Q1

Given R1(L,M,N,O,P,Q,R,S,T)
FDs = { LRP->Q,LR->ST,M->LO,MR->N}

a):

Closure of LPR = LPRQST which are not all, then LRP->Q violates BCNF.
Closure of LR = LRST which are not all, then LR->ST violates BCNF.
Closure of M = MLO which are not all, then M->LO violates BCNF.
Closure of MR = MRN which are not all, then MR->N violates BCNF.

b):

Closure of LPR = LPRQST.
LPR->Q violates BCNF for R1
Split R1 into
R2(L,P,R,Q,S,T) {LPR->Q,LR->ST}
R3(L,M,N,O,P,R) {M->LO,MR->N}

Closure of LR = LRST
LR->ST violates BCNF for R2
Split R2 into
R4(L,R,S,T) {LR->ST}
R5(L,P,R,Q) {LPR->Q}

Closure of M = MLO
M->LO violates BCNF for R3
Split R3 into
R6(M,L,O) {M->LO}
R7(M,N,P,R) {MR->N}

Closure of MR = MRNLO
MR->N violates BCNF for R7
Split R7 into
R8(M,R,N) {MR->N}
R9(M,P,R) {empty}

Final decomposition
R8(M,N,R) {MR->N}
R9(M,P,R) {empty}
R6(L,M,O) {M->LO}
R5(L,P,Q,R) {LPR->Q}
R4(L,R,S,T) {LR->ST}

Q2

a): Find minimal basis

Step 1: rewrite FD such that there is only one attribute on RHS.
AB->C
AB->D
ACDE->B
ACDE->F
B->A
B->C
B->D
CD->A
CD->F
CDE->F
CDE->G
EB->D

Step 2: minimize LHS
B->C
B->D
CDE->B
CD->F
B->A
B->C
B->D
CD->A
CD->F
CD->F
CDE->G
B->D

Step 3: remove redundant FDs and get minimal basis
CDE->B
B->C
CD->A
CD->F
CDE->G
B->D

b): Find all keys

Step 1: find attributes that are not on RHS of minimal basis
EH (All key should contain EH)

Step 2: find attributes that are on RHS but not on LHS
AFG (No key contain AFG)

Step 3: find keys
Closure of EHB = ABCDEFGH which is key
Closure of EHC = EHC which is not key
Closure of EHD = EHD which is not key
Closure of EHCD = ABCDEFGH which is key
Keys: EHB, EHCD

c): Employ decomposition to find 3NF relations

Recall revised minimal basis:
CDE->B
B->CD
CD->AF
CDE->G

For each FD:X->Y in minimal basis, define new relation with schema (XY)
R1(CDEB)
R2(BCD)
R3(CDAF)
R4(CDEG)

Since BCD in BCDE, get new relations
R1(CDEB)
R2(CDAF)
R3(CDEG)

Since no relation is a superkey for R, add a relation whose schema is some key
R1(CDEB)
R2(CDAF)
R3(CDEG)
R4(HB)

d):

Since closure of B = BCD, then B->CD violates BCNF for R1(CDEB).
Then allow redundancy.