USTC-CG/2024 课程作业 实验报告

实验 3	**图像融合 Poisson Image Editing**
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功能实现 Features Implemented

作业要求部分 Required Features

Uniform & Cotangent 权重计算

实现位置：`node_mesh_weight_cotangent.cpp` & `node_mesh_weight_uniform.cpp`

不同 weight 通过一个展平的 $n \times n$ 矩阵 (`Buffer1f`) 传递给「极小曲面计算」节点。

Cotangent Weight:

$$ \begin{aligned} \text{cot}(\alpha) &= \frac{\text{edge1} \cdot \text{edge2}}{\left| \text{edge1} \times \text{edge2} \right|} \\ \text{cot}(\beta) &= \frac{\text{edge3} \cdot \text{edge4}}{\left| \text{edge3} \times \text{edge4} \right|} \end{aligned} \Rightarrow \text{weight} = -\text{cot}(\alpha) - \text{cot}(\beta) $$

```
// node_mesh_weight_cotangent.cpp
pxr::VtArray<float> result(n * n);

for (const auto& vertex : mesh->vertices()) {
    if (vertex.is_boundary()) {
        result[vertex.idx() * n + vertex.idx()] = 1.0f;
        continue;
    }

    auto sum = 0.0f;
    for (const auto& half_edge : vertex.outgoing_halfedges()) {
        auto vertex2 = half_edge.to();
        auto vertex3 = half_edge.prev().opp().to();
        auto vertex4 = half_edge.next().to();

        auto edge1 = mesh->point(vertex2) - mesh->point(vertex3);
        auto edge2 = mesh->point(vertex) - mesh->point(vertex3);
        auto edge3 = mesh->point(vertex2) - mesh->point(vertex4);
        auto edge4 = mesh->point(vertex) - mesh->point(vertex4);

        // cot = cos() / sin() => dot product / cross product
        auto cot1 = edge1.dot(edge2) / (edge1.cross(edge2).norm());
        auto cot2 = edge3.dot(edge4) / (edge3.cross(edge4).norm());

        cot1 = abs(cot1);
        cot2 = abs(cot2);
```

```
        sum += cot1 + cot2;
        result[vertex.idx() * n + vertex2.idx()] = -cot1 - cot2;
    }
    result[vertex.idx() * n + vertex.idx()] = sum;
}
```

极小曲面计算

实现位置：`node_min_surf.cpp`

将传递进来的 weight 重新展开成 `Eigen::SparseMatrix`，并通过 `Eigen::SparseLU` 求解线性方程组。

Laplace 方程的建立和求解：

- 边界条件： $$ u = g\quad x \in \Omega \Rightarrow v'_i = v_i \quad \text{if } i \in \text{boundary} $$
- 平均曲率为零： $$ \Delta u = 0 \quad x \in \Omega \Rightarrow \sum_{j \in N(i)} w_{ij} (v_i - v_j) = 0 $$

考虑建立一个大型稀疏矩阵，其中每一行、列对应一个顶点，每个元素对应一个权重; 上述条件中的每个方程对应一个行向量 (边界点不参与平均曲率的计算，意味着每个节点对应一个方程)

```
// node_min_surf.cpp
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        if (weight[i * n + j] != 0)
            coefficients.emplace_back(i, j, weight[i * n + j]);

for (const auto& vertex : mesh->vertices()) {
    auto i = vertex.idx();
    PolyMesh::Point p = mesh->point(vertex);
    if (vertex.is_boundary()) {
        b_x[i] = p[0];
        b_y[i] = p[1];
        b_z[i] = p[2];
        continue;
    }
    else {
        b_x[i] = 0;
        b_y[i] = 0;
        b_z[i] = 0;
    }
}

// Generate sparse matrix out of coefficients
Eigen::SparseMatrix<double> A(n, n);
A.setZero();
A.setFromTriplets(coefficients.begin(), coefficients.end());
// Solve the linear system
Eigen::SparseLU<Eigen::SparseMatrix<double>> solver;
solver.compute(A);
```

```
    if (solver.info() != Eigen::Success) {
        throw std::runtime_error("Failed to decompose the matrix.");
    }
```

Tutte 参数化

实现位置：`node_boundary_mapping.cpp`

思路：首先找到一个边界点; 从这个边界点开始寻找下一个，直至回到自身。(由于 Openmesh 边界只存储了一个 half edge, 上述遍历总能保证单向)

```
// node_boundary_mapping.cpp
auto find_boundary_loop(PolyMesh* mesh)
{
    OpenMesh::SmartVertexHandle start_vertex;
    for (auto vertex : mesh->vertices()) {
        if (vertex.is_boundary()) {
            start_vertex = vertex;
            break;
        }
    }

    std::vector<PolyMesh::VertexHandle> boundary_loop;
    auto current_vertex = start_vertex;
    do {
        boundary_loop.push_back(current_vertex);
        for (auto vertex : current_vertex.vertices()) {
            if (vertex.is_boundary()) {
                current_vertex = vertex;
                break;
            }
        }
    } while (current_vertex != start_vertex);

    return boundary_loop;
}
```

接下来只需要实现一个从 $[0,1]$ 到 $\mathbb{R}^2 \subset \mathbb{R}^3$ 的映射即可。(正方形、圆形)

纹理映射

把上述节点按照如下思路连接起来：

```
graph LR
    A[Read USD] --> B[Mesh Weight]
    A --> C[Boundary Mapping]
    B --> D[Min Surface]
    C --> D
    D --> E[Extract UV]
```

```
    A --> F[Mix UV]
    E --> F
    F --> G[Add Texture]
    G --> H[Write USD]
```

额外功能 Extra Features

Least Squares & Shape-preserving 权重计算

实现位置：`node_mesh_wls.cpp` & `node_mesh_weight_shape_preserving.cpp`

Least Squares Weight:

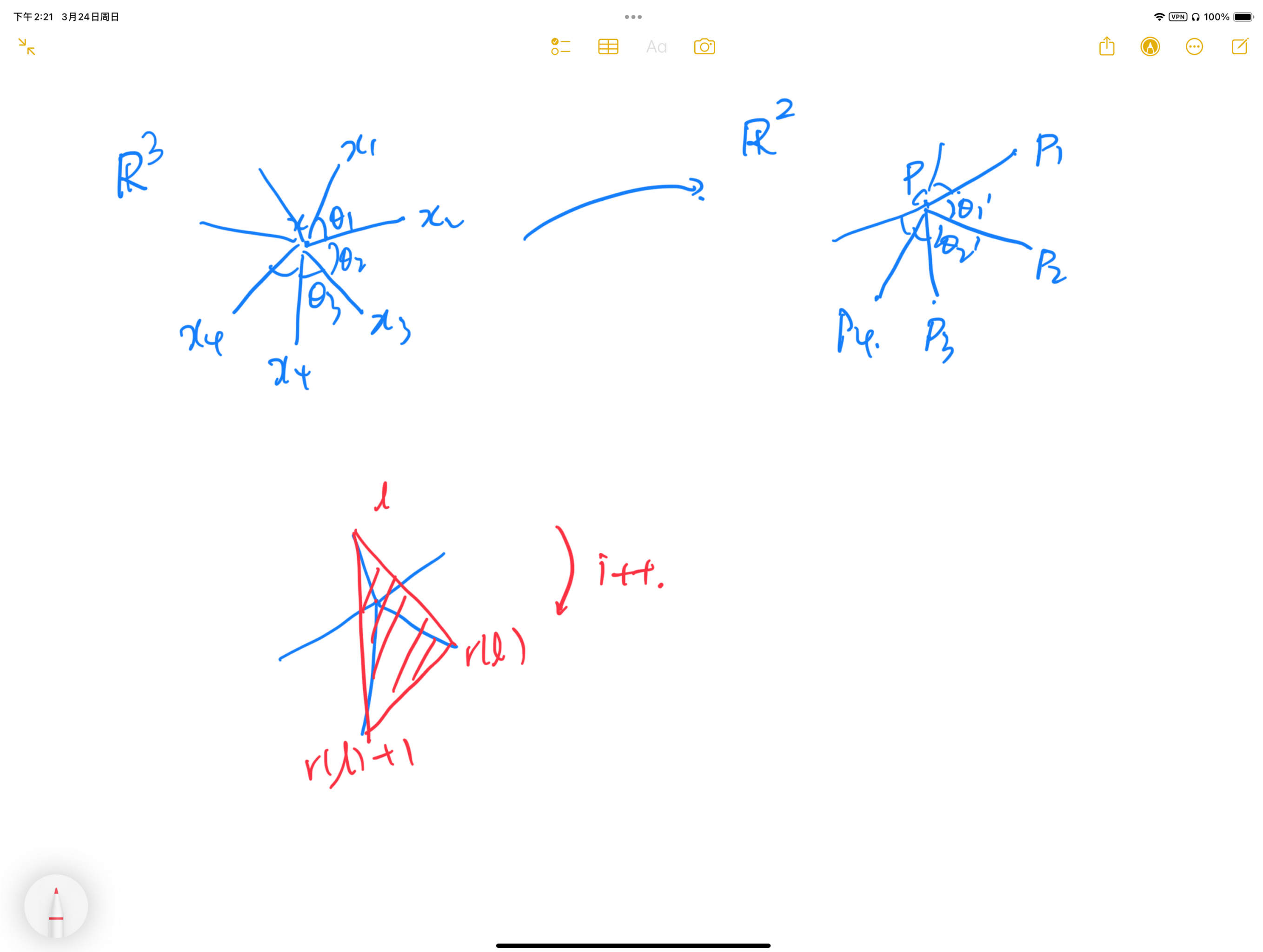
$$ \displaystyle w_{ij} = \frac{1}{\left| \boldsymbol{v}_i - \boldsymbol{v}_j \right|^2} $$

可提前做归一化处理，也可以直接令 $w_{ii} = \sum_{j\neq i} w_{_ij}$

Shape-preserving Weight:

实现思路较为繁琐，做一点简单的拆解：

对于任意的内点，对它的 1-邻域做如下保角度比例的变换到$\mathbb{R}^2$上：



在这样的变换前后，保证

$$ \theta_1' / \theta_1 = \theta_2' / \theta_2 = \cdots = \theta_n' / \theta_n $$

(对应实现思路：记录 `theta_sum`，然后将每个角度变换为 `2 * PI * theta / theta_sum`. 变换后每点的坐标为 $\left(r \cdot \cos(\theta'), r \cdot \sin(\theta') \right)$)

变换后各个 weight 计算方法为如下过程的累加：

对于任意的 $l \in N(i)$, 寻找这样一个 $r(l) \in N(i)$, 使得

$ P $ 在 $P_l, P_{r(l)}, P_{r(l) + 1} $ 三点构成的三角形中。

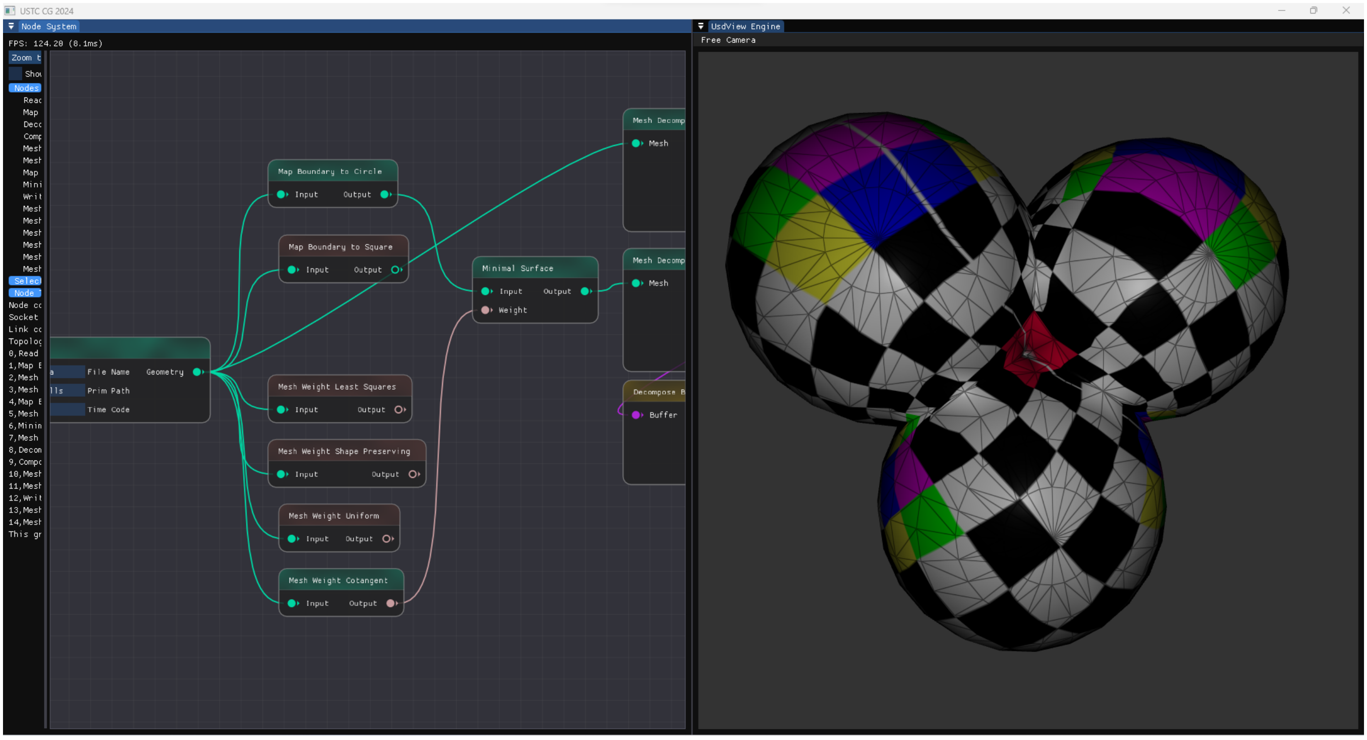
此时 PP_l, $PP_{r(l)}$, $PP_{r(l) + 1}$ 三条线段将三角形分割为三部分。 此时给这三个点增加权重，分别为与其不相邻的小三角形占总面积的比例。

```
std::vector<float> angles, lengths, id_list;
auto angles_sum = 0.0f;
for (const auto& half_edge : vertex.outgoing_halfedges()) {
    auto next_edge = half_edge.next();
    auto edge_1 = mesh->point(next_edge.to()) - mesh->point(vertex);
    auto edge_2 = mesh->point(half_edge.to()) - mesh->point(vertex);

    auto cos_angle = edge_1.dot(edge_2) / (edge_1.norm() * edge_2.norm());
    auto angle = acos(cos_angle);
    angles.push_back(angle);
    angles_sum += angle;

    lengths.push_back(edge_2.norm());
    id_list.push_back(half_edge.to().idx());
}

std::vector<Eigen::Vector2f> neighbors;
auto theta = 0.0f;
for (auto i = 0; i < angles.size(); i++) {
    neighbors.push_back(Eigen::Vector2f(lengths[i] * cos(theta),
lengths[i] * sin(theta)));
    theta += 2 * M_PI * angles[i] / angles_sum;
}

Eigen::Vector2f p(0.0f, 0.0f);

for (auto i = 0; i < angles.size(); i++) {
    auto p1 = neighbors[i];
    for (auto j = 0; j < angles.size(); j++) {
        auto next = (j + 1) % angles.size();
        if (j != i && next != i) {
            auto p2 = neighbors[j];
            auto p3 = neighbors[next];

            auto S = tri_area(p1, p2, p3);
            auto sigma1 = tri_area(p, p2, p3) / S;
            auto sigma2 = tri_area(p1, p, p3) / S;
            auto sigma3 = tri_area(p1, p2, p) / S;
```

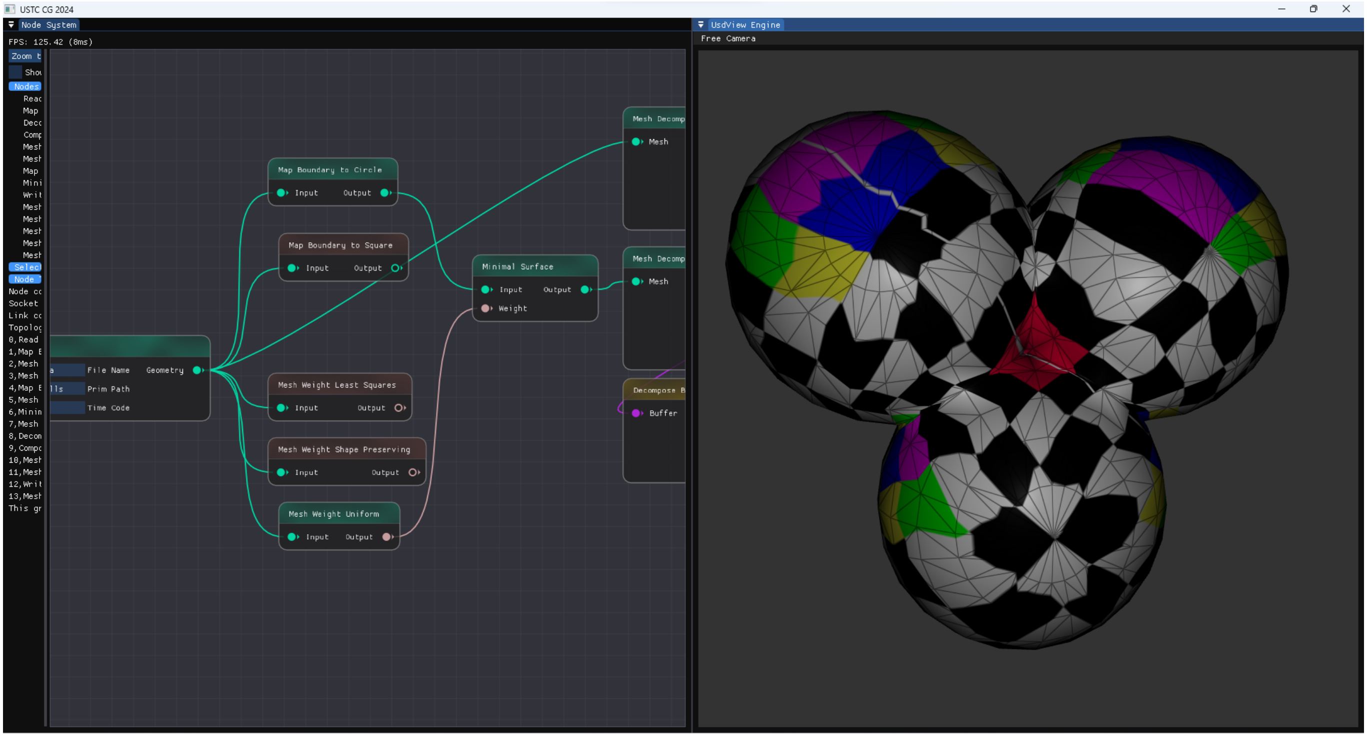
```
                if (sigma1 + sigma2 + sigma3 < 1.001) {
                    result[vertex.idx() * n + vertex.idx()] += 1.0f;
                    result[vertex.idx() * n + id_list[i]] += -sigma1;
                    result[vertex.idx() * n + id_list[j]] += -sigma2;
                    result[vertex.idx() * n + id_list[next]] += -sigma3;
                }
            }
        }
    }
```

运行截图 Screenshots

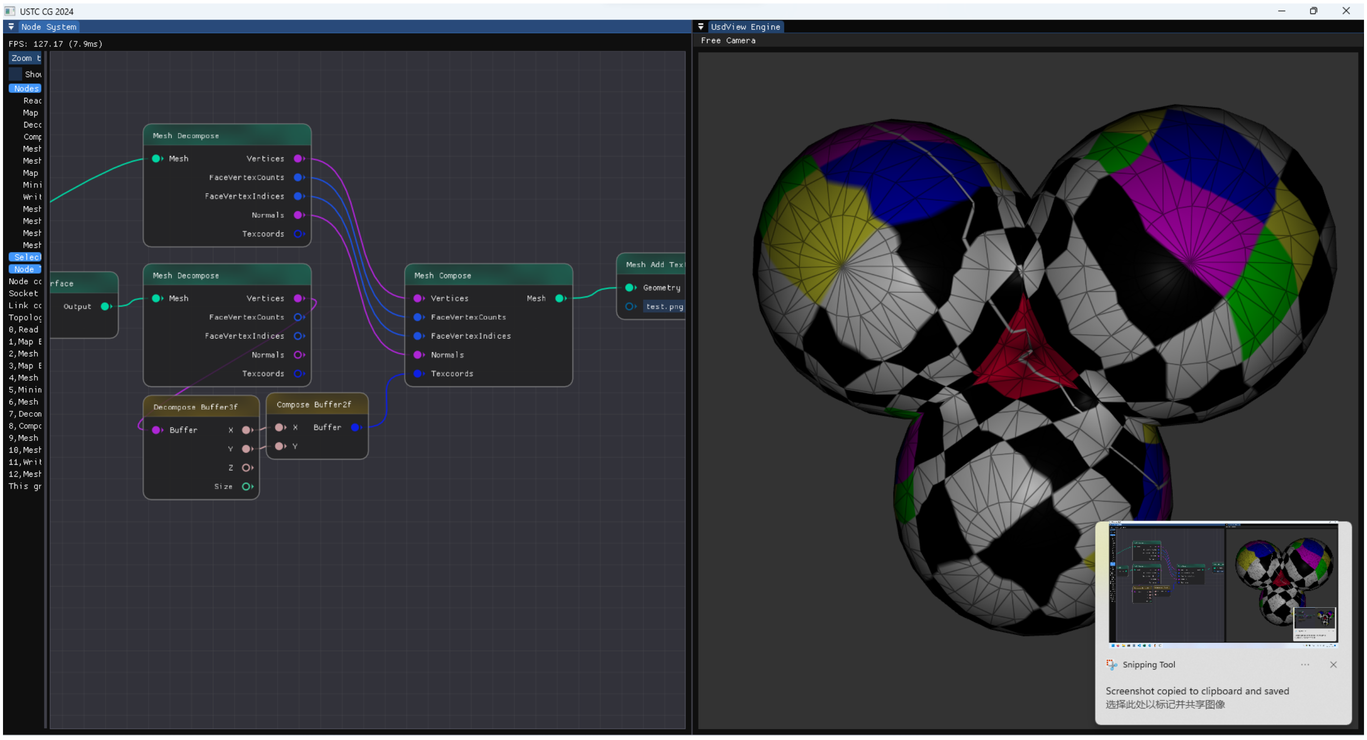
圆形边界 & Cotangent 权重：



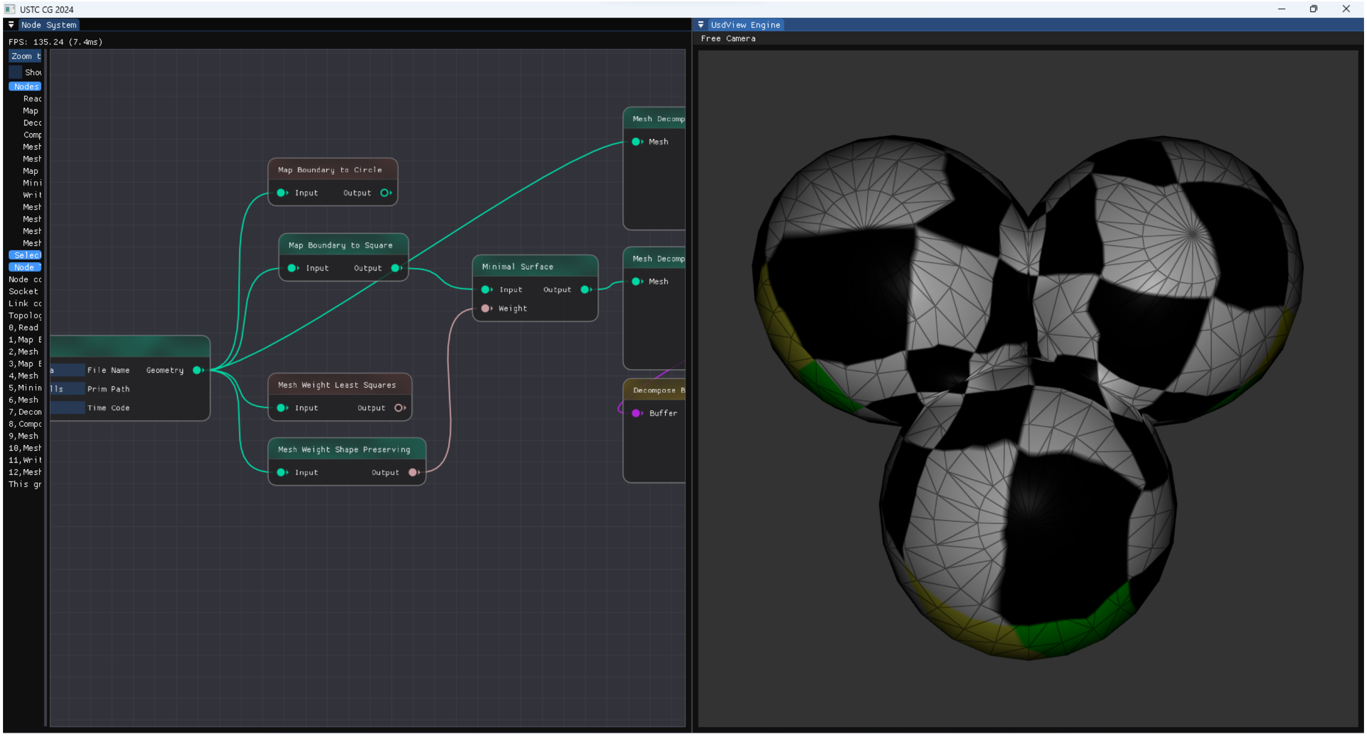
圆形边界 & Uniform 权重：



圆形边界 & Shape-preserving 权重：



正方形边界 & Shape-preserving 权重：


USTC CG 2024
Node System
FPS: 135.24 (7.4ms)
UsdView Engine
Free Camera
Map Boundary to Circle
Input
Output
Map Boundary to Square
Input
Output
Minimal Surface
Input
Output
Weight
File Name
Geometry
Prim Path
Time Code
Mesh Weight Least Squares
Input
Output
Mesh Weight Shape Preserving
Input
Output
Mesh
Mesh
Buffer