Lab report:
Measuring Young's modulus and Poisson's ratio of a metal wire

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Abstract

Both Young's modulus and Poisson's ratio reflect important properties of a material. In this experiment, we take a metal wire as a target to present a way to measure both quantities.

Keywords: Young's modulus, Poisson's ratio

Introduction

Young's modulus is an important mechanical parameter of a material that reflects the amount of resistance to deformation. Stress is defined as the ratio of the tensile force F to the original cross-section A, and the ratio of ΔL to the original length L is defined as the longitudinal line strain, In the elastic range, this could be described as a linear relationship named Hooke's Law:

$$\frac{F}{A} = E \cdot \frac{\Delta L}{L} \tag{1}$$

Only longitudinal strain in the material is considered in the equation above, we define the transverse line strain as the ratio of the transverse variation Δd to the transverse length d. Experiment shows that they follow a linear relationship:

$$\frac{\Delta d}{d} = -\mu \cdot \frac{\Delta L}{L} \tag{2}$$

The negative sign shows that longitudinal stretching leads to transverse shrinkage and vice versa.

Notice that the Δd is way too small to measure, we can measure the resistivity $\rho \propto 1/d^2$.

In this experiment, we also need to use an unbalanced bridge to measure the small change in resistance, illustrated as follows:

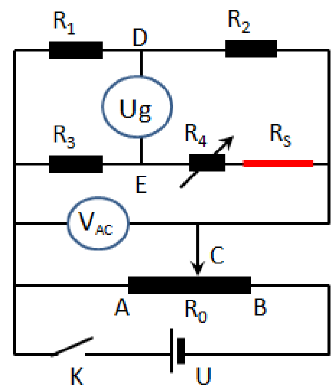

Figure 1: unbalanced bridge used to measure resistance

When U_g is balanced out, the bridge follows that: $R_1/R_2 = R_3/(R_4 + R_s)$, where the R_s is changing when the wire stretches. We can use this equation to calculate the resistance R_s: When given a change of $\Delta R_s \leq (R_4 + R_s) \cdot 1\%$, the bridge approximately follows:

$$U_g \approx \frac{U_{AC}}{4} \cdot \frac{\Delta R_s}{R_4 + R_s} \tag{3}$$

Materials

Connect the circuit as the unbalanced bridge shows, where $R_1 = R_2 = R_3 = 51.00\Omega$, and connect R_4 into a resistance box.

Mesaure both U_g and U_{AC}, keep $U_{AC} \sim (0.3V, 0.5V)$, change R_4 to make the bridge balanced (when $\mid U_g \mid < 0.020mV$ is considered balanced).

To measure Young's modulus, the change in length also needs to be measured with a reading microscope pointing at the solder joint $\mathbb{W}$.

Results

$$
\begin{aligned}
D &= 0.2mm \\
R_4 + R_s &= 51.00\Omega \\
L_1 + L_2 &= 121.2cm \\
L_2 &= 26.8cm \\
V_{AC} &= 0.400V \\
R_4 &= 14.97\Omega
\end{aligned}
\tag{4}
$$

Images are attached in a separate paper.

Discussion

$$
\begin{aligned}
R_s &= \rho \frac{L}{A} \\
\frac{\Delta d}{d} &= -\mu \cdot \frac{\Delta L}{L} \\
\frac{\Delta R_s}{R_s} &= (1+2\mu)\frac{\Delta L}{L} \\
U_g &= \frac{U_{AC}}{4} \cdot \frac{\Delta R_s}{R_4 + R_s} \\
\Delta R_s &= 4(R_4 + R_s) \cdot \frac{U_g}{U_{AC}}
\end{aligned}
\tag{5}
$$

For that we can conclude:

$$
U_g = \frac{U_{AC}}{4} \cdot \frac{R_s \cdot (1+2\mu) \cdot \dfrac{\Delta L}{L}}{R_4 + R_s} = \frac{U_{AC} R_s (1+2\mu)}{4(R_4 + R_s)} \Delta L
\tag{6}
$$

Young's modulus:

$$
E = \frac{\pi R^2 k_{\Delta L \sim m}}{g(L_1 + L_2)} = 2.288 \times 10^{-11} E/Pa
\tag{7}
$$

Poisson's ratio:

$$
\mu = \frac{1}{2}\left(\frac{4k(R_s + R_4)}{R_s U_{AC}} - 1\right) = 0.237
\tag{8}
$$

The accepted value of Young's modulus E is $2.0E/10^{11}pa$, accepted value of poisson's ratio is 0.3.

References

Most contents were translated from the handout.

Images

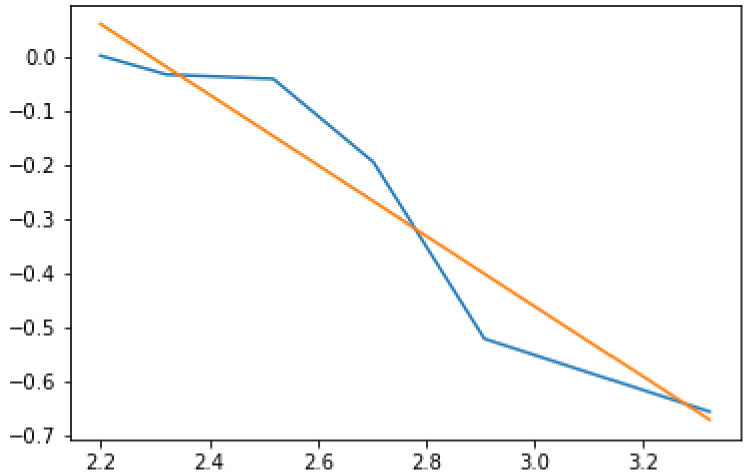

Figure 2: $U_g \sim \Delta L$

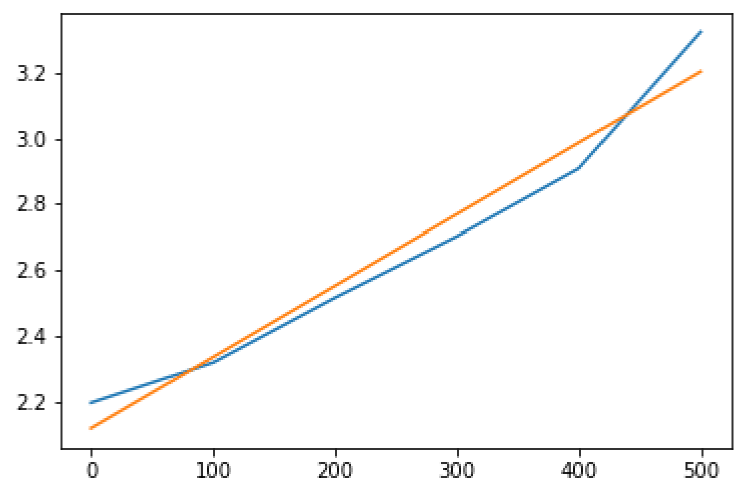

Figure 3: $\Delta L \sim \Delta m$