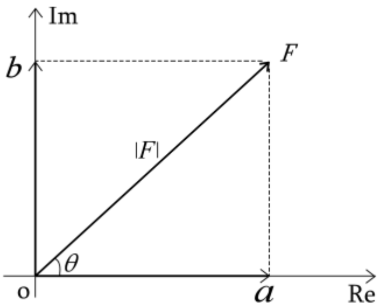
6-2 复数

1. 复数的表示形式



$F = a + \mathrm{j}b$ 代数式

$(j = \sqrt{-1}$ 为虚数单位$)$

$F = |F|e^{\mathrm{j}\theta}$ 指数式

三角函数式

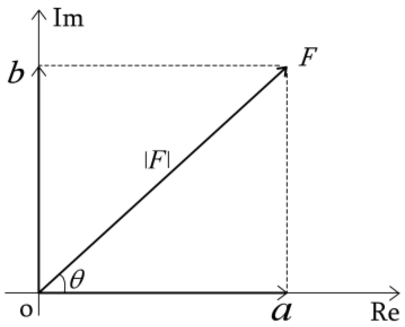
$F = |F|e^{\mathrm{j}\theta} = |F|(\cos\theta + \mathrm{j}\sin\theta) = a + \mathrm{j}b$

$F = |F|e^{\mathrm{j}\theta} = |F|\angle\theta$ 极坐标式

几种表示方法的关系：

$F = a + \mathrm{j}b$

$F = |F|e^{\mathrm{j}\theta} = |F|\angle\theta$



$$\begin{cases} a = |F|\cos\theta \\ b = |F|\sin\theta \end{cases}$$

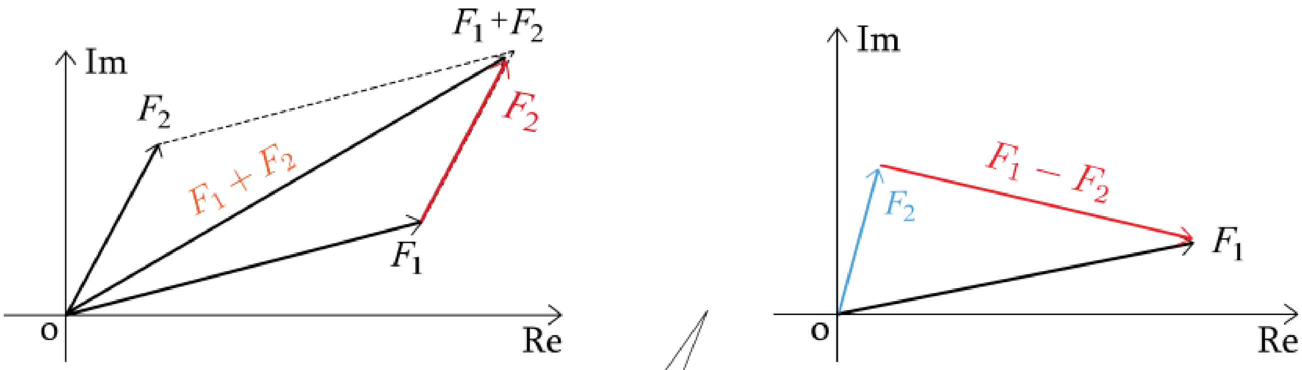
$$\begin{cases} |F| = \sqrt{a^2 + b^2} \\ \theta = \arctan\left(\frac{b}{a}\right) \end{cases}$$

2. 复数运算

①加减运算　采用代数式

若　$F_1 = a_1 + \mathrm{j}b_1,\ F_2 = a_2 + \mathrm{j}b_2$

则　$F_1 \pm F_2 = (a_1 \pm a_2) + \mathrm{j}(b_1 \pm b_2)$



图解法

②乘除运算 —— 采用极坐标式

$$F_1 = |F_1|\underline{/\theta_1},\ F_2 = |F_2|\underline{/\theta_2}$$

$$F_1 \cdot F_2 = |F_1|e^{\mathrm{j}\theta_1} \cdot |F_2|e^{\mathrm{j}\theta_2} = |F_1||F_2|e^{\mathrm{j}(\theta_1+\theta_2)}$$

$$= |F_1||F_2|\underline{/\theta_1 + \theta_2}$$

模相乘
幅角相加

$$\frac{F_1}{F_2} = \frac{|F_1|\underline{/\theta_1}}{|F_2|\underline{/\theta_2}} = \frac{|F_1|e^{\mathrm{j}\theta_1}}{|F_2|e^{\mathrm{j}\theta_2}} = \frac{F_1}{F_2}e^{\mathrm{j}(\theta_1-\theta_2)}$$

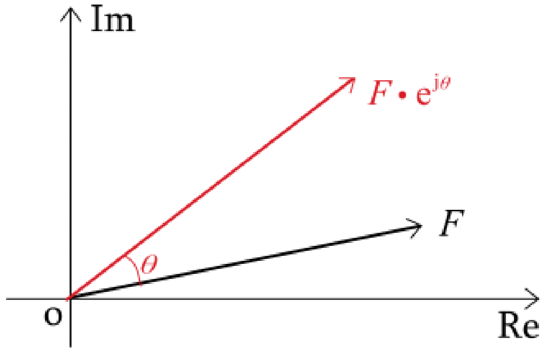
$$= \frac{|F_1|}{|F_2|}\underline{/\theta_1 - \theta_2}$$

模相除
幅角相减

③ 旋转因子

$$e^{j\theta}=\cos\theta+j\sin\theta=1\underline{/\theta}$$

$$F\cdot e^{j\theta}$$

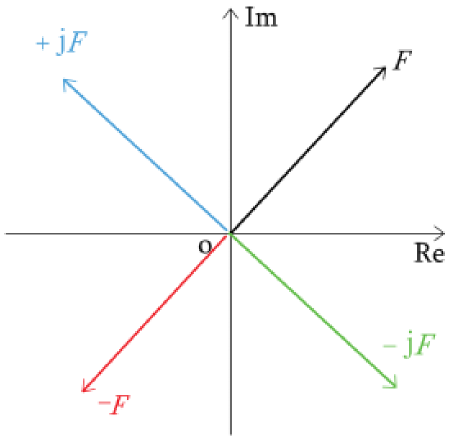


特殊旋转因子

$$\theta=\frac{\pi}{2},\quad e^{j\frac{\pi}{2}}=\cos(\frac{\pi}{2})+j\sin(\frac{\pi}{2})=+j$$

$$\theta=-\frac{\pi}{2},\quad e^{j(-\frac{\pi}{2})}=\cos(-\frac{\pi}{2})+j\sin(-\frac{\pi}{2})=-j$$

$$\theta=\pm\pi,\quad e^{j(\pm\pi)}=\cos(\pm\pi)+j\sin(\pm\pi)=-1$$



所以 $+j,-j,-1$ 都可以看成旋转因子