Biostat 200C Homework 4

Due 11:59PM May 23rd

Q1. Balanced one-way ANOVA random effects model

Consider the balanced one-way ANOVA random effects model with a levels and n observations in each level

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}, \quad i = 1, \ldots, a, \quad j = 1, \ldots, n.$$

where α_i are iid from $N(0, \sigma_\alpha^2)$, ϵ_{ij} are iid from $N(0, \sigma_\epsilon^2)$.

1. Derive the ANOVA estimate for μ, σ_α^2, and σ_ϵ^2. Specifically show that

$$\begin{aligned}
\mathbb{E}(\bar{y}_{\cdot\cdot}) &= \mathbb{E}\left(\frac{\sum_{ij} y_{ij}}{na}\right) = \mu \\
\mathbb{E}(\text{SSE}) &= \mathbb{E}\left[\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i\cdot})^2\right] = a(n-1)\sigma_\epsilon^2 \\
\mathbb{E}(\text{SSA}) &= \mathbb{E}\left[\sum_{i=1}^{a}\sum_{j=1}^{n}(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2\right] = (a-1)(n\sigma_\alpha^2 + \sigma_\epsilon^2),
\end{aligned}$$

   which can be solved to obtain ANOVA estimate

$$\begin{aligned}
\widehat{\mu} &= \frac{\sum_{ij} y_{ij}}{na}, \\
\widehat{\sigma}_\epsilon^2 &= \frac{\text{SSE}}{a(n-1)}, \\
\widehat{\sigma}_\alpha^2 &= \frac{\text{SSA}/(a-1) - \widehat{\sigma}_\epsilon^2}{n}.
\end{aligned}$$

2. Calculate the three estimates for the `pulp` example in class, check if your results match with the R output.

Q2. ELMR Exercise 11.1 (p251)

The `ratdrink` data consist of 5 weekly measurements of body weight for 27 rats. The first 10 rats are on a control treatment while 7 rats have thyroxine added to their drinking water and 10 rats have thiouracil added to their water.

```
help("ratdrink")
```

1. Plot the data showing how weight increases with age on a single panel, taking care to distinguish the three treatment groups. Now create a three-panel plot, one for each group. Discuss what can be seen.
2. Fit a linear longitudinal model with a random slope and intercept for each rat. Each treatment group should have a different mean line. Give interpretation for the following estimates:

- The fixed effect intercept term.
- The interaction between thiouracil and week.

- The intercept random effect SD (standard deviation).

3. Check whether there is a significant treatment effect.
4. Construct diagnostic plots showing the residuals against the fitted values and a QQ plot of the residuals. Comment on the plots.
5. Construct confidence intervals for the parameters of the model. Which random effect terms may not be significant? Is the thyroxine group significantly different from the control group?