AMS528 final Submission

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1 prob. a

We solve the Riemann problem under the initial condition. $u(x,0) = \left\{ \begin{array}{cc} \text{-1} & \text{x} \leq 0 \\ 1 & \text{x>0} \end{array} \right\}$

Then we will have solution in this form $u(x,t) = \left\{ \begin{array}{cc} \text{-1} & \text{x} \leq \text{4t} \\ \frac{x}{4t} & -4t < x < 4t \\ 1 & \text{x}\geq\text{4t} \end{array} \right\}$

It is rarefaction fan solution.

If we have $u(x,0) = \left\{ \begin{array}{cc} 1 & \text{x} \leq 0 \\ \text{-1} & \text{x>0} \end{array} \right\}$

Then we will have solution $u(x,t) = \left\{ \begin{array}{cc} 1 & \text{x} \leq 0 \\ \text{-1} & \text{x>0} \end{array} \right\}$

It is shock.

Godunov scheme is to define piecewise constant initial condition $\hat{u}(x,t_n)$ with the value U_j^non the grid cell $x_{j-1/2} < x < x_{j+1/2}$.And we can totally solve it in the interval $[t_n, t_{n+1}]$ by using conservation law.Then averaging the exact solution at t_{n+1},we get another sequence of Riemann problem and we continue doing the procedure.

Because we integrate along x=j+1/2 and j-1/2 while u is a constant at the two points from t_n to t_{n+1},we can define flux function according to R-H condition. $F(u_l,u_r) = \left\{ \begin{array}{cc} f(u_l) & if \frac{f(u_r)-f(u_l)}{u_r-u_l} \geq 0 \\ f(u_r) & if \frac{f(u_r)-f(u_l)}{u_r-u_l} < 0 \end{array} \right\}$

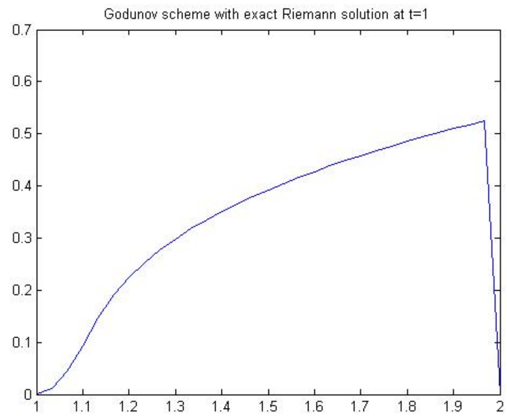
2 prob. b

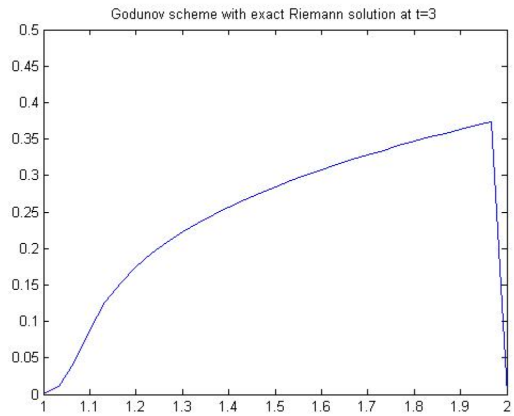
For the stability requirement, we have that $|\frac{\Delta t f'(x)}{\Delta x} \leq 1|$ where$f'(x) = 4u^3$ and $u_{max} = 1$. And it is also the requiremrnt for CFL,then we have $\frac{\Delta t}{\Delta x} \leq \frac{1}{4}$

3 prob. c

Here I choose $\Delta x = \frac{1}{30}, \frac{\Delta t}{\Delta x} = \frac{1}{5}$, due to the stability requirement.

Furthermore, because the value at x=1 and x=2 are 0,we only consider the domain [1,2].

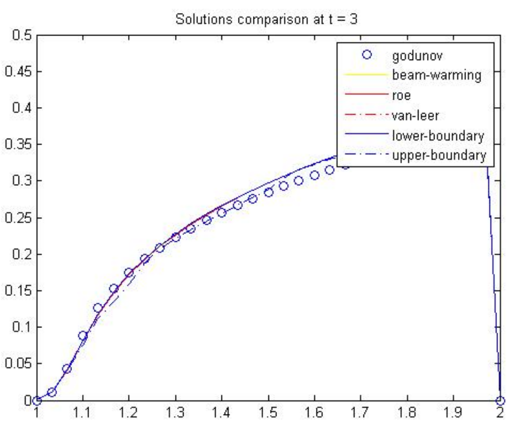


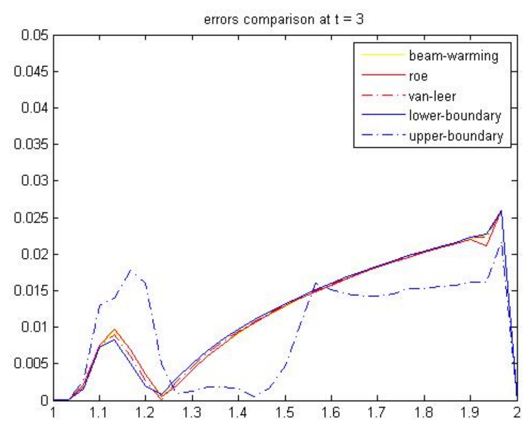


From the graphs, we see that the initial value moves fast to right ,because at t=1,the initial sine function has deformed to the above left graph.
And then,the value in the domain damps but the shape remains.Value near x=2 is 0.5 at t=1 and 0.35 at t=3.

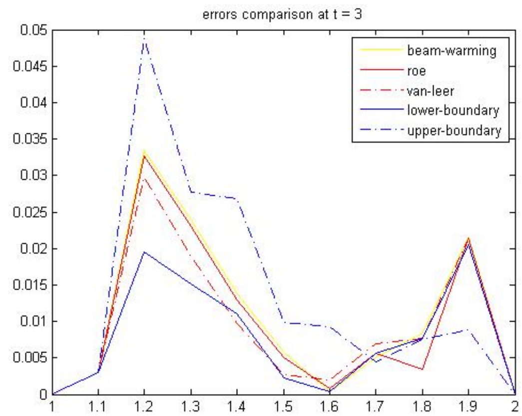
4 prob.d

I choose Beam-Warming method as another flux limiter method and the comparison for different methods is given below.





We can see that at t=3. All these methods almost perform the same.
From the error graph,we see that the largest error appears near x=2. However, the error for upper-boundary method has a smooth distribution rather than concentrate near x=2.



When using $\Delta x = \frac{1}{10}$,we see that upper-boundary method has a large near x=1 and lower-boundary method does not give a large error in the whole domain.