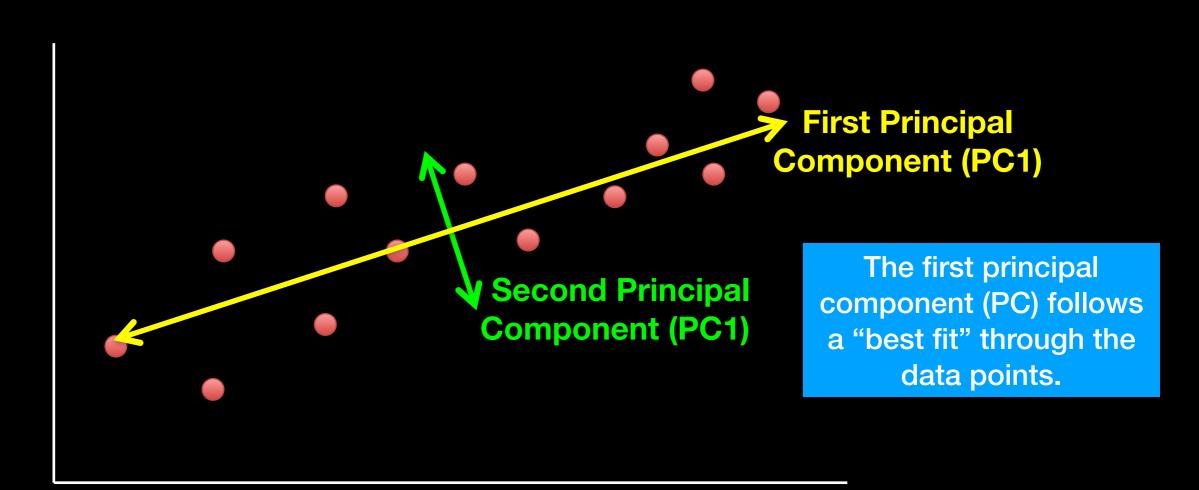




PCA: Principal Component Analysis

PCA projects the features onto the principal components.

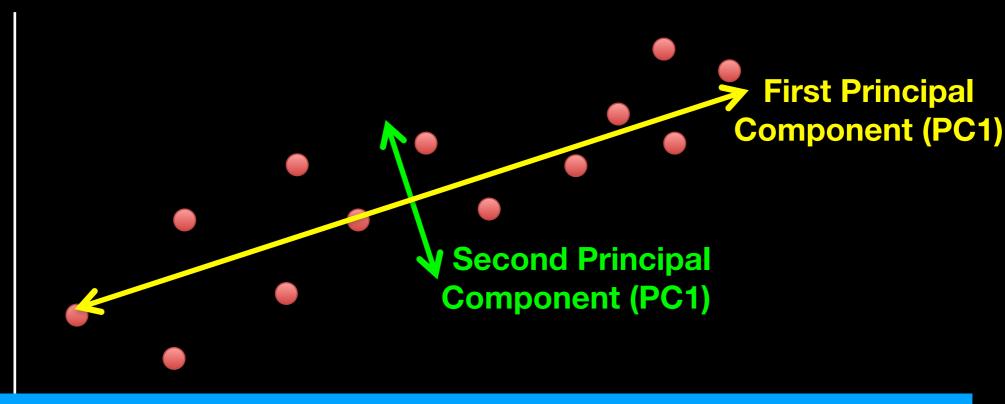
The motivation is to reduce the features dimensionality while only losing a small amount of information.



PCA: Principal Component Analysis

PCA projects the features onto the principal components.

The motivation is to reduce the features dimensionality while only losing a small amount of information.



Principal components are new low dimensional axis (or surfaces) closest to the observations

Recap: PCA objectives

- To reduce dimensionality
- To visualize multidimensional data
- To choose the most useful variables (features)
- To identify groupings of objects (e.g. genes/samples)
- To identify outliers

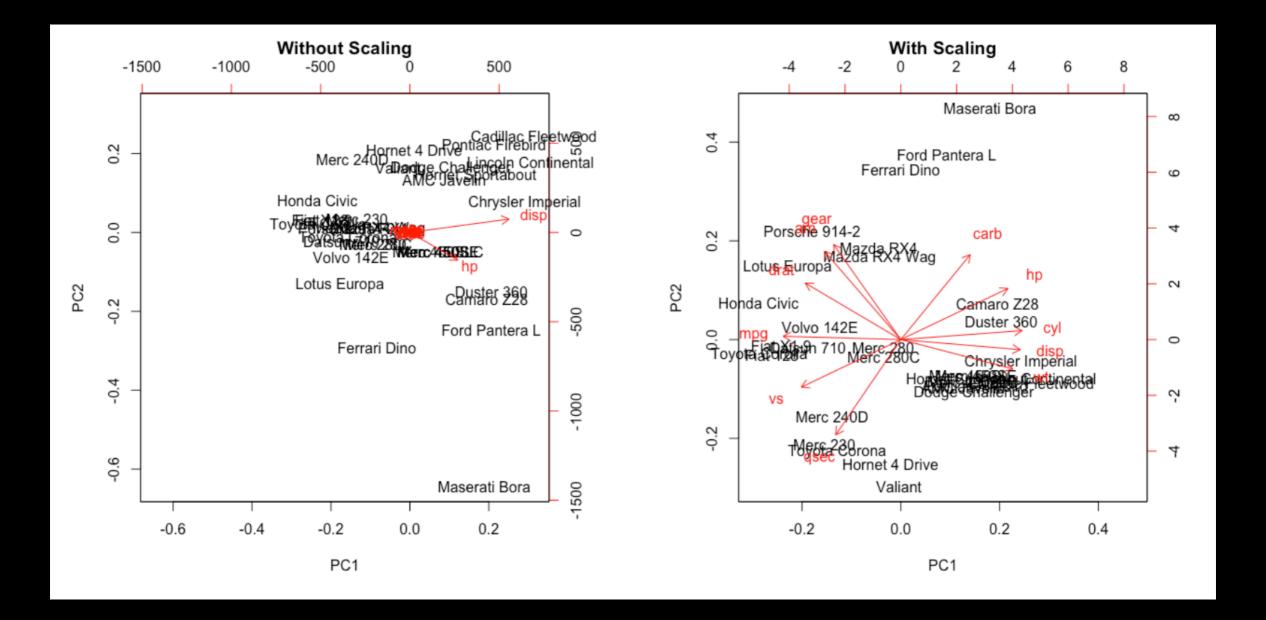
Practical PCA issue: Scaling

```
> data(mtcars)
> head(mtcars)
                   mpg cyl disp
                                 hp drat wt
                                                 qsec vs am gear carb
Mazda RX4
                  21.0
                            160 110 3.90 2.620 16.46
                                                               4
                                                                    4
Mazda RX4 Wag
                  21.0
                            160 110 3.90 2.875 17.02
                                                                    4
Datsun 710
                  22.8
                                 93 3.85 2.320 18.61 1
                            108
Hornet 4 Drive
                  21.4
                            258 110 3.08 3.215 19.44
                                                                    1
Hornet Sportabout
                  18.7
                            360 175 3.15 3.440 17.02
Valiant
                                105 2.76 3.460 20.22
                  18.1
# Means and standard deviations vary a lot
> round(colMeans(mtcars), 2)
                disp
          cyl
                              drat
                                                                          carb
                         hp
   mpg
                                       wt
                                             qsec
                                                                  gear
                                                      VS
                                                             am
20.09
         6.19 230.72 146.69
                              3.60
                                     3.22
                                            17.85
                                                    0.44
                                                           0.41
                                                                  3.69
                                                                          2.81
> round(apply(mtcars, 2, sd), 2)
          cyl
                disp
                         hp
                                                                          carb
                              drat
                                       wt
                                             qsec
   mpg
                                                      ٧S
                                                             am
                                                                  gear
                                                           0.50
         1.79 123.94 68.56
                                             1.79
  6.03
                              0.53
                                     0.98
                                                    0.50
                                                                  0.74
                                                                          1.62
```

Practical PCA issue: Scaling

prcomp(x, scale=FALSE)

prcomp(x, scale=TRUE)



Your turn!

Unsupervised Learning Mini-Project

Input: read, View/head,

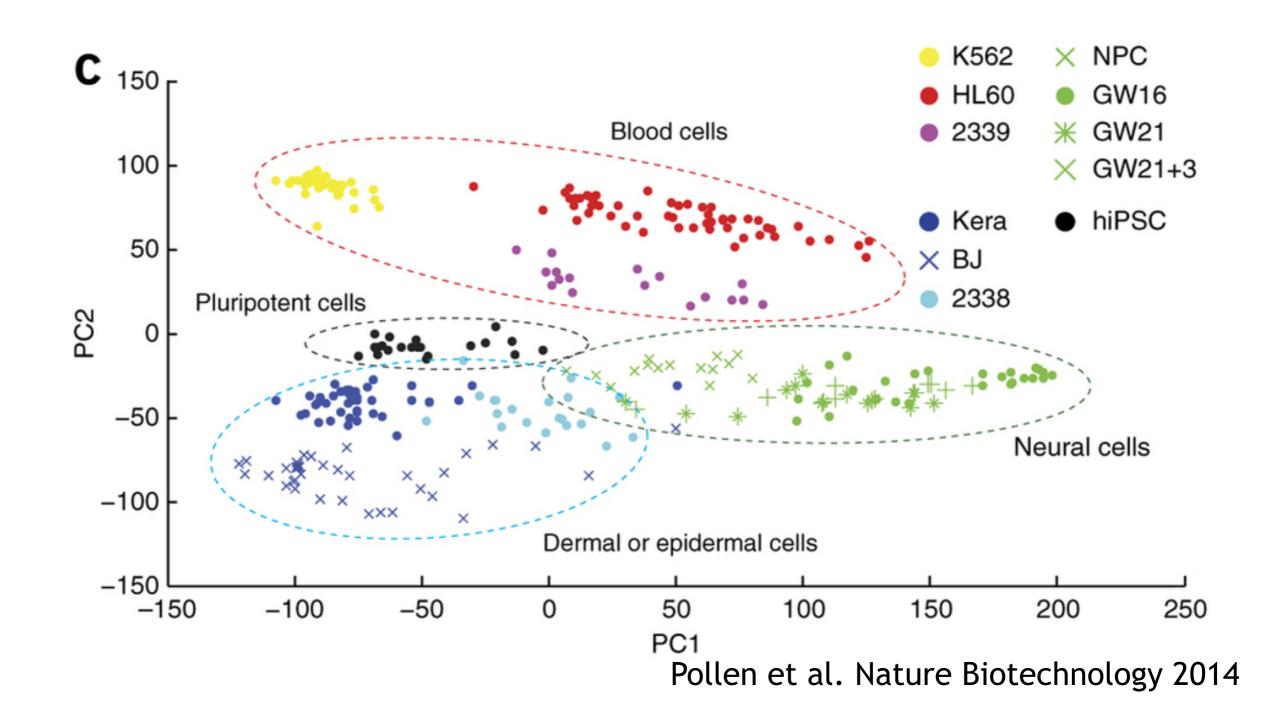
PCA: prcomp,

Cluster: kmeans, hclust Compare: plot, table, etc.

Reference Slides

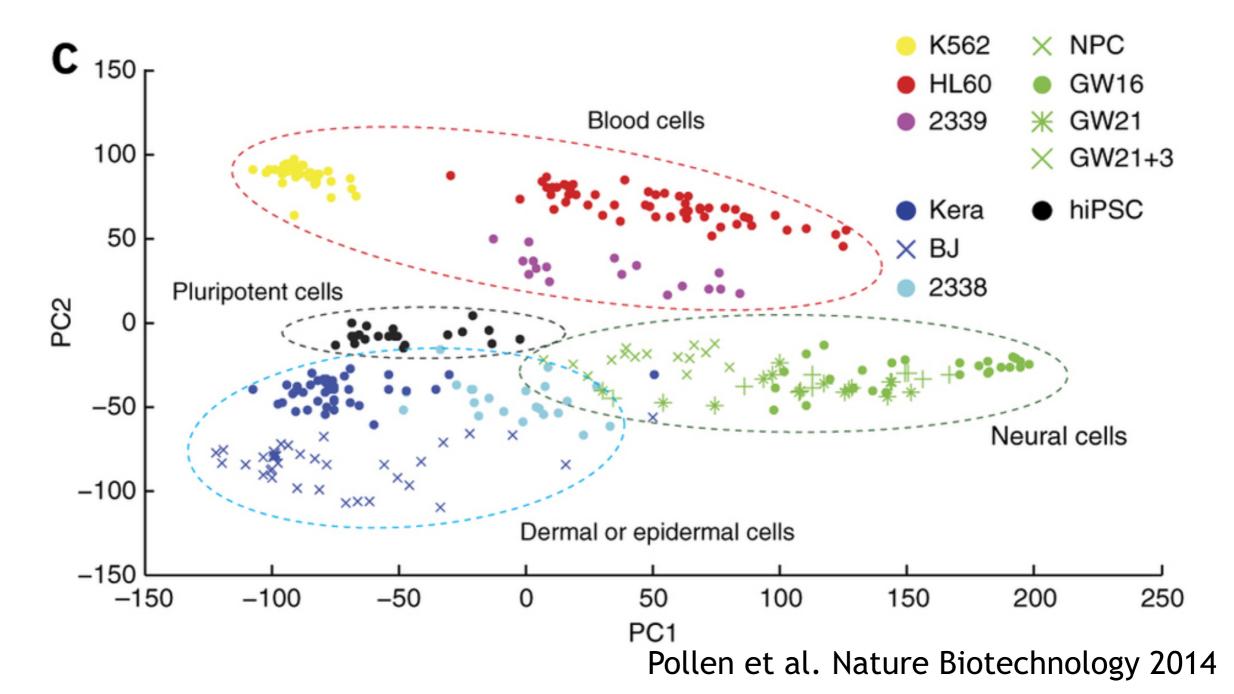
This PCA plot shows clusters of cell types.

This graph was drawn from single-cell RNA-seq. There were about 10,000 transcribed genes in each cell.



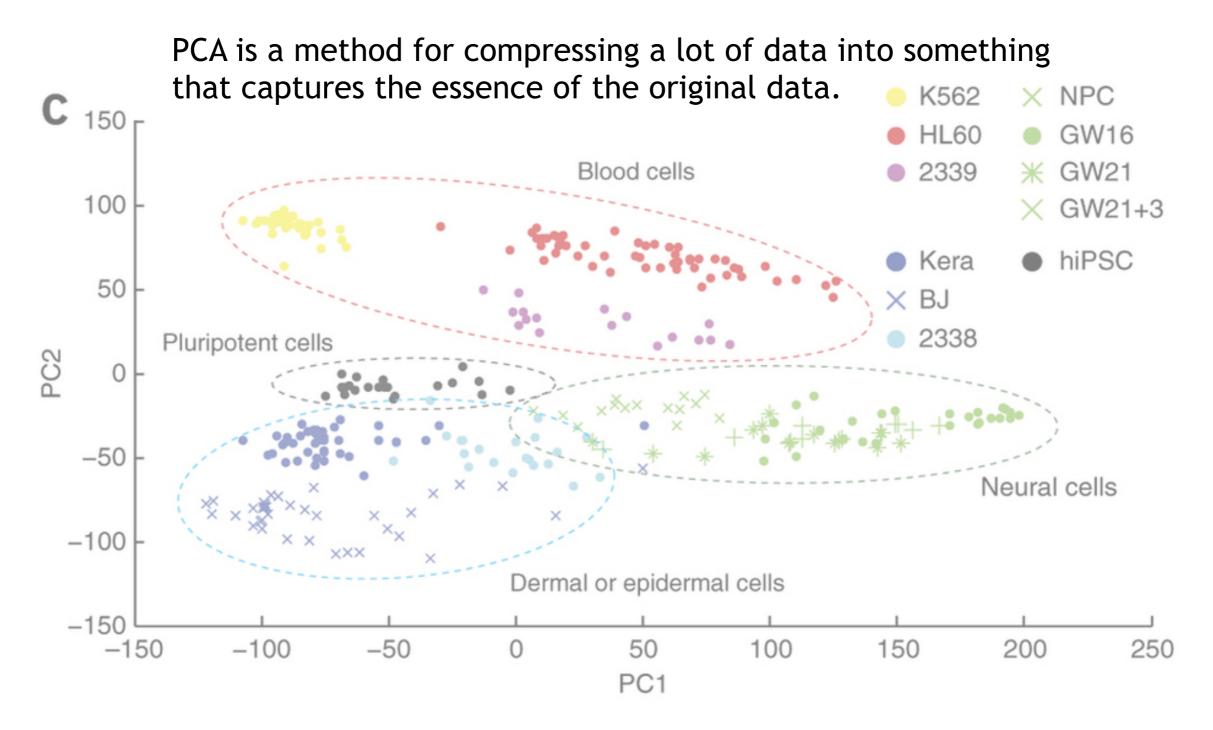
This PCA plot shows clusters of cell types.

Each dot represents a single-cell and its transcription profile The general idea is that cells with similar transcription should cluster.



This PCA plot shows clusters of cell types.

How does transcription from 10,000 genes get compressed to a single dot on a graph?



What does PCA aim to do?

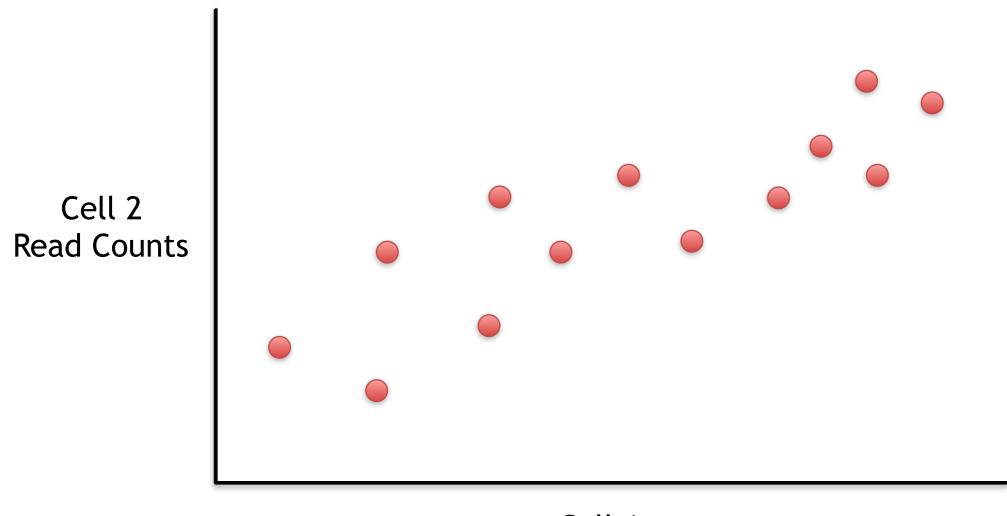
- PCA takes a dataset with a lot of dimensions (i.e. lots of cells) and flattens it to 2 or 3 dimensions so we can look at it.
 - It tries to find a meaningful way to flatten the data by focusing on the things that are different between cells. (much, much more on this later)

A PCA example

Again, we'll start with just two cells Here's the data:

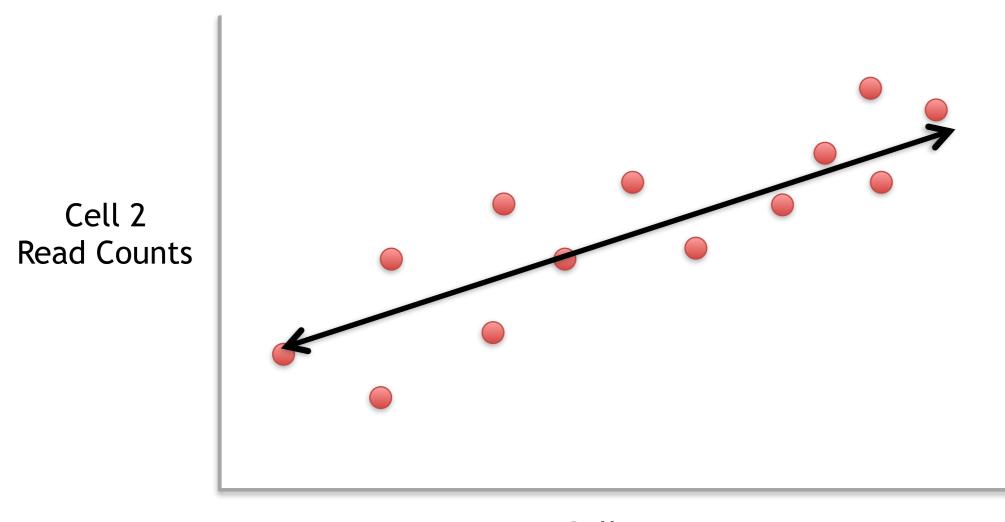
| Gene | Cell1 reads | Cell2 reads |
|-------|-------------|-------------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| (etc) | (etc) | (etc) |

Here is a 2-D plot of the data from 2 cells.



Cell 1 Read Counts

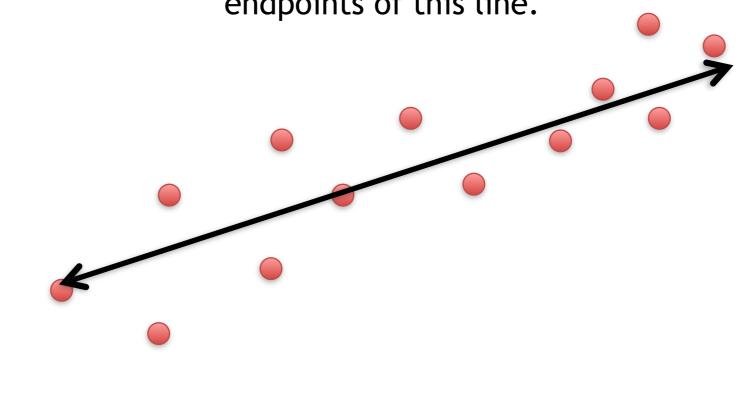
Generally speaking, the dots are spread out along a diagonal line.



Cell 1 Read Counts

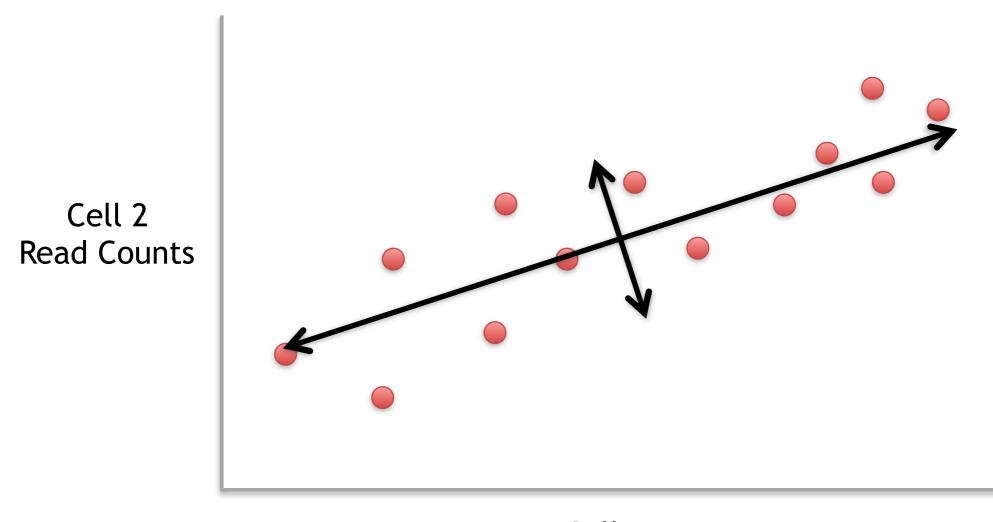
Generally speaking, the dots are spread out along a diagonal line.

Another way to think about this is that the maximum variation in the data is between the two endpoints of this line.



Cell 1 Read Counts

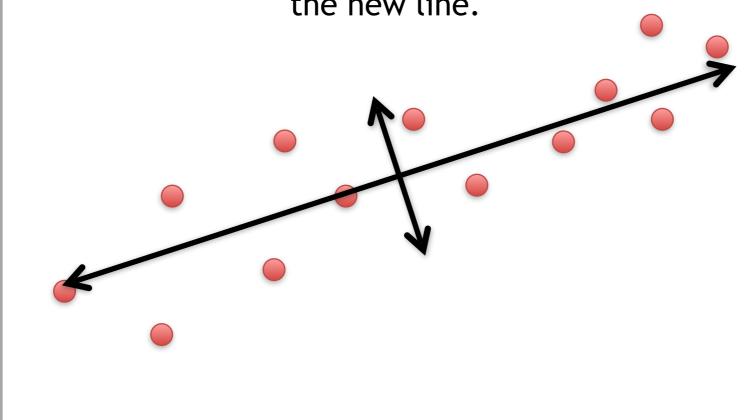
Cell 2 Read Counts Generally speaking, the dots are also spread out a little above and below the first line.



Cell 1 Read Counts

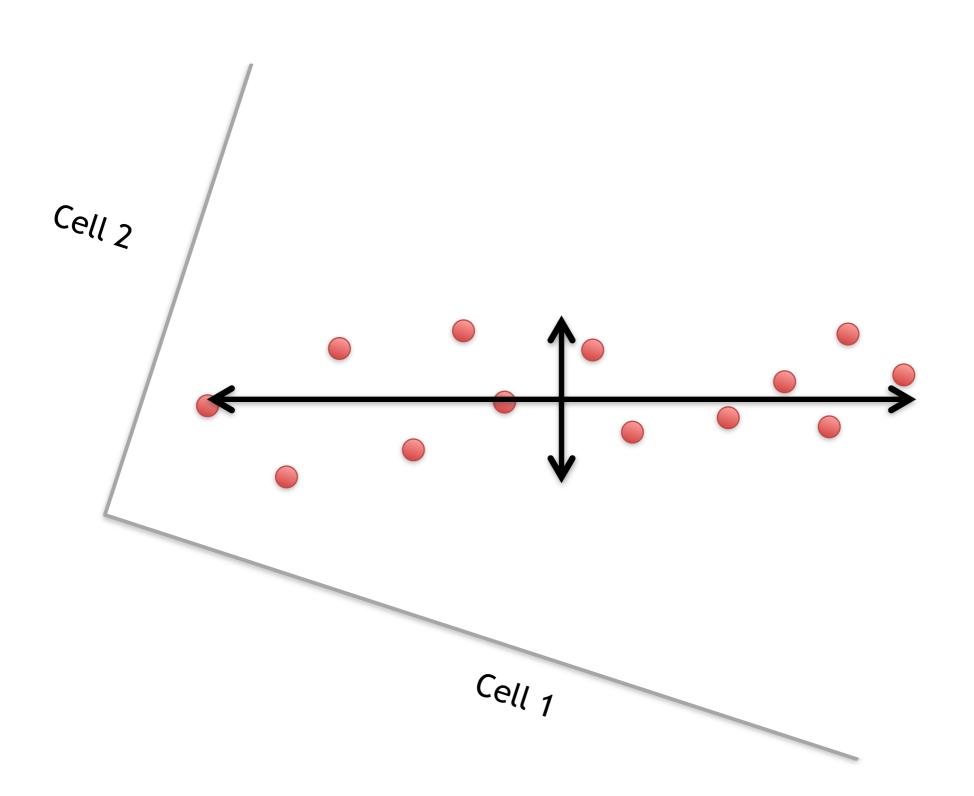
Generally speaking, the dots are also spread out a little above and below the first line.

Another way to think about this is that the 2nd largest amount of variation is at the endpoints of the new line.

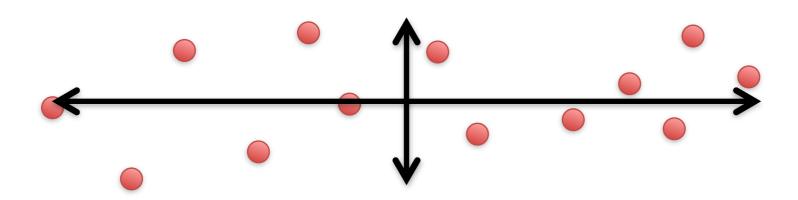


Cell 1 Read Counts

Cell 2 Read Counts

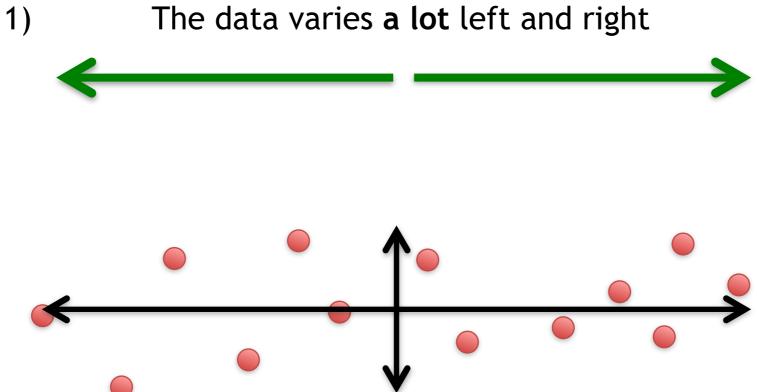


This makes the left/right, above/below variation easier to see.

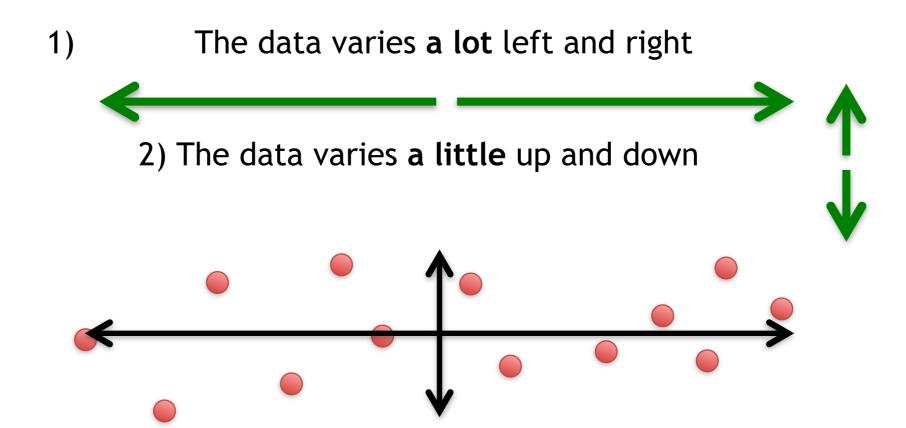


This makes the left/right, above/below variation easier to see.

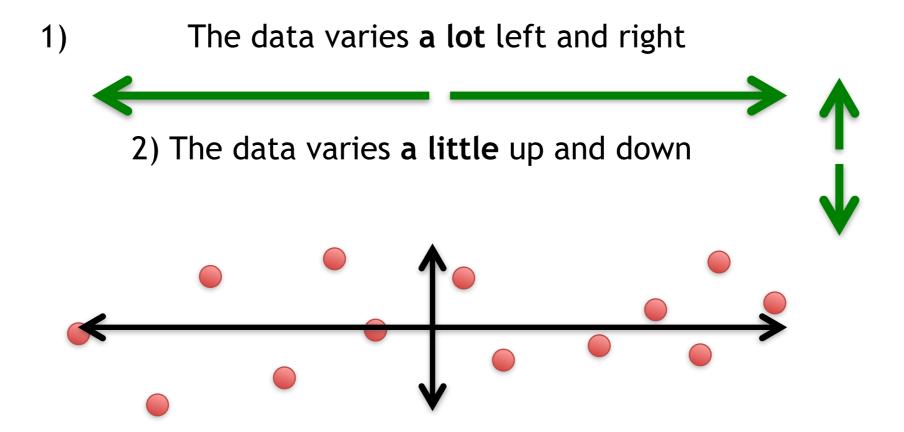
easier to see.



This makes the left/right, above/below variation easier to see.



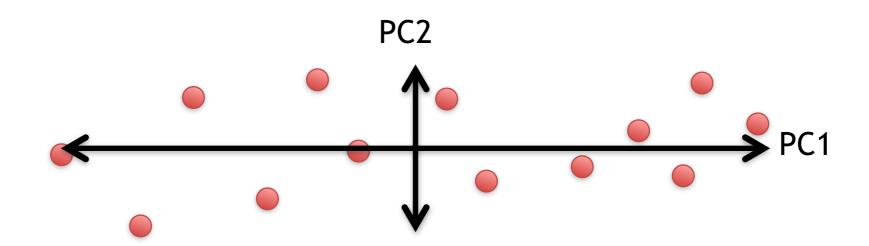
This makes the left/right, above/below variation easier to see.



Note: All of the points can be drawn in terms of left/right + up/down, just like any other 2-D graph.

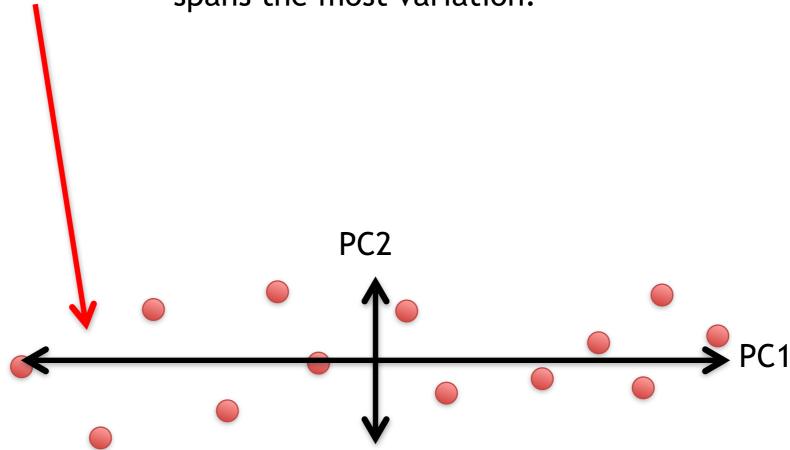
That is to say, we do not need another line to describe "diagonal" variation - we've already captured the two directions that can have variation.

These two "new" (or "rotated") axes that describe the variation in the data are "Principal Components" (PCs)



These two "new" axes that describe the variation in the data are "Principal Components" (PCs)

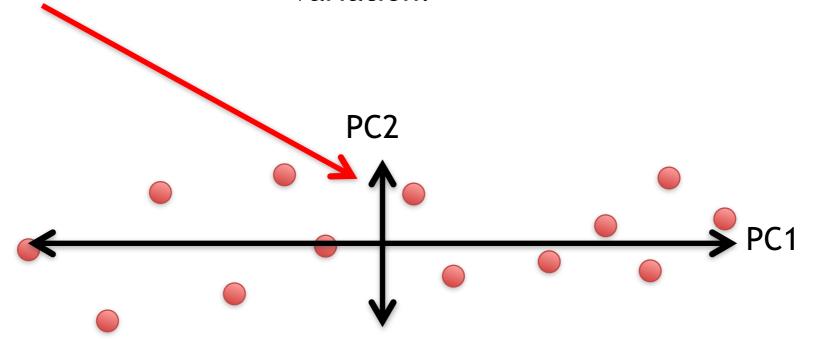
PC1 (the first principal component) is the axis that spans the most variation.



These two "new" axes that describe the variation in the data are "Principal Components" (PCs)

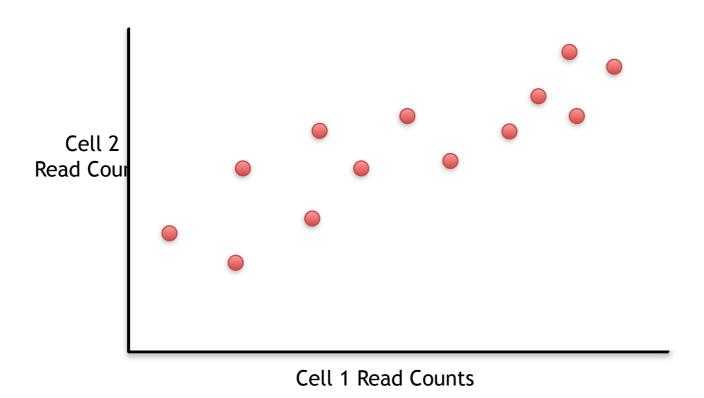
PC1 (the first principal component) is the axis that spans the most variation.

PC2 is the axis that spans the second most variation.



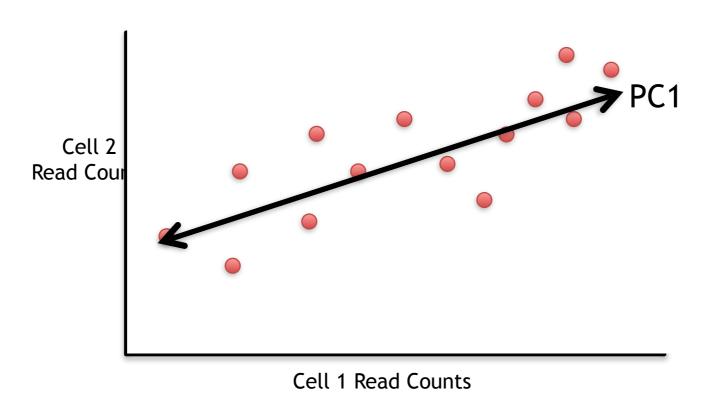
General ideas so far...

 For each gene, we plotted a point based on how many reads were from each cell.



General ideas so far...

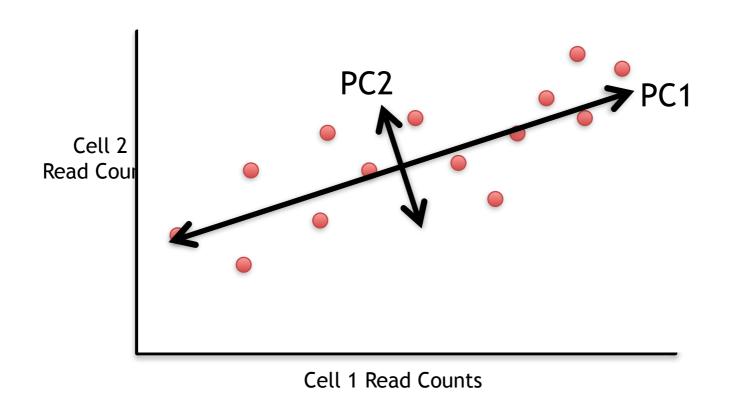
 For each gene, we plotted a point based on how many reads were from each cell.



PC1 captures the direction where most of the variation is.

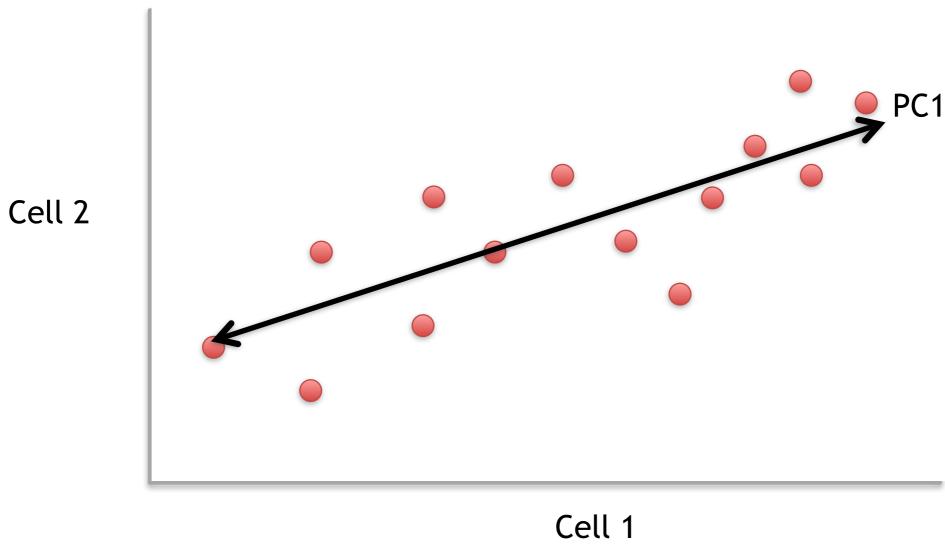
General ideas so far...

 For each gene, we plotted a point based on how many reads were from each cell.

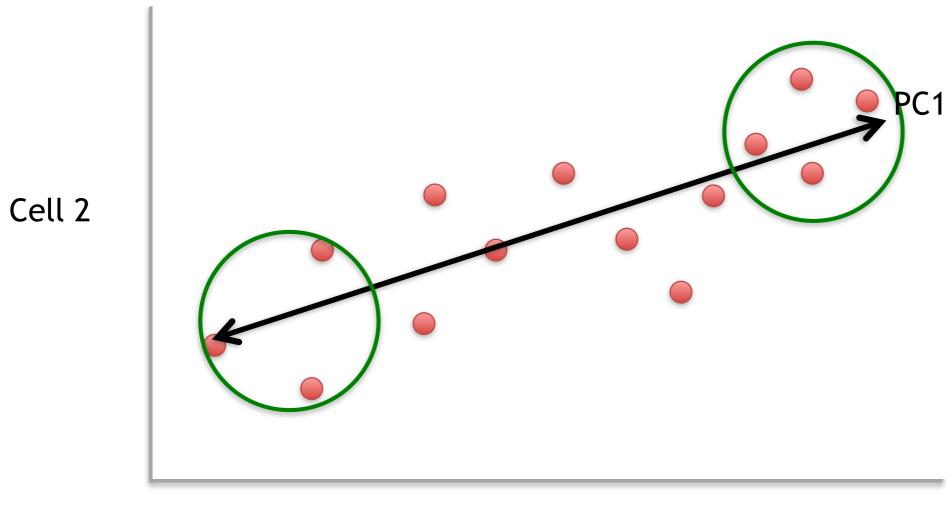


- PC1 captures the direction where most of the variation is.
- PC2 captures the direction with the 2nd most variation.

For now, let's focus on PC1

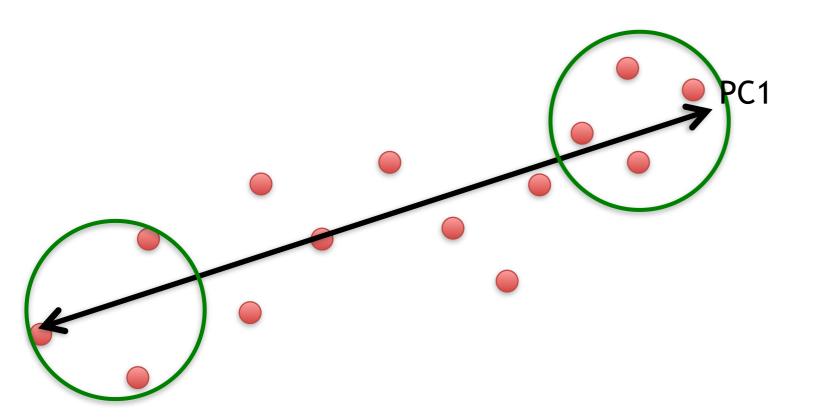


The length and direction of PC1 is mostly determined by the circled genes.

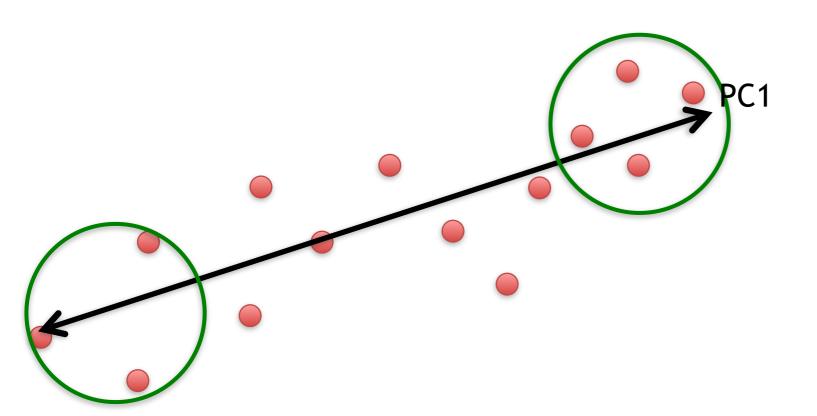


Cell 1

The length and direction of PC1 is mostly determined by the circled genes.

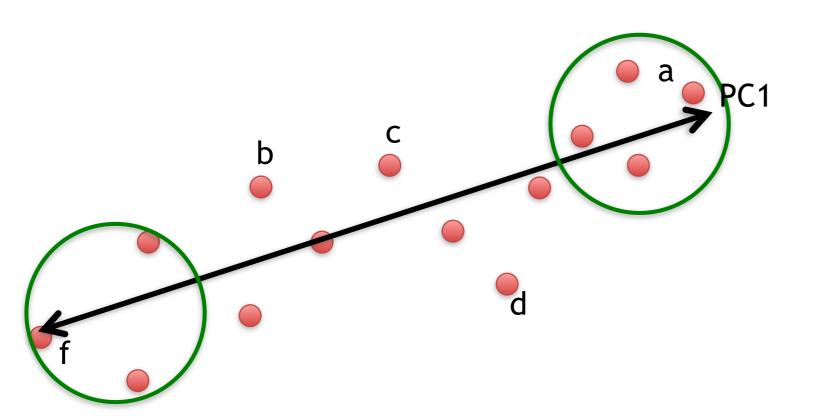


The length and direction of PC1 is mostly determined by the circled genes.



We can score genes based on how much they influence PC1.

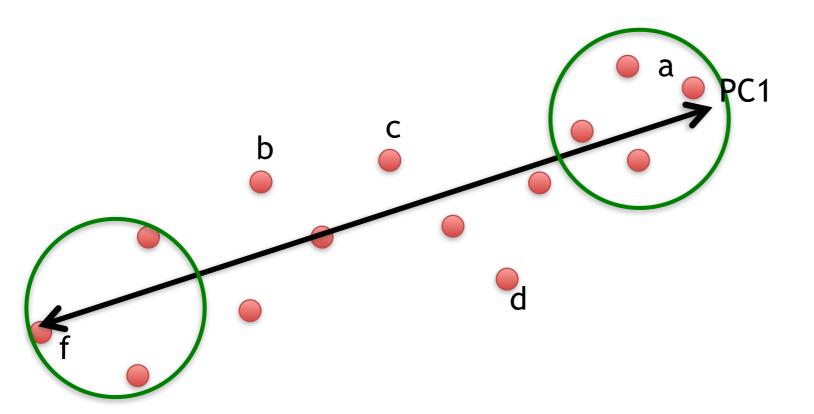
The length and direction of PC1 is mostly determined by the circled genes.



We can score genes based on how much they influence PC1.

| Gene | Influence on PC1 |
|------|---------------------|
| a | high |
| b | low |
| С | low |
| d | low |
| е | high |
| f | high |
| ••• | ••• |

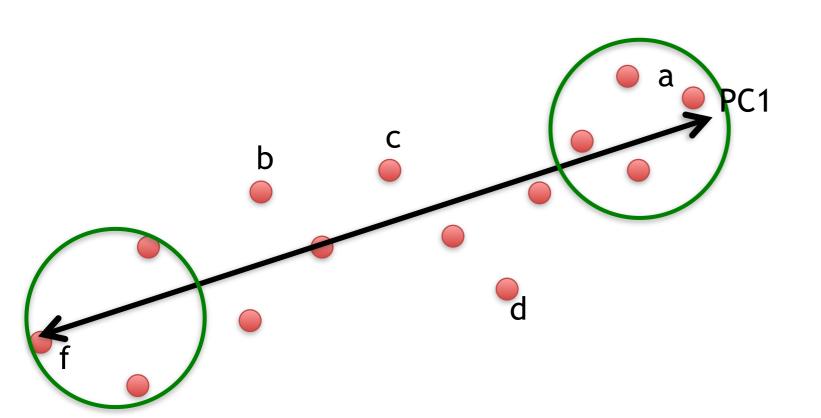
The length and direction of PC1 is mostly determined by the circled genes.



Some genes have more influence on PC1 than others.

| Gene | Influence on PC1 |
|------|---------------------|
| a | high |
| b | low |
| С | low |
| d | low |
| е | high |
| f | high |
| ••• | ••• |

The length and direction of PC1 is mostly determined by the circled genes.



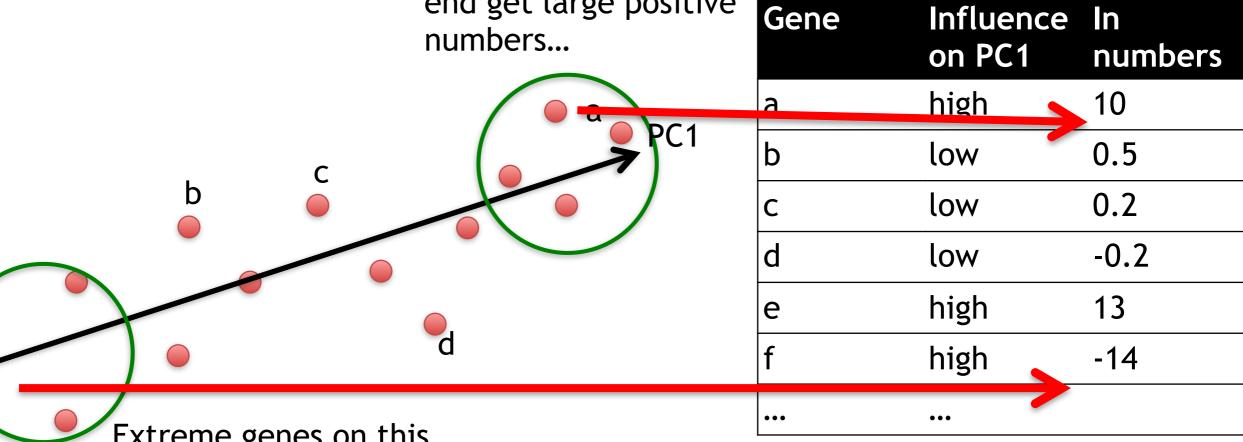
Some genes have more influence on PC1 than others.

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | ••• | |

Genes with little influence on PC1 get values close to zero, and genes with more influence get numbers further from zero.

Some genes have more influence on PC1 than others.

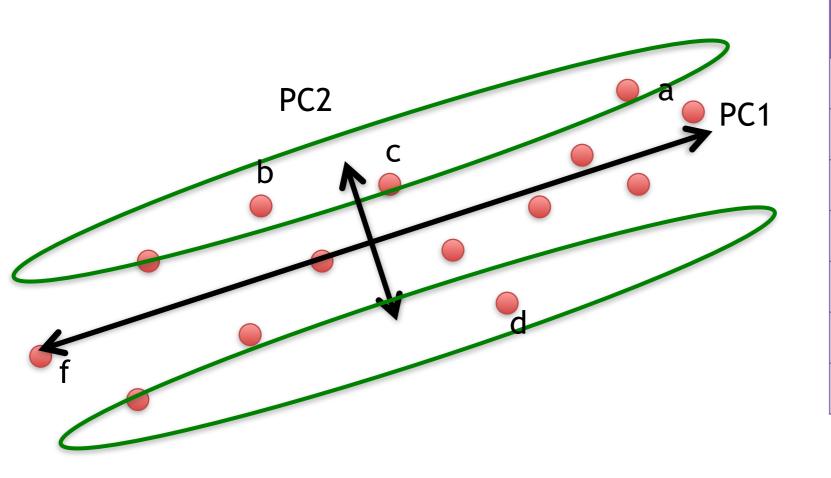
Extreme genes on this end get large positive numbers...



Extreme genes on this end get large negative numbers...

Genes with little influence on PC1 get values close to zero, and genes with more influence get numbers further from zero.

Genes that influence PC2



| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Our two PCs

PC1 PC2

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | ••• | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Combining the read counts for all genes in a cell to get a single value.

PC1 PC2

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | ••• | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC1

PC2

| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | ••• | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC1

PC2

| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |

| Gene | Influence on PC1 | |
|------|------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Cell1 PC1 score = (read count * influence) + ... for all genes

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC1

PC2

| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |
| | | |

| Gene | Influence on PC1 | |
|------|---|-------------------|
| a | high | 10 |
| b | low | 0. <mark>5</mark> |
| С | low | 0. <mark>2</mark> |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | <u>, , , , , , , , , , , , , , , , , , , </u> | |
| | | |

Cell1 PC1 score = (10 * 10) + ...

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC1

PC2

| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 18 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Cell1 PC1 score = (10 * 10) + (0 * 0.5) + ...

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC1

PC2

| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | ••• | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

Cell1 PC1 score =
$$(10 * 10) + (0 * 0.5) + ... etc... = 12$$

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC1

PC2

| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| e | high | 13 |
| f | high | -14 |
| | ••• | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 1 0 |
| С | high | 8 |
| d | high | -12 |
| e | low | 0.2 |
| f | low | -0.1 |
| | ••• | |

Cell1 PC1 score =
$$(10 * 10) + (0 * 0.5) + ... etc... = 12$$

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC₁

PC2

| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |

| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| e | high | 13 |
| f | high | -14 |
| | | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | løw | -0.1 |
| | | |

Cell1 PC1 score = (10 * 10) + (0 * 0.5) + ... etc... = 12 Cell1 PC2 score = (10 * 3) + (0 * 10) + ...

Combining the read counts for all genes in a cell to get a single value.

The original read counts

PC1

PC2

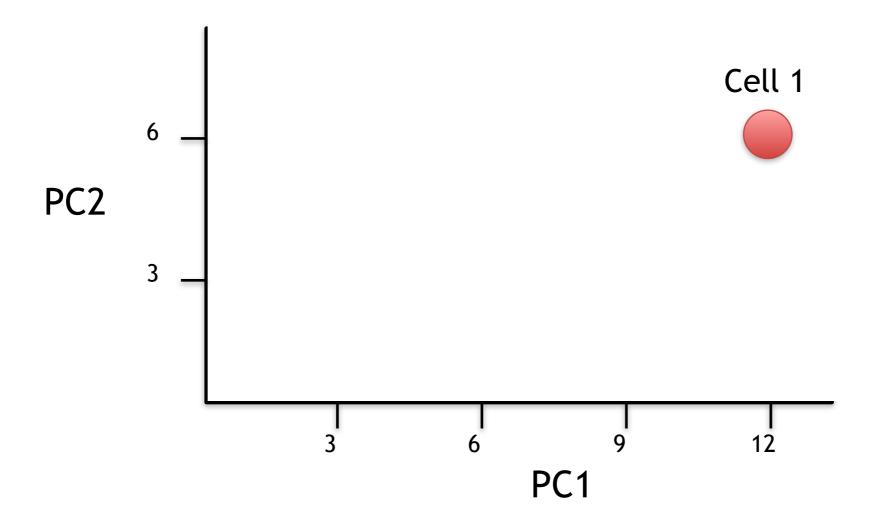
| Gene | Cell1 | Cell2 |
|------|-------|-------|
| a | 10 | 8 |
| b | 0 | 2 |
| С | 14 | 10 |
| d | 33 | 45 |
| е | 50 | 42 |
| f | 80 | 72 |
| g | 95 | 90 |
| h | 44 | 50 |
| i | 60 | 50 |
| etc | etc | etc |
| | | |

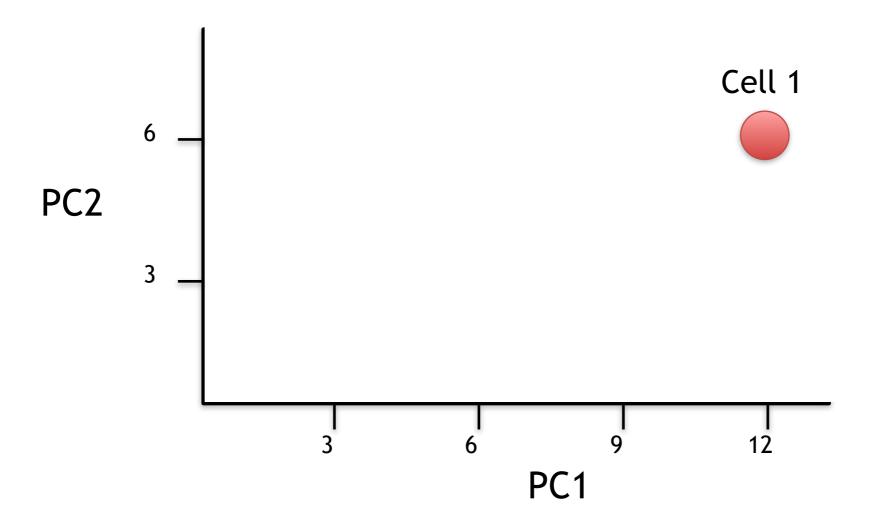
| Gene | Influence on PC1 | |
|------|---------------------|------|
| a | high | 10 |
| b | low | 0.5 |
| С | low | 0.2 |
| d | low | -0.2 |
| е | high | 13 |
| f | high | -14 |
| ••• | ••• | |

| Gene | Influence on PC2 | In numbers |
|------|---------------------|---------------|
| a | medium | 3 |
| b | high | 10 |
| С | high | 8 |
| d | high | -12 |
| е | low | 0.2 |
| f | low | -0.1 |
| ••• | ••• | |

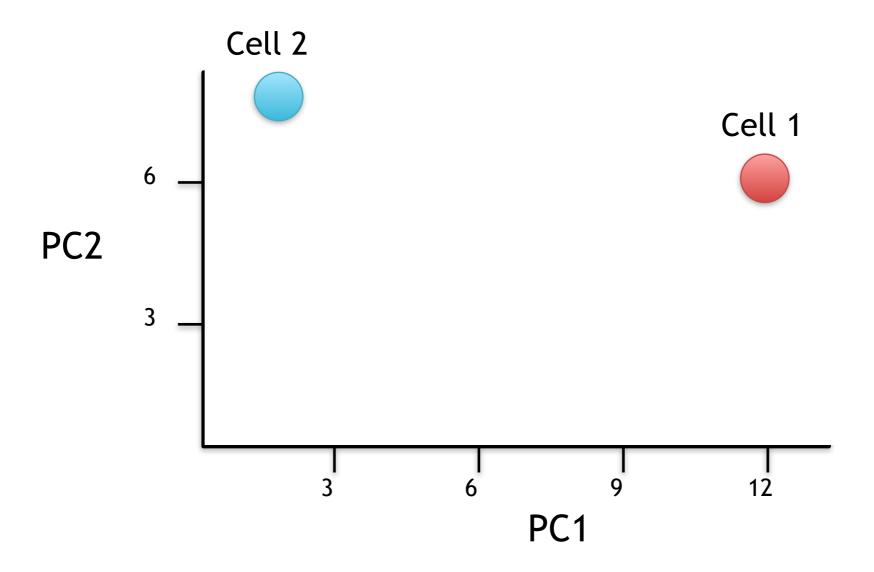
Cell1 PC1 score = (10 * 10) + (0 * 0.5) + ... etc... = 12

Cell1 PC2 score = (10 * 3) + (0 * 10) + ... etc... = 6

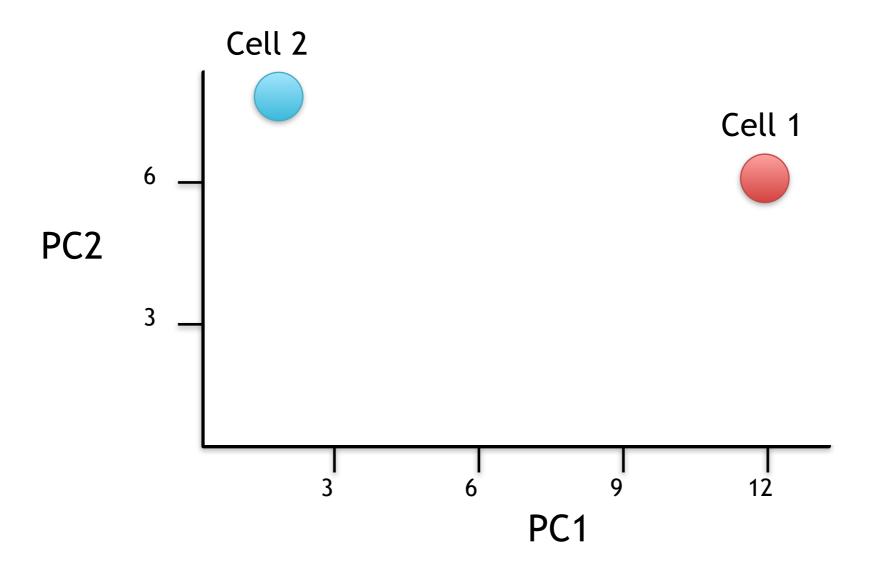




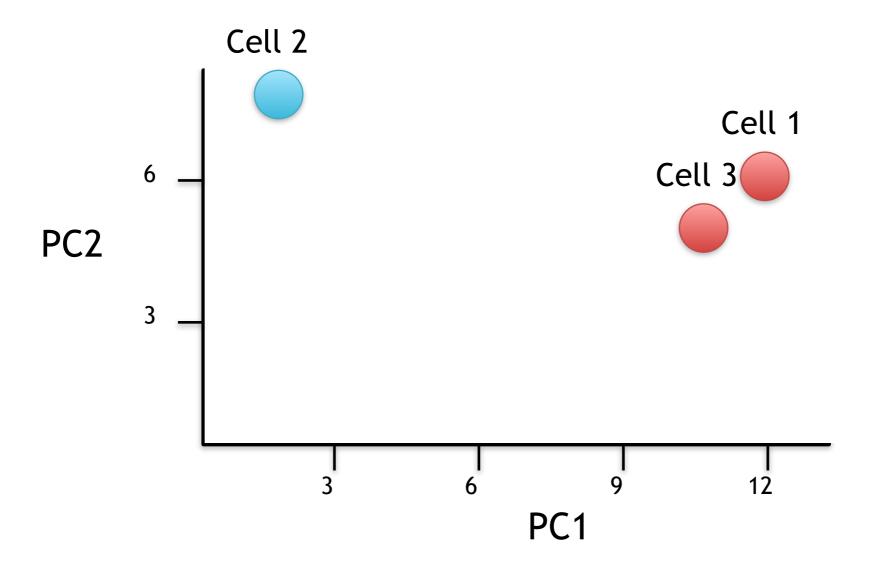
Now calculate scores for Cell2



Now calculate scores for Cell2

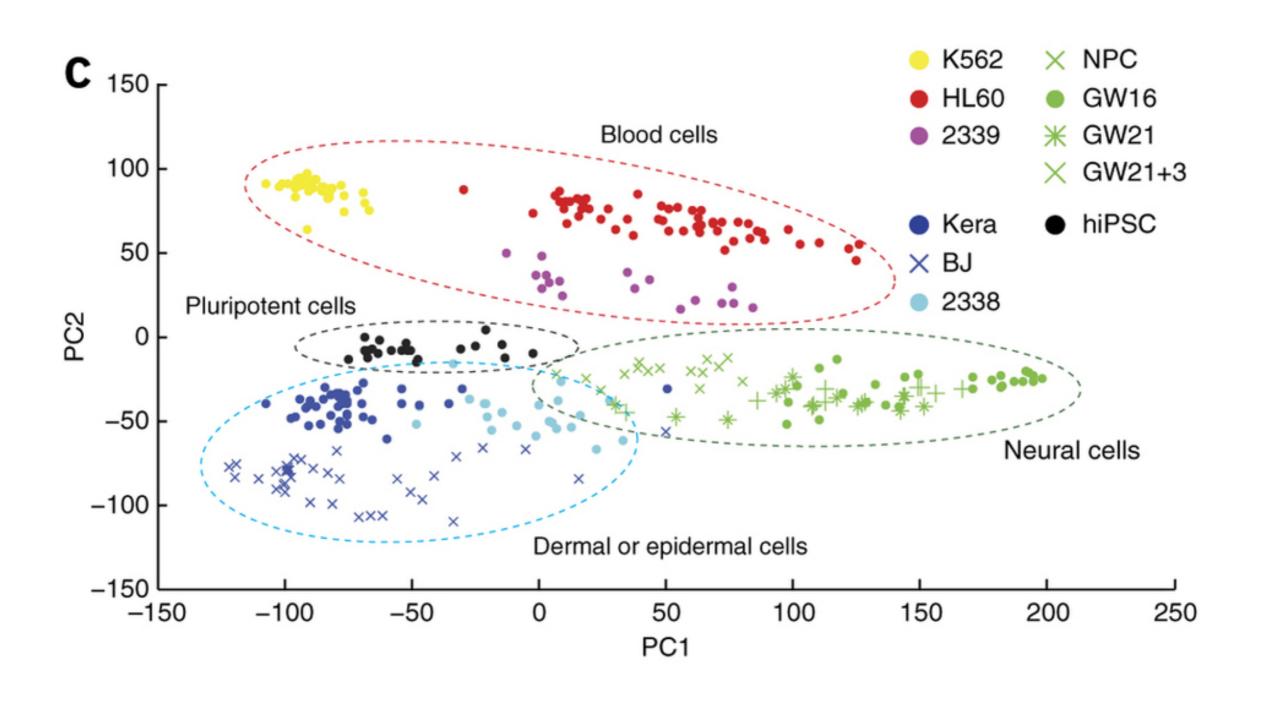


If we sequenced a third cell, and its transcription was similar to cell 1, it would get scores similar to cell 1's.



If we sequenced a third cell, and its transcription was similar to cell 1, it would get scores similar to cell 1's.

Hooray! We know how they plotted all of the cells!!!



Back to lab Focus on Section 3 to 6...

Unsupervised Learning Mini-Project

Input: read, View/head,

PCA: prcomp,

Cluster: kmeans, hclust Compare: plot, table, etc.

[Muddy Point Assessment]

BONUS: Predictive Modeling with PCA Components

We can use our PCA and clustering models to predict the potential malignancy of new samples:

```
## Predicting Malignancy Of New samples

url <- "https://tinyurl.com/new-samples-CSV"
new <- read.csv(url)
npc <- predict(wisc.pr, newdata=new)

plot(wisc.pr$x[,1:2], col= (diagnosis+1))
points(npc[,1], npc[,2], col="blue", pch=16)</pre>
```

[Muddy Point Assessment]