

For relation $LMNOPQRST$

$$W = \{LRP \rightarrow Q, LR \rightarrow ST, M \rightarrow LO, MR \rightarrow N\}$$

First from these FDs we know that M, R and P must be in the key since they only appear on the left side of the FD.

$$MRP^+ = \{M, R, P, L, O, Q, N, S, T\}$$

The key is MRP.

a. The FDs that violate BCNF are

All of them.

b. Employ BCNF decomposition until the relations are lossless, non-redundant

We start with $LRP \rightarrow Q$.

$$LRP^+ = \{L, R, P, Q, S, T\}$$

So $R_1 = (L, R, P, Q, S, T)$ and $R_2 = (L, M, N, O, P, R)$

After decomposition, the FDs projected will be $LPR \rightarrow Q$ and $LR \rightarrow ST$. The rest of the FD will stay since they have M on the LHS.

R_1 does not satisfy the BCNF since $LR \rightarrow ST$ and LR is not the super-key.

Decompose R_1 via $LR \rightarrow ST$

$$LR^+ = \{L, R, S, T\}$$

So $R_1 = (L, R, S, T)$ $R_3 = (L, R, P, Q)$ and both satisfies BCNF since only one FD is projected on each.

Onto R_2 , we decompose via $MR \rightarrow N$

$$MR^+ = \{M, R, N, L, O\}$$

So $R_2 = (M, R, N, L, O)$ $R_4 = (M, P, R)$

No FD is projected onto R_4 since MRP is only on the left side of FDs.

All FD goes to R_2

R_2 does not satisfy BCNF because $M \rightarrow LO$

Decompose R_2 via $M \rightarrow LO$

$$M^+ = \{M, L, O\}$$

So $R_2 = (M, L, O)$ and $R_5 = (M, R, N)$

Again, the FD projects to R_2 is $M \rightarrow LO$ and $R_4 MR \rightarrow N$.

Both satisfy BCNF.

In conclusion,

$$R_1 = (L, P, Q, R, S, T) \quad R_2 = (L, M, O) \quad R_3 = (L, P, Q, R) \quad R_4 = (M, P, R) \quad R_5 = (M, N, R)$$