

# 数值代数实验报告 2

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## GitHub仓库

[https://github.com/tiankaima/numerical\\_algebra](https://github.com/tiankaima/numerical_algebra)

## 实验平台

```
> uname -a
Linux tiankai-omenlaptop15en1002ax 6.1.53-1-MANJARO #1 SMP PREEMPT_DYNAMIC Wed Sep 13
14:10:57 UTC 2023 x86_64 GNU/Linux
> cmake --version
cmake version 3.27.6
> clang --version
clang version 16.0.6
Target: x86_64-pc-linux-gnu
Thread model: posix
```

## 目录结构

- CustomMath\_lib存放了具体的算法实现
- Doctest\_tests存放了单元测试
- homeworks存放了作业的源代码，并且在main.cpp对每次作业进行了调用
- main.cpp是主程序
- Mathematica存放了.nb文件，用于生成测试数据
- writeups存放了实验报告的源代码，比如本文

## Linux 平台编译

```
> mkdir build
> cd build
> cmake ..
> make
```

## 运行

```
./numerical_algebra
```

## Windows 平台编译

使用 Visual Studio 打开numerical\_algebra.sln

## 问题描述

### 2.1

估计 5 到 20 阶 Hilbert 矩阵的  $\infty$  范数条件数

### 2.2

设

$$A_n = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 1 \\ -1 & 1 & 0 & 0 & \cdots & 1 \\ - & -1 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & \cdots & -1 & -1 & 1 & 1 \\ -1 & \cdots & -1 & -1 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{n \times n} \quad (1)$$

随机选取  $x \in \mathbb{R}^n$ ，并计算出  $b = A_n x$ ，然后用列主元高斯消去法求解该方程组。用 p69 页公式估计  $n$  从 5 到 30 时计算解  $\hat{x}$  的精度，并与真实相对误差作比较。要求输出真实相对误差和估计的相对误差上界。

## 程序介绍

- 复用了上次作业的全部代码。重构了部分逻辑使所有代码使用unsigned long 以及long double类型, 确保精度问题。
- 修改了上次提交时选主元消去结果误差巨大的问题（单独考虑对角线全1的情况）
- 矩阵的1-范数估计的实现方法在CustomMath\_lib/InfinityNorm/InfinityNorm.cpp中, 分别提供了两种办法计算 $\|A\|_1$  和 $\|A^{-T}\|_1$
- 本次作业提交在homeworks/homework\_2.cpp中

## 运行结果

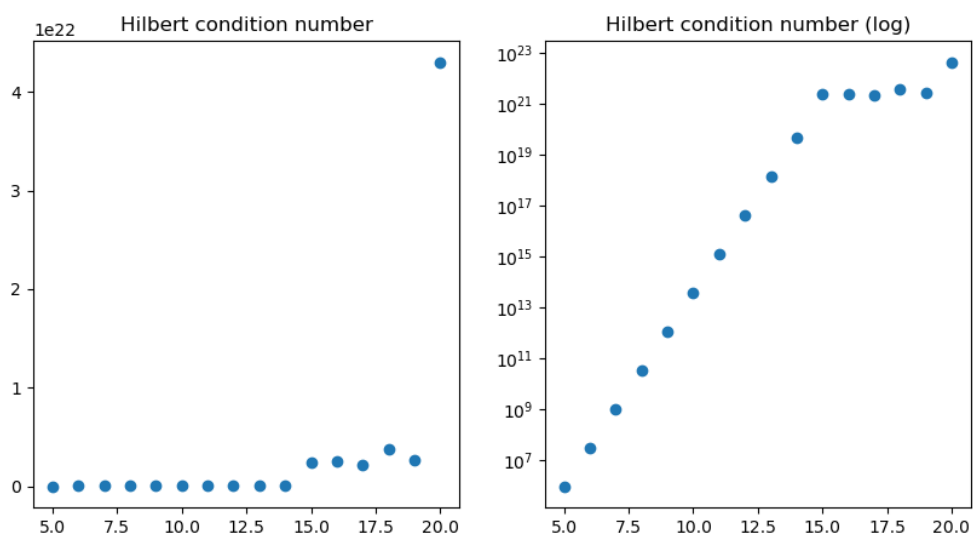
```
----- Q 2.1 -----
n = 5, condition_number = 943656.0000000018689
n = 6, condition_number = 29070279.00000236591
n = 7, condition_number = 985194886.5019417926
n = 8, condition_number = 33872791096.51495649
n = 9, condition_number = 1099654542182.044117
n = 10, condition_number = 35357438794311.81662
n = 11, condition_number = 1233700677314114.901
n = 12, condition_number = 41152583629496310.3
n = 13, condition_number = 1323308355530996361
n = 14, condition_number = 4.514480476929424986e+19
n = 15, condition_number = 2.372051355568131468e+21
n = 16, condition_number = 2.476114807729196188e+21
n = 17, condition_number = 2.128317989845009304e+21
n = 18, condition_number = 3.706607164001239419e+21
n = 19, condition_number = 2.656854800573346314e+21
n = 20, condition_number = 4.307025616349245435e+22
----- Q 2.2 -----
n = 5
real diff = 0
calculated diff = 0
diff in diff = 0
n = 6
real diff = 0
calculated diff = 0
diff in diff = 0
n = 7
real diff = 0
calculated diff = 0
diff in diff = 0
n = 8
real diff = 0
calculated diff = 0
diff in diff = 0
n = 9
real diff = 0
calculated diff = 0
diff in diff = 0
n = 10
real diff = 0
calculated diff = 0
diff in diff = 0
n = 11
real diff = 0
calculated diff = 0
diff in diff = 0
n = 12
real diff = 1.734723475976807094e-18
calculated diff = 6.612014234494321208e-18
diff in diff = 4.877290758517514114e-18
n = 13
real diff = 2.775557561562891351e-17
calculated diff = 1.010805595288567532e-16
diff in diff = 7.332498391322783968e-17
n = 14
real diff = 0
calculated diff = 0
diff in diff = 0
n = 15
real diff = 2.860979249076398383e-16
```

calculated diff = 7.249472996104535398e-16  
diff in diff = 4.388493747028137015e-16  
n = 16  
real diff = 5.559768866249354567e-16  
calculated diff = 1.567047177758013681e-15  
diff in diff = 1.011070291133078224e-15  
n = 17  
real diff = 1.004799640629047946e-15  
calculated diff = 3.496455435560005585e-15  
diff in diff = 2.491655794930957639e-15  
n = 18  
real diff = 6.14552570651475634e-16  
calculated diff = 2.351142373483552314e-15  
diff in diff = 1.73658980283207668e-15  
n = 19  
real diff = 2.217842437587711137e-15  
calculated diff = 9.386353512628392564e-15  
diff in diff = 7.168511075040681427e-15  
n = 20  
real diff = 3.291862266251029277e-15  
calculated diff = 1.90430239171948247e-14  
diff in diff = 1.575116165094379542e-14  
n = 21  
real diff = 5.239604687546524397e-15  
calculated diff = 1.492253799195121321e-14  
diff in diff = 9.682933304404688812e-15  
n = 22  
real diff = 2.762792910639579795e-14  
calculated diff = 1.056866738955917202e-13  
diff in diff = 7.805874478919592225e-14  
n = 23  
real diff = 2.01765896295918722e-14  
calculated diff = 8.778261422489434963e-14  
diff in diff = 6.760602459530247742e-14  
n = 24  
real diff = 8.520916509357622023e-15  
calculated diff = 2.945177052358312678e-14  
diff in diff = 2.093085401422550476e-14  
n = 25  
real diff = 1.933140316207661783e-13  
calculated diff = 7.61381409311802854e-13  
diff in diff = 5.680673776910366756e-13  
n = 26  
real diff = 5.375343117701421744e-13  
calculated diff = 1.327550673943842739e-12  
diff in diff = 7.900163621737005642e-13  
n = 27  
real diff = 6.018842530046321184e-14  
calculated diff = 1.970167956926096132e-13  
diff in diff = 1.368283703921464014e-13  
n = 28  
real diff = 4.199517687703756253e-13  
calculated diff = 1.313682704452139813e-12  
diff in diff = 8.937309356817641882e-13  
n = 29  
real diff = 3.691738107736230547e-12  
calculated diff = 1.133693937670085629e-11  
diff in diff = 7.645201268964625745e-12  
n = 30  
real diff = 4.616569611648569034e-13  
calculated diff = 1.056317222467293025e-12  
diff in diff = 5.946602613024361212e-13

## 结果分析

方便分析数据，使用python绘制了两份图表：

### Hilbert 矩阵的条件范数



### 估计的相对误差 / 真实误差

