Image Noise and Filtering (III)

Dr. Xiqun Lu

College of Computer Science

Zhejiang University

Nonlinear Total Variation Filtering [1]

- The constrained Minimization Problem

$$\min \int_{\Omega} \sqrt{u_x^2 + u_y^2}\, dxdy$$

- Subject to constraints involving *u*

Conservation of Energy:

$$\int_{\Omega} u\, dxdy = \int_{\Omega} u_0\, dxdy$$

$$\boxed{u_0 = u + n}$$

$$\int_{\Omega} \frac{1}{2}\left(u - u_0\right)^2 dxdy = \sigma^2$$

Euler-Lagrange Equation

$$\boxed{\int_{\Omega}\{(u_x^2+u_y^2)^{1/2}+\lambda_1(u\text{-}u_0)+\lambda_2\frac{1}{2}\left((u-u_0)^2-\sigma^2\right)\}\mathrm{dxdy}}$$

$$\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)+\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right)-\lambda_1-\lambda_2(u-u_0)=0$$

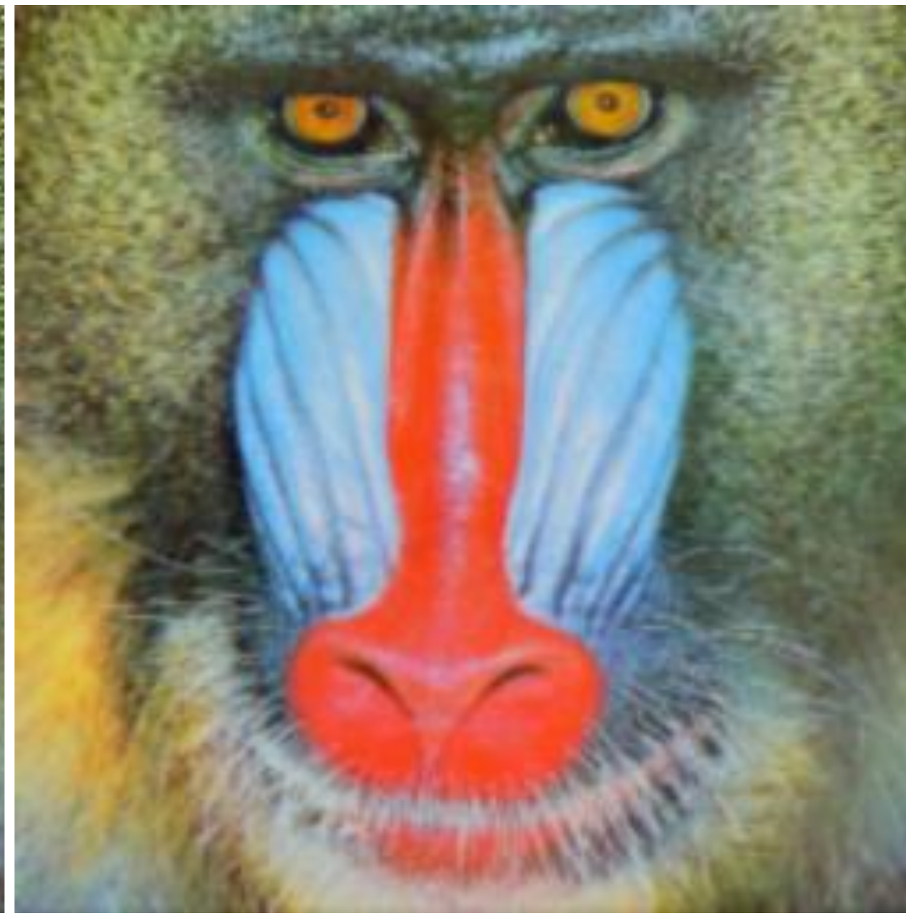
- The gradient descent algorithm

$$u^{(i+1)}=u^{(i)}+dt\left(\frac{u_{xx}}{\sqrt{u_x^2+u_y^2}}+\frac{u_{yy}}{\sqrt{u_x^2+u_y^2}}+\lambda(u^{(0)}-u^{(i)})\right)$$

Started with $u^{(0)} = u_0$

Experimental Results (I)

(a) The input image



(b) The filtered image (iters = 20, dt = 0.01, λ = 0.5)

Experimental Results (II)

(b) The filtered image (iters = 20, dt = 0.01, λ = 0.5)

(c) The filtered image (iters = 500, dt = 0.01, λ = 0.5)

Nonlinear Total Variation [4]

(a) Lena (Gaussian noise mean =0 std = 10)
Iteration number =100, ISNR = 3.1698 dB

(b) Lena (Gaussian noise mean =0 std = 20)
ISNR = 7.9477 dB

References

- [1] L. Rudin, S. Osher and E. Fatemi, “Nonlinear total variation based noise removal,” Physical D, 60:259-268, 1992.

Thank You!

Dr. Xiqun Lu

xqlu@zju.edu.cn