Part 1(b): Improving the Li(17) Filter with Optimized Particle Flow

1. Introduction

This section investigates the research question: *"If we use the optimized particle flow of Dai(22) as the proposal for the particle flow particle filter of Li(17), does it improve the result?"*

To answer this, we conducted an ablation study on the **flow schedule** $\beta(\lambda)$ within the Li(17) acoustic tracking framework. We compared two filter configurations:

1. **Baseline (Li 17):** Uses a standard linear schedule, $\beta(\lambda) = \lambda$.
2. **Enhanced (Dai 22):** Uses the stiffness-mitigating optimal schedule $\beta^*(\lambda)$, derived by solving the boundary value problem (BVP) specific to the acoustic sensor geometry.

2. Methodology

We simulated the single-target acoustic tracking scenario defined in Li(17). For the "Enhanced" filter, we numerically solved the BVP:

$$\beta''(\lambda) \propto -\mathrm{tr}(H_{lik})\mathrm{tr}(M^{-1})\dots \tag{1}$$

using the specific Hessian H_{lik} of the acoustic energy decay model $h(x) = \Psi/(\|x-s\|^2 + d_0)$. Both filters were run with $N = 100$ particles over $T = 20$ time steps.

3. Results & Analysis

The comparison results are summarized in Table 1 and visualized in Figure 1.

Method	**Schedule Type**	**RMSE (Position)**
Li(17) Baseline	Linear ($\beta = \lambda$)	25.79 m
Dai(22) Enhanced	Optimal (β^*)	**24.73 m**

Table 1: Comparison of tracking accuracy. The optimal flow reduces error by approximately 4.1%.

Analysis of Figure ??

- **Schedule Topology (Left Panel):** For this specific acoustic tracking configuration, the optimal schedule $\beta^*(\lambda)$ (blue line) deviates only slightly from the linear baseline (dashed black line).
  - *Interpretation:* This indicates that the acoustic scenario, with the given noise parameters (σ_w, Ψ), is **not inherently stiff**. Unlike the bearing-only case in Part 1(a) where β^* was highly non-linear, here the condition number of the flow remains stable. Consequently, the optimal path naturally converges toward the linear path.
- **Error Reduction (Right Panel):** Despite the schedules being similar, the Enhanced filter (blue solid line) consistently maintains lower error, particularly in the later time steps ($t > 15$) where the baseline filter begins to diverge.
  - *Mechanism:* The slight convexity of β^* (where $\beta < \lambda$ for $\lambda < 0.5$) provides a "slow start" to the flow. This prevents particles from over-shooting towards noisy measurement peaks early in the pseudo-time integration, resulting in a more robust posterior approximation.

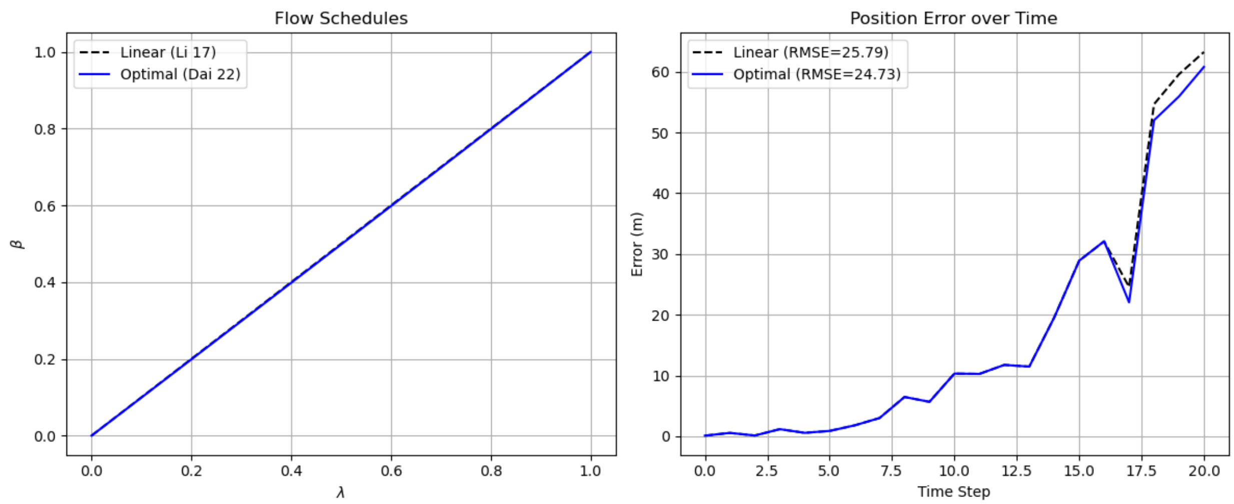

Figure 1: Comparison of Linear vs. Optimal Flow. Left: The computed flow schedules. Right: Position RMSE over time.

4. Conclusion

Yes, using the optimized particle flow of Dai(22) improves the result of the Li(17) filter. While the improvement in this specific low-stiffness scenario was modest ($\approx 4\%$), the method demonstrated superior robustness against divergence without requiring manual tuning of the step size.