Motion Estimation
— Optical Flow (II)

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Outline

- Optical Flow (Pixel-level)
  - What is optical flow?
  - Lucas-Kanade algorithm (LK) [2]
  - Horn-Schunck algorithm (HS) [3]

- BMA (Block-level)
  - The principle of BMA
  - Full search scheme
  - Three step search [4]
  - New three step search [5]
  - Four step search [6]
  - Diamond search scheme [7]

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- Optical Flow (Pixel-level)
  - What is optical flow?
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Horn-Schunck Algorithm (HS) [3]

- The basic assumption of HS algorithm — piece-wise smooth flow field
  - But flow field has *discontinuities* at the boundaries of objects in the scene
- Embed the optical flow constraint into a *regularization* framework

$$\boxed{E_{HS}(u,v) = \int_{\Omega} \{(I_x u + I_y v + I_t)^2 + \alpha(|\nabla u|^2 + |\nabla v|^2)\} dxdy}$$

Brightness Constancy **Spatial Coherency**

$$|\nabla u|^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = u_x^2 + u_y^2, \quad |\nabla v|^2 = \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 = v_x^2 + v_y^2$$

2D Euler Lagrange

- 2D Euler Lagrange: the functional

$$S = \iint_{\Omega} L(x, y, f, f_x, f_y)\,dxdy$$

is minimized only if *f* satisfies the partial differential equation

$$\frac{\partial L}{\partial f} - \frac{\partial}{\partial x}\frac{\partial L}{\partial f_x} - \frac{\partial}{\partial y}\frac{\partial L}{\partial f_y} = 0$$

$$L(u, v, u_x, u_y, v_x, v_y) = \left(I_x u + I_y v + I_t\right)^2 + \alpha\left(u_x^2 + u_y^2 + v_x^2 + v_y^2\right)$$

- In Horn-Schunck

$$\frac{\partial L}{\partial u} = 2\left(I_x u + I_y v + I_t\right)I_x$$

$$\frac{\partial L}{\partial u_x} = 2\alpha u_x \qquad \frac{\partial}{\partial x}\frac{\partial L}{\partial u_x} = 2\alpha u_{xx}$$

$$\frac{\partial L}{\partial u_y} = 2\alpha u_y \qquad \frac{\partial}{\partial y}\frac{\partial L}{\partial u_y} = 2\alpha u_{yy}$$

Linear PDE

- The Euler-Lagrange PDE for Horn-Schunck is

$$(I_x u + I_y v + I_t)I_x - \alpha(u_{xx} + u_{yy}) = (I_x u + I_y v + I_t)I_x - \alpha\Delta u = 0$$

$$(I_x u + I_y v + I_t)I_y - \alpha(v_{xx} + v_{yy}) = (I_x u + I_y v + I_t)I_y - \alpha\Delta v = 0$$

- u_{xx} and u_{yy} can be obtained by a Laplacian operator:

$$\Lambda = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

- In the end, we solve a large linear equation

$$\begin{pmatrix} I_x^2 + \alpha\Lambda & I_x I_y \\ I_y I_x & I_y^2 + \alpha\Lambda \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = -\begin{pmatrix} I_x I_t \\ I_y I_t \end{pmatrix}$$

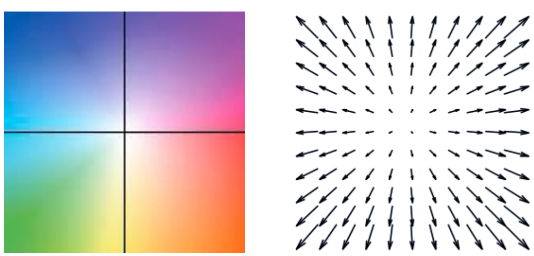
How to solve a large linear system Ax=b?

$$\boxed{\begin{pmatrix} I_x^2 + \alpha\Lambda & I_x I_y \\ I_y I_x & I_y^2 + \alpha\Lambda \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = -\begin{pmatrix} I_x I_t \\ I_y I_t \end{pmatrix}}$$

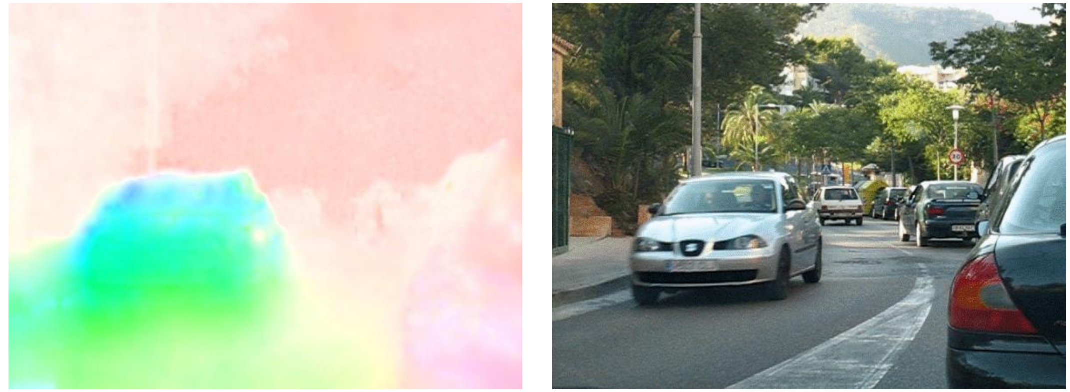
- With $\alpha > 0$, this system is positive definite!
- You can solve with your favorite iterative solver
  - Gauss-Seidel, successive over-relaxation (SOR)

Example

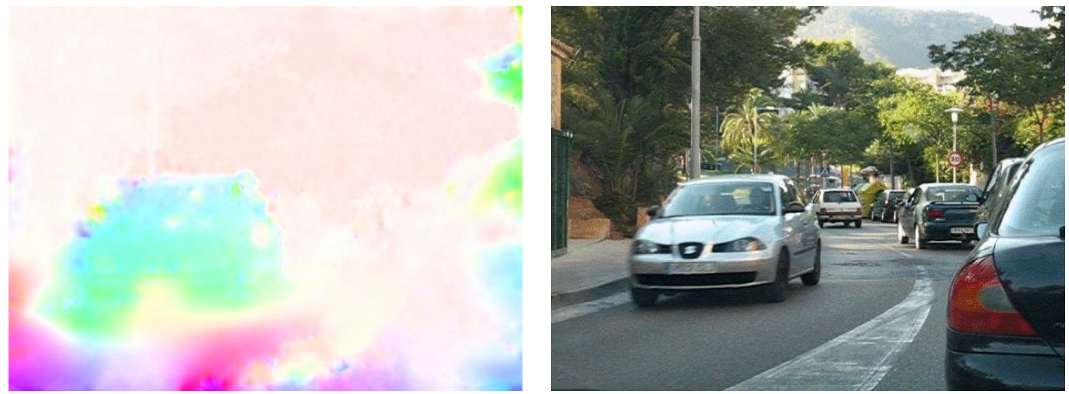
Input two frames



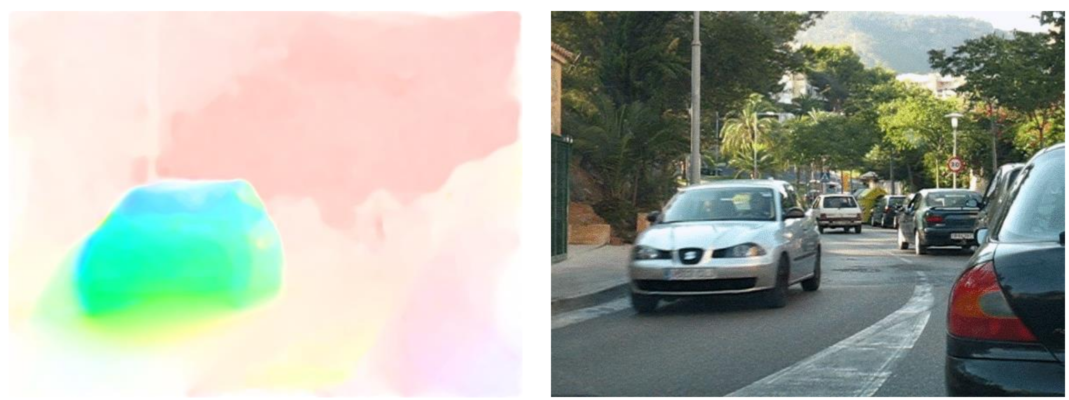
Flow visualization



Horn-Schunck



Coarse-to-fine LK



Coarse-to-fine LK with median filtering

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Thank You!

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