Probability Theory

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Abstract

1 Introduction

1.1 How to write Mathematics

1.1.1 多项式定理

$$(x_1 + x_2 + \cdots + x_r)^n = \sum_{\substack{(n_1, \cdots, n_r): \\ n_1 + \cdots + n_r = n}} \binom{n}{n_1, n_2, \cdots, n_r} x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$$

上式的求和号是对满足 $n_1 + n_2 + \cdots + n_r = n$ 的所有非负整数向量 $(n_1, n_2, \ldots, n_r)$ 求和

这个式子描述了如何展开一个多项式的幂次方 $(x_1 + x_2 + \cdots + x_r)^n$。让我们逐步解释每个部分：

1. 左边的表达式：

$$(x_1 + x_2 + \cdots + x_r)^n$$

这表示将 $x_1, x_2, \ldots, x_r$ 的和的 n 次幂展开。

2. 右边的表达式：

$$\sum_{\substack{(n_1, \ldots, n_r): \\ n_1 + n_2 + \cdots + n_r = n}} \binom{n}{n_1, n_2, \ldots, n_r} x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$$

这部分是展开式的详细形式，我们来分解它：

求和符号 $\sum$：表示对右侧的项进行求和。

条件 $(n_1, \ldots, n_r) : n_1 + n_2 + \cdots + n_r = n$：这个条件确保 $n_1, n_2, \ldots, n_r$ 是非负整数，并且它们的和是 n。

多项式系数 $\binom{n}{n_1, n_2, \ldots, n_r}$：这个符号 $\binom{n}{n_1, n_2, \ldots, n_r}$ 称为多项式系数或者多项式的 multinomial coefficient。它表示从 n 个不同项中选取 n_1 个 x_1，n_2 个 x_2，…，n_r 个 x_r 的方法数。其计算公式为：

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1! n_2! \cdots n_r!}$$

各项 $x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$：这些项表示在多项式展开中每一项的具体形式，其中 $x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$ 表示 x_1 的幂为 n_1，x_2 的幂为 n_2，依此类推，直到 x_r 的幂为 n_r。

因此，整体来说，这个式子的右边表示了把多项式 $(x_1 + x_2 + \cdots + x_r)^n$ 展开成所有可能的 $x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$ 形式的和，其中 $n_1, n_2, \ldots, n_r$ 是非负整数，它们的总和为 n。

LaTeX is great at typesetting mathematics. Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n} \sum_{i}^{n} X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.

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References