AMS-528 Numerical Analysis (III)

Final Exam: take-home assignment

Reports in electronic form should be sent to roman.samulyak@stonybrook.edu before 11:30 am on Monday, May 19.

The in-class exam is on May 19, 2014, 11:45 - 1:45 PM, in our classroom.

Consider the nonlinear conservation law

$$u_t + (u^4)_x = 0$$

(a). Find the Riemann solution and design the Godunov scheme.

(b). Analyze the CFL condition.

(c). Given the initial condition

$$u(x,0) = \begin{cases} \sin^2 \pi(x-1) & \text{if } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

find the numerical solution in an appropriate domain range from $t = 0$ to $t = 6$ using the Godunov scheme with exact Rieman solution.

(d). Design a second order scheme using the flux limiter method and solve for the same initial condition. Compare solutions obtained with:

(1) Roe's limiter (upper boundary of the 2nd order TVD region),

(2) van Leer's limiter,

(3) limiter coinsiding with the lower boundary of the 2nd order TVD region, ($\phi(\theta) = \theta$ if $0 \le \theta \le 1$, and $\phi(\theta) = 1$ if $\theta > 1$)

(4) limiter coinsiding with the upper boundary of the TVD region, ($\phi(\theta) = 2\theta$ if $0 \le \theta \le 1$, and $\phi(\theta) = 2$ if $\theta > 1$). In all cases, $\phi(\theta) = 0$ if $\theta < 0$.