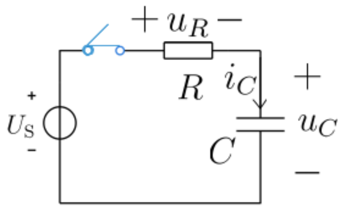
5-8 一阶电路分析的三要素法



$u_C(0_-)=U_0$

$$u_{\mathrm{C}}(t)=U_{\mathrm{S}}+(U_0-U_{\mathrm{S}})\mathrm{e}^{-\frac{1}{RC}t}$$

$$u_{\mathrm{C}}(\infty)=U_{\mathrm{S}}\qquad u_{\mathrm{C}}(0_+)=U_0$$

$$\boxed{u_{\mathrm{C}}(t)=u_{\mathrm{C}}(\infty)+[u_{\mathrm{C}}(0_+)-u_{\mathrm{C}}(\infty)]\,\mathrm{e}^{-\frac{t}{RC}}}$$

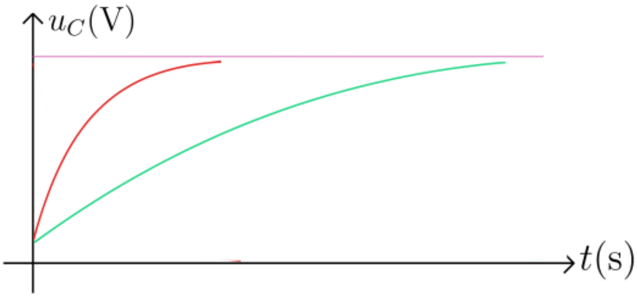
$\tau=RC$ 称为时间常数 表征过渡过程的快慢

τ 越小，过渡过程越快，反之越慢。

$$u_{\mathrm{C}}(t)=\boxed{u_{\mathrm{C}}(\infty)}+[\boxed{u_{\mathrm{C}}(0_+)}-u_{\mathrm{C}}(\infty)]\,\mathrm{e}^{-\frac{t}{\boxed{\tau}}}$$

要素二　要素一　要素三

（稳态值）（初始值）（时间常数）



$$u_{\mathrm{C}}(t)=u_{\mathrm{C}}(\infty)+[u_{\mathrm{C}}(0_+)-u_{\mathrm{C}}(\infty)]\,\mathrm{e}^{-\frac{t}{\tau}}\qquad \tau=RC$$

$$i_{\mathrm{L}}(t)=i_{\mathrm{L}}(\infty)+[i_{\mathrm{L}}(0_+)-i_{\mathrm{L}}(\infty)]\,\mathrm{e}^{-\frac{t}{\tau}}\qquad \tau=\frac{L}{R}$$

对于一阶电路，一般可以不求解微分方程，可直接应用三要素公式求解。

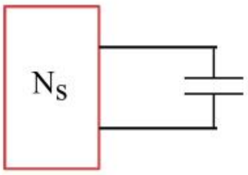
三要素公式 $f(t)=f(\infty)+[f(0_+)-f(\infty)]\,\mathrm{e}^{-\frac{t}{\tau}}$

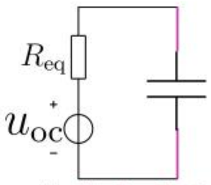
$f(0_+)$为初始值　　$\mathrm{f}(\infty)$为稳态值　　τ为时间常数

对于含电容一阶电路　$\tau=RC$

对于含电感一阶电路　$\tau=\dfrac{L}{R}$

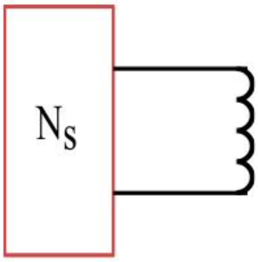
三要素法应用的步骤

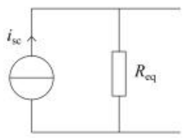




戴维宁等效

稳态时 $u_c(\infty) = u_{oc}$ $\quad \tau = R_{eq}C$





诺顿等效

稳态时 $i_L(\infty) = i_{sc}$ $\quad \tau = \dfrac{L}{R_{eq}}$

第1步：求初始值$u_C(0_+)$或$i_L(0_+)$

根据 $u_C(0_+) = u_C(0_-)$ $\quad i_L(0_+) = i_L(0_-)$

第2步：求稳态值$u_C(\infty)$或$i_L(\infty)$

根据 $u_c(\infty) = u_{oc}$ $\quad i_L(\infty) = i_{sc}$

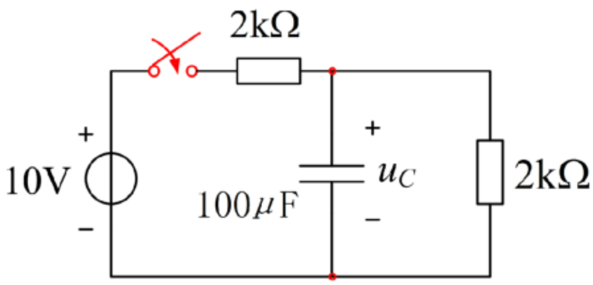
第3步：求时间常数τ

根据 $\tau = R_{eq}C$ $\quad \tau = \dfrac{L}{R_{eq}}$

R_{eq}为戴维宁或诺顿等效电路的等效电阻。

最后：将三个要素代入三要素公式。

例1：



电路原来已达稳态，t=0时开关闭合，求$u_C(t)$。

$$\boxed{u_C(0_+) = u_C(0_-) = 0\text{V}}$$

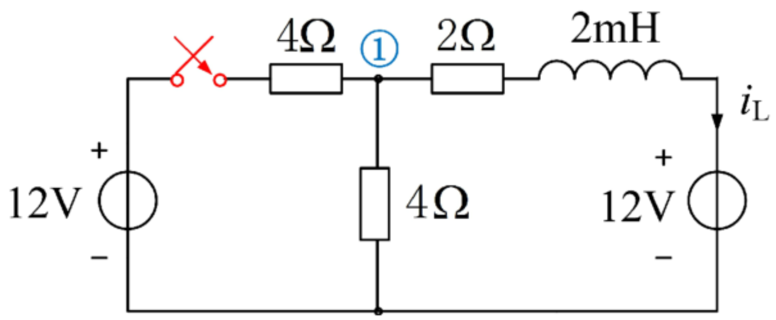
$$\boxed{u_C(\infty) = 10 \times \frac{2}{2+2} = 5\text{V}}$$

$$R_{eq} = 1\text{k}\Omega$$

$$\boxed{\tau = R_{eq}C = 1000 \times 100 \times 10^{-6} = 0.1\text{s}}$$

$$u_C(t) = u_C(\infty) + [u_C(0_+) - u_C(\infty)]\,\mathrm{e}^{-\frac{t}{\tau}} = 5 - 5\mathrm{e}^{-10t}\text{V}$$

例2:



电路原已达到稳态，t=0时开关闭合，求t>0时的$i_L(t)$

$$\boxed{i_L(0_+) = i_L(0_-) = -\frac{12}{2+4} = -2\text{A}}$$

t=∞时,电感相当于短路,

$$R_{eq} = \frac{4\times 4}{4+4} + 2 = 4\Omega$$

$$\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{2}\right)u_{n1} = \frac{12}{4}+\frac{12}{2} \qquad u_{n1} = 9\text{V}$$

$$\boxed{\tau = \frac{L}{R_{eq}} = \frac{2\times 10^{-3}}{4} = 0.5\times 10^{-3}\text{s}}$$

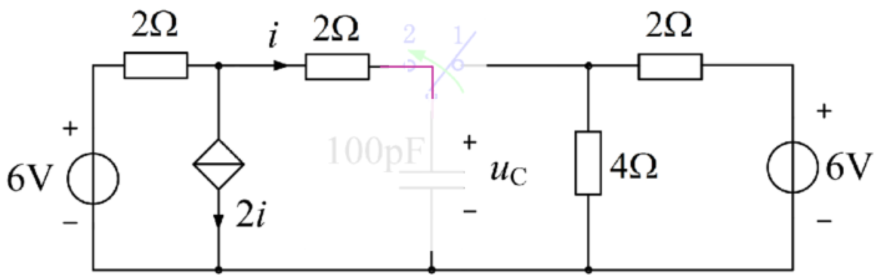
$$\boxed{i_L(\infty) = \frac{u_{n1}-12}{2} = -1.5\text{A}}$$

$$i_L(t) = i_L(\infty) + [i_L(0_+) - i_L(\infty)+]\, e^{-\frac{t}{\tau}} = -1.5 - 0.5e^{-2000t}\text{A}$$

注意：

此题课件视频中有误，电感电流初始值和随时间变化的表达式以课件文档为准（正确）

例3:

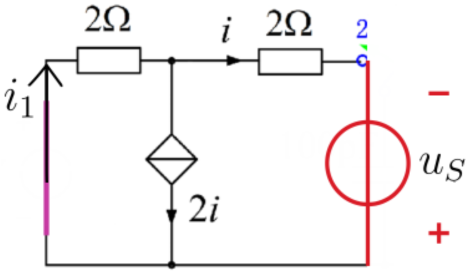


电路原已达稳态，t=0时，开关由1打向2，求$u_C(t)$。

$$u_C(0_+) = u_C(0_-) = \frac{4}{4+2}\times 6 = 4\text{V}$$

t=∞时,电容相当于开路，$i = 0\text{A}$

$$u_c(\infty) = u_{oc} = 6\text{V}$$



根据KCL $i_1 = 2i + i = 3i$

根据KVL $2i_1 + 2i - U_S = 0 \quad U_S = 8i \quad R_{eq} = \frac{U_S}{i} = 8\Omega$

$$\tau = R_{eq}C = 8\times 100\times 10^{-12}\text{s} = 8\times 10^{-10}\text{s}$$

$$u_C(t) = u_C(\infty) + [u_C(0_+) - u_C(\infty)]\,\text{e}^{-\frac{t}{\tau}} = 6 - 2e^{-1.25\times 10^9 t}\text{V}$$

注意：

此题课件视频中有误，电容电压随时间变化的表达式以课件文档为准（正确）。