Finite Element Methods

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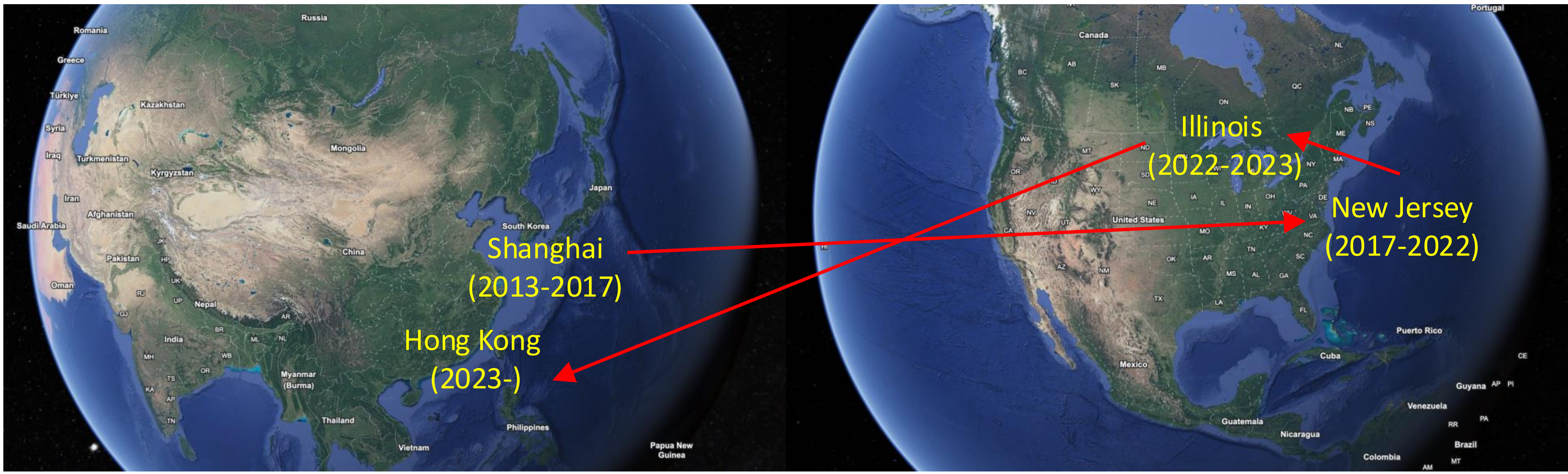
Outline

- **Introduction to FEM**
  - Basics of FEM
  - History of FEM
  - Example of FEM Simulations
- **Course logistics**
  - Syllabus
  - Grading

A little bit about the instructor…



Instructor:

- **Tianju XUE**

Email: cetxue@ust.hk

Major author of **JAX-FEM**: an open-source finite element library.

JAX-FEM

https://github.com/deepmodeling/jax-fem

What is FEM?

*"The **finite element method** (FEM) is a popular method for **numerically solving differential equations** arising in engineering and mathematical modeling."*

WIKIPEDIA
The Free Encyclopedia

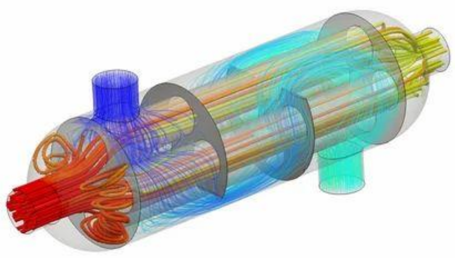
Caveat: FEM is a **numerical discretization** method, not a **modelling** approach!

Sadly, people often say *"finite element modelling"*, which is misleading…

Question: what is a discretization error, and what is a modelling error?

We use PDEs to understand the world

Heat equation

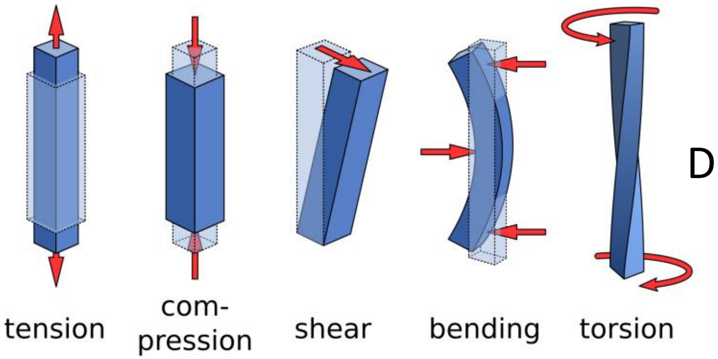


A heat exchanger

Navier–Stokes equation

Fluid flow

Linear elasticity equation



Deformed solids

Basics of FEM

Strong form

⇕

Weak form

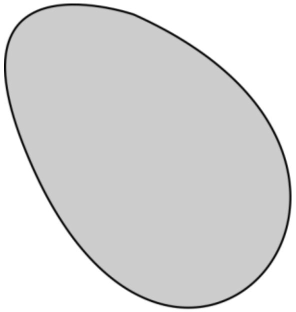
⇓

Galerkin approximation

⇕

Matrix form

Roadmap of linear FEM



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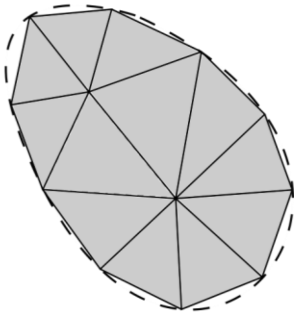

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Step 1: Discretion of a continuous system

Step 2: Determination of finite element matrices from the geometry, material, and loading data

Step 3: Assembly of the finite element matrices

Step 4: Displacement and traction boundary conditions

Step 5: Solve the resulting equation system and interpretation of the results (post-processing)

Basic procedures of FEM
(for solving a linear elasticity problem)

Basics of FEM

Strong form

⇕

Weak form

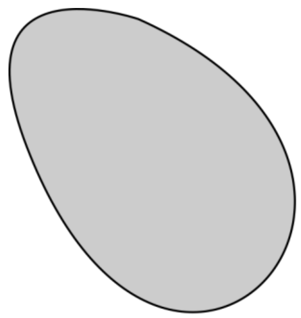
⇓

Galerkin approximation

⇕

Matrix form

Roadmap of linear FEM



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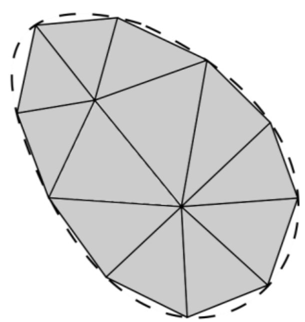

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Question 1: What if the problem is **nonlinear**?

Question 2: How do we solve **inverse** problems (e.g., structural topology optimization)?

Question 3: Can modern **deep learning** techniques help enhance classic FEM?

We will answer these questions in this course!

History of FEM

1956 The first paper on the FEM published by Turner, M. J., Clough, R. W., Martin H. C. and Topp, L. J. in 1956.

1967 First book on the FEM published by Dr. O.C. Zienkiewicz, called 'The Finite Element Method'.

1968 Ergatoudis, B. M. Irons and O. C. Zienkiewicz introduced "isoparametric' mapping for quadrilateral elements.

Late 1960s NASTRAN is a finite element analysis program developed for NASA under United States government funding for the aerospace industry.

1970 ANalysis SYStem, known as ANSYS was founded in 1970 by Dr. John Swanson

1976 LS-DYNA originated from the 3D FEA program DYNA3D, developed by Dr. John O. Hallquist at Lawrence Livermore National Laboratory (LLNL) in 1976.

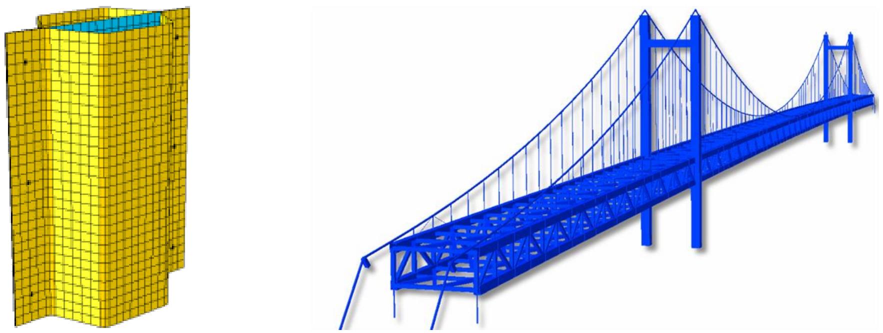
1978 Abaqus company was founded in 1978 by Dr. David Hibbitt.

1986 The eponymous company COMSOL developing the software was founded in 1986 by Svante Littmarck and Farhad Saeidi in Stockholm, Sweden.

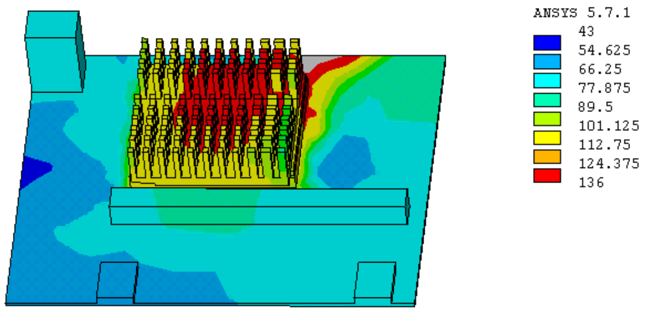
Post 2000 After 2000, more modern multi-scale simulation software (Digimat, MultiMech, etc.) started to grow.

Suggested reading: Liu, W.K., Li, S. and Park, H.S., 2022. Eighty years of the finite element method: Birth, evolution, and future. Archives of Computational Methods in Engineering, pp.1-23.

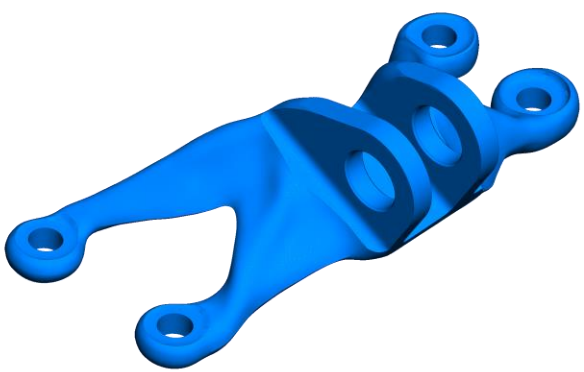
Examples of FEM simulations



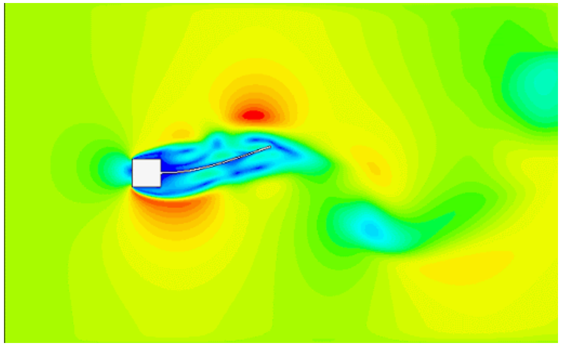
Structural analysis



Heat transfer



Shape optimization



Fluid flow

FEM software

Popular commercial FEM software

Popular open-source FEM software

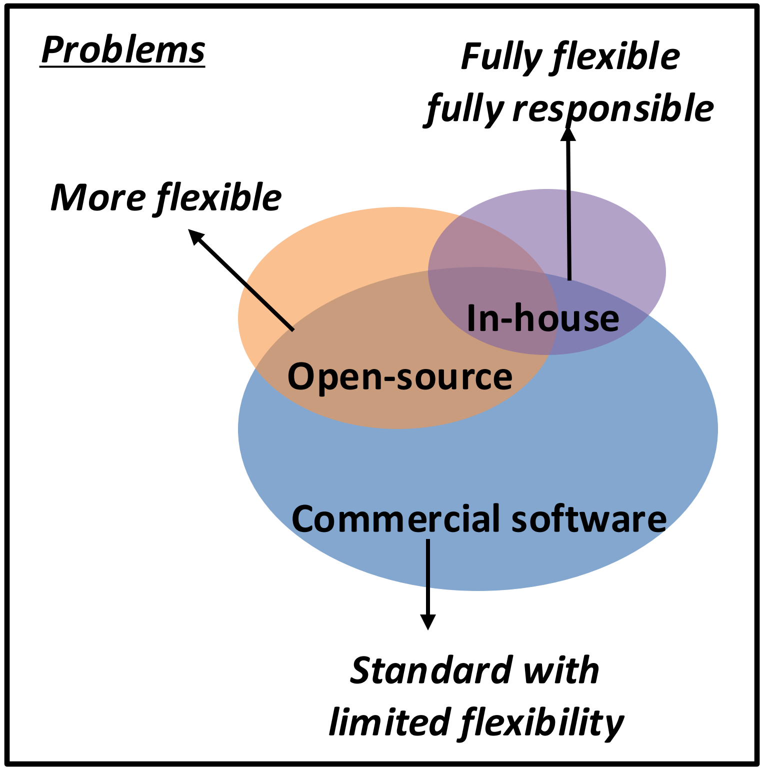
FEM software

Time required for solving a problem

In-house code
- – Writing from scratch

Open-source libraries
- – JAX-FEM
- – FEniCS

Commercial software
- – Ansys
- – Abaqus
- – COMSOL



Very important: This course does **NOT** teach you how to use commercial software.

The focus will be the essence of FEM itself (writing from scratch and use JAX-FEM).

Why should we promote open research?

yannlecun 01/25/2025

To people who see the performance of DeepSeek and think:
"China is surpassing the US in AI."
You are reading this wrong.
The correct reading is:
"Open source models are surpassing proprietary ones."

DeepSeek has profited from open research and open source (e.g. PyTorch and Llama from Meta)
They came up with new ideas and built them on top of other people's work.
Because their work is published and open source, everyone can profit from it.
That is the power of open research and open source.

Yann LeCun
Turing Award Winner

A design example with computer simulation

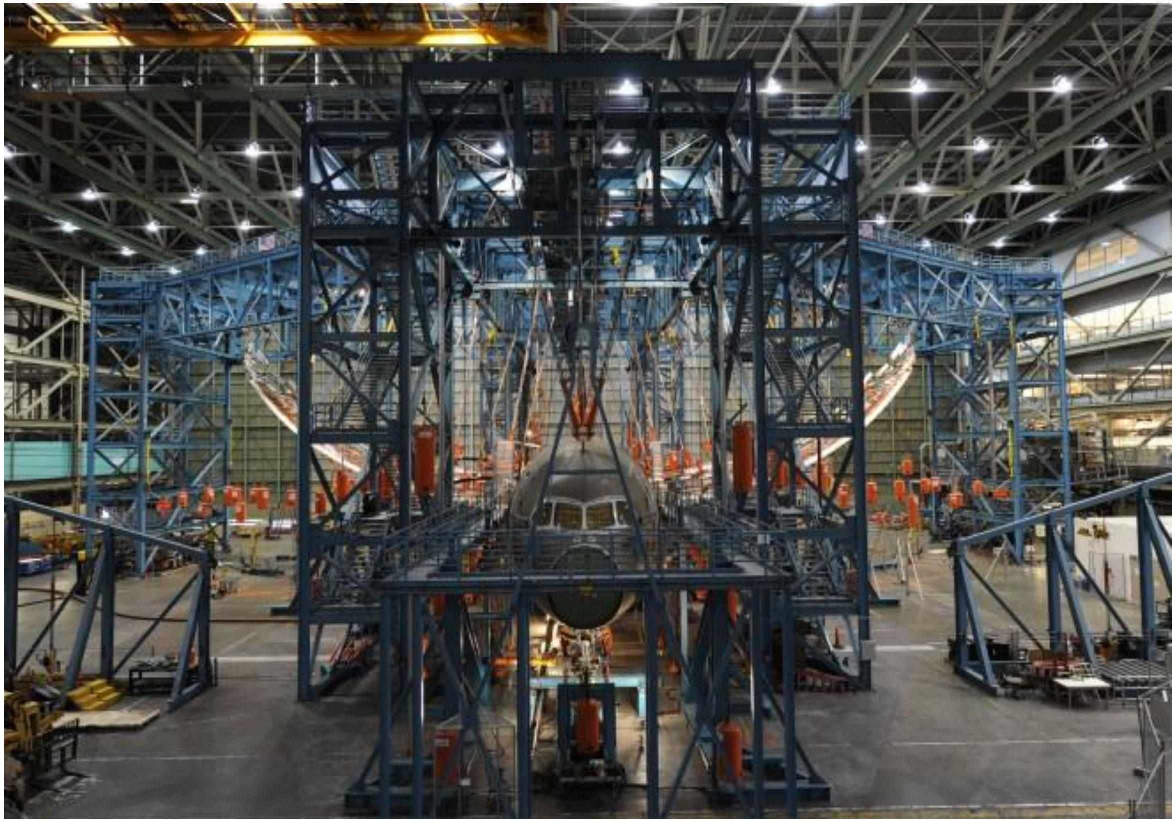
Boeing 787 Development: Simulation + Design

The Boeing 787 was targeted to be 20% more efficient than its predecessor 767. Notably the Boeing 787 is the first commercial plane to be largely comprised on composites. Composites (including carbon fiber reinforced) make up 50% of the weight and 80% of the volume

Boeing 787 Design Tools: Experiments

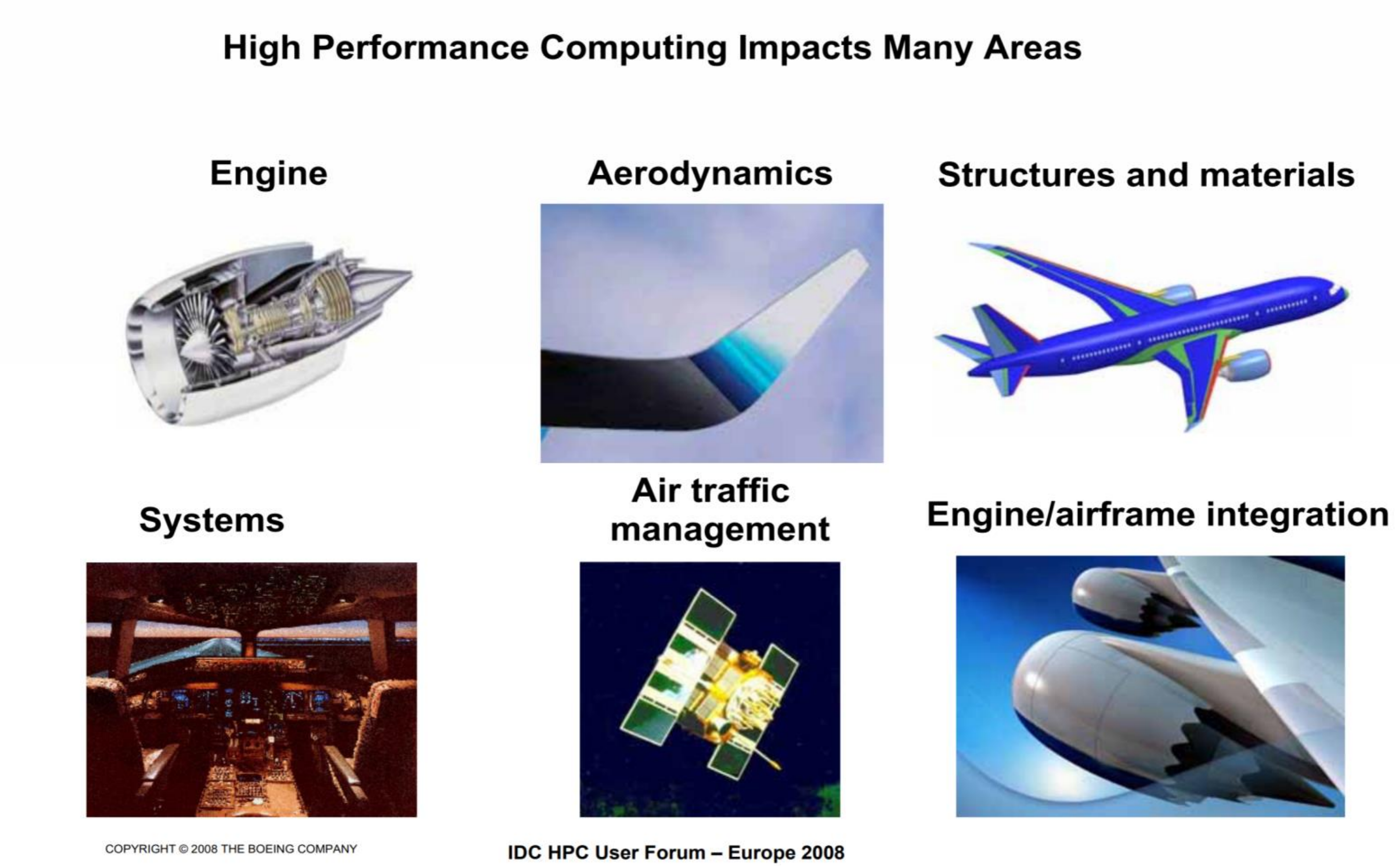
Boeing test the flexural properties of the 787 wings. Experiments can be expensive, especially for such an expensive machine.

Wings being flexed



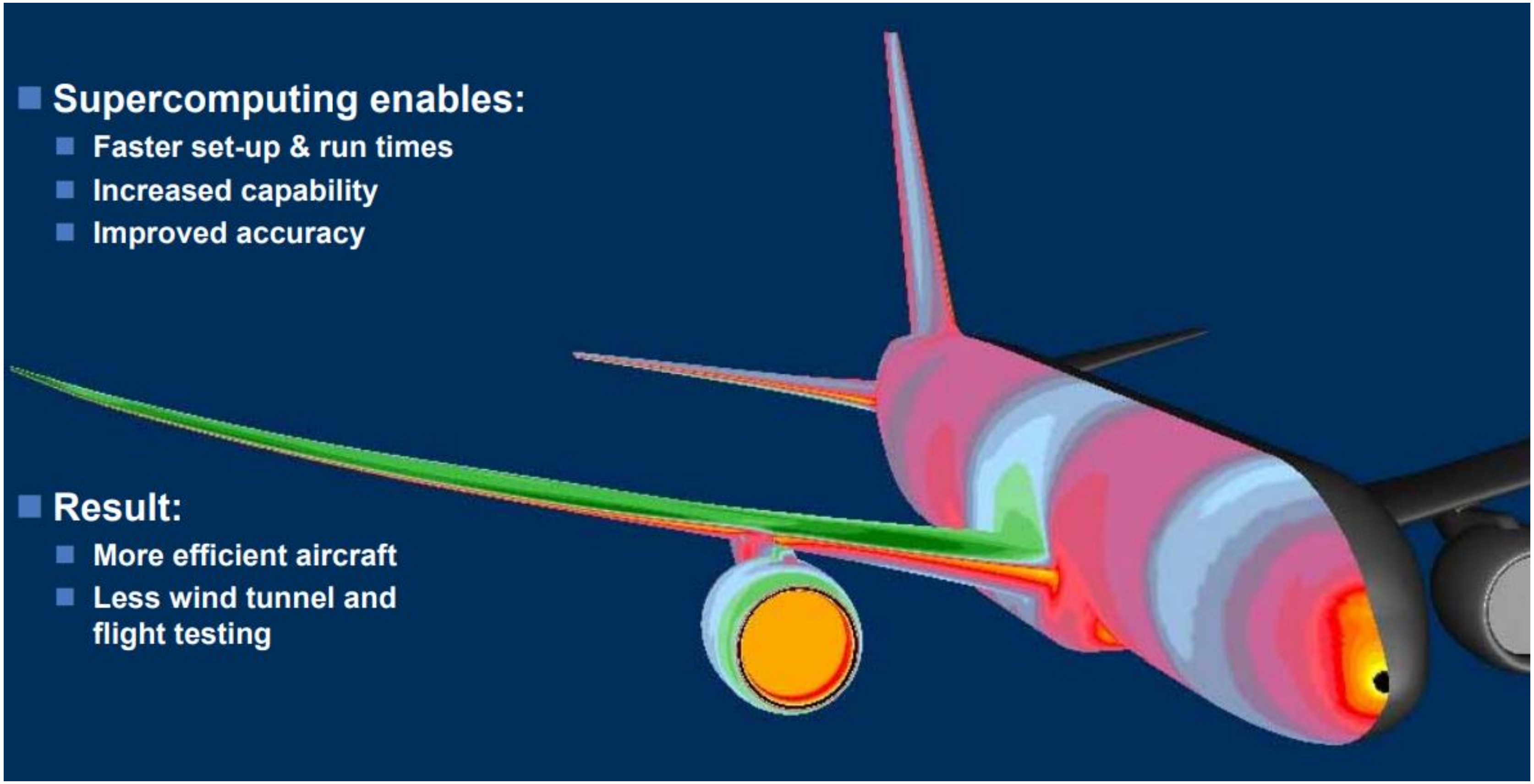
https://www.wired.com/2010/03/boeing-787-passes-incredible-wing-flex-test/

Boeing 787 Design Tools: HPC



https://www.hpcuserforum.com/presentations/Germany/Boeing%20Ball%20IDC%20pdf.pdf

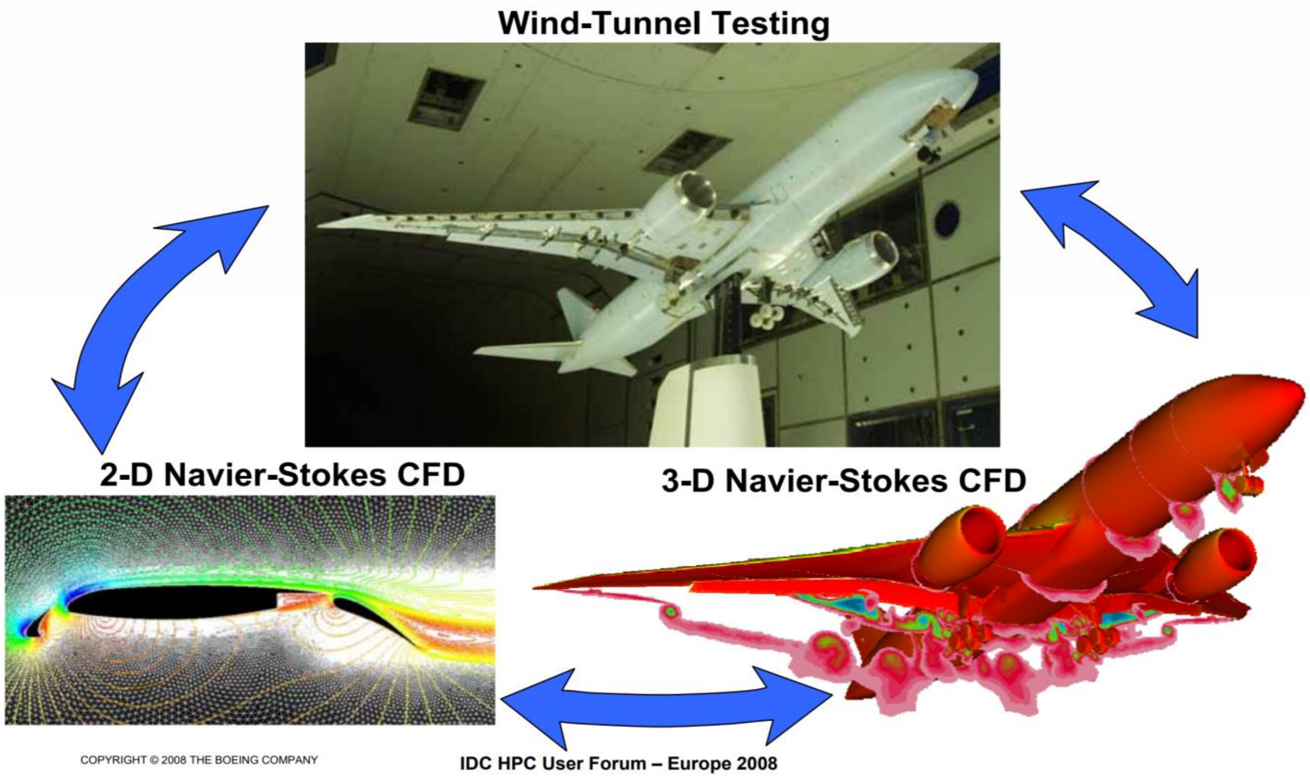
Boeing 787 Design Tools: HPC



https://www.hpcuserforum.com/presentations/Germany/Boeing%20Ball%20IDC%20pdf.pdf

Boeing 787 Design Tools: Experiments + Simulations

Simulations can leverage how many experimental tests Boeing must do. Here we can see how CFD simulations can complement wind-tunnel testing.



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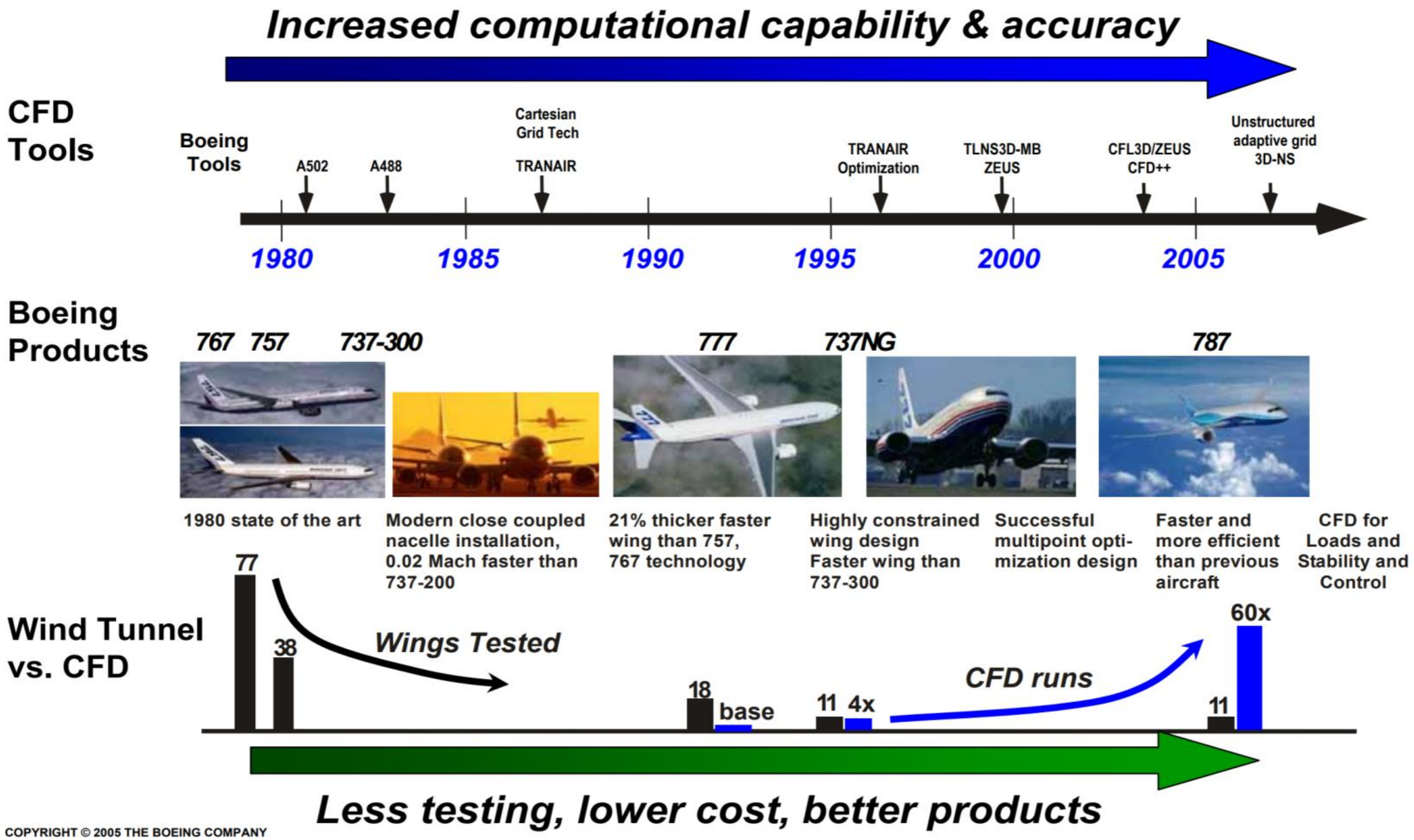
Boeing 787 Design Tools: Computational Efficiency



https://www.hpcuserforum.com/presentations/Germany/Boeing%20Ball%20IDC%20pdf.pdf

Syllabus

Date	
5 Feb 2025	Lec01: Introduction; Mathematical preliminary
12 Feb 2025	Lec02: 1D linear problem; Basic error analysis
19 Feb 2025	Lec03: 2D linear problem; Element technology
26 Feb 2025	Lec04: 2D linear problem; Numerical integration; Constraints (Lagrange multiplier, degree elimination, etc.)
5 Mar 2025	Lec05: 2D linear problem; Theoretical analysis
12 Mar 2025	Lec06: Vector-valued problem (linear elasticity); Spurious solutions (volume locking, shear locking, hourglassing, etc.)
19 Mar 2025	Lec07: Beams, plates, and shells
26 Mar 2025	Lec08: Nonlinear problem (Newton's method)
9 April 2025	Midterm exam; Lec09: Time-dependent problems; modal analysis
16 April 2025	Lec10: Inverse problems; Structural topology optimization
23 April 2025	Lec11: JAX, automatic differentiation and JAX-FEM
30 April 2025	Lec12: FFT methods
7 May 2025	Final project presentation

Useful information

Lectures

- Room 5510
- Wednesday: 19:00 – 21:50

Grading (Tentative)

- Participation: 10%
- Assignments: 30%
- Midterm Exam: 30%
- Final Project: 30%

Textbooks

[1] Fish, Jacob, and Ted Belytschko. *A first course in finite elements*. Vol. 1. John Wiley & Sons Limited, 2007.

[2] Hughes, Thomas JR. *The finite element method: linear static and dynamic finite element analysis*. Courier Corporation, 2003.

[3] Belytschko, Ted, Wing Kam Liu, Brian Moran, and Khalil Elkhodary. *Nonlinear finite elements for continua and structures.* John wiley & sons, 2014.