6.2

$$H = \omega(q^2 + p^2)/2$$

$$f = \frac{\partial H}{\partial p} = \omega p$$

$$g = -\frac{\partial H}{\partial q} = -\omega q$$

Define new coordinates:

$$q = rcos\phi$$

$$p = -rsin\phi$$

The Jacobian of this transformation is:

$$J = \begin{Bmatrix} \frac{\partial q}{\partial r} & \frac{\partial q}{\partial \phi} \\ \frac{\partial p}{\partial r} & \frac{\partial p}{\partial \phi} \end{Bmatrix} = \begin{Bmatrix} cos\phi & -rsin\phi \\ -sin\phi & -rcos\phi \end{Bmatrix} => J^{-1} = \frac{1}{r} \begin{Bmatrix} rcos\phi & -rsin\phi \\ -sin\phi & -cos\phi \end{Bmatrix} = \begin{Bmatrix} \frac{\partial r}{\partial q} & \frac{\partial r}{\partial p} \\ \frac{\partial \phi}{\partial q} & \frac{\partial \phi}{\partial p} \end{Bmatrix}$$

Plug this into the Vlasov Equation:

$$\begin{aligned} VE &= \frac{\partial \Psi}{\partial t} + f\frac{\partial \Psi}{\partial q} + q\frac{\partial \Psi}{\partial p} \\ &= \frac{\partial \Psi}{\partial t} + \omega p \left[\frac{\partial \Psi}{\partial r} \cos\phi - \frac{\partial \Psi}{\partial \phi} \frac{\sin\phi}{r} \right] - \omega q \left[\frac{\partial \Psi}{\partial r} \sin\phi - \frac{\partial \Psi}{\partial \phi} \frac{\cos\phi}{r} \right] \\ &= \frac{\partial \Psi}{\partial t} + \omega \frac{\partial \Psi}{\partial r} \left(-r\sin\phi\cos\phi + r\cos\phi\sin\phi \right) + \omega \frac{\partial \Psi}{\partial \phi} \left(r\sin\phi\sin\phi/r + r\cos\phi\cos\phi/r \right) \\ &= \frac{\partial \Psi}{\partial t} + \omega \frac{\partial \Psi}{\partial \phi} = 0 \end{aligned}$$