$$V = \hat{V}(t) e^{j\omega_{rf} t} \qquad \Delta\omega = \omega_r - \omega_{rf}$$

$$\frac{d\hat{V}}{dt} + \left[\frac{\omega_r}{2Q} - j\Delta\omega\right]\hat{V} = \frac{R\omega_r}{2Q}(\hat{I}_G - \hat{I}_B)$$

$$\hat{I}_D = \hat{I}_{D0} + g_1(\hat{V}_T - \hat{V}) + g_2\int^{t} dt_1 (\hat{V}_T - \hat{V}(t_1))$$

$\hat{V}_T$ is the constant target voltage. Assume $\hat{I}_B$ constant diff

$$\ddot{\hat{V}} + (\alpha_c - j\Delta\omega)\dot{\hat{V}} = -\alpha_D \dot{\hat{V}} - \alpha_I(\hat{V} - \hat{V}_T)$$

$$u = \hat{V} - \hat{V}_T$$

$$\ddot{u} + (\alpha_c + \alpha_D - j\Delta\omega)\dot{u} + \alpha_I u = 0$$

$$u = e^{\lambda t}$$

$$2\lambda = -(\alpha_D + \alpha_c - j\Delta\omega) \pm \left[(\alpha_D + \alpha_c - j\Delta\omega)^2 - 4\alpha_I\right]^{\frac{1}{2}}$$

$$[\,] = (\alpha_D + \alpha_c)^2 - 2j\Delta\omega(\alpha_D + \alpha_c) - 4\alpha_I - \Delta\omega^2$$

if $(\alpha_D + \alpha_c)^2 - \Delta\omega^2 < 0$, $\alpha_I = \dfrac{-j(\alpha_c + \alpha_D)\Delta\omega}{2}$

other wise $\alpha_I = \frac{1}{4}(\alpha_D + \alpha_c - j\Delta\omega)^2$