Summer 2018 MSAN 502 Homework 1, due 7/12 11:59pm

Each problem should be started on a separate piece of paper

1. Explain why each of the three elementary row operations does not affect the solution set of a linear system.

2. Find values for a, b, and c such that the parabola $y = ax^2 + bx + c$ passes through the points (1,1), (2,4), and (-1,1).

For Problems 3-7, solve the linear systems

3. $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right]$

4. $\left[\begin{array}{cccc|c} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$

5. $\left[\begin{array}{ccc|c} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 5 \end{array}\right]$

6. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{array}\right]$

7. $\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$

8. $\begin{aligned} v + 3w - 2x &= 0 \\ 2u + v - 4w + 3x &= 0 \\ 2u + 3v + 2w - x &= 0 \\ -4u - 3v + 5w - 4x &= 0 \end{aligned}$

9. Determine the values of a for which the following system has no solutions, exactly one solution, or infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & a^2-3 & a \end{array}\right]$$

10. What condition, if any, must a, b, c satisfy for the linear system $\left\{\begin{array}{rrrl} x & +3y & +z & = a \\ -x & -2y & +z & = b \\ 3x & +7y & -z & = c \end{array}\right.$ to be consistent?

11. Find scalars c_i for which the equation $c_1(-1,0,2) + c_2(2,2,-2) + c_3(1,-2,1) = (-6,12,4)$ holds.

12. (10) Let $u = (2,-2,3), v = (1,-3,4), w = (3,6,-4)$
    (a) Find x that satisfies the equation $2u - v + x = 7x + w$.
    (b) Find the distance between u and v.
    (c) Find $\|u\| - \|v\|$

(d) Find $u \cdot v$

(e) Simplify $(u - v) \cdot (u + 2v)$ first, then compute.

(f) Find a unit vector that is in the same direction to v.

(g) Find the angle between u and v

13. Let u, v be the same vectors as in Problem 12.

(a) Find all vectors orthogonal to u.

(b) Find all vectors orthogonal to both u and v.

14. True or False. Give reasons or counterexample when false.

(a) All pivots in the row echelon form must occur in different columns.

(b) If a linear system has more unknowns than equations, then it must have infinitely many solutions.

(c) In $\mathbb{R}^3$, if u and v are both orthogonal to w, then u and v are parallel.

(d) If u and v are orthogonal, then $\|u - v\| = \sqrt{2}$.

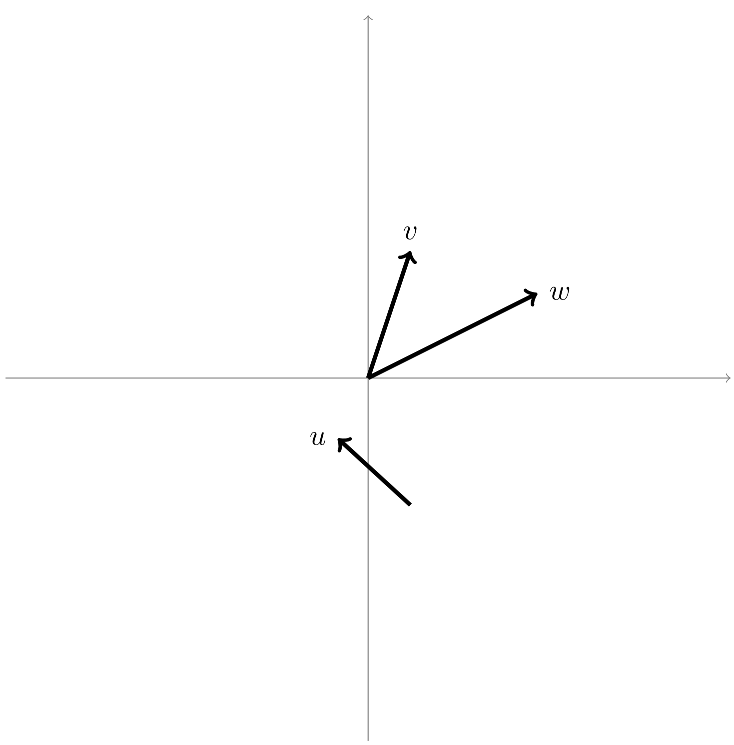
(e) A system of linear equations cannot have exactly 2 solutions.

(f) If $u \cdot v = u \cdot w$, then $v = w$.

15. Print this page, and **Draw and label** the following 5 vectors on the graph below:

$$v + w, \quad v - w, \quad v + u, \quad w + 2v, \quad , w - 2u$$

Since vectors can float, **all 5 vectors that you draw should start at the origin** for the purpose of grading.



Python problems

A. Familiarize yourself with the `numpy.linalg` package. Define a 10×10 matrix A whose entries are the numbers from 1 to 100 ordered consecutively going across rows (so $a_{1,1} = 1, a_{2,1} = 11, a_{10,1} = 91$, etc). Let $v = (1, 2, ...10)$, and $b = (1, 1, ...1)$. Compute Av and solve $Ax = b$ and $Ax = v$ using `linalg.solve`. Verify your solution by computing Ax.

B. You may get some error in doing above. You decide to "shake" your matrix by some noise, after all data usual comes in noisy. Use the `numpy.random` package to build a 10×10 matrix $R(\epsilon)$ whose entries are i.i.d. Gaussian random variables that follow $\mathrm{N}(0, \epsilon^2)$. Verify now that you "can" compute the questions above for various values of $\epsilon > 0$ for the new matrix $A + R(\epsilon)$.

C. Write a python function `BackSub(A, b)` that does the backward substitution from scratch. The input A must be an upper triangular square matrix whose diagonals are all nonzero, and b must be a vector of appropriate dimension. Raise an error if inputs do not satisfy the requirements. Only 1 for loop is needed. It should return the unique solution of solving $Ax = b$. Test your function on

   (a) Problem 3 (b) a 3×3 matrix that is not upper triangular (c) a 3×2 matrix (d) Problem 5