

数值代数实验报告 4

马天开

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1 问题描述

1.1 考虑两点边值问题

$$\begin{cases} \epsilon \frac{d^2 y}{dx^2} + \frac{dy}{dx} = a, 0 < a < 1, \\ y(0) = 0, y(1) = 1 \end{cases}$$

容易知道它的精确解为

$$y = \frac{1-a}{1-e^{-\frac{1}{\epsilon}}} (1 - e^{-\frac{x}{\epsilon}}) + ax$$

为了把微分方程离散化, 把 $[0, 1]$ 区间 n 等分, 令 $h = \frac{1}{n}, x_i = ih, i = 1, \dots, n-1$, 得到差分方程

$$\epsilon \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} + \frac{y_{i+1} - y_i}{h} = a$$

简化为

$$(\epsilon + h)y_{i+1} - (2\epsilon + h)y_i + \epsilon y_{i-1} = ah^2$$

离散化后得到线性方程组 $Ay = b$, 其中

$$A = \begin{bmatrix} -(2\epsilon + h) & (\epsilon + h) & 0 & 0 & \cdots & 0 \\ \epsilon & -(2\epsilon + h) & (\epsilon + h) & 0 & \cdots & 0 \\ 0 & \epsilon & -(2\epsilon + h) & (\epsilon + h) & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & \epsilon & -(2\epsilon + h) & (\epsilon + h) \\ 0 & 0 & \cdots & 0 & \epsilon & -(2\epsilon + h) \end{bmatrix}$$

注意 A 为 $n-1$ 阶矩阵, 将线性方程组与差分方程进行对比得出正确的 b 向量 (尤其注意第一行和最后一行)。

对 $\epsilon = 1, a = 1/2, n = 100$, 分别用 Jacobi 迭代法, G-S 迭代法和 SOR 迭代法求线性方程组的解, 要求 4 位有效数字, 然后比较迭代次数, 运行时间与精确解的误差。迭代法终止条件为 $\|x^{(k+1)} - x^{(k)}\| < 10^{-6}$ 。

对 $\epsilon = 0.1, 0.01, 0.0001$, 考虑同样的问题。要求输出计算结果, 收敛所需要的迭代次数和运行时间。

1.2 考虑偏微分方程

$$-\Delta u + g(x, y)u = f(x, y) \quad (x, y) \in [0, 1] \times [0, 1]$$

在 $[0, 1] \times [0, 1]$ 边界上 $u = 1$. 沿 x 方向和 y 方向均匀剖分 N 等份, 令 $h = 1/N$, 并设应用中心差分离散化后得到差分方程的代数方程组为

$$-u_{i-1,j} - u_{i,j-1} + (4 + h^2 g(ih, jh))u_{i,j} - u_{i+1,j} - u_{i,j+1} = h^2 f(ih, jh)$$

取 $g(x, y) = \exp(xy)$, $f(x, y) = x + y$, 分别用 Jacobi 迭代法, G-S 迭代法和 SOR 迭代法求解上述代数方程组, 要求输出解的最小分量, 并比较 $N = 20, 40, 60$ 时收敛所需要的迭代次数和运行时间, 迭代终止条件为 $\|u^{(k+1)} - u^{(k)}\|_2 < 10^{-7}$.

要求仿照下面写的 Jacobi 迭代格式的推导过程推导处 G-S 迭代和 SOR 迭代的格式, 在用 SOR 迭代法求解的过程中, 请对不同的 N 使用合适的松弛因子 ω , 并在程序输出中打印松弛因子的值。观察运行结果后选取合适的。(代码中不需要体现选取过程, 只需给出即可)。

注意本题中的三个迭代法的算法需要重新写, 不能用矩阵的通用算法!!!

2 算法说明

1、Jacobi 迭代法

考虑其迭代格式 $Dx_{(k+1)} = (L + U)x_k + b$ 。对第 i 行有

$$D_{ii}x_i^{(k+1)} = \sum_{j \neq i} (L + U)_{ij}x_j^{(k)} + b_i$$

将 D, L, U 还原成代数方程组的 Jacobi 迭代式

$$D_{ii}x_i^{(k+1)} = L_{i1}x_1^{(k)} + \cdots + L_{ii-1}x_{i-1}^{(k)} + U_{ii+1}x_{i+1}^{(k)} + \cdots + U_{in}x_n^{(k)} + b_i$$

即只有与向量 b 的下标相同的位置替换成 x^{k+1} 。

由此类比推广至矩阵 (或者可以直接将矩阵拉直成向量), 知代数方程组 (1) 的 Jacobi 迭代格式为

$$(4 + h^2 g(ih, jh))u_{i,j}^{(k+1)} = u_{i-1,j}^{(k)} + u_{i,j-1}^{(k)} + u_{i+1,j}^{(k)} + u_{i,j+1}^{(k)} + h^2 f(ih, jh)$$

2、G-S 迭代法

$$(4 + h^2 g(ih, jh))u_{i,j}^{(k+1)} = u_{i-1,j}^{(k+1)} + u_{i,j-1}^{(k+1)} + u_{i+1,j}^{(k)} + u_{i,j+1}^{(k)} + h^2 f(ih, jh)$$

3、SOR 迭代法

$$u_{i,j}^{(k+1)} = \frac{\omega}{4 + h^2 g(ih, jh)} (u_{i-1,j}^{(k+1)} + u_{i,j-1}^{(k+1)} + u_{i+1,j}^{(k)} + u_{i,j+1}^{(k)} + h^2 f(ih, jh)) + (1 - \omega)u_{i,j}^{(k)}$$

3 程序介绍

并没有什么特别值得介绍的, 也许。这次抢了点 ddl 有的地方可能乱一点, 见谅。

4 运行结果

----- Q 4.1 -----

eps = 1

[0,0.0128705,0.0256626,0.0383772,0.0510151,0.063577,0.0760636,0.0884757,0.100814,0.113079,0.12521,0.137484,0.149749,0.162014,0.174279,0.186544,0.198809,0.211074,0.223339,0.235604,0.247869,0.260134,0.272399,0.284664,0.296929,0.309194,0.321459,0.333724,0.345989,0.358254,0.370519,0.382784,0.395049,0.407314,0.419579,0.431844,0.444109,0.456374,0.468639,0.480904,0.493169,0.505434,0.517699,0.529964,0.542229,0.554494,0.566759,0.579024,0.591289,0.603554,0.615819,0.628084,0.640349,0.652614,0.664879,0.677144,0.689409,0.701674,0.713939,0.726204,0.738469,0.750734,0.762999,0.775264,0.787529,0.799794,0.812059,0.824324,0.836589,0.848854,0.861119,0.873384,0.885649,0.897914,0.910179,0.922444,0.934709,0.946974,0.959239,0.971504,0.983769,0.996034,1.008299,1.020564,1.032829,1.045094,1.057359,1.069624,1.081889,1.094154,1.106419,1.118684,1.130949,1.143214,1.155479,1.167744,1.179999,1.192264,1.204529,1.216794,1.229059,1.241324,1.253589,1.265854,1.278119,1.290384,1.302649,1.314914,1.327179,1.339444,1.351709,1.363974,1.376239,1.388504,1.400769,1.413034,1.425299,1.437564,1.449829,1.462094,1.474359,1.486624,1.498889,1.511154,1.523419,1.535684,1.547949,1.560214,1.572479,1.584744,1.597009,1.609274,1.621539,1.633804,1.646069,1.658334,1.670599,1.682864,1.695129,1.707394,1.719659,1.731924,1.744189,1.756454,1.768719,1.780984,1.793249,1.805514,1.817779,1.830044,1.842309,1.854574,1.866839,1.879104,1.891369,1.903634,1.915899,1.928164,1.940429,1.952694,1.964959,1.977224,1.989489,2.001754,2.014019,2.026284,2.038549,2.050814,2.063079,2.075344,2.087609,2.099874,2.112139,2.124404,2.136669,2.148934,2.161199,2.173464,2.185729,2.197994,2.210259,2.222524,2.234789,2.247054,2.259319,2.271584,2.283849,2.296114,2.308379,2.320644,2.332909,2.345174,2.357439,2.369704,2.381969,2.394234,2.406499,2.418764,2.431029,2.443294,2.455559,2.467824,2.480089,2.492354,2.504619,2.516884,2.529149,2.541414,2.553679,2.565944,2.578209,2.590474,2.602739,2.615004,2.627269,2.639534,2.651799,2.664064,2.676329,2.688594,2.700859,2.713124,2.725389,2.737654,2.749919,2.762184,2.774449,2.786714,2.798979,2.811244,2.823509,2.835774,2.848039,2.860304,2.872569,2.884834,2.897099,2.909364,2.921629,2.933894,2.946159,2.958424,2.970689,2.982954,2.995219,3.007484,3.019749,3.032014,3.044279,3.056544,3.068809,3.081074,3.093339,3.105604,3.117869,3.130134,3.142399,3.154664,3.166929,3.179194,3.191459,3.203724,3.215989,3.228254,3.240519,3.252784,3.265049,3.277314,3.289579,3.301844,3.314109,3.326374,3.338639,3.350904,3.363169,3.375434,3.387699,3.399964,3.412229,3.424494,3.436759,3.449024,3.461289,3.473554,3.485819,3.498084,3.510349,3.522614,3.534879,3.547144,3.559409,3.571674,3.583939,3.596204,3.608469,3.620734,3.632999,3.645264,3.657529,3.669794,3.682059,3.694324,3.706589,3.718854,3.731119,3.743384,3.755649,3.767914,3.780179,3.792444,3.804709,3.816974,3.829239,3.841504,3.853769,3.866034,3.878299,3.890564,3.902829,3.915094,3.927359,3.939624,3.951889,3.964154,3.976419,3.988684,4.000949,4.013214,4.025479,4.037744,4.049999,4.062264,4.074529,4.086794,4.099059,4.111324,4.123589,4.135854,4.148119,4.160384,4.172649,4.184914,4.197179,4.209444,4.221709,4.233974,4.246239,4.258504,4.270769,4.283034,4.295299,4.307564,4.319829,4.332094,4.344359,4.356624,4.368889,4.381154,4.393419,4.405684,4.417949,4.430214,4.442479,4.454744,4.467009,4.479274,4.491539,4.503804,4.516069,4.528334,4.540599,4.552864,4.565129,4.577394,4.589659,4.601924,4.614189,4.626454,4.638719,4.650984,4.663249,4.675514,4.687779,4.699999,4.712264,4.724529,4.736794,4.749059,4.761324,4.773589,4.785854,4.798119,4.810384,4.822649,4.834914,4.847179,4.859444,4.871709,4.883974,4.896239,4.908504,4.920769,4.933034,4.945299,4.957564,4.969829,4.982094,4.994359,5.006624,5.018889,5.031154,5.043419,5.055684,5.067949,5.080214,5.092479,5.104744,5.117009,5.129274,5.141539,5.153804,5.166069,5.178334,5.190599,5.202864,5.215129,5.227394,5.239659,5.251924,5.264189,5.276454,5.288719,5.300984,5.313249,5.325514,5.337779,5.350044,5.362309,5.374574,5.386839,5.399104,5.411369,5.423634,5.435899,5.448164,5.460429,5.472694,5.484959,5.497224,5.509489,5.521754,5.534019,5.546284,5.558549,5.570814,5.583079,5.595344,5.607609,5.619874,5.632139,5.644404,5.656669,5.668934,5.681199,5.693464,5.705729,5.717994,5.730259,5.742524,5.754789,5.767054,5.779319,5.791584,5.803849,5.816114,5.828379,5.840644,5.852909,5.865174,5.877439,5.889704,5.901969,5.914234,5.926499,5.938764,5.951029,5.963294,5.975559,5.987824,6.000089,6.012354,6.024619,6.036884,6.049149,6.061414,6.073679,6.085944,6.098209,6.110474,6.122739,6.135004,6.147269,6.159534,6.171799,6.184064,6.196329,6.208594,6.220859,6.233124,6.245389,6.257654,6.269919,6.282184,6.294449,6.306714,6.318979,6.331244,6.343509,6.355774,6.368039,6.380304,6.392569,6.404834,6.417099,6.429364,6.441629,6.453894,6.466159,6.478424,6.490689,6.502954,6.515219,6.527484,6.539749,6.552014,6.564279,6.576544,6.588809,6.601074,6.613339,6.625604,6.637869,6.650134,6.662399,6.674664,6.686929,6.699194,6.711459,6.723724,6.735989,6.748254,6.760519,6.772784,6.785049,6.797314,6.809579,6.821844,6.834109,6.846374,6.858639,6.870904,6.883169,6.895434,6.907699,6.919964,6.932229,6.944494,6.956759,6.969024,6.981289,6.993554,7.005819,7.018084,7.030349,7.042614,7.054879,7.067144,7.079409,7.091674,7.103939,7.116204,7.128469,7.140734,7.152999,7.165264,7.177529,7.189794,7.202059,7.214324,7.226589,7.238854,7.251119,7.263384,7.275649,7.287914,7.300179,7.312444,7.324709,7.336974,7.349239,7.361504,7.373769,7.386034,7.398299,7.410564,7.422829,7.435094,7.447359,7.459624,7.471889,7.484154,7.496419,7.508684,7.520949,7.533214,7.545479,7.557744,7.569999,7.582264,7.594529,7.606794,7.619059,7.631324,7.643589,7.655854,7.668119,7.680384,7.692649,7.704914,7.717179,7.729444,7.741709,7.753974,7.766239,7.778504,7.790769,7.803034,7.815299,7.827564,7.839829,7.852094,7.864359,7.876624,7.888889,7.901154,7.913419,7.925684,7.937949,7.950214,7.962479,7.974744,7.987009,7.999274,8.011539,8.023804,8.036069,8.048334,8.060599,8.072864,8.085129,8.097394,8.109659,8.121924,8.134189,8.146454,8.158719,8.170984,8.183249,8.195514,8.207779,8.220044,8.232309,8.244574,8.256839,8.269104,8.281369,8.293634,8.305899,8.318164,8.330429,8.342694,8.354959,8.367224,8.379489,8.391754,8.404019,8.416284,8.428549,8.440814,8.453079,8.465344,8.477609,8.489874,8.502139,8.514404,8.526669,8.538934,8.551199,8.563464,8.575729,8.587994,8.599999,8.612264,8.624529,8.636794,8.649059,8.661324,8.673589,8.685854,8.698119,8.710384,8.722649,8.734914,8.747179,8.759444,8.771709,8.783974,8.796239,8.808504,8.820769,8.833034,8.845299,8.857564,8.869829,8.882094,8.894359,8.906624,8.918889,8.931154,8.943419,8.955684,8.967949,8.980214,8.992479,9.004744,9.017009,9.029274,9.041539,9.053804,9.066069,9.078334,9.090599,9.102864,9.115129,9.127394,9.139659,9.151924,9.164189,9.176454,9.188719,9.200984,9.213249,9.225514,9.237779,9.250044,9.262309,9.274574,9.286839,9.299104,9.311369,9.323634,9.335899,9.348164,9.360429,9.372694,9.384959,9.397224,9.409489,9.421754,9.434019,9.446284,9.458549,9.470814,9.483079,9.495344,9.507609,9.519874,9.532139,9.544404,9.556669,9.568934,9.581199,9.593464,9.605729,9.617994,9.630259,9.642524,9.654789,9.667054,9.679319,9.691584,9.703849,9.716114,9.728379,9.740644,9.752909,9.765174,9.777439,9.789704,9.801969,9.814234,9.826499,9.838764,9.851029,9.863294,9.875559,9.887824,9.899999,9.912264,9.924529,9.936794,9.949059,9.961324,9.973589,9.985854,9.998119,10.010384,10.022649,10.034914,10.047179,10.059444,10.071709,10.083974,10.096239,10.108504,10.120769,10.133034,10.145299,10.157564,10.169829,10.182094,10.194359,10.206624,10.218889,10.231154,10.243419,10.255684,10.267949,10.280214,10.292479,10.304744,10.317009,10.329274,10.341539,10.353804,10.366069,10.378334,10.390599,10.402864,10.415129,10.427394,10.439659,10.451924,10.464189,10.476454,10.488719,10.500984,10.513249,10.525514,10.537779,10.550044,10.562309,10.574574,10.586839,10.599104,10.611369,10.623634,10.635899,10.648164,10.660429,10.672694,10.684959,10.697224,10.709489,10.721754,10.734019,10.746284,10.758549,10.770814,10.783079,10.795344,10.807609,10.819874,10.832139,10.844404,10.856669,10.868934,10.881199,10.893464,10.905729,10.917994,10.930259,10.942524,10.954789,10.967054,10.979319,10.991584,10.100814,10.113079,10.12521,10.137484,10.149749,10.162014,10.174279,10.186544,10.198809,10.211074,10.223339,10.235604,10.247869,10.260134,10.272399,10.284664,10.296929,10.309194,10.321459,10.333724,10.345989,10.358254,10.370519,10.382784,10.395049,10.407314,10.419579,10.431844,10.444109,10.456374,10.468639,10.480904,10.493169,10.505434,10.517699,10.529964,10.542229,10.554494,10.566759,10.579024,10.591289,10.603554,10.615819,10.628084,10.640349,10.652614,10.664879,10.677144,10.689409,10.701674,10.713939,10.726204,10.738469,10.750734,10.762999,10.775264,10.787529,10.799794,10.812059,10.824324,10.836589,10.848854,10.861119,10.873384,10.885649,10.897914,10.910179,10.922444,10.934709,10.946974,10.959239,10.971504,10.983769,10.996034,11.008299,11.020564,11.032829,11.045094,11.057359,11.069624,11.081889,11.094154,11.106419,11.118684,11.130949,11.143214,11.155479,11.167744,11.179999,11.192264,11.204529,11.216794,11.229059,11.241324,11.253589,11.265854,11.278119,11.290384,11.302649,11.314914,11.327179,11.339444,11.351709,11.363974,11.376239,11.388504,11.400769,11.413034,11.425299,11.437564,11.449829,11.462094,11.474359,11.486624,11.498889,11.511154,11.523419,11.535684,11.547949,11.560214,11.572479,11.584744,11.597009,11.609274,11.621539,11.633804,11.646069,11.658334,11.670599,11.682864,11.695129,11.707394,11.719659,11.731924,11.744189,11.756454,11.768719,11.780984,11.793249,11.805514,11.817779,11.830044,11.842309,11.854574,11.866839,11.879104,11.891369,11.903634,11.915899,11.928164,11.940429,11.952694,11.964959,11.977224,11.989489,12.001754,12.014019,12.026284,12.038549,12.050814,12.063079,12.075344,12.087609,12.099874,12.112139,12.124404,12.136669,12.148934,12.161199,12.173464,12.185729,12.197994,12.210259,12.222524,12.234789,12.247054,12.259319,12.271584,12.283849,12.296114,12.308379,12.320644,12.332909,12.345174,12.35743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[0.0504528,0.0967736,0.139338,0.178488,0.214533,0.247755,0.278413,0.306738,0.332942,0.357219,0.378413,0.404528,0.430643,0.456758,0.482873,0.508988,0.535103,0.561218,0.587333,0.613448,0.639563,0.665678,0.691793,0.717908,0.744023,0.770138,0.796253,0.822368,0.848483,0.874598,0.900713,0.926828,0.952943,0.979058,1.005173,1.031288,1.057403,1.083518,1.109633,1.135748,1.161863,1.187978,1.214093,1.240208,1.266323,1.292438,1.318553,1.344668,1.370783,1.396898,1.423013,1.449128,1.475243,1.501358,1.527473,1.553588,1.579703,1.605818,1.631933,1.658048,1.684163,1.710278,1.736393,1.762508,1.788623,1.814738,1.840853,1.866968,1.893083,1.919198,1.945313,1.971428,1.997543,2.023658,2.049773,2.075888,2.101993,2.128108,2.154223,2.180338,2.206453,2.232568,2.258683,2.284798,2.310913,2.337028,2.363143,2.389258,2.415373,2.441488,2.467603,2.493718,2.519833,2.545948,2.572063,2.598178,2.624293,2.650408,2.676523,2.702638,2.728753,2.754868,2.780983,2.807098,2.833213,2.859328,2.885443,2.911558,2.937673,2.963788,2.989903,3.016018,3.042133,3.068248,3.094363,3.120478,3.146593,3.172708,3.198823,3.224938,3.251053,3.277168,3.303283,3.329398,3.355513,3.381628,3.407743,3.433858,3.459973,3.486088,3.512203,3.538318,3.564433,3.590548,3.616663,3.642778,3.668893,3.695008,3.721123,3.747238,3.773353,3.799468,3.825583,3.851698,3.877813,3.903928,3.930043,3.956158,3.982273,4.008388,4.034503,4.060618,4.086733,4.112848,4.138963,4.165078,4.191193,4.217308,4.243423,4.269538,4.295653,4.321768,4.347883,4.373998,4.400113,4.426228,4.452343,4.478458,4.504573,4.530688,4.556803,4.582918,4.609033,4.635148,4.661263,4.687378,4.713493,4.739608,4.765723,4.791838,4.817953,4.844068,4.870183,4.896298,4.922413,4.948528,4.974643,5.000758,5.026873,5.052988,5.079103,5.105218,5.131333,5.157448,5.183563,5.209678,5.235793,5.261908,5.288023,5.314138,5.340253,5.366368,5.392483,5.418598,5.444713,5.470828,5.496943,5.523058,5.549173,5.575288,5.601403,5.627518,5.653633,5.679748,5.705863,5.731978,5.758093,5.784208,5.810323,5.836438,5.862553,5.888668,5.914783,5.940898,5.967013,5.993128,6.019243,6.045358,6.071473,6.097588,6.123703,6.149818,6.175933,6.202048,6.228163,6.254278,6.280393,6.306508,6.332623,6.358738,6.384853,6.410968,6.437083,6.463198,6.489313,6.515428,6.541543,6.567658,6.593773,6.619888,6.646003,6.672118,6.698233,6.724348,6.750463,6.776578,6.802693,6.828808,6.854923,6.881038,6.907153,6.933268,6.959383,6.985498,7.011613,7.037728,7.063843,7.089958,7.116073,7.142188,7.168303,7.194418,7.220533,7.246648,7.272763,7.298878,7.324993,7.351108,7.377223,7.403338,7.429453,7.455568,7.481683,7.507798,7.533913,7.560028,7.586143,7.612258,7.638373,7.664488,7.690603,7.716718,7.742833,7.768948,7.795063,7.821178,7.847293,7.873408,7.899523,7.925638,7.951753,7.977868,8.003983,8.030098,8.056213,8.082328,8.108443,8.134558,8.160673,8.186788,8.212903,8.239018,8.265133,8.291248,8.317363,8.343478,8.369593,8.395708,8.421823,8.447938,8.474053,8.500168,8.526283,8.552398,8.578513,8.604628,8.630743,8.656858,8.682973,8.709088,8.735203,8.761318,8.787433,8.813548,8.839663,8.865778,8.891893,8.918008,8.944123,8.970238,8.996353,9.022468,9.048583,9.074698,9.100813,9.126928,9.153043,9.179158,9.205273,9.231388,9.257503,9.283618,9.309733,9.335848,9.361963,9.388078,9.414193,9.440308,9.466423,9.492538,9.518653,9.544768,9.570883,9.596998,9.623113,9.649228,9.675343,9.701458,9.727573,9.753688,9.779803,9.805918,9.832033,9.858148,9.884263,9.910378,9.936493,9.962608,9.988723,10.014838,10.040953,10.067068,10.093183,10.119298,10.145413,10.171528,10.197643,10.223758,10.249873,10.275988,10.302103,10.328218,10.354333,10.380448,10.406563,10.432678,10.458793,10.484908,10.511023,10.537138,10.563253,10.589368,10.615483,10.641598,10.667713,10.693828,10.719943,10.746058,10.772173,10.798288,10.824403,10.850518,10.876633,10.902748,10.928863,10.954978,10.981093,11.007208,11.033323,11.059438,11.085553,11.111668,11.137783,11.163898,11.189913,11.216028,11.242143,11.268258,11.294373,11.320488,11.346603,11.372718,11.398833,11.424948,11.451063,11.477178,11.503293,11.529408,11.555523,11.581638,11.607753,11.633868,11.659983,11.686098,11.712213,11.738328,11.764443,11.790558,11.816673,11.842788,11.868903,11.895018,11.921133,11.947248,11.973363,11.999478,12.025593,12.051708,12.077823,12.103938,12.130053,12.156168,12.182283,12.208398,12.234513,12.260628,12.286743,12.312858,12.338973,12.365088,12.391203,12.417318,12.443433,12.469548,12.495663,12.521778,12.547893,12.574008,12.599993,12.626008,12.652023,12.678038,12.704053,12.730068,12.756083,12.782098,12.808113,12.834128,12.860143,12.886158,12.912173,12.938188,12.964203,12.990218,13.016233,13.042248,13.068263,13.094278,13.120293,13.146308,13.172323,13.198338,13.224353,13.250368,13.276383,13.302398,13.328413,13.354428,13.380443,13.406458,13.432473,13.458488,13.484503,13.510518,13.536533,13.562548,13.588563,13.614578,13.640593,13.666608,13.692623,13.718638,13.744653,13.770668,13.796683,13.822698,13.848713,13.874728,13.900743,13.926758,13.952773,13.978788,14.004803,14.030818,14.056833,14.082848,14.108863,14.134878,14.160893,14.186908,14.212923,14.238938,14.264953,14.290968,14.316983,14.342998,14.369013,14.395028,14.421043,14.447058,14.473073,14.499088,14.525103,14.551118,14.577133,14.603148,14.629163,14.655178,14.681193,14.707208,14.733223,14.759238,14.785253,14.811268,14.837283,14.863298,14.889313,14.915328,14.941343,14.967358,14.993373,15.019388,15.045403,15.071418,15.097433,15.123448,15.149463,15.175478,15.201493,15.227508,15.253523,15.279538,15.305553,15.331568,15.357583,15.383598,15.409613,15.435628,15.461643,15.487658,15.513673,15.539688,15.565703,15.591718,15.617733,15.643748,15.669763,15.695778,15.721793,15.747808,15.773823,15.799838,15.825853,15.851868,15.877883,15.903898,15.929913,15.955928,15.981943,16.007958,16.033973,16.059988,16.085993,16.112008,16.138023,16.164038,16.190053,16.216068,16.242083,16.268098,16.294113,16.320128,16.346143,16.372158,16.398173,16.424188,16.450203,16.476218,16.502233,16.528248,16.554263,16.580278,16.606293,16.632308,16.658323,16.684338,16.710353,16.736368,16.762383,16.788398,16.814413,16.840428,16.866443,16.892458,16.918473,16.944488,16.970503,16.996518,17.022533,17.048548,17.074563,17.100578,17.126593,17.152608,17.178623,17.204638,17.230653,17.256668,17.282683,17.308698,17.334713,17.360728,17.386743,17.412758,17.438773,17.464788,17.490803,17.516818,17.542833,17.568848,17.594863,17.620878,17.646893,17.672908,17.698923,17.724938,17.750953,17.776968,17.802983,17.828998,17.855013,17.881028,17.907043,17.933058,17.959073,17.985088,18.011103,18.037118,18.063133,18.089148,18.115163,18.141178,18.167193,18.193208,18.219223,18.245238,18.271253,18.297268,18.323283,18.349298,18.375313,18.401328,18.427343,18.453358,18.479373,18.505388,18.531403,18.557418,18.583433,18.609448,18.635463,18.661478,18.687493,18.713508,18.739523,18.765538,18.791553,18.817568,18.843583,18.869598,18.895613,18.921628,18.947643,18.973658,18.999673,19.025688,19.051703,19.077718,19.103733,19.129748,19.155763,19.181778,19.207793,19.233808,19.259823,19.285838,19.311853,19.337868,19.363883,19.389898,19.415913,19.441928,19.467943,19.493958,19.519973,19.545988,19.571993,19.598008,19.624023,19.650038,19.676053,19.702068,19.728083,19.754098,19.780113,19.806128,19.832143,19.858158,19.884173,19.910188,19.936203,19.962218,19.988233,20.014248,20.040263,20.066278,20.092293,20.118308,20.144323,20.170338,20.196353,20.222368,20.248383,20.274398,20.300413,20.326428,20.352443,20.378458,20.404473,20.430488,20.456503,20.482518,20.508533,20.534548,20.560563,20.586578,20.612593,20.638608,20.664623,20.690638,20.716653,20.742668,20.768683,20.794698,20.820713,20.846728,20.872743,20.898758,20.924773,20.950788,20.976803,21.002818,21.028833,21.054848,21.080863,21.106878,21.132893,21.158908,21.184923,21.210938,21.236953,21.262968,21.288983,21.314998,21.341013,21.367028,21.393043,21.419058,21.445073,21.471088,21.497103,21.523118,21.549133,21.575148,21.601163,21.627178,21.653193,21.679208,21.705223,21.731238,21.757253,21.783268,21.809283,21.835298,21.861313,21.887328,21.913343,21.939358,21.965373,21.991388,22.017403,22.043418,22.069433,22.095448,22.121463,22.147478,22.173493,22.199508,22.225523,22.251538,22.277553,22.303568,22.329583,22.355598,22.381613,22.407628,22.433643,22.459658,22.485673,22.511688,22.537703,22.563718,22.589733,22.615748,22.641763,22.667778,22.693793,22.719808,22.745823,22.771838,22.797853,22.823868,22.849883,22.875898,22.901913,22.927928,22.953943,22.979958,23.005973,23.031988,23.057993,23.084008,23.110023,23.136038,23.162053,23.188068,23.214083,23.240098,23.266113,23.292128,23.318143,23.344158,23.370173,23.396188,23.422203,23.448218,23.474233,23.500248,23.526263,23.552278,23.578293,23.604308,23.630323,23.656338,23.682353,23.708368,23.734383,23.760398,23.786413,23.812428,23.838443,23.864458,23.890473,23.916488,23.942503,23.968518,23.994533,24.020548,24.046563,24.072578,24.098593,24.124608,24.150623,24.176638,24.202653,24.228668,24.254683,24.280698,24.306713,24.332728,24.358743,24.384758,24.410773,24.436788,24.462803,24.488818,24.514833,24.540848,24.566863,24.592878,24.618893,24.644908,24.670923,24.696938,24.722953,24.748968,24.774983,24.800998,24.827013,24.853028,24.879043,24.905058,24.931073,24.957088,24.983103,25.009118,25.035133,25.061148,25.087163,25.113178,25.139193,25.165208,25.191223,25.217238,25.243253,25.269268,25.295283,25.321298,25.347313,25.373328,25.399343,25.425358,25.451373,25.477388,25.503403,25.529418,25.555433,25.581448,25.607463,25.633478,25.659493,25.685508,25.711523,25.737538,25.763553,25.789568,25.815583,25.841598,25.867613,25.893628,25.919643,25.945658,25.971673,25.997688,26.023703,26.049718,26.075733,26.101748,26.127763,26.153778,26.179793,26.205808,26.231823,26.257838,26.283853,26.309868,26.335883,26.361898,26.387913,26.413928,26.439943,26.465958,26.491973,26.517988,26.543993,26.569998,26.596013,26.622028,26.648043,26.674058,26.699993,26.726008,26.752023,26.778038,26.804053,26.830068,26.856083,26.882098,26.908113,26.934128,26.960143,26.986158,27.012173,27.038188,27.064203,27.090218,27.116233,27.142248,27.168263,27.194278,27.220293,27.246308,27.272323,27.298338,27.324353,27.350368,27.376383,27.402398,27.428413,27.454428,27.480443,27.506458,27.532473,27.558488,27.584503,27.610518,27.636533,27.662548,27.688563,27.714578,27.740593,27.766608,27.792623,27.818638,27.844653,27.870668,27.896683,27.922698,27.948713,27.974728,27.999993,28.026008,28.052023,28.078038,28.104053,28.130068,28.156083,28.182098,28.208113,28.234128,28.260143,28.286158,28.312173,28.338188,28.364203,28.390218,28.416233,28.442248,28.468263,28.494278,28.520293,28.546308,28.572323,28.598338,28.624353,28.650368,28.676383,28.702398,28.728413,28.754428,28.780443,28.806458,28.832473,28.858488,28.884503,28.910518,28.936533,28.962548,28.988563,29.014578,29.040593,29.066608,29.092623,29.118638,29.144653,29.170668,29.196683,29.222698,29.248713,29.274728,29.300743,29.326758,29.352768,29.378783,29.404798,29.430813,29.456828,29.482843,29.508858,29.534873,29.560888,29.586903,29.612918,29.638933,29.664948,29.690963,29.716978,29.742993,29.769008,29.795023,29.821038,29.847053,29.873068,29.899083,29.925098,29.951113,29.977128,30.003143,30.029158,30.055173,30.081188,30.107203,30.133218,30.159233,30.185248,30.211263,30.237278,30.263293,30.289308,30.315323,30.341338,30.367353,30.393368,30.419383,30.445398,30.471413,30.497428,30.523443,30.549458,30.575473,30.601488,30.627503,30.653518,30.679533,30.705548,30.731563,30.757578,30.783593,30.809608,30.835623,30.861638,30.887653,30.913668,30.939683,30.965698,30.991713,31.017728,31.043743,31.069758,31.095773,31.121788,31.147803,31.173818,31.199833,31.225848,31.251863,31.277878,31.303893,31.329908,31.355923,31.381938,31.407953,31.433968,31.459983,31.485998,31.512013,31.538028,31.564043,31.590058,31.616073,31.642088,31.668103,31.694118,31.720133,31.746148,31.772163,31.798178,31.824193,31.850208,31.876223,31.902238,31.928253,31.954268,31.980283,32.006298,32.032313,32.058328,32.084343,32.110358,32.136373,32.162388,32.188403,32.214418,32.240433,32.266448,32.292463,32.318478,32.344493,32.370508,32.396523,32.422538,32.448553,32.474568,32.500583,32.526598,32.552613,32.578628,32.604643,32.630658,32.656673,32.682688,32.708703,32.734718,32.760733,32.786748,32.812763,32.838778,32.864793,32.890808,32.916823,32.942838,32.968853,32.994868,33.020883,33.046898,33.072913,33.098928,33.124943,33.150958,33.176973,33.202988,3
```

106 iterations

1250 s

diff = 0.502493

[0.500049,0.509951,0.515,0.52,0.525,0.53,0.535,0.54,0.545,0.55,0.555,0.56,0.565,0.57,0.575,0.58,

----- Q 4.2 -----

N = 20

Jacobi Iteration:

1207 iterations

3880 s

0.974077

Gauss-Seidel Iteration:

633 iterations

3351 s

0.974077

SOR Iteration ($\omega=1.8$):

88 iterations

524 s

0.974077

N = 40

Jacobi Iteration:

4410 iterations

60575 s

0.976126

Gauss-Seidel Iteration:

2316 iterations

55730 s

0.976127

SOR Iteration ($\omega=1.9$):

186 iterations

4880 s

0.976127

N = 60

Jacobi Iteration:

9494 iterations

262795 s

0.976783

Gauss-Seidel Iteration:

4993 iterations

222763 s

0.976784

SOR Iteration ($\omega=1.9$):

205 iterations

11014 s

0.976785

5 结果分析

第一问画了个图，虽然没什么用，但是还是放上来了。

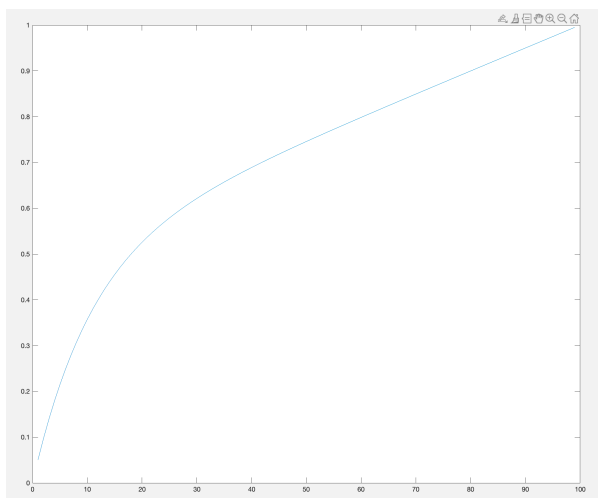


图 1: 计算数据

对比解析解绘制的图像：

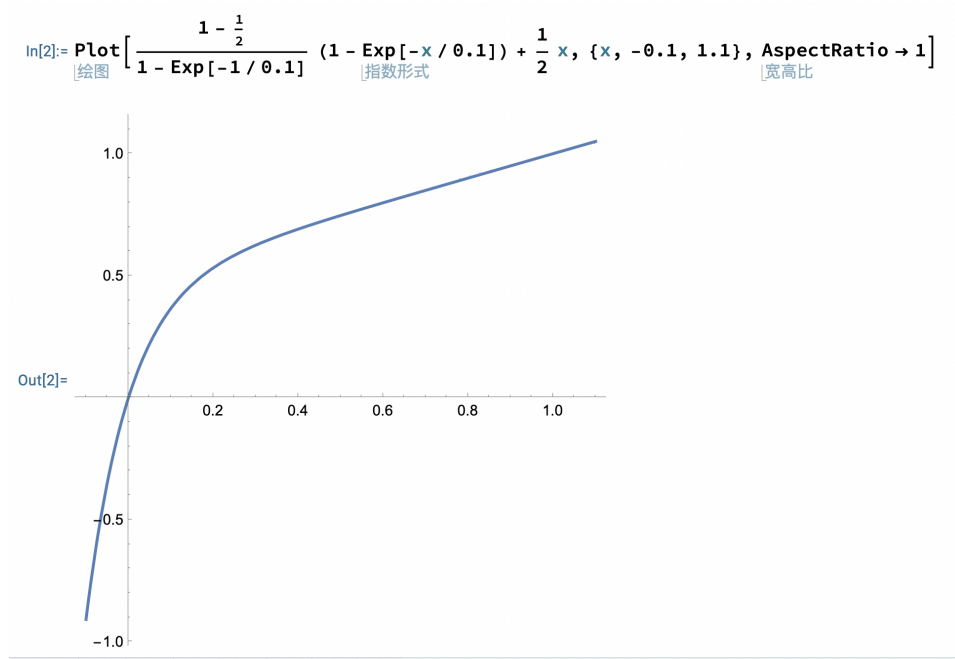


图 2: 解析解