Project Submission Instructions

Please complete and save your project in pdf file. Before submission, please use the following naming convention:

`firstname-lastname`-P4.pdf

Your pdf file should be submitted to the e-mail address below:

`BIOS276.31r8w8ndztx2a19n@u.box.com`

Please do not use my personal e-mail address. After submission you should receive confirmation of successful uploading. I will let you know if I am missing your project by the day following the due date.

Please describe all your work in clear terms, before implementing `R` code. Each question should include a description of your approach with clear indication of where I can find the associated source code. Your code should be attached to your assignment or uploaded online to a repository I can freely access.

Problem. Consider Bayesian inference for data arising from counting processes. Specifically, given a set of covariates $X_i \in \mathbb{R}^p$, and an associated set of regression coefficients $\beta \in \mathbb{R}^p$, assume

$$\begin{aligned} Y_i \mid \lambda_i &\sim_{ind} \text{Poisson}(e^{\lambda_i}) \\ \lambda_i \mid \beta, \tau &\sim_{ind} N(X_i'\beta, \tau) \\ \beta &\sim N\left(0, (X'X)^{-1} g\right) \\ \tau &\sim IG(1,1); \end{aligned}$$

where $X \in \mathbb{R}^{n \times p}$ is a full rank covariate matrix, and $g > 0$ is a prior hyperparameter.

a. Describe an MCMC strategy aimed at obtaining samples from the posterior distribution

$$p(\beta, \tau \mid Y)$$

b. Describe an HMC strategy aimed at obtaining samples from the posterior distribution

$$p(\beta, \tau \mid Y).$$

In your algorithm update $\beta, \tau \mid \lambda_{1:n}, Y$ using Gibbs, and $\lambda_{1:n} \mid Y, \beta, \tau$ using HMC.

c. Test algorithms in (a) and (b) on the following data:

```
y  = c(18,17,15,20,10,20,25,13,12)
x1 = gl(3,1,9)
x2 = gl(3,3)
dat = data.frame(y, x1, x2)
```

Comment on tuning, convergence and mixing.