hw3

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Chapter 5

Question 1

- Because condensed identity coefficients all pertain to the pair i and j Thus, in the absence of inbreeding, all nonzero condensed identity coefficients can be expressed in terms of ordinary kinship coefficients.
- Also, because individual i and j are non-inbred relatives so Δ_1 to Δ_6 are equal to 0. And $\Delta_7, \Delta_8, and\ \Delta_9$ are the probabilities of interests for relative pairs in outbred, which indicated $\Delta_7 + \Delta_8 + \Delta_9 = 1$.

part 1:

- Since Δ_7 can be characterized as the probability that two individuals share IBD alleles. Δ_7 is also called the 'coefficient of fraternity'. So Δ_7 can be represented as the Global kinship coefficients from their parents' allele pair $\Phi_{km} * \Phi_{ln}$ plus the other way around $\Phi_{kn} * \Phi_{lm}$. Then we get:

$$\Delta_7 = \Phi_{km} * \Phi_{ln} + \Phi_{kn} * \Phi_{lm}$$

part 2:

- As mentioned above because individual i and j are non-inbred relatives so Δ_1 to Δ_6 are equal to 0. so the relation:

$$\begin{aligned}
\Phi_{ij} &= \Delta_1 + \frac{1}{2}(\Delta_3 + \Delta_5 + \Delta_7) + \frac{1}{4}\Delta_8, \quad and\ \Delta_1 \sim \Delta_6 = 0 \\
&= \frac{1}{2}\Delta_7 + \frac{1}{4}\Delta_8 \\
\frac{1}{4}\Delta_8 &= \Phi_{ij} - \frac{1}{2}\Delta_7 \\
\Delta_8 &= 4\Phi_{ij} - 2\Delta_7
\end{aligned}$$

part 3:

- As mentioned above, as $\Delta_7, \Delta_8, and\ \Delta_9$ are the probabilities of interests for relative pairs in outbred, which indicated $\Delta_7 + \Delta_8 + \Delta_9 = 1$, the relation:

$$\Delta_9 = 1 - \Delta_7 - \Delta_8$$

Question 2

- From the question we knew that:

$$1 = \Delta_7 + \Delta_8 + \Delta_9$$

$$\begin{aligned}\Phi_{ij} &= \frac{1}{2}\Delta_7 + \frac{1}{4}\Delta_8 \\ &= \frac{1}{4}\Phi_{km} + \frac{1}{4}\phi_{kn} + \frac{1}{4}\Phi_{lm} + \frac{1}{4}\Phi_{ln}\end{aligned}$$

$$\Delta_7 = \Phi_{km} * \Phi_{ln} + \Phi_{kn} * \Phi_{lm}$$

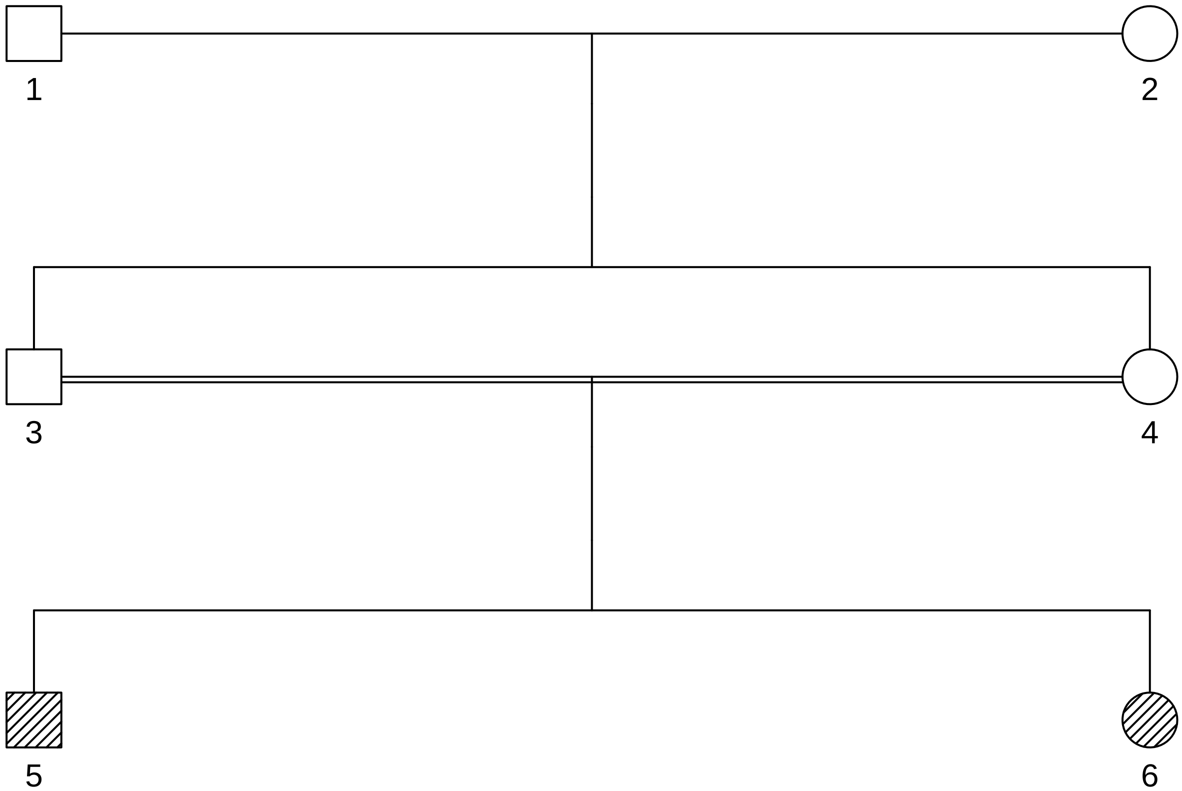
- Prove by $(a+b)^2 \geq 4ab$:

$$\begin{aligned}(4\Phi_{ij})^2 &= (\underbrace{\Phi_{km} + \Phi_{kn}}_{a} + \underbrace{\Phi_{lm} + \Phi_{ln}}_{b})^2 \\ &\geq 4(\Phi_{km} + \Phi_{kn})(\Phi_{lm} + \Phi_{ln}) \\ &\geq 4(\Phi_{km}\Phi_{kn} + \Phi_{lm}\Phi_{ln}) \\ &\geq 4\Delta_7\end{aligned}$$

$$\begin{aligned}\Delta_8 &= 4\Phi_{ij} - 2\Delta_7 \\ &\leq 4\Phi_{ij} \\ {\Delta_8}^2 &\geq 4\Delta_7 \\ \\ &\quad Due\ to\ noninbred\ relatives: \\ \Delta_9 &\leq 1 \\ \\ &\quad In\ the\ end\ we\ got: \\ {\Delta_8}^2 &\geq 4\Delta_7\Delta_9\end{aligned}$$

Quesion 3

```
x = fullSibMating(1)
plot(x, hatched = 5:6)
```



$$
\begin{aligned}
&\Delta_1 = \frac{1}{2}(\frac{1}{2})^3 = \frac{1}{16},\ which\ parents\ only\ shared\ one\ allel\ by\ descent = \frac{1}{2} \\
&\Delta_2 = \frac{1}{2}(\frac{1}{2})^3 = \frac{1}{32},\ which\ parents\ shared\ two\ allels\ by\ descent = \frac{1}{4}
\end{aligned}
$$

$$
\begin{aligned}
&\quad For\ \Delta_3 \sim \Delta_6,\ which\ needs\ parents\ to\ share\ one\ allel \\
&\Delta_3 = \frac{1}{2} * 2 * (\frac{1}{2})^3 = \frac{1}{8} \\
&\Delta_4 = \frac{1}{2} * \frac{1}{2} * (\frac{1}{2})^3 = \frac{1}{32} \\
&\Delta_5 = \frac{1}{2} * 2 * (\frac{1}{2})^3 = \frac{1}{8} \\
&\Delta_6 = \frac{1}{2} * \frac{1}{2} * (\frac{1}{2})^3 = \frac{1}{32}
\end{aligned}
$$

$$
\begin{aligned}
&\quad From\ textbook\ which\ proved: \\
&\Delta_8 = 4 * \psi_8 = \frac{5}{16}
\end{aligned}
$$

$$
\begin{aligned}
&\quad Plug\ above\ resultes\ into\ the\ equation: \\
&\Phi_{ij} = \Delta_1 + \frac{1}{2}(\Delta_3 + \Delta_5 + \Delta_7) + \frac{1}{4}\Delta_8 \\
&\quad = \frac{1}{16} + \frac{1}{2}(\frac{1}{8} + \frac{1}{8} + \Delta_7) + \frac{1}{4}(\frac{5}{16}) \\
&\Delta_7 = \frac{7}{32}
\end{aligned}
$$

$$
\begin{aligned}
&\quad Because\ \Delta_1 \sim \Delta_9\ sums\ up\ to\ 1: \\
&\Delta_9 = 1 - \Delta_8 - \Delta_7 - \Delta_6 - \Delta_5 - \Delta_4 - \Delta_3 - \Delta_2 - \Delta_1 \\
&\quad = 1 - \frac{5}{16} - \frac{7}{32} - \frac{1}{32} - \frac{1}{8} - \frac{1}{32} - \frac{1}{8} - \frac{1}{32} - \frac{1}{16} \\
&\quad = \frac{1}{16}
\end{aligned}
$$

- let's check the answers

```r
delta = identityCoefs(x, ids = 5:6)
tibble::tibble(delta)
```

```
## # A tibble: 9 x 1
##    delta
##    <dbl>
## 1 0.0625
## 2 0.0312
## 3 0.125
## 4 0.0312
## 5 0.125
## 6 0.0312
## 7 0.219
## 8 0.312
## 9 0.0625
```

- Worked with Peter Yeh.