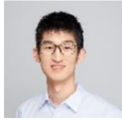
深蓝学院
shenlanxueyuan.com

Foundations of Robotics

Lec 2: Configuration Space



主讲人 滕瀚哲
美国加州大学河滨分校
ARCS实验室博士

Outline

1. Configuration Space
2. Degrees of Freedom
3. Topology and Representation
4. Homework

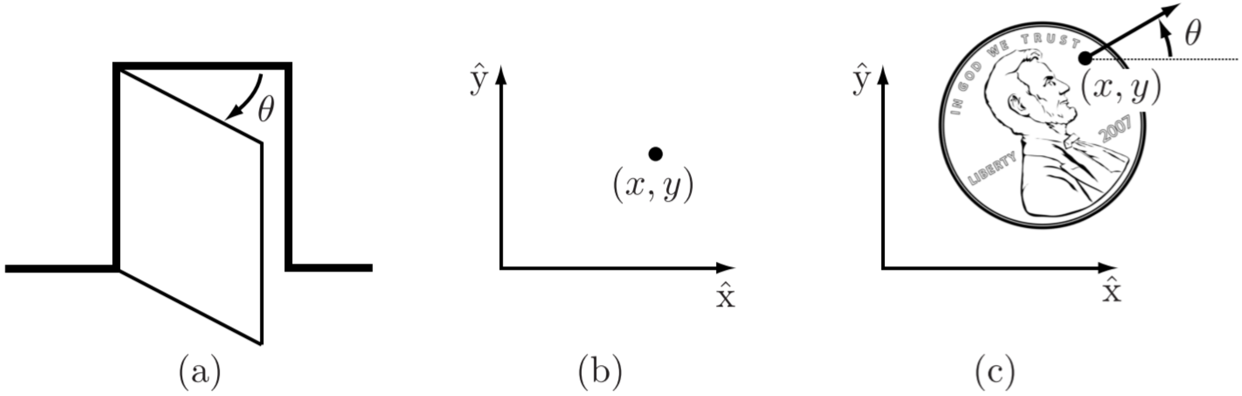
Outline

Configuration Space

A robot is mechanically constructed by **joints** and **links**.

The most fundamental question one may ask: What is the current state of the robot?

Configuration: A complete specification of the position of every point of the robot.

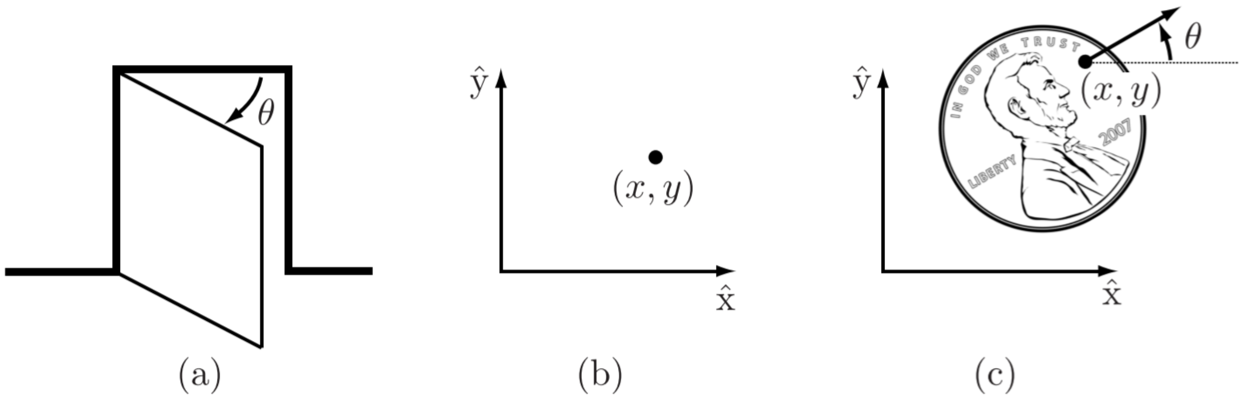


Configuration Space

Configuration: A complete specification of the position of every point of the robot.

Configuration Space (C-space): The n-dimensional space containing all possible configurations of the robot.

➢ The configuration of a robot is represented <u>by a point</u> in its C-space.



Outline

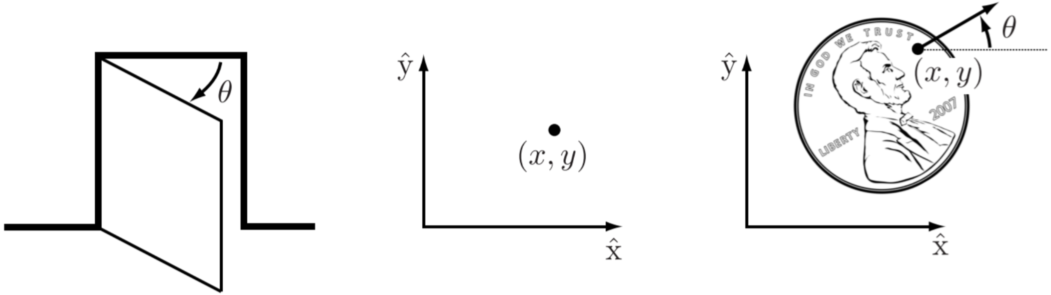
Degrees of Freedom

Configuration: A complete specification of the position of every point of the robot.

Configuration Space (C-space): The n-dimensional space containing all possible configurations of the robot.

Degrees of Freedom (dof): The minimum number n of real-valued coordinates needed to represent the configuration is the number of degrees of freedom (dof) of the robot.

- Alternatively, it is the dimension of the C-space.

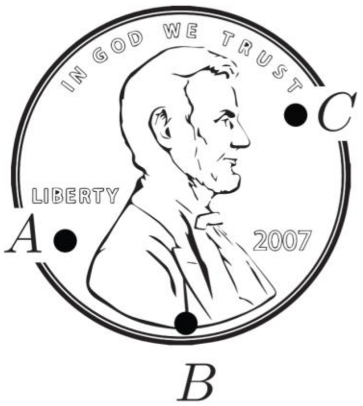


(a) (b) (c)

Degrees of Freedom

What is the dof for general rigid bodies in 2D space?

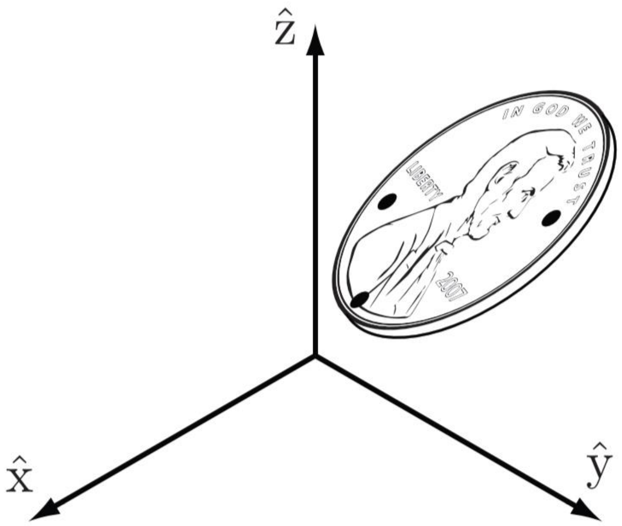
- Let's investigate this by <u>picking points</u> on rigid bodies.



Point	Coordinates	Independent Constraints	Real DoF Introduced
A	2	0	2
B	2	1	1
C	2	2	0
D	2	2	0
Total	**-**	**-**	**3**

Degrees of Freedom

What is the dof for general rigid bodies in 3D space?



Point	Coordinates	Independent Constraints	Real DoF Introduced
A	3	0	3
B	3	1	2
C	3	2	1
D	3	3	0
Total	**-**	**-**	**6**

Degrees of Freedom

We have been applying the following general rule for determining the number of degrees of freedom of a system:

degrees of freedom = (sum of freedoms of the points) –
(number of independent constraints)

Alternatively:

degrees of freedom = (number of variables) –
(number of independent equations)

A better solution for general robots consisting of many rigid bodies:

degrees of freedom = (sum of freedoms of the bodies) –
(number of independent constraints)

→ Grübler’s Formula!

Grübler's Formula

Grübler's Formula

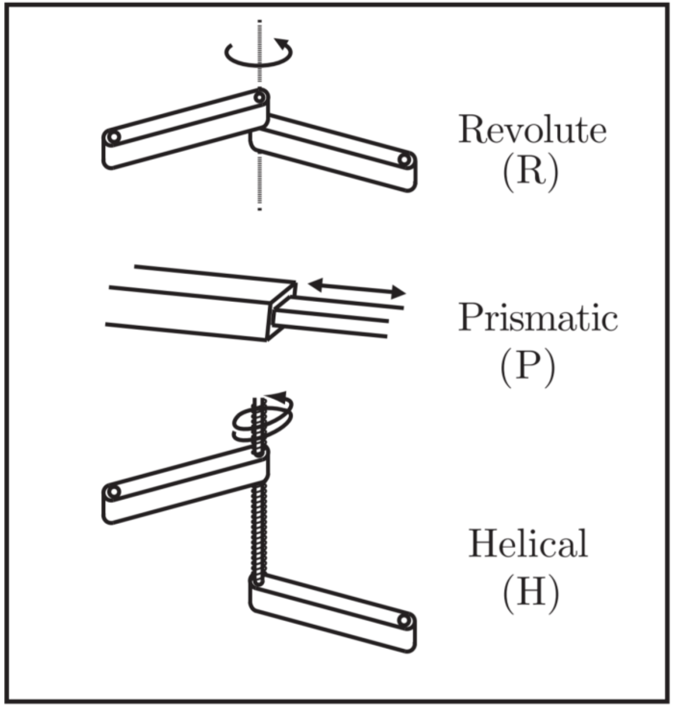
degrees of freedom = (sum of freedoms of the bodies) –
(number of independent constraints)

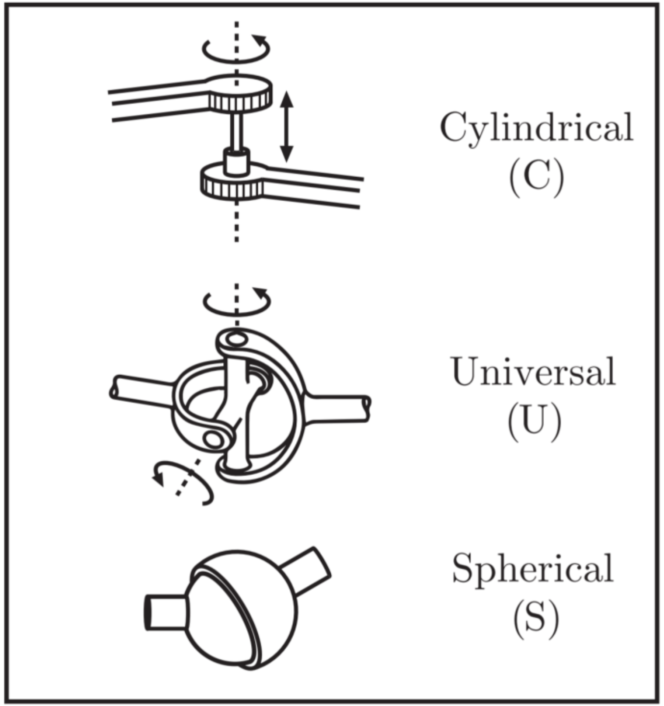
The main idea of applying Grübler's Formula:

(1) identify <u>links</u> and <u>joints</u> in the system,
(2) compare them with our known joint-link systems, and
(3) plug into the formula accordingly.

Note: we use the terms "links" and "bodies" interchangeably.

Typical Joints





Typical Joints

Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

Grübler's Formula

Recall that

dof = (sum of freedoms of the bodies) – (number of independent constraints)

$$dof = m(N-1) - \sum_{i=1}^{J} c_i$$

$$= m(N-1) - \sum_{i=1}^{J} (m - f_i)$$

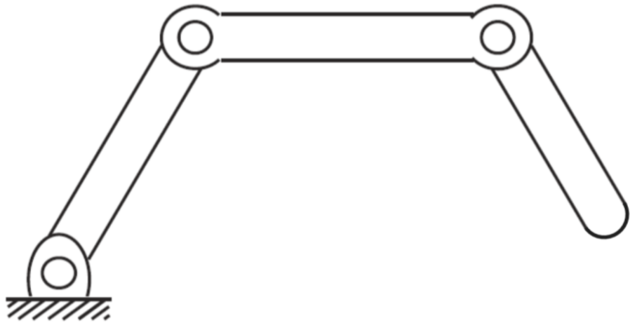
$$\boxed{= m(N-1-J) + \sum_{i=1}^{J} f_i}$$

Grübler's Formula!

Notation	Definition
N	number of bodies, including ground
J	number of joints
m	dof of a rigid body (m=3 for planar bodies, m=6 for spatial bodies)
c_i	number of constraints introduced by joint i
f_i	number of freedoms introduced by joint i

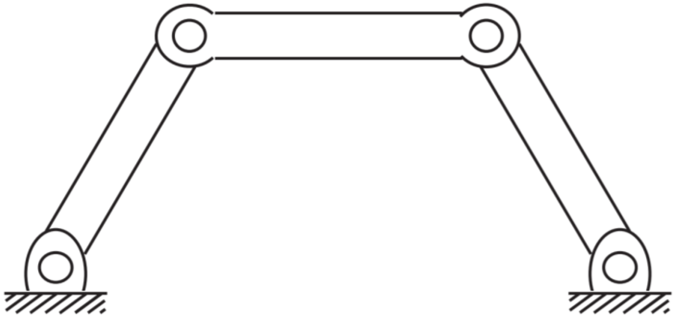
Grübler's Formula

$$dof = m(N - 1 - J) + \sum_{i=1}^{J} f_i$$



3R serial "open-chain" robot arm

$$dof = 3(4 - 1 - 3) + 3 = 3$$

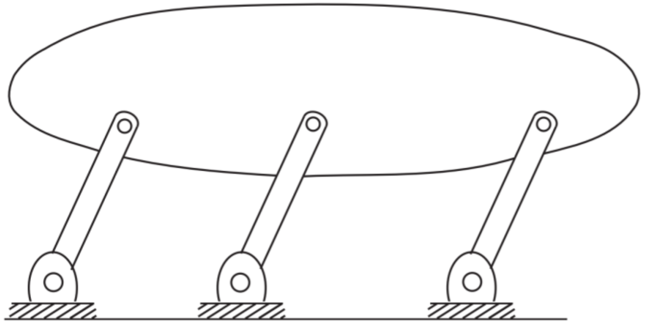


Four-bar "closed-chain" mechanism

$$dof = 3(4 - 1 - 4) + 4 = 1$$

Grübler's Formula

$$dof = m(N - 1 - J) + \sum_{i=1}^{J} f_i$$



A parallelogram linkage

$$dof = 3(5 - 1 - 6) + 6 = 0?$$

No, the dof should be 1.
Grübler's Formula does not apply.

Notes on Grübler's Formula

- The formula holds only if all joint constraints are <u>independent</u>, and a <u>linear</u> analysis of mobility suffices.
  - ➢ Alternatively, you may remove redundant links and compute again.

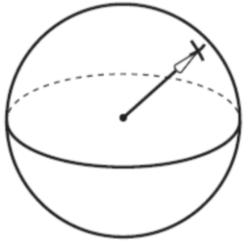
- In other words, it holds in most generic cases, but a mechanism may have <u>more</u> dof or <u>fewer</u> dof in some special cases.

Outline

Configuration Space Topology

- Until now we have been focusing on one important aspect of a robot's C-space: its dimension, or the number of degrees of freedom.
- However, the shape of the space is also important.

A point moving on a plane



A point moving on the surface of a sphere

- Both are two dimensional, but they do not have the same shape.
  - ➢ The plane extends infinitely while the sphere wraps around.

Configuration Space Topology

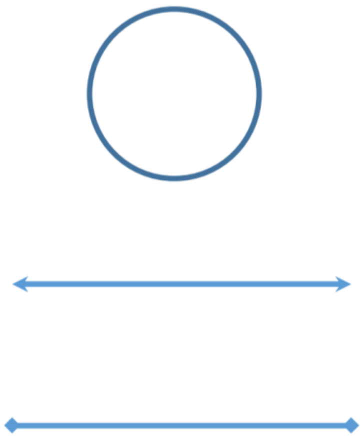
- Until now we have been focusing on one important aspect of a robot's C-space: its dimension, or the number of degrees of freedom.
- However, the shape of the space is also important.
- Formally, it is described by the topology of the space.

Two spaces are **topologically equivalent** if one can be continuously deformed into the other without cutting or gluing.

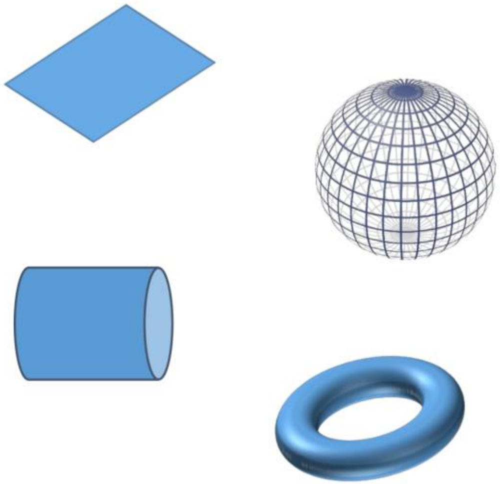
Configuration Space Topology

- Topologically distinct: not topologically equivalent

Topologically-distinct
one-dimensional spaces

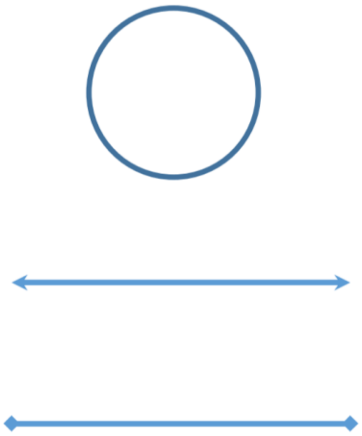


Topologically-distinct
two-dimensional spaces

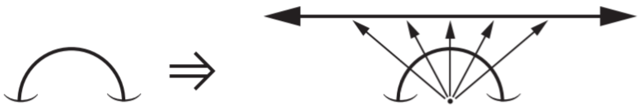


Configuration Space Topology

- Topologically-distinct one-dimensional spaces



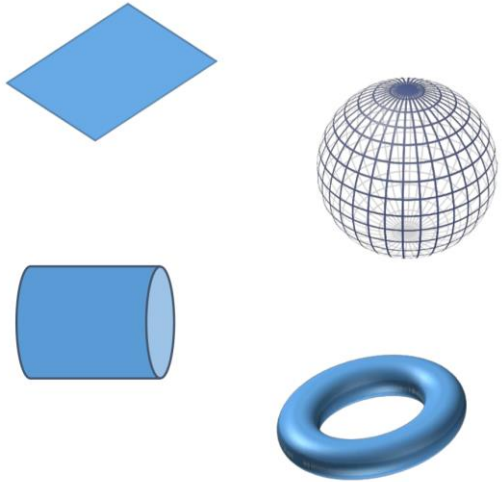
Notation	Meaning	1D Topology
S or S^1	circle	circle
$\mathbb{E}$ or $\mathbb{E}^1$	1D Euclidean space	line
$\mathbb{R}$ or $\mathbb{R}^1$	real number	line
$[a, b] \subset \mathbb{R}^1$	closed interval	closed interval
$(a, b) \subset \mathbb{R}^1$	open interval	line*



*An open interval can be continuously deformed into a line.

Configuration Space Topology

- Topologically-distinct two-dimensional spaces



Notation	2D Topology
$\mathbb{E}^2$ or $\mathbb{R}^2$	plane
S^2	sphere
$T^2 = S^1 \times S^1$	torus*
$\mathbb{E}^1 \times S^1$	cylinder*

*Some C-spaces can be expressed as the Cartesian product of two or more spaces of lower dimension.

Configuration Space Representation

- To perform computations, we must have a numerical representation of the space, consisting of a set of real numbers.
  - We need to choose the type of representation.
  - The topology of a space is independent of our choice of the representation; it is a fundamental property of the space itself.

- Explicit representation
  - A choice of n coordinates or parameters
  - Example: n-vector for Euclidean spaces, e.g., $(x, y, z) \in \mathbb{R}^3$
  - Example: latitude-longitude representation for the surface of a sphere

- Implicit representation
  - Uses the coordinates of the higher-dimensional space, but subjects to constraints
  - Example: loop-closure equations for a closed-chain mechanism
  - Example: $x^2 + y^2 + z^2 = 1$ for the surface of a sphere

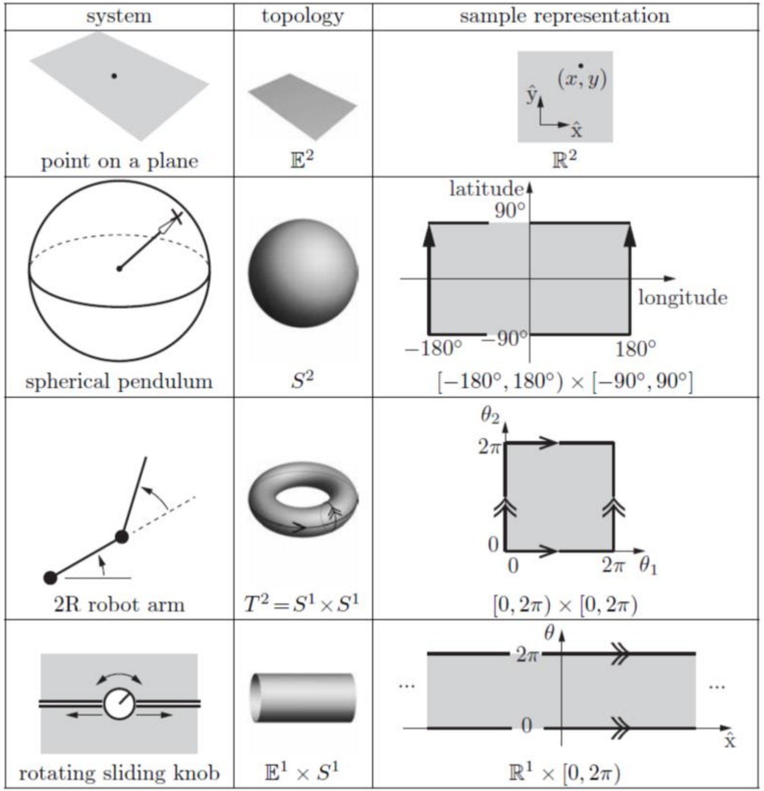
Configuration Space Representation

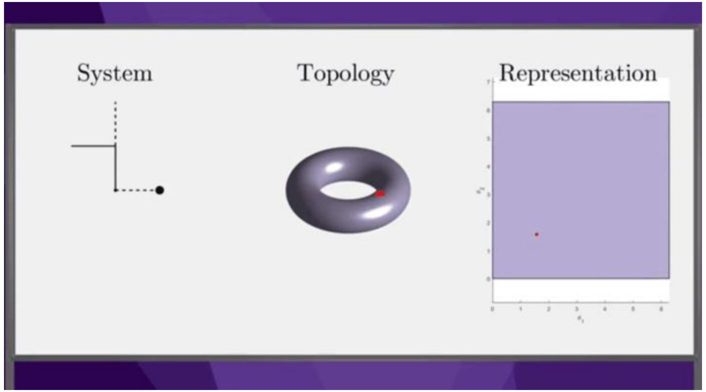
Note: using explicit representation for non-Euclidean spaces can cause singularities!

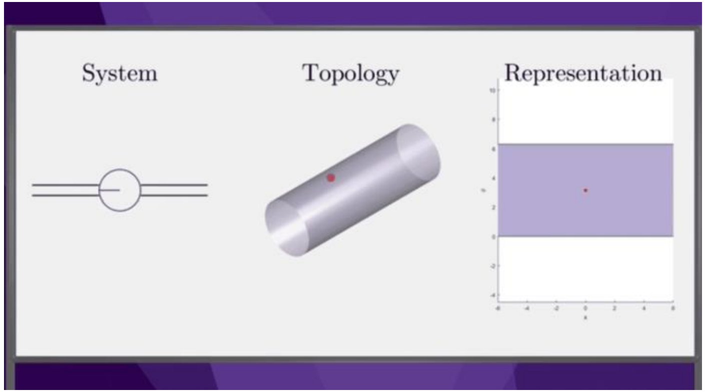
- Example: the latitude-longitude representation of a 2D unit sphere embedded in a 3D space, i.e. $[-180°, 180°) \times [-90°, 90°]$, can have singularities at the North and South Poles, due to the fact that a sphere does not have the same topology as a plane.
- Example: See the following representations for the space of 3D rotations.

Representation	Explicit/Implicit	Singularity	# of Parameters	# of Constraints
Roll-pitch-yaw angles	Explicit	Yes	3	0
Euler angles	Explicit	Yes	3	0
Exponential coordinates	Explicit	Yes	3	0
Axis-angle	Implicit	Yes	4	1
Unit quaternions	Implicit	No	4	1
Rotation matrices	Implicit	No	9	6

Configuration Space Representation





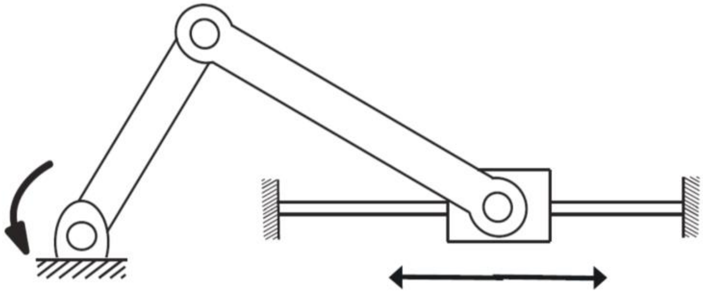


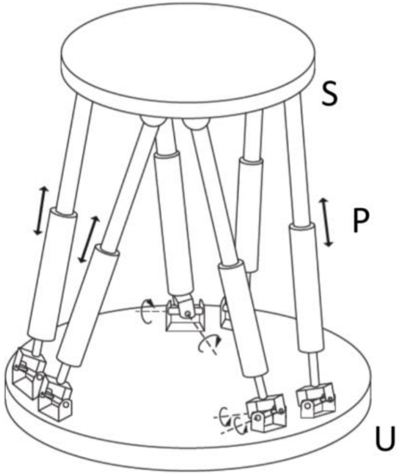
[1] https://www.youtube.com/playlist?list=PLggLP4f-rq02vX0OQQ5vrCxbJrzamYDfx

Outline

Homework

(1) Find the degrees of freedom of the following mechanisms





(2) Textbook Exercises: 2.1, 2.9(a-b), 2.11(a-b), 2.29

(3) Lab Assignments: Open-loop control of the Turtlebot robot

- In Gazebo simulation, write a script using ROS (in Python or C++) to make the robot run along the sides of a square.

Thanks for Listening！