ECE 6143 Homework 3

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1 Problem 1

1.1

$$\begin{aligned} k(u,v) &= \alpha k_1(u,v) + \beta k_2(u,v) \\ &= \sqrt{\alpha}\phi_1^T(u)\sqrt{\alpha}\phi_1(v) + \sqrt{\beta}\phi_2^T(u)\sqrt{\beta}\phi_2(v) \\ &= [\sqrt{\alpha}\phi_1^T(u), \sqrt{\beta}\phi_2^T(u)] \begin{bmatrix} \sqrt{\alpha}\phi_1(v) \\ \sqrt{\beta}\phi_2(v) \end{bmatrix} \end{aligned}$$

1.2

$$\begin{aligned} k(u,v) &= k_1(u,v)k_2(u,v) \\ &= \phi_1^T(u)\phi_1(v)\phi_2^T(u)\phi_2(v) \\ &= \sum_i f_i(u)f_i(v) \sum_j g_j(u)g_j(v) \\ &= \sum_{ij} f_i(u)g_j(u)f_i(v)g_j(v) \\ &= \phi_3^T(u)\phi_3(v) \end{aligned}$$

Where $\phi_3^T(x) = [f_1(x)g_1(x), ..., f_i(x)g_j(x), ...]$

1.3

Since $f : X \to X$, let $f(u) = a, f(v) = b$

$$\begin{aligned} k(u,v) &= k_1(f(u), f(v)) \\ &= k_1(a,b) \end{aligned}$$

1.4

Since $g : X \to R$, let $g(u) = a, g(v) = b$

$$\begin{aligned} k(u,v) &= g(u)g(v) \\ &= ab \\ &= [a]^T[b] \end{aligned}$$

1.5

$$
\begin{aligned}
k(u,v) &= f(k_1(u,v)) \\
&= \alpha_0 k_1^0(u,v) + \alpha_1 k_1^1(u,v) + \alpha_2 k_1^2(u,v) + ... + \alpha_n k_1^n(u,v) \\
&= \left[\sqrt{\alpha_0} k_1^{0/2}(u,v), \sqrt{\alpha_1} k_1^{1/2}(u,v), ..., \sqrt{\alpha_n} k_1^{n/2}(u,v)\right] \begin{bmatrix} \sqrt{\alpha_0} k_1^{0/2}(u,v) \\ \sqrt{\alpha_1} k_1^{1/2}(u,v) \\ ... \\ \sqrt{\alpha_n} k_1^{n/2}(u,v) \end{bmatrix}
\end{aligned}
$$

1.6

$$
\begin{aligned}
k(u,v) &= \exp(k_1(u,v)) \\
&= \sum_{i=1}^{\infty} \frac{k_1^i(u,v)}{i!} \text{ (Applying Taylor Expansion)}
\end{aligned}
$$

Then the reason will be the same as section 1.5

1.7

Define $\phi(x) = (\frac{\pi\sigma^2}{4})^{-d/4} \exp(-2||x-z||^2/\sigma^2)$, which is an infinite dimensional function over $z \in \mathbb{R}^d$

$$
\begin{aligned}
k(u,v) &= \int_z \phi(u)\phi(v) \mathrm{d}z \\
&= \int_z (\frac{\pi\sigma^2}{4})^{-d/4} \exp(-\frac{2||u-z||^2}{\sigma^2})(\frac{\pi\sigma^2}{4})^{-d/4} \exp(-\frac{2||v-z||^2}{\sigma^2}) \mathrm{d}z \\
&= (\frac{\pi\sigma^2}{4})^{-d/2} \exp\left(\frac{2}{\sigma^2}(-u^T u - v^T v)\right) \int_z \exp \frac{2}{\sigma^2}\left(2(u+v)^T z - 2z^T z\right) \mathrm{d}z
\end{aligned}
$$

Denote $w = (u+v)/2$, we have $w^T w = \frac{1}{4}(u^T u + 2u^T v + v^T v)$

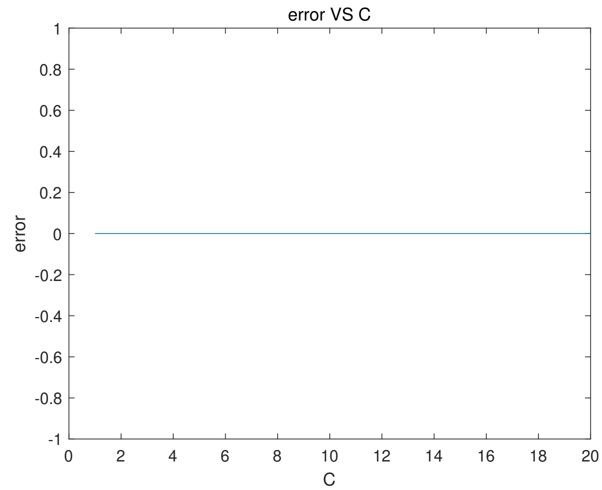
$$
\begin{aligned}
k(u,v) &= (\frac{\pi\sigma^2}{4})^{-d/2} \exp\left(\frac{2}{\sigma^2}(-u^T u - v^T v)\right) \int_z \exp \frac{2}{\sigma^2}\left(4w^T z - 2z^T z\right) \mathrm{d}z \\
&= (\frac{\pi\sigma^2}{4})^{-d/2} \exp\left(\frac{2}{\sigma^2}(-u^T u - v^T v)\right) \exp \frac{2}{\sigma^2}(2w^T w) \int_z \exp \frac{2}{\sigma^2}\left(4w^T z - 2z^T z - 2w^T w\right) \mathrm{d}z \\
&= (\frac{\pi\sigma^2}{4})^{-d/2} \exp\left(\frac{2}{\sigma^2}(-u^T u - v^T v + 2w^T w)\right) \int_z \exp\left(-\frac{4}{\sigma^2}||z-w||^2\right) \mathrm{d}z \\
&= (\frac{\pi\sigma^2}{4})^{-d/2} \exp\left(\frac{2}{\sigma^2}(-\frac{1}{2})(u^T u + v^T v - 2u^T v)\right) (\frac{\pi\sigma^2}{4})^{d/2} \\
&= \exp(-\frac{||u-v||^2}{\sigma^2})
\end{aligned}
$$

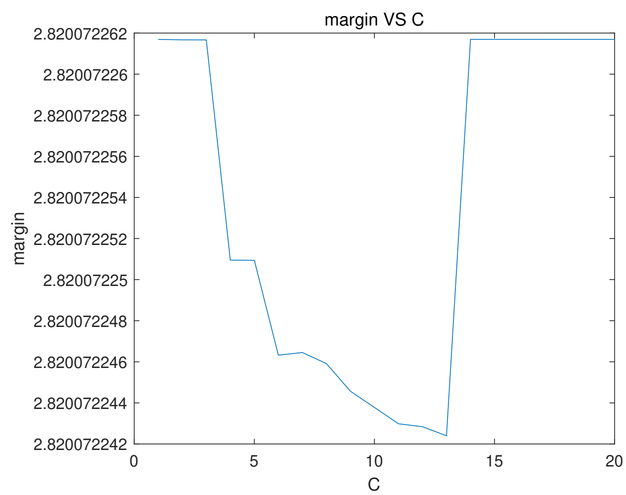
So proved

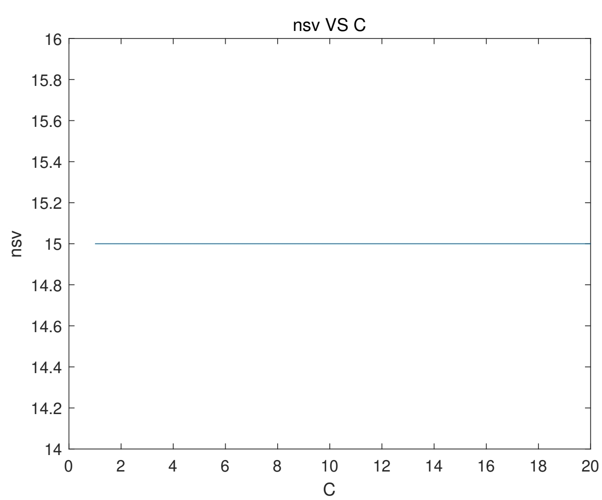
2 Problem 2

For alpha values, please refer to alpha_linear.mat, alpha_poly.mat, and alpha_rbf.mat

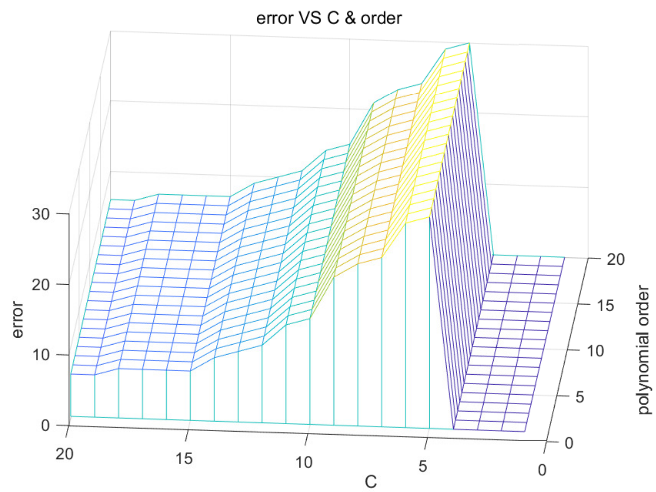
2.1 Linear Kernel

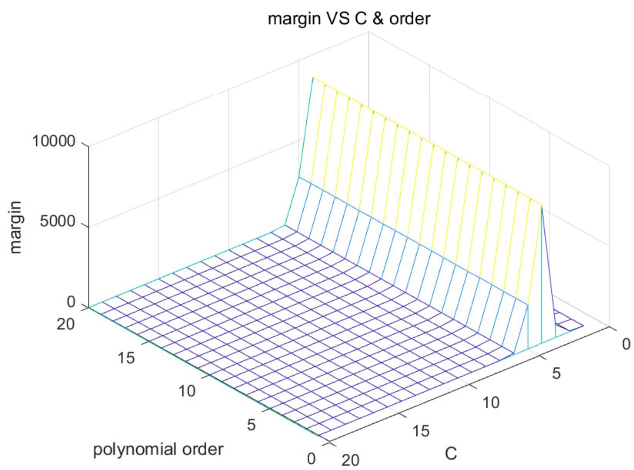


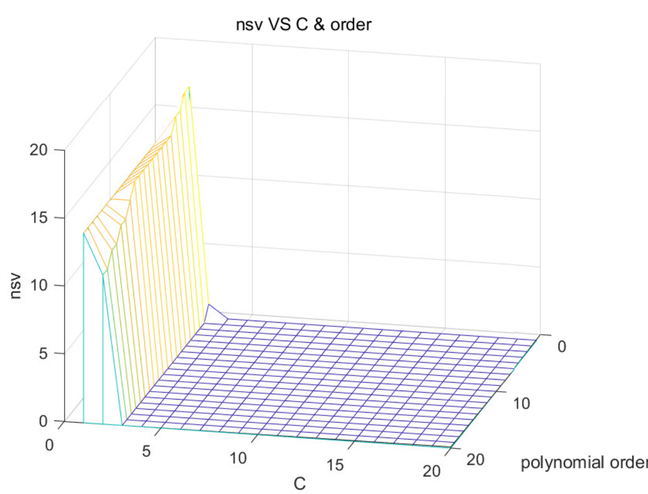




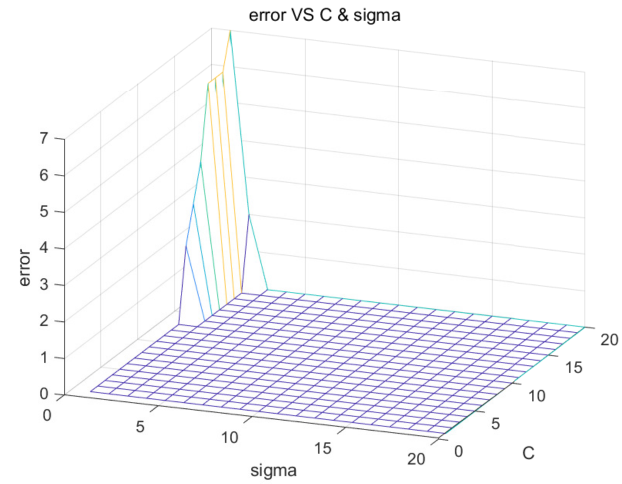
2.2 Polynomial Kernel

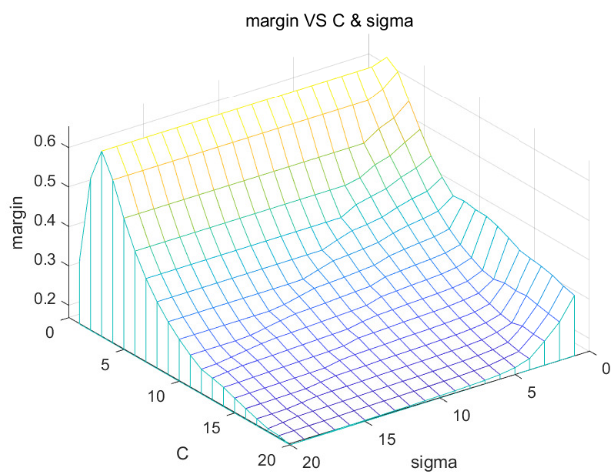


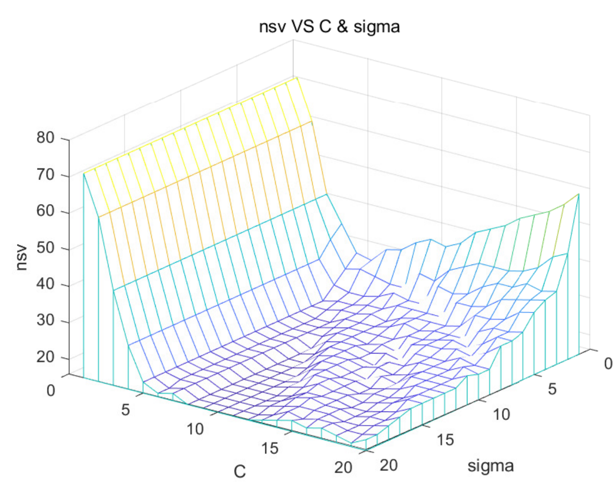




2.3 RBF Kernel







3 p3

Applying PCA, the top three eigenvectors with corresponding eigen values are being used. We can see that the edges are being saved while patterns on the teapot is begin abandoned.

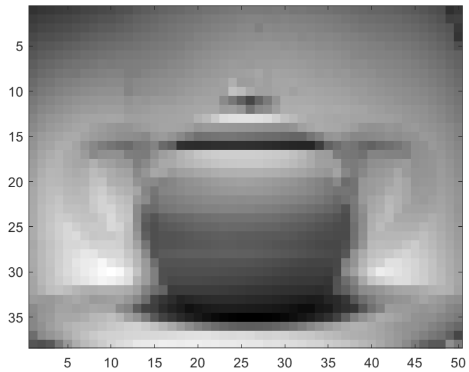


Figure 2: mean figure

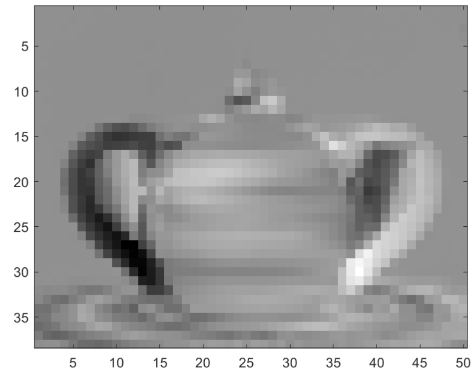


Figure 3: eigenvector 1

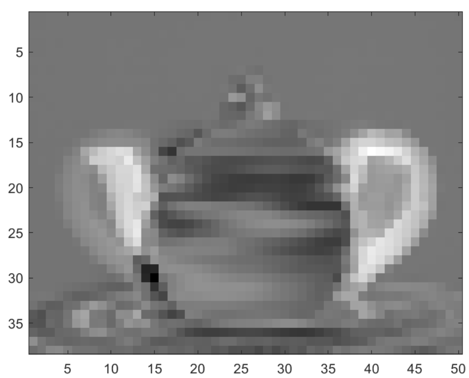


Figure 4: eigenvector 2

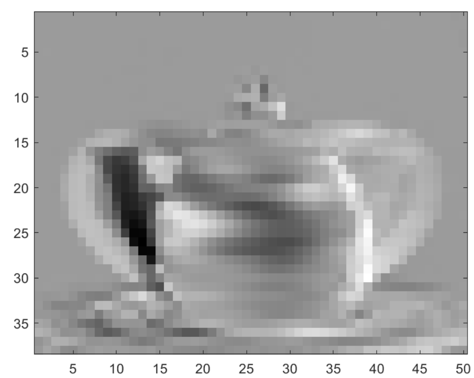


Figure 5: eigenvector 3

Figure 6: original figure

Figure 7: after pca

Figure 8: original figure

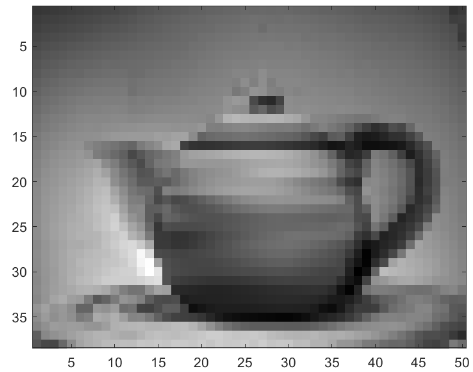


Figure 9: after pca

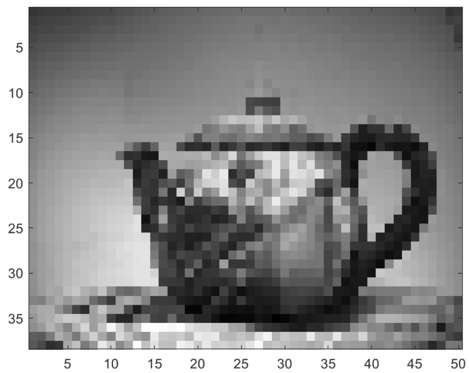


Figure 10: original figure

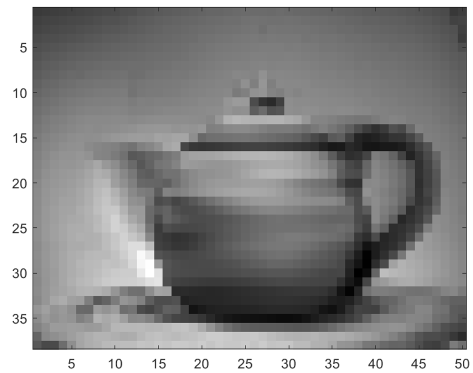


Figure 11: after pca

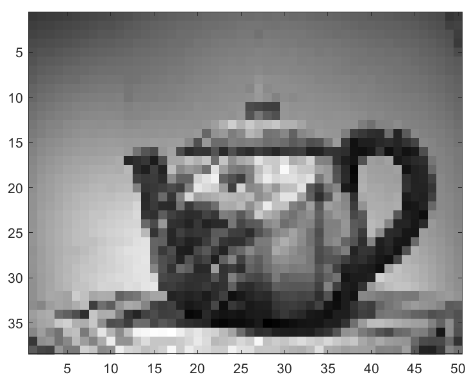


Figure 12: original figure

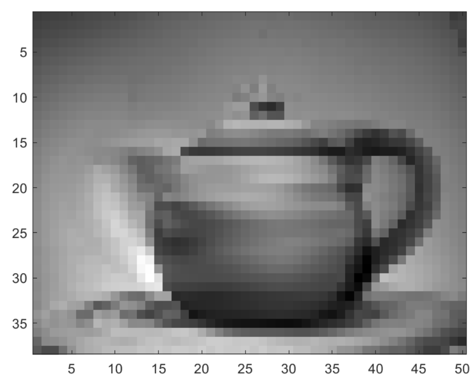


Figure 13: after pca

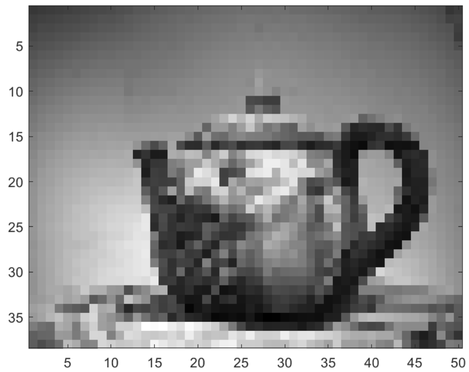


Figure 14: original figure

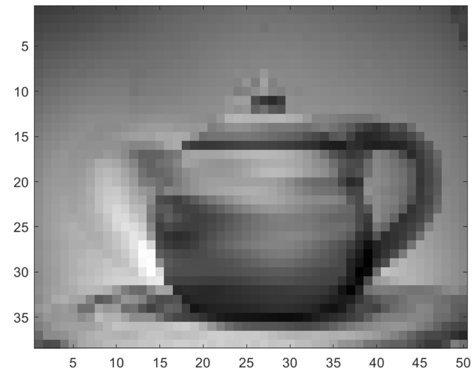


Figure 15: after pca

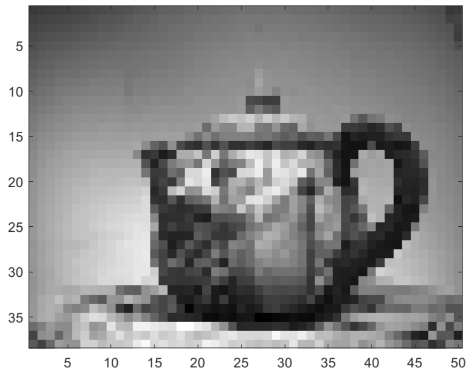


Figure 16: original figure

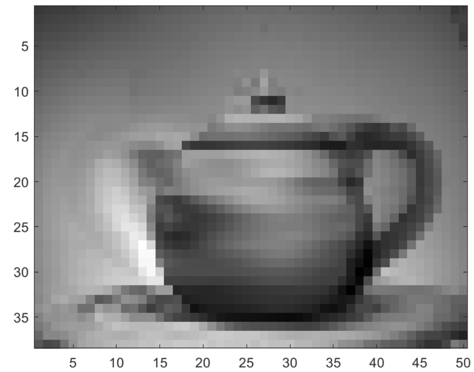


Figure 17: after pca

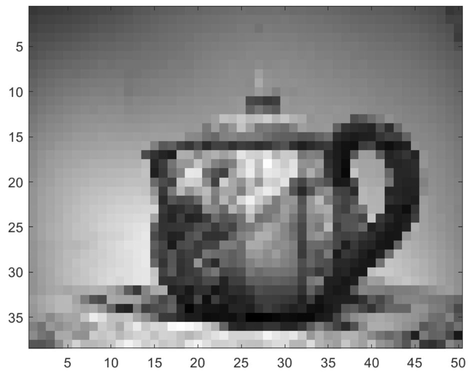


Figure 18: original figure

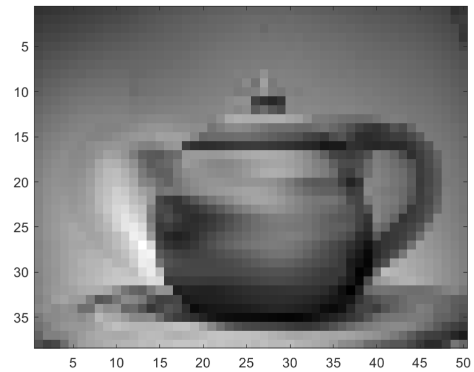


Figure 19: after pca

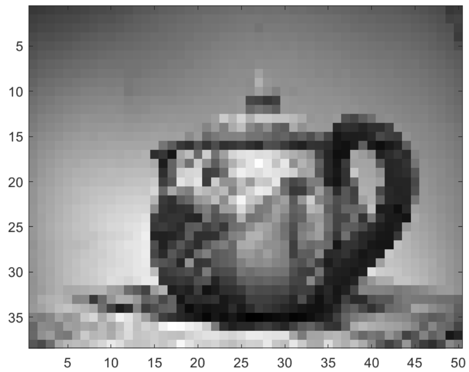


Figure 20: original figure

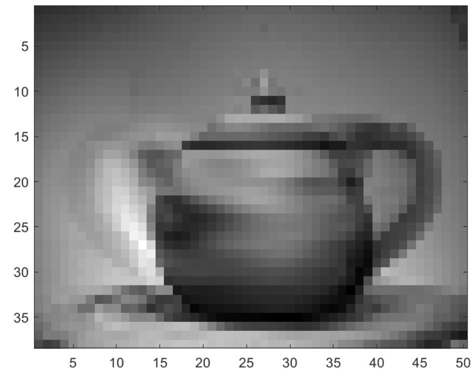


Figure 21: after pca

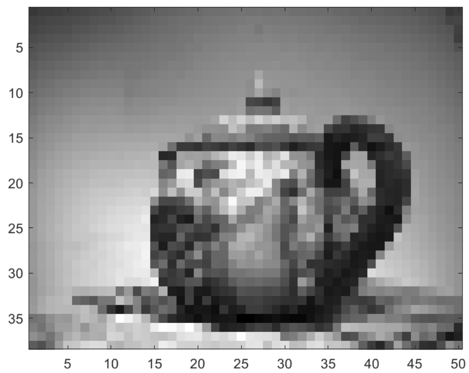


Figure 22: original figure

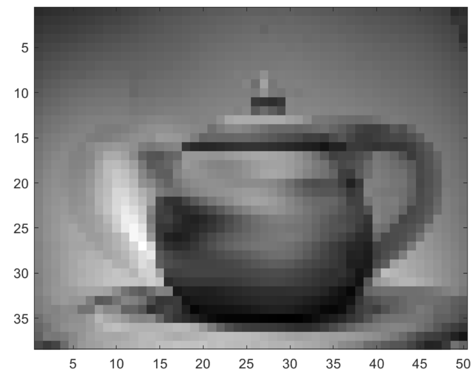


Figure 23: after pca

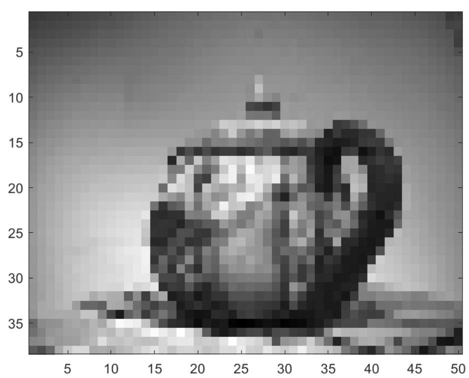


Figure 24: original figure

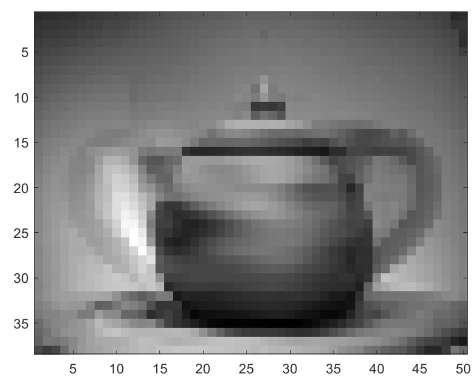


Figure 25: after pca