Wang_T_2

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1.1 Question A

```
In [62]: import numpy as np
```

```
In [495]: def REF_basic(A):
              # Coerce the data type to be float so to make fraction computation
              B=A.copy().astype(np.float)
              m,n = B.shape
              # Make sure the dimension of input to be n ∈ (n+1), othersie raise error
              if m!=n-1:
                  raise TypeError('This is not a n*(n+1) model')
              for i in range(m): # Iterate from the first column
                  # Make sure there is a pivot for each column
                  if np.max(abs(B[i:,i])) == 0:
                      raise TypeError('Matrix is not invertible')
                  if B[i,i] == 0:# Swap the row to make there is a leading number
                      change = np.argmax(abs(B[i:,i]))+i
                      temp = B[i].copy()
                      B[i] = B[change]
                      B[change]=temp
                  for j in range(i+1,m):
                      B[j]=B[j]-B[i]*B[j,i]/B[i,i] # Do the forward elimination
              return B
```

(a) Test the function in HW1-7

```
In [496]: test1=np.array([[2,2,2,0],[-2,5,2,1],[8,1,4,-1]])
          REF_basic(test1)
```

```
---------------------------------------------------------------------------

TypeError                                 Traceback (most recent call last)

<ipython-input-496-05dc646d408d> in <module>()
  1 test1=np.array([[2,2,2,0],[-2,5,2,1],[8,1,4,-1]])
```

```
----> 2 REF_basic(test1)


    <ipython-input-495-3a781e51dec4> in REF_basic(A)
      6     for i in range(m): # Iterate from the first column
      7         if np.max(abs(B[i:,i])) == 0: # Make sure there is a pivot for each column
----> 8             raise TypeError('Matrix is not invertible')
      9         if B[i,i] == 0:
     10             change = np.argmax(abs(B[i:,i]))+i # Swap the row to make there is a le


    TypeError: Matrix is not invertible
```

(b) Test the function in the given array

```
In [497]: test2=np.array([[1,2,3,4,6],[1,2,5,2,-2],[1,1,5,5,1],[1,4,1,5,0]])
          print(test2)
          print(REF_basic(test2))
```

```
[[ 1  2  3  4  6]
 [ 1  2  5  2 -2]
 [ 1  1  5  5  1]
 [ 1  4  1  5  0]]
[[ 1.   2.   3.   4.   6. ]
 [ 0.   2.  -2.   1.  -6. ]
 [ 0.   0.   1.   1.5 -8. ]
 [ 0.   0.   0.  -5.   8. ]]
```

1.2 Question B

```
In [528]: # Define the function BackSub
          def BackSub(A,b):
              m,n = A.shape
              if len(A.shape)!=2: # A must be a matrix
                  raise TypeError('Input is not a matrix')
              # A must be a square matrix
              if m != n:
                  raise TypeError('Input is not a square matrix')
              # A must be an upper triangle square matrix
              if np.allclose(A, np.triu(np.round(A,6))) == 0:
                  raise TypeError('Input is not an upper triangle square matrix')
              if np.linalg.det(A)==0: # A's diagonals are all nonzero
                  raise TypeError('Input is not an invertible matrix')
              if len(b.shape)!=1: # b must be a vector
                  raise TypeError('The second input is not a vector')
              if b.shape[0]!=A.shape[0] :# dimension of b must equal to a
```

```
            raise TypeError('Two inputs do not match')
        else:
            # Start the backwards elimination from the last column
            for i in range(len(b)-1,-1,-1):
                b[i]=b[i]/A[i,i]
                A[i,i]=1
                # In each column, do the elimination from the diagnal upwards
                for j in range(i-1,-1,-1):
                    b[j]=b[j]-A[j,i]*b[i]
                    A[j,i]=0
        return b

    def Mysolve(A,b):
        row,column = A.shape
        B=np.column_stack((A,b))
        C=REF_basic(B)
        D = C[:,:column]
        e = C[:,column]
        return BackSub(D,e)
```

Randomly generate some linear systems to test the function `Mysolve`
Test 1

```
In [499]: import random
          test1 = np.array(np.random.randint(0,10,9))
          test1 = test1.reshape(3,3)
          b1 = np.array(np.random.randint(0,10,3))
          print (test1)
          print (b1)
```

```
[[2 6 6]
 [8 9 0]
 [0 5 5]]
[3 9 8]
```

```
In [500]: Mysolve(test1,b1)
```

```
Out[500]: array([-3.3       ,  3.93333333, -2.33333333])
```

```
In [501]: np.linalg.solve(test1,b1)
```

```
Out[501]: array([-3.3       ,  3.93333333, -2.33333333])
```

Test 2

```
In [502]: np.random.seed(222)
          test2 = np.array(np.random.randint(0,10,16))
```

```
test2 = test2.reshape(4,4)
b2 = np.array(np.random.randint(0,10,4))
print (test2)
print (b2)
```

```
[[6 3 2 5]
 [6 8 8 4]
 [7 8 8 0]
 [4 3 3 2]]
[1 9 9 8]
```

```
In [503]: Mysolve(test2,b2)
```

```
Out[503]: array([  2.46666153, -14.81664357,  13.78331408,   0.61666667])
```

```
In [504]: np.linalg.solve(test2,b2)
```

```
Out[504]: array([  2.46666667, -14.81666667,  13.78333333,   0.61666667])
```

Test 3

```
In [505]: test3 = np.array(np.random.randint(0,10,25))
          test3 = test3.reshape(5,5)
          b3 = np.array(np.random.randint(0,10,5))
          print (test3)
          print (b3)
          print(Mysolve(test3,b3))
          print(np.linalg.solve(test3,b3))
```

```
[[0 3 7 7 7]
 [9 4 5 9 4]
 [9 5 5 4 8]
 [4 8 0 4 0]
 [0 1 3 7 5]]
[1 2 6 4 2]
[ 0.15808574  0.42805316 -0.90165032 -0.01419142  0.87524752]
[ 0.15808581  0.42805281 -0.90165017 -0.01419142  0.87524752]
```

```
In [506]: Mysolve(test3,b3)
```

```
Out[506]: array([ 0.15808574,  0.42805316, -0.90165032, -0.01419142,  0.87524752])
```

```
In [507]: np.linalg.solve(test3,b3)
```

```
Out[507]: array([ 0.15808581,  0.42805281, -0.90165017, -0.01419142,  0.87524752])
```

In all three tests, my function generates the same results as np.linalg.solve does.

1.3 Quesiton C

Test the complexity of the function.

```
In [529]: import time
          np.random.seed(12321)
          k=range(11)
          T=[]
          for i in k:
              test = np.array(np.random.randint(0,10,(2**i)**2))
              test = test.reshape(2**i,2**i)
              b = np.array(np.random.randint(0,10,2**i))
              t=0
              for j in range(0,7):
                  start = time.time()
                  Mysolve(test,b)
                  end = time.time()
                  t=t+end-start
                  j=j+1
              T.append(np.mean(t))
          print(T)
```

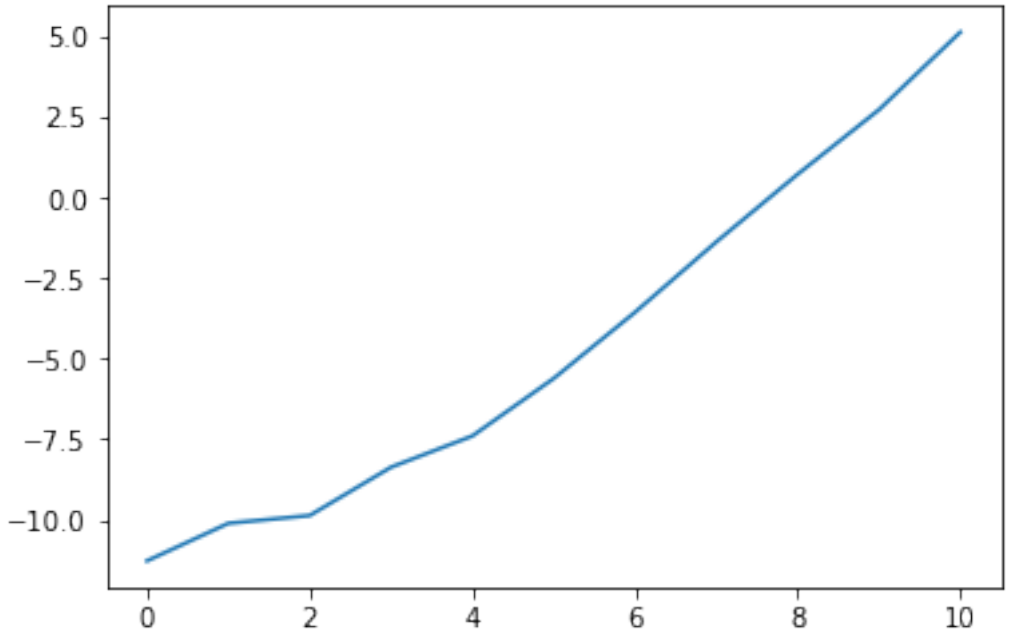
```
/Users/simpson/anaconda3/lib/python3.6/site-packages/numpy/linalg/linalg.py:1874: RuntimeWarni
  r = _umath_linalg.det(a, signature=signature)


[0.00040340423583984375, 0.0009050369262695312, 0.0010700225830078125, 0.0030281543731689453, 
```

Plot the time spent and the dataset scale

```
In [547]: import matplotlib.pyplot as plt
          plt.plot(k,np.log2(T))
          plt.show
```

```
Out[547]: <function matplotlib.pyplot.show>
```




```
In [541]: print ("%ds is the total time spend to solve those systems."%(sum(T)*7))

309s is the total time spend to solve those systems.
```

This line seems to be like a straight line, this is reasonable since the matrix size is 2^k and when we take the log we should get a linear relationship.