

# 数值代数实验报告 4

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## 1 问题描述

### 1.1 考虑两点边值问题

$$\begin{cases} \epsilon \frac{d^2 y}{dx^2} + \frac{dy}{dx} = a, 0 < a < 1, \\ y(0) = 0, y(1) = 1 \end{cases}$$

容易知道它的精确解为

$$y = \frac{1-a}{1-e^{-\frac{1}{\epsilon}}} (1 - e^{-\frac{x}{\epsilon}}) + ax$$

为了把微分方程离散化, 把  $[0, 1]$  区间  $n$  等分, 令  $h = \frac{1}{n}, x_i = ih, i = 1, \dots, n-1$ , 得到差分方程

$$\epsilon \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} + \frac{y_{i+1} - y_i}{h} = a$$

简化为

$$(\epsilon + h)y_{i+1} - (2\epsilon + h)y_i + \epsilon y_{i-1} = ah^2$$

离散化后得到线性方程组  $Ay = b$ , 其中

$$A = \begin{bmatrix} -(2\epsilon + h) & (\epsilon + h) & 0 & 0 & \cdots & 0 \\ \epsilon & -(2\epsilon + h) & (\epsilon + h) & 0 & \cdots & 0 \\ 0 & \epsilon & -(2\epsilon + h) & (\epsilon + h) & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & \epsilon & -(2\epsilon + h) & (\epsilon + h) \\ 0 & 0 & \cdots & 0 & \epsilon & -(2\epsilon + h) \end{bmatrix}$$

注意  $A$  为  $n-1$  阶矩阵, 将线性方程组与差分方程进行对比得出正确的  $b$  向量 (尤其注意第一行和最后一行)。

对  $\epsilon = 1, a = 1/2, n = 100$ , 分别用 Jacobi 迭代法, G-S 迭代法和 SOR 迭代法求线性方程组的解, 要求 4 位有效数字, 然后比较迭代次数, 运行时间与精确解的误差。迭代法终止条件为  $\|x^{(k+1)} - x^{(k)}\| < 10^{-6}$ 。

对  $\epsilon = 0.1, 0.01, 0.0001$ , 考虑同样的问题。要求输出计算结果, 收敛所需要的迭代次数和运行时间。

## 1.2 考虑偏微分方程

$$-\Delta u + g(x, y)u = f(x, y) \quad (x, y) \in [0, 1] \times [0, 1]$$

在  $[0, 1] \times [0, 1]$  边界上  $u = 1$ . 沿  $x$  方向和  $y$  方向均匀剖分  $N$  等份, 令  $h = 1/N$ , 并设应用中心差分离散化后得到差分方程的代数方程组为

$$-u_{i-1,j} - u_{i,j-1} + (4 + h^2 g(ih, jh))u_{i,j} - u_{i+1,j} - u_{i,j+1} = h^2 f(ih, jh)$$

取  $g(x, y) = \exp(xy)$ ,  $f(x, y) = x + y$ , 分别用 Jacobi 迭代法, G-S 迭代法和 SOR 迭代法求解上述代数方程组, 要求输出解的最小分量, 并比较  $N = 20, 40, 60$  时收敛所需要的迭代次数和运行时间, 迭代终止条件为  $\|u^{(k+1)} - u^{(k)}\|_2 < 10^{-7}$ .

要求仿照下面写的 Jacobi 迭代格式的推导过程推导处 G-S 迭代和 SOR 迭代的格式, 在用 SOR 迭代法求解的过程中, 请对不同的  $N$  使用合适的松弛因子  $\omega$ , 并在程序输出中打印松弛因子的值. 观察运行结果后选取合适的。(代码中不需要体现选取过程, 只需给出即可)。

注意本题中的三个迭代法的算法需要重新写, 不能用矩阵的通用算法!!!

## 2 算法说明

### 1、Jacobi 迭代法

考虑其迭代格式  $Dx_{(k+1)} = (L + U)x_k + b$ . 对第  $i$  行有

$$D_{ii}x_i^{(k+1)} = \sum_{j \neq i} (L + U)_{ij}x_j^{(k)} + b_i$$

将  $D, L, U$  还原成代数方程组的 Jacobi 迭代式

$$D_{ii}x_i^{(k+1)} = L_{i1}x_1^{(k)} + \cdots + L_{ii-1}x_{i-1}^{(k)} + U_{ii+1}x_{i+1}^{(k)} + \cdots + U_{in}x_n^{(k)} + b_i$$

即只有与向量  $b$  的下标相同的位置替换成  $x^{k+1}$ 。

由此类比推广至矩阵 (或者可以直接将矩阵拉直成向量), 知代数方程组 (1) 的 Jacobi 迭代格式为

$$(4 + h^2 g(ih, jh))u_{i,j}^{(k+1)} = u_{i-1,j}^{(k)} + u_{i,j-1}^{(k)} + u_{i+1,j}^{(k)} + u_{i,j+1}^{(k)} + h^2 f(ih, jh)$$

### 2、G-S 迭代法

$$(4 + h^2 g(ih, jh))u_{i,j}^{(k+1)} = u_{i-1,j}^{(k+1)} + u_{i,j-1}^{(k+1)} + u_{i+1,j}^{(k)} + u_{i,j+1}^{(k)} + h^2 f(ih, jh)$$

### 3、SOR 迭代法

$$u_{i,j}^{(k+1)} = \frac{\omega}{4 + h^2 g(ih, jh)} (u_{i-1,j}^{(k+1)} + u_{i,j-1}^{(k+1)} + u_{i+1,j}^{(k)} + u_{i,j+1}^{(k)} + h^2 f(ih, jh)) + (1 - \omega)u_{i,j}^{(k)}$$

## 3 程序介绍

并没有什么特别值得介绍的, 也许。这次抢了点 ddl 有的地方可能乱一点, 见谅。

## 4 运行结果

----- Q 4.1 -----

eps = 1

[0,0.0128705,0.0256626,0.0383772,0.0510151,0.063577,0.0760636,0.0884757,0.100814,0.113079,0.12521,0.137484,0.149749,0.162014,0.174279,0.186544,0.198809,0.211074,0.223339,0.235604,0.247869,0.260134,0.272399,0.284664,0.296929,0.309194,0.321459,0.333724,0.345989,0.358254,0.370519,0.382784,0.395049,0.407314,0.419579,0.431844,0.444109,0.456374,0.468639,0.480904,0.493169,0.505434,0.517699,0.529964,0.542229,0.554494,0.566759,0.579024,0.591289,0.603554,0.615819,0.628084,0.640349,0.652614,0.664879,0.677144,0.689409,0.701674,0.713939,0.726204,0.738469,0.750734,0.762999,0.775264,0.787529,0.799794,0.812059,0.824324,0.836589,0.848854,0.861119,0.873384,0.885649,0.897914,0.910179,0.922444,0.934709,0.946974,0.959239,0.971504,0.983769,0.996034,1.008299,1.020564,1.032829,1.045094,1.057359,1.069624,1.081889,1.094154,1.106419,1.118684,1.130949,1.143214,1.155479,1.167744,1.179999,1.192264,1.204529,1.216794,1.229059,1.241324,1.253589,1.265854,1.278119,1.290384,1.302649,1.314914,1.327179,1.339444,1.351709,1.363974,1.376239,1.388504,1.400769,1.413034,1.425299,1.437564,1.449829,1.462094,1.474359,1.486624,1.498889,1.511154,1.523419,1.535684,1.547949,1.560214,1.572479,1.584744,1.597009,1.609274,1.621539,1.633804,1.646069,1.658334,1.670599,1.682864,1.695129,1.707394,1.719659,1.731924,1.744189,1.756454,1.768719,1.780984,1.793249,1.805514,1.817779,1.830044,1.842309,1.854574,1.866839,1.879104,1.891369,1.903634,1.915899,1.928164,1.940429,1.952694,1.964959,1.977224,1.989489,2.001754,2.014019,2.026284,2.038549,2.050814,2.063079,2.075344,2.087609,2.099874,2.112139,2.124404,2.136669,2.148934,2.161199,2.173464,2.185729,2.197994,2.210259,2.222524,2.234789,2.247054,2.259319,2.271584,2.283849,2.296114,2.308379,2.320644,2.332909,2.345174,2.357439,2.369704,2.381969,2.394234,2.406499,2.418764,2.431029,2.443294,2.455559,2.467824,2.480089,2.492354,2.504619,2.516884,2.529149,2.541414,2.553679,2.565944,2.578209,2.590474,2.602739,2.615004,2.627269,2.639534,2.651799,2.664064,2.676329,2.688594,2.700859,2.713124,2.725389,2.737654,2.749919,2.762184,2.774449,2.786714,2.798979,2.811244,2.823509,2.835774,2.848039,2.860304,2.872569,2.884834,2.897099,2.909364,2.921629,2.933894,2.946159,2.958424,2.970689,2.982954,2.995219,3.007484,3.019749,3.032014,3.044279,3.056544,3.068809,3.081074,3.093339,3.105604,3.117869,3.130134,3.142399,3.154664,3.166929,3.179194,3.191459,3.203724,3.215989,3.228254,3.240519,3.252784,3.265049,3.277314,3.289579,3.301844,3.314109,3.326374,3.338639,3.350904,3.363169,3.375434,3.387699,3.399964,3.412229,3.424494,3.436759,3.449024,3.461289,3.473554,3.485819,3.498084,3.510349,3.522614,3.534879,3.547144,3.559409,3.571674,3.583939,3.596204,3.608469,3.620734,3.632999,3.645264,3.657529,3.669794,3.682059,3.694324,3.706589,3.718854,3.731119,3.743384,3.755649,3.767914,3.780179,3.792444,3.804709,3.816974,3.829239,3.841504,3.853769,3.866034,3.878299,3.890564,3.902829,3.915094,3.927359,3.939624,3.951889,3.964154,3.976419,3.988684,4.000949,4.013214,4.025479,4.037744,4.049999,4.062264,4.074529,4.086794,4.099059,4.111324,4.123589,4.135854,4.148119,4.160384,4.172649,4.184914,4.197179,4.209444,4.221709,4.233974,4.246239,4.258504,4.270769,4.283034,4.295299,4.307564,4.319829,4.332094,4.344359,4.356624,4.368889,4.381154,4.393419,4.405684,4.417949,4.430214,4.442479,4.454744,4.467009,4.479274,4.491539,4.503804,4.516069,4.528334,4.540599,4.552864,4.565129,4.577394,4.589659,4.601924,4.614189,4.626454,4.638719,4.650984,4.663249,4.675514,4.687779,4.699999,4.712264,4.724529,4.736794,4.749059,4.761324,4.773589,4.785854,4.798119,4.810384,4.822649,4.834914,4.847179,4.859444,4.871709,4.883974,4.896239,4.908504,4.920769,4.933034,4.945299,4.957564,4.969829,4.982094,4.994359,5.006624,5.018889,5.031154,5.043419,5.055684,5.067949,5.080214,5.092479,5.104744,5.117009,5.129274,5.141539,5.153804,5.166069,5.178334,5.190599,5.202864,5.215129,5.227394,5.239659,5.251924,5.264189,5.276454,5.288719,5.300984,5.313249,5.325514,5.337779,5.350044,5.362309,5.374574,5.386839,5.399104,5.411369,5.423634,5.435899,5.448164,5.460429,5.472694,5.484959,5.497224,5.509489,5.521754,5.534019,5.546284,5.558549,5.570814,5.583079,5.595344,5.607609,5.619874,5.632139,5.644404,5.656669,5.668934,5.681199,5.693464,5.705729,5.717994,5.730259,5.742524,5.754789,5.767054,5.779319,5.791584,5.803849,5.816114,5.828379,5.840644,5.852909,5.865174,5.877439,5.889704,5.901969,5.914234,5.926499,5.938764,5.951029,5.963294,5.975559,5.987824,6.000089,6.012354,6.024619,6.036884,6.049149,6.061414,6.073679,6.085944,6.098209,6.110474,6.122739,6.135004,6.147269,6.159534,6.171799,6.184064,6.196329,6.208594,6.220859,6.233124,6.245389,6.257654,6.269919,6.282184,6.294449,6.306714,6.318979,6.331244,6.343509,6.355774,6.368039,6.380304,6.392569,6.404834,6.417099,6.429364,6.441629,6.453894,6.466159,6.478424,6.490689,6.502954,6.515219,6.527484,6.539749,6.552014,6.564279,6.576544,6.588809,6.601074,6.613339,6.625604,6.637869,6.650134,6.662399,6.674664,6.686929,6.699194,6.711459,6.723724,6.735989,6.748254,6.760519,6.772784,6.785049,6.797314,6.809579,6.821844,6.834109,6.846374,6.858639,6.870904,6.883169,6.895434,6.907699,6.919964,6.932229,6.944494,6.956759,6.969024,6.981289,6.993554,7.005819,7.018084,7.030349,7.042614,7.054879,7.067144,7.079409,7.091674,7.103939,7.116204,7.128469,7.140734,7.152999,7.165264,7.177529,7.189794,7.202059,7.214324,7.226589,7.238854,7.251119,7.263384,7.275649,7.287914,7.299999,7.312264,7.324529,7.336794,7.349059,7.361324,7.373589,7.385854,7.398119,7.410384,7.422649,7.434914,7.447179,7.459444,7.471709,7.483974,7.496239,7.508504,7.520769,7.533034,7.545299,7.557564,7.569829,7.582094,7.594359,7.606624,7.618889,7.631154,7.643419,7.655684,7.667949,7.680214,7.692479,7.704744,7.717009,7.729274,7.741539,7.753804,7.766069,7.778334,7.790599,7.802864,7.815129,7.827394,7.839659,7.851924,7.864189,7.876454,7.888719,7.900984,7.913249,7.925514,7.937779,7.950044,7.962309,7.974574,7.986839,7.999104,8.011369,8.023634,8.035899,8.048164,8.060429,8.072694,8.084959,8.097224,8.109489,8.121754,8.134019,8.146284,8.158549,8.170814,8.183079,8.195344,8.207609,8.219874,8.232139,8.244404,8.256669,8.268934,8.281199,8.293464,8.305729,8.317994,8.330259,8.342524,8.354789,8.367054,8.379319,8.391584,8.403849,8.416114,8.428379,8.440644,8.452909,8.465174,8.477439,8.489704,8.501969,8.514234,8.526499,8.538764,8.551029,8.563294,8.575559,8.587824,8.599999,8.612264,8.624529,8.636794,8.649059,8.661324,8.673589,8.685854,8.698119,8.710384,8.722649,8.734914,8.747179,8.759444,8.771709,8.783974,8.796239,8.808504,8.820769,8.833034,8.845299,8.857564,8.869829,8.882094,8.894359,8.906624,8.918889,8.931154,8.943419,8.955684,8.967949,8.980214,8.992479,9.004744,9.017009,9.029274,9.041539,9.053804,9.066069,9.078334,9.090599,9.102864,9.115129,9.127394,9.139659,9.151924,9.164189,9.176454,9.188719,9.200984,9.213249,9.225514,9.237779,9.250044,9.262309,9.274574,9.286839,9.299104,9.311369,9.323634,9.335899,9.348164,9.360429,9.372694,9.384959,9.397224,9.409489,9.421754,9.434019,9.446284,9.458549,9.470814,9.483079,9.495344,9.507609,9.519874,9.532139,9.544404,9.556669,9.568934,9.581199,9.593464,9.605729,9.617994,9.630259,9.642524,9.654789,9.667054,9.679319,9.691584,9.703849,9.716114,9.728379,9.740644,9.752909,9.765174,9.777439,9.789704,9.801969,9.814234,9.826499,9.838764,9.851029,9.863294,9.875559,9.887824,9.899999,9.912264,9.924529,9.936794,9.949059,9.961324,9.973589,9.985854,9.998119,10.010384,10.022649,10.034914,10.047179,10.059444,10.071709,10.083974,10.096239,10.108504,10.120769,10.133034,10.145299,10.157564,10.169829,10.182094,10.194359,10.206624,10.218889,10.231154,10.243419,10.255684,10.267949,10.280214,10.292479,10.304744,10.317009,10.329274,10.341539,10.353804,10.366069,10.378334,10.390599,10.402864,10.415129,10.427394,10.439659,10.451924,10.464189,10.476454,10.488719,10.500984,10.513249,10.525514,10.537779,10.550044,10.562309,10.574574,10.586839,10.599104,10.611369,10.623634,10.635899,10.648164,10.660429,10.672694,10.684959,10.697224,10.709489,10.721754,10.734019,10.746284,10.758549,10.770814,10.783079,10.795344,10.807609,10.819874,10.832139,10.844404,10.856669,10.868934,10.881199,10.893464,10.905729,10.917994,10.930259,10.942524,10.954789,10.967054,10.979319,10.991584,10.003849,10.016114,10.028379,10.040644,10.052909,10.065174,10.077439,10.089704,10.101969,10.114234,10.126499,10.138764,10.151029,10.163294,10.175559,10.187824,10.199999,10.212264,10.224529,10.236794,10.249059,10.261324,10.273589,10.285854,10.298119,10.310384,10.322649,10.334914,10.347179,10.359444,10.371709,10.383974,10.396239,10.408504,10.420769,10.433034,10.445299,10.457564,10.469829,10.482094,10.494359,10.506624,10.518889,10.531154,10.543419,10.555684,10.567949,10.580214,10.592479,10.604744,10.617009,10.629274,10.641539,10.653804,10.666069,10.678334,10.690599,10.702864,10.715129,10.727394,10.739659,10.751924,10.764189,10.776454,10.788719,10.800984,10.813249,10.825514,10.837779,10.850044,10.862309,10.874574,10.886839,10.899104,10.911369,10.923634,10.935899,10.948164,10.960429,10.972694,10.984959,10.997224,11.009489,11.021754,11.034019,11.046284,11.058549,11.070814,11.083079,11.095344,11.107609,11.119874,11.132139,11.144404,11.156669,11.168934,11.181199,11.193464,11.205729,11.218004,11.230269,11.242534,11.254799,11.267064,11.279329,11.291594,11.303859,11.316124,11.328389,11.340654,11.352919,11.365184,11.377449,11.389714,11.401979,11.414244,11.426509,11.438774,11.451039,11.463304,11.475569,11.487834,11.499999,11.512264,11.524529,11.536794,11.549059,11.561324,11.573589,11.585854,11.598119,11.610384,11.622649,11.634914,11.647179,11.659444,11.671709,11.683974,11.696239,11.708504,11.720769,11.733034,11.745299,11.757564,11.769829,11.782094,11.794359,11.806624,11.818889,11.831154,11.843419,11.855684,11.867949,11.880214,11.892479,11.904744,11.917009,11.929274,11.941539,11.953804,11.966069,11.978334,11.990599,12.002864,12.015129,12.027394,12.039659,12.051924,12.064189,12.076454,12.088719,12.100984,12.113249,12.125514,12.137779,12.150044,12.162309,12.174574,12.186839,12.199104,12.211369,12.223634,12.235899,12.248164,12.2604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29169 s
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[0.0504528,0.0967736,0.139338,0.178488,0.214533,0.247755,0.278413,0.306738,0.332942,0.357219,0.377488,0.397757,0.418026,0.438295,0.458564,0.478833,0.499102,0.519371,0.53964,0.559909,0.580178,0.600447,0.620716,0.640985,0.661254,0.681523,0.701792,0.722061,0.74233,0.762599,0.782868,0.803137,0.823406,0.843675,0.863944,0.884213,0.904482,0.924751,0.94502,0.965289,0.985558,1.005827,1.026096,1.046365,1.066634,1.086903,1.107172,1.127441,1.14771,1.167979,1.188248,1.208517,1.228786,1.249055,1.269324,1.289593,1.309862,1.330131,1.3504,1.370669,1.390938,1.411207,1.431476,1.451745,1.472014,1.492283,1.512552,1.532821,1.55309,1.573359,1.593628,1.613897,1.634166,1.654435,1.674704,1.694973,1.715242,1.735511,1.75578,1.776049,1.796318,1.816587,1.836856,1.857125,1.877394,1.897663,1.917932,1.938201,1.95847,1.978739,1.999008,2.019277,2.039546,2.059815,2.080084,2.100353,2.120622,2.140891,2.16116,2.181429,2.201698,2.221967,2.242236,2.262505,2.282774,2.303043,2.323312,2.343581,2.36385,2.384119,2.404388,2.424657,2.444926,2.465195,2.485464,2.505733,2.526002,2.546271,2.56654,2.586809,2.607078,2.627347,2.647616,2.667885,2.688154,2.708423,2.728692,2.748961,2.76923,2.7895,2.809769,2.830038,2.850307,2.870576,2.890845,2.911114,2.931383,2.951652,2.971921,2.99219,3.012459,3.032728,3.052997,3.073266,3.093535,3.113804,3.134073,3.154342,3.174611,3.19488,3.215149,3.235418,3.255687,3.275956,3.296225,3.316494,3.336763,3.357032,3.377301,3.39757,3.417839,3.438108,3.458377,3.478646,3.498915,3.519184,3.539453,3.559722,3.579991,3.60026,3.620529,3.640798,3.661067,3.681336,3.701605,3.721874,3.742143,3.762412,3.782681,3.80295,3.823219,3.843488,3.863757,3.884026,3.904295,3.924564,3.944833,3.965102,3.985371,4.00564,4.025909,4.046178,4.066447,4.086716,4.106985,4.127254,4.147523,4.167792,4.188061,4.20833,4.228599,4.248868,4.269137,4.289406,4.309675,4.329944,4.350213,4.370482,4.390751,4.41102,4.431289,4.451558,4.471827,4.492096,4.512365,4.532634,4.552903,4.573172,4.593441,4.61371,4.633979,4.654248,4.674517,4.694786,4.715055,4.735324,4.755593,4.775862,4.796131,4.8164,4.836669,4.856938,4.877207,4.897476,4.917745,4.938014,4.958283,4.978552,4.998821,5.01909,5.039359,5.059628,5.079897,5.100166,5.120435,5.140704,5.160973,5.181242,5.201511,5.22178,5.242049,5.262318,5.282587,5.302856,5.323125,5.343394,5.363663,5.383932,5.404201,5.42447,5.444739,5.465008,5.485277,5.505546,5.525815,5.546084,5.566353,5.586622,5.606891,5.62716,5.647429,5.667698,5.687967,5.708236,5.728505,5.748774,5.769043,5.789312,5.809581,5.82985,5.850119,5.870388,5.890657,5.910926,5.931195,5.951464,5.971733,5.991999,6.012268,6.032537,6.052806,6.073075,6.093344,6.113613,6.133882,6.154151,6.17442,6.194689,6.214958,6.235227,6.255496,6.275765,6.296034,6.316303,6.336572,6.356841,6.37711,6.397379,6.417648,6.437917,6.458186,6.478455,6.498724,6.518993,6.539262,6.559531,6.5798,6.59999,6.620259,6.640528,6.660797,6.681066,6.701335,6.721604,6.741873,6.762142,6.782411,6.80268,6.822949,6.843218,6.863487,6.883756,6.904025,6.924294,6.944563,6.964832,6.985101,7.00537,7.025639,7.045908,7.066177,7.086446,7.106715,7.126984,7.147253,7.167522,7.187791,7.20806,7.228329,7.248598,7.268867,7.289136,7.309405,7.329674,7.349943,7.370212,7.390481,7.41075,7.431019,7.451288,7.471557,7.491826,7.512095,7.532364,7.552633,7.572902,7.593171,7.61344,7.633709,7.653978,7.674247,7.694516,7.714785,7.735054,7.755323,7.775592,7.795861,7.81613,7.8364,7.856669,7.876938,7.897207,7.917476,7.937745,7.958014,7.978283,7.998552,8.018821,8.03909,8.059359,8.079628,8.099897,8.120166,8.140435,8.160704,8.180973,8.201242,8.221511,8.24178,8.262049,8.282318,8.302587,8.322856,8.343125,8.363394,8.383663,8.403932,8.424201,8.44447,8.464739,8.485008,8.505277,8.525546,8.545815,8.566084,8.586353,8.606622,8.626891,8.64716,8.667429,8.687698,8.707967,8.728236,8.748505,8.768774,8.789043,8.809312,8.829581,8.84985,8.870119,8.890388,8.910657,8.930926,8.951195,8.971464,8.991733,9.011999,9.032268,9.052537,9.072806,9.093075,9.113344,9.133613,9.153882,9.174151,9.19442,9.214689,9.234958,9.255227,9.275496,9.295765,9.316034,9.336303,9.356572,9.376841,9.39711,9.417379,9.437648,9.457917,9.478186,9.498455,9.518724,9.538993,9.559262,9.579531,9.5998,9.620069,9.640338,9.660607,9.680876,9.701145,9.721414,9.741683,9.761952,9.782221,9.80249,9.822759,9.843028,9.863297,9.883566,9.903835,9.924104,9.944373,9.964642,9.984911,10.00518,10.025449,10.045718,10.065987,10.086256,10.106525,10.126794,10.147063,10.167332,10.187601,10.20787,10.228139,10.248408,10.268677,10.288946,10.309215,10.329484,10.349753,10.369999,10.390268,10.410537,10.430806,10.451075,10.471344,10.491613,10.511882,10.532151,10.55242,10.572689,10.592958,10.613227,10.633496,10.653765,10.674034,10.694303,10.714572,10.734841,10.75511,10.775379,10.795648,10.815917,10.836186,10.856455,10.876724,10.896993,10.917262,10.937531,10.9578,10.978069,10.998338,10.100607,10.120876,10.141145,10.161414,10.181683,10.201952,10.222221,10.24249,10.262759,10.283028,10.303297,10.323566,10.343835,10.364104,10.384373,10.404642,10.424911,10.44518,10.465449,10.4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,10.688408,10.708677,10.728946,10.749215,10.769484,10.789753,10.809999,10.830268,10.850537,10.870806,10.891075,10.911344,10.931613,10.951882,10.972151,10.99242,10.100607,10.120876,10.141145,10.161414,10.181683,10.201952,10.222221,10.24249,10.262759,10.283028,10.303297,10.323566,10.343835,10.364104,10.384373,10.404642,10.424911,10.44518,10.465449,10.485718,10.505987,10.526256,10.546525,10.566794,10.587063,10.607332,10.627601,10.64787,10.668139,10.688408,10.708677,10.728946,10.749215,10.769484,10.789753,10.809999,10.830268,10.850537,10.870806,10.891075,10.911344,10.931613,10.951882,10.972151,10.99242,10.100607,10.120876,10.141145,10.161414,10.181683,10.201952,10.222221,10.24249,10.262759,10.283028,10.303297,10.323566,10.343835,10.364104,10.384373,10.404642,10.424911,10.44518,10.465449,10.485718,10.505987,10.526256,10.546525,10.566794,10.587063,10.607332,10.627601,10.64787,10.668139,10.688408,10.708677,10.728946,10.749215,10.769484,10.789753,10.809999,10.830268,10.850537,10.870806,10.891075,10.911344,10.931613,10.951882,10.972151,10.99242,10.100607,10.120876,10.141145,10.161414,10.181683,10.201952,10.222221,10.24249,10.262759,10.283028,10.303297,10.323566,10.343835,10.364104,10.384373,10.404642,10.424911,10.44518,10.465449,10.485718,10.505987,10.526256,10.546525,10.566794,10.587063,10.607332,10.627601,10.64787,10.668139,10.688408,10.708677,10.728946,10.749215,10.769484,10.789753,10.809999,10.830268,10.850537,10.870806,10.891075,10.911344,10.931613,10.951882,10.972151,10.99242,10.100607,10.120876,10.141145,10.161414,10.181683,10.201952,10.222221,10.24249,10.262759,10.283028,10.303297,10.323566,10.343835,10.364104,10.384373,10.404642,10.424911,10.44518,10.465449,10.485718,10.505987,10.526256,10.546525,10.566794,10.587063,10.607332,10.627601,10.64787,10.668139,10.688408,10.708677,10.728946,10.749215,10.769484,10.789753,10.809999,10.830268,10.850537,10.870806,10.891075,10.911344,10.931613,10.951882,10.972151,10.99242,10.100607,10.120876,10.141145,10.161414,10.181683,10.201952,10.222221,10.24249,10.262759,10.283028,10.303297,10.323566,10.343835,10.364104,10.384373,10.404642,10.424911,10.44518,10.465449,10.485718,10.505987,10.526256,10.546525,10.566794,10.587063,10.607332,10.627601,10.64787,10.668139,10.688408,10.708677,10.728946,10.749215,10.769484,10.789753,10.809999,10.830268,10.850537,10.870806,10.8
```

106 iterations

1250 s

diff = 0.502493

[0.500049,0.509951,0.515,0.52,0.525,0.53,0.535,0.54,0.545,0.55,0.555,0.56,0.565,0.57,0.575,0.58,

----- Q 4.2 -----

N = 20

Jacobi Iteration:

1207 iterations

3880 s

0.974077

Gauss-Seidel Iteration:

633 iterations

3351 s

0.974077

SOR Iteration ( $\omega=1.8$ ):

88 iterations

524 s

0.974077

N = 40

Jacobi Iteration:

4410 iterations

60575 s

0.976126

Gauss-Seidel Iteration:

2316 iterations

55730 s

0.976127

SOR Iteration ( $\omega=1.9$ ):

186 iterations

4880 s

0.976127

N = 60

Jacobi Iteration:

9494 iterations

262795 s

0.976783

Gauss-Seidel Iteration:

4993 iterations

222763 s

0.976784

SOR Iteration ( $\omega=1.9$ ):

205 iterations

11014 s

0.976785

## 5 结构分析

第一问画了个图，虽然没什么用，但是还是放上来了。

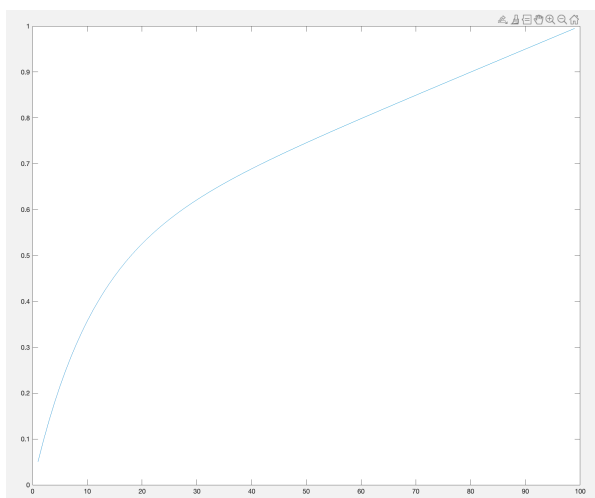


图 1: 计算数据

对比解析解绘制的图像：

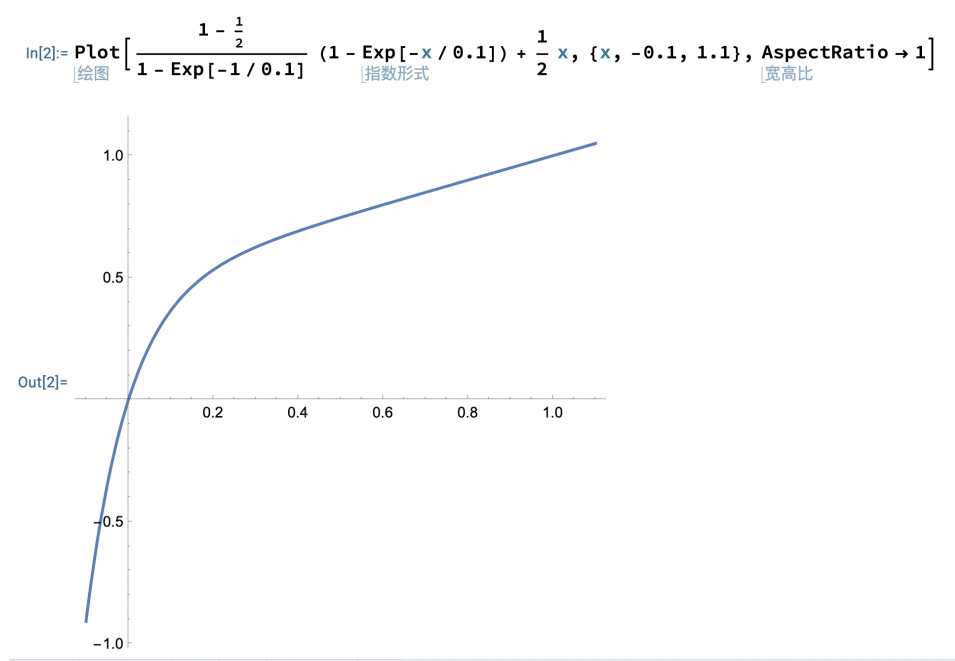


图 2: 解析解