数值代数实验报告 8

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1 问题描述

1.1 SVD 迭代

参考课本 7.6.2 节 (P234-240)SVD 迭代完成 SVD 算法 7.6.3, 并对附件 svddata.txt 中的矩阵作 SVD 分解 $A = P\Sigma Q$。并计算 $PP^T - I, QQ^T - I, P\Sigma Q - A$ 的绝对值最大的元素，依次用 ep, eq, et 表示。

输出格式为:
迭代次数: x
奇异值从小到大:
ep = xx
eq = xx
et = xx

A=PTQ
T= [矩阵]
P= [矩阵]
Q= [矩阵]

2 算法说明

必须实现的算法有:

1. SVD 迭代 ⇒ `SVDMethod/SVDMethod`

其中 SVD 分解中 2x2 的实现参考了: `https://www.math.ucla.edu/~cffjiang/research/svd/svd.pdf`，在此表示感谢。

3 运行结果

也可在 `homeworks/reports/data/report_8_output.txt` 中查看。

```
iterations: 61053
time spent: 2837150us
eigenvalues = [ -8.666483 -4.981007 -1.140182 -0.703989 0.375993 0.880006 1.898629
2.602050 3.144497 5.947023 32.297933 214.310227 ]
ep = 0.000000
eq = 0.000000
et = 0.000215
T=
[[ 214.310227
[          0.375993
[                   -0.703989
[                             1.898629
[                                      2.602050
[                                               0.880006
[                                                        -1.140182
[                                                                 3.144497
[                                                                          5.947023
],
[                                                                                   -4.981007
],
[                                                                                             -8.66
],
[
],
[
[
[
[
[
[
[
[
[
[
[
[
[
[
[
[
P=
```

```
[[ -0.201521 -0.290610 -0.190181 -0.254386 -0.201216 -0.262533 -0.243492 -0.159834
-0.216831 -0.092888 -0.158165 -0.147892 -0.147795 -0.158657 -0.218459 -0.159834 -0.242183
-0.114773 -0.093357 -0.122810 -0.194173 -0.113186 -0.245103 -0.212867 -0.235342 -0.032580
-0.159496 -0.153544 ],
[ -0.240911 0.091983 0.165372 0.042807 0.062961 -0.316217 0.066812 0.225143 0.082553
-0.148776 0.296058 0.092212 -0.183622 0.217375 0.083576 0.225143 -0.210723 -0.103036
-0.136730 -0.127714 0.389903 0.200737 -0.236992 -0.228177 -0.260677 0.009042 0.073778
0.131944 ],
[ 0.197714 0.124192 0.059551 -0.062472 -0.011076 0.080930 -0.299423 -0.017047 -0.337266
0.277632 -0.001570 0.277625 0.142855 -0.029391 0.089154 -0.017047 -0.353302 0.044776
0.217531 -0.242887 0.151998 0.220644 -0.031675 0.133882 0.006444 -0.089931 -0.391654
0.242128 ],
[ 0.086592 -0.231328 0.081913 -0.097873 -0.044376 -0.088771 0.420548 -0.076500 -0.017163
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-0.104771 -0.036389 0.192274 -0.020685 0.043867 -0.217018 0.113708 0.092602 -0.102224
-0.030344 ],
[ -0.123676 0.090646 -0.164856 -0.035850 -0.054280 -0.038613 -0.002685 -0.282975 -0.253231
0.088804 0.275850 -0.177341 -0.111085 0.322421 -0.177225 -0.282975 0.061941 0.098210
-0.108534 -0.052621 0.435183 -0.299806 0.250617 -0.028294 0.242395 0.119175 -0.047212
0.100385 ],
[ 0.404673 0.267761 -0.067119 -0.019984 -0.035964 -0.091020 0.006447 0.090883 0.054218
0.212876 -0.214334 0.163920 -0.254242 -0.197156 -0.045151 0.090883 -0.118267 -0.408227
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[ -0.348918 0.171102 -0.001016 0.150389 -0.077997 -0.035229 -0.123297 -0.169319 0.340394
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-0.462953 ],
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[ 0.139859 0.051203 0.173717 0.081373 0.051263 -0.077748 -0.062964 -0.442066 0.235113
```

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```
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```