MTH013

《Calculus for Science and Engineering》

Module Teacher: Yun Lu

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Office: MB343C

Office Hours: 15:00-17:00 Tuesday

Teaching Arrangements

- Lecture 1: Tuesday 9:00-11:00 FB195
- Lecture 2: Friday 15:00-17:00 FB195

网络平台 LMO

- XJTLU-LMO

  -Module Handbook 课程手册

  -Teaching Plan 教学计划

  -Lecture Slides 课件

  -Solutions to Exercise Booklet 习题解答

  -Reading Materials 课外阅读材料

Module Assessments 课程考核

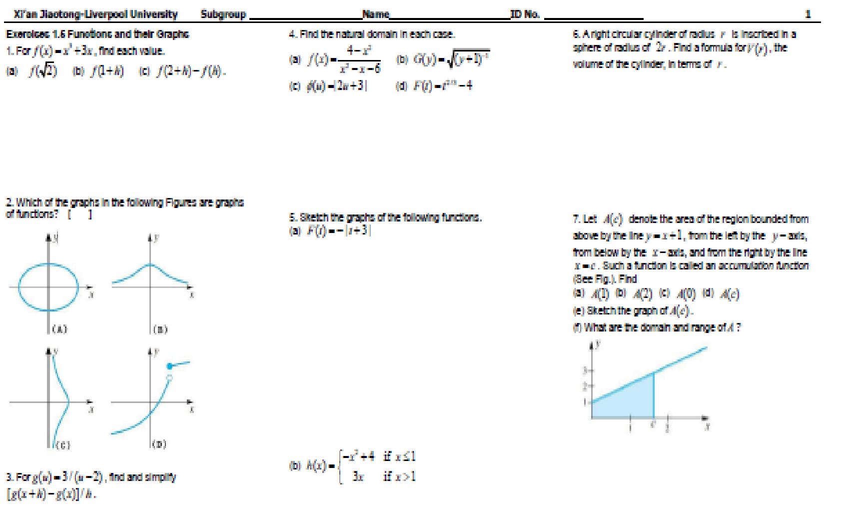
- In-Semester Exam (15%, Week 7)
  期中考试占15%
- Course Work (10%, weekly homework and two online tests in week 4, week 11)
  平时成绩占10%
- Final Examination (75%)
  期末考试占75%

Pass >= 40

Earn 5 Credits

Exercise Booklet (Weekly Homework)

- Submitted by assignment links in LMO
- Deadline: 23:00 Monday
- Solutions release time: Tuesday
- Contribution to final marks (5%)



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Exercises 1.6 Functions and their Graphs

1. For $f(x)=x^3+3x$, find each value.
(a) $f(\sqrt{2})$ (b) $f(1+h)$ (c) $f(2+h)-f(h)$.

2. Which of the graphs in the following Figures are graphs of functions? [ ]

(A) (B) (C) (D)

3. For $g(u)=3/(u-2)$, find and simplify $[g(x+h)-g(x)]/h$.

4. Find the natural domain in each case.
(a) $f(x)=\dfrac{4-x^2}{x^2-x-6}$ (b) $G(y)=\sqrt{(y+1)^{-1}}$
(c) $\phi(u)=|2u+3|$ (d) $F(t)=t^{2/3}-4$

5. Sketch the graphs of the following functions.
(a) $F(t)=-|t+3|$
(b) $h(x)=\begin{cases}-x^2+4 & \text{if } x\le 1\\ 3x & \text{if } x>1\end{cases}$

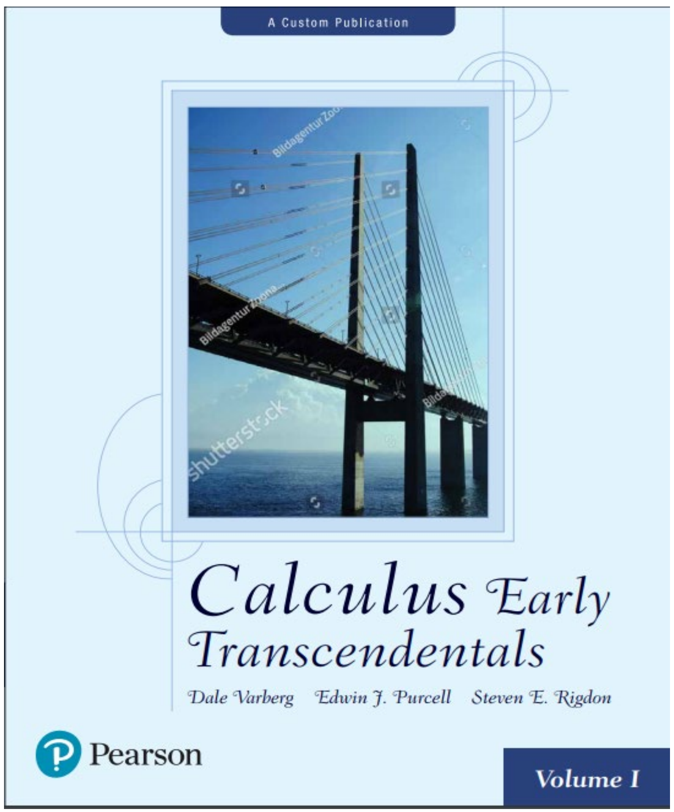
6. A right circular cylinder of radius r is inscribed in a sphere of radius of $2r$. Find a formula for $V(r)$, the volume of the cylinder, in terms of r.

7. Let $A(c)$ denote the area of the region bounded from above by the line $y=x+1$, from the left by the $y-$axis, from below by the $x-$axis, and from the right by the line $x=c$. Such a function is called an accumulation function (See Fig.). Find
(a) $A(1)$ (b) $A(2)$ (c) $A(0)$ (d) $A(c)$
(e) Sketch the graph of $A(c)$.
(f) What are the domain and range of A?

W2	W3	W5	W6	W7	W9	W10	W11	W12	W13
P1-3	P4-6	P7-9	P10-11	P12-14	P17-20	P21-22	P23-26	P27-29	P30-33

Textbook 教材

- 《Calculus Early Transcendentals》 D. Varberg etc.
- 《微积分》，上下册，同济大学数学系



Learning Suggestions

Change Learning Behavior 转变学习方式

Passive Learning → Active Learning

被动学习→主动学习

自觉的转变自己的学习方法和学习习惯作为开始阶段的第一要务

- Focus on deep understanding of concepts, methods and conclusions

  注重数学概念、方法和结论的深入理解

- For detail in a step-by-step of operation and reasoning

  运算和推理的步骤

- Expression of mathematical language

  数学语言的规范表达

- 阅读教材
- 数学语言
- 专业词汇
- 课前预习
- 学会记笔记
- 及时复习
- 认真完成作业
- 及时解惑

Chapter 1 Preliminaries

- 1.5 Functions and Their Graphs
- 1.6 Operations on Functions
- 1.7 Exponential and Logarithmic Functions
- 1.8 The Trigonometric Functions
- 1.9 The Inverse Trigonometric Functions

1.5 Functions and Their Graphs

Definition of Functions (函数的定义):

Let D and R be nonempty sets (非空集合). A function from D to R is a rule (法则) f of correspondence that assigns a unique element $y \in R$ to each element $x \in D$.

(1) x is called the independent variable (自变量) and y the dependent variable (因变量), and .

(2) We say "y is a function of x" and symbolically:

$$y = f(x) \quad x \in D.$$

Function Notation

- A single letter like f (or g or F) is used to name a function. Then $f(x)$, read "f of x" or "f at x", denotes the value that f assigns to x.

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Domain and Range

The domains frequently appeared are intervals（区间）such as
$[a,b],(a,b),(a,\infty),\cdots$

- The set D is called the domain(定义域) of the function and denoted by $D(f)$ or D_f .
- The set of all the values that y can take is called the range(值域) of the function and symbolically $R(f)$, or R_f , that is,

$$R(f)=\{y\colon\ y=f(x),x\in D(f)\}$$

- To specify （说明）a function completely, we must state,
  (1) The rule of correspondence (对应法则),
  (2) The domain of the function (定义域).

two key elements of a function（函数的两个要素）.

- When no domain is specified for a function, we assume that it is the largest set of real numbers for which the rule for the function makes sense. This is called the natural domain (自然定义域).

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Graphs of Functions

The graph of $y = f(x),\ x \in [a,b]$ is the set in the plane $\mathbb{R}^2$:

$$\{(x,y) : y = f(x),\ x \in [a,b]\}.$$

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Even Functions and Odd Functions: Symmetry of graph

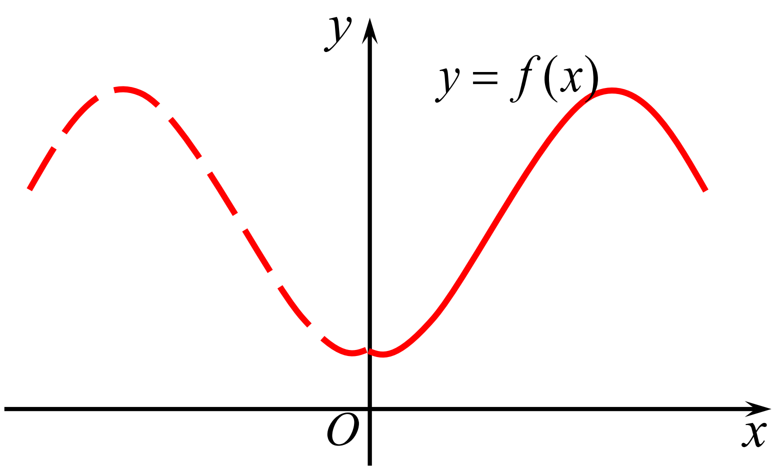
Domain D is symmetric with respect to the origin.

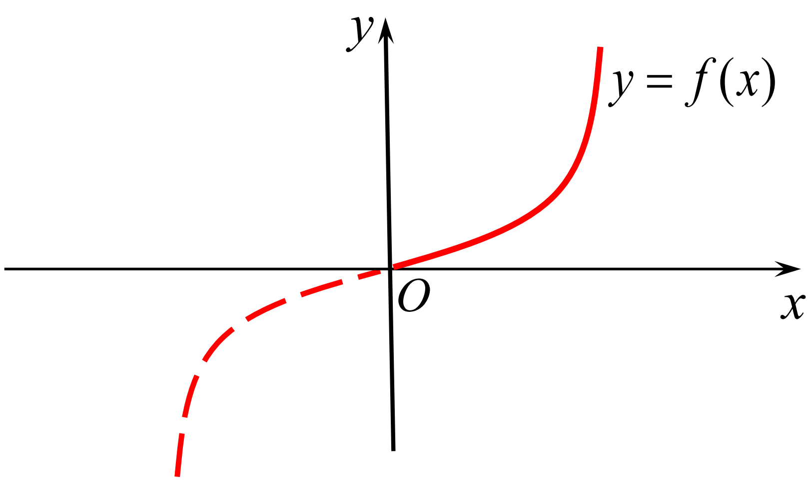
Even function of x if $f(-x) = f(x)$, $x \in D$.

Odd function of x if $f(-x) = -f(x)$, $x \in D$.

The graph of an even function is symmetric about the y-axis.

The graph of an odd function is symmetric about the origin.





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Monotonic Functions 单调函数

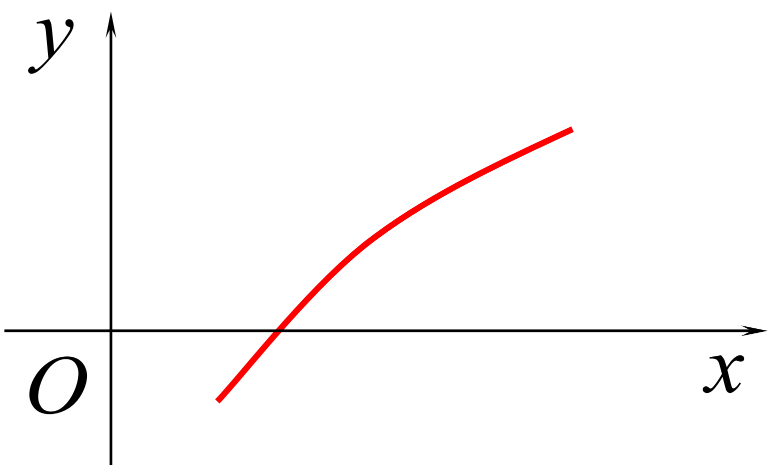
The function $f(x)$ is defined on some interval I,

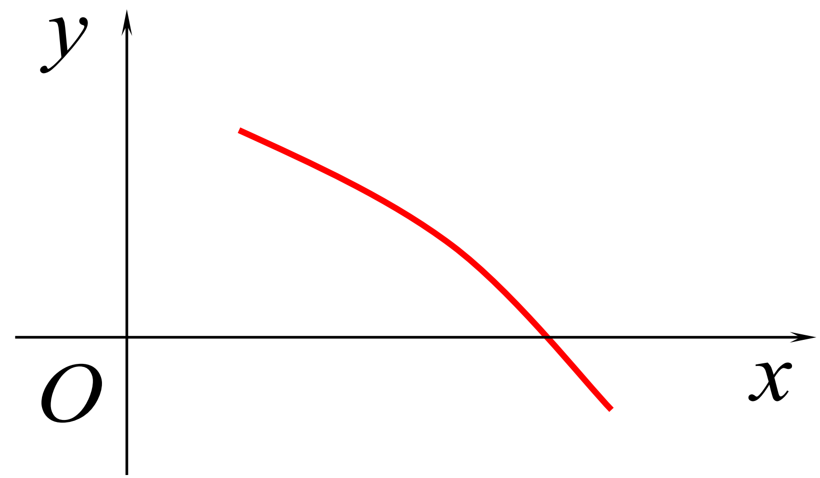
增函数

f is increasing on I if $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

减函数

f is decreasing on I if $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



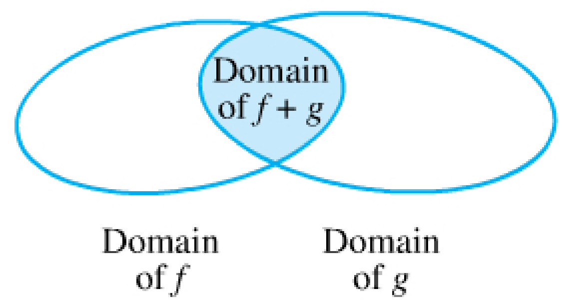


1.6 Operations on Functions（函数的运算）

(1) Sums(和), Differences(差), Products(积), Quotients(商), and Powers(幂)

Given two functions f and g, D is the intersection(交) of the two domains of functions f and g. We define

$$(f+g)(x)=f(x)+g(x) \quad x\in D$$



$$(f-g)(x)=f(x)-g(x) \quad x\in D$$

$$(f\cdot g)(x)=f(x)\cdot g(x) \quad x\in D$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)} \quad x\in D \text{ and } g(x)\neq 0$$

$$f^n(x)=[f(x)]^n,\ x\in D(f)$$

(2) Composition of Functions 复合函数

Suppose that $f(x)$ and $g(x)$ be two functions.

$$g \circ f(x) := g(f(x))$$

is called the composition of g with f.

Note the domain of the composite function is :

$$D(g \circ f) = \{x \mid x \in D(f),\ f(x) \in D(g)\}$$

$D(g \circ f)$ need not be the same as $D(f)$, usually, $D(g \circ f) \subseteq D(f)$。

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Decomposition of a given function 函数的分解

Complex Example: Decompose the following function as the composition of some simple functions.

$$y = \ln\left(\sin\frac{1}{\sqrt{x^2+1}}\right)$$

$$\Rightarrow y = \ln u,\ u = \sin v,\ v = t^{-\frac{1}{2}},\ t = x^2+1$$

(3) Inverse Functions 反函数

Difinition (one-to-one function 一一对应函数)

A function is said to be one-to-one (1-1) if distinct values of x always lead to distinct values of $y = f(x)$; that is

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2). \text{ (i.e.,} f(x_1) = f(x_2) \Rightarrow x_1 = x_2.\text{)}$$

If a function f is one-to-one, then we can define the inverse of f.

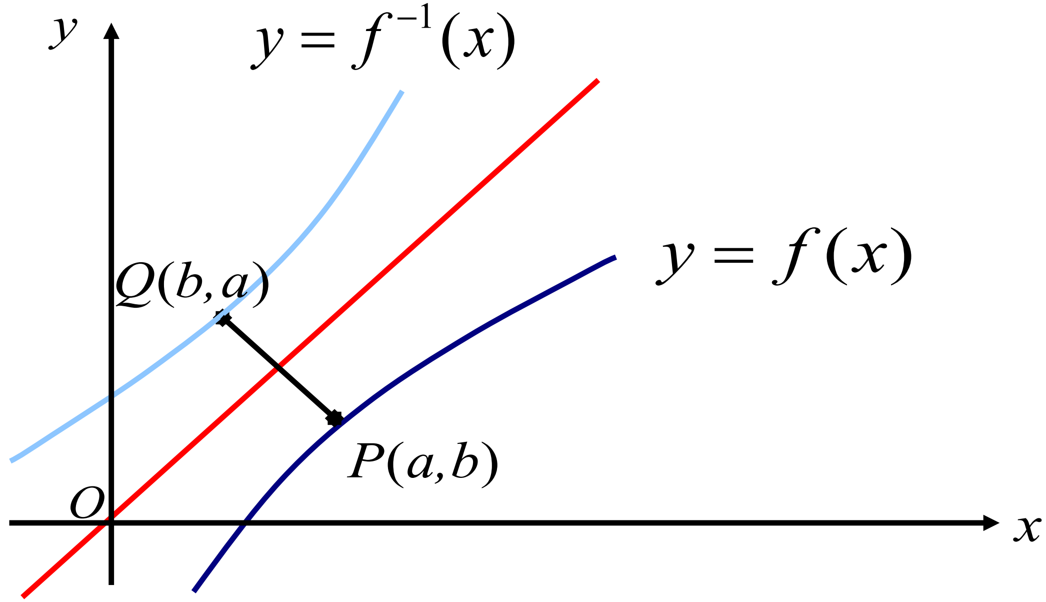
If the function f is one to one, then the function (denoted by f^{-1}) that assigns x to each y in $R(f)$ as follows is called the inverse of f :

$$\text{If } y = f(x), \text{ then } x = f^{-1}(y).$$

The inverse function have the domain $D(f^{-1}) = R(f)$ and the range $R(f^{-1}) = D(f)$. And $f(f^{-1}(x)) = x$; $f^{-1}(f(x)) = x$.

Warning: $f^{-1}(x) \neq 1/f(x)$, that is, $f^{-1}(x) \neq [f(x)]^{-1}$

The Graph of Inverse



The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are symmetric about the line $y = x$.

A sufficient condition (充分条件) for the Existence of Inverse Function **(Theorem A, *P*.40):**

> If f is a monotonic function on its domain, then f has an inverse. And f and f^{-1} have the same kind of monotonicity.

1.7 Exponential and Logarithmic Functions
（指数函数和对数函数）

Exponential Function(指数函数) $f(x) = a^x$, $a > 0$ and $a \neq 1$.

Domain : $(-\infty, +\infty)$; Range : $(0, +\infty)$.

Theorem A Properties of Exponents (指数运算性质)

If $a > 0$, $b > 0$ and x and y are real numbers, then

(1) $a^x a^y = a^{x+y}$ (2) $\dfrac{a^x}{a^y} = a^{x-y}$

(3) $(a^x)^y = a^{xy}$ (4) $a^{-x} = \dfrac{1}{a^x}$

(5) $(ab)^x = a^x b^x$ (6) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Logarithmic Function (对数函数)

If $a > 0$ and $a \neq 1$, we define $y = \log_a x$ to be the inverse of the function $x = a^y$; that is, $y = \log_a x \Leftrightarrow x = a^y$.

Domain: $(0, +\infty)$; Range: $(-\infty, +\infty)$

Theorem B Properties of Logarithms (对数运算性质)

If a, b, and c are positive numbers, where $a \neq 1$, and if x is any real number, then

(1) $\log_a 1 = 0$

(2) $\log_a bc = \log_a b + \log_a c$

(3) $\log_a \dfrac{b}{c} = \log_a b - \log_a c$

(4) $\log_a b^x = x \log_a b$

Other Identities $\log_a a = 1$, $\log_a a^b = b$, $a^{\log_a b} = b$.

Base changing formula(换底公式) for logarithms

$$\boxed{\log_a b = \frac{\log_c b}{\log_c a}, \quad c > 0, c \neq 1.}$$

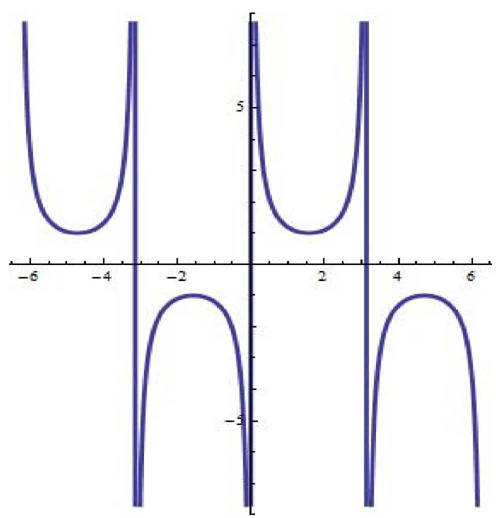
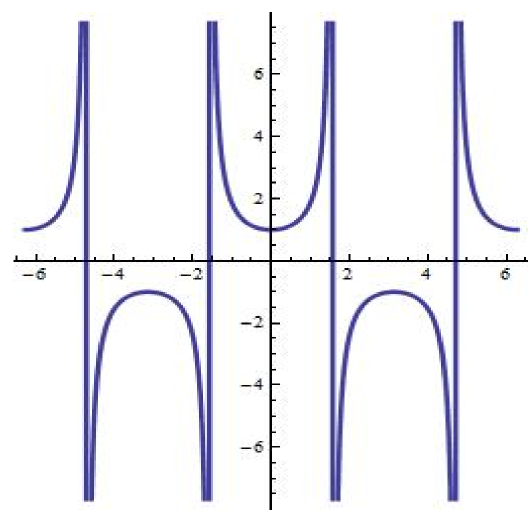
1.8 The Trigonometric Functions (三角函数）

function	domain	period	monotonic interval	odd/even
$\sin t$	R	2π	$[-\pi/2\,,\ \pi/2\,]$	odd
$\cos t$	R	2π	$[\,0\,,\pi\,]$	even
$\tan t$	$R\setminus\left(k\pi+\pi/2\right)$	π	$(-\pi/2\,,\ \pi/2\,)$	odd

(t is in radian 弧度)　　（单调区间）

$$\tan t=\frac{\sin t}{\cos t}\text{（正切）}\qquad \cot t=\frac{\cos t}{\sin t}\text{（余切）}$$

$$\sec t=\frac{1}{\cos t}\text{（正割）}\qquad \csc t=\frac{1}{\sin t}\text{（余割）}$$

Function	Cosecant(余割） function	Secant(正割) function
Notation	$y = \csc x = \dfrac{1}{\sin x}$	$y = \sec x = \dfrac{1}{\cos x}$
Domain	$R \setminus \{n\pi \mid n \in Z\}$	$R \setminus \{n\pi + \dfrac{\pi}{2} \mid n \in Z\}$
Range	$R \setminus (-1,1)$	$R \setminus (-1,1)$
Graph of Function		

Odd–even identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Cofunction identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Addition identities

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Double-angle identities

$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

Half-angle identities

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

Sum identities

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

Product identities

$$\sin x \sin y = -\tfrac{1}{2}[\cos(x + y) - \cos(x - y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x + y) + \cos(x - y)]$$

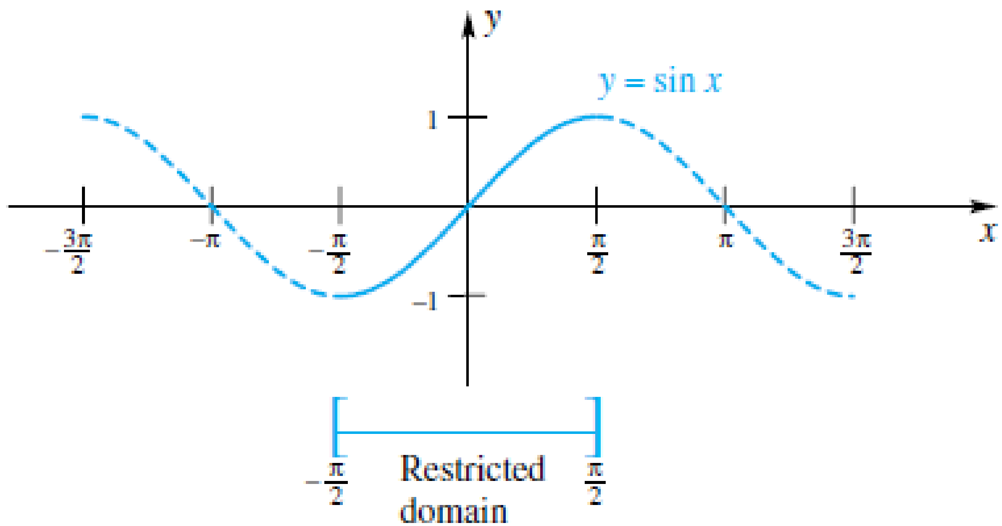
$$\sin x \cos y = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)]$$

1.9 The Inverse Trigonometric Functions (反三角函数)

The function $y = \sin x \ \ (x \in (-\infty, \infty))$ has no its inverse.

If we restrict its domains to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Then the function

$y = \sin x \ \ (x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right])$ has an inverse $x = \sin^{-1} y \ \ (y \in [-1, 1])$.



$$x = \sin^{-1} y \iff y = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

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1. Inverse for sine: $y = \sin^{-1} x \ \ (x \in [-1, 1])$

or $y = \arcsin x \ \ (x \in [-1, 1])$

Warning:

$\sin^{-1} x \neq (\sin x)^{-1}$, $\sin^{-1} x \neq \dfrac{1}{\sin x}$

(1) Domain: $[-1, 1]$

(2) Range: $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

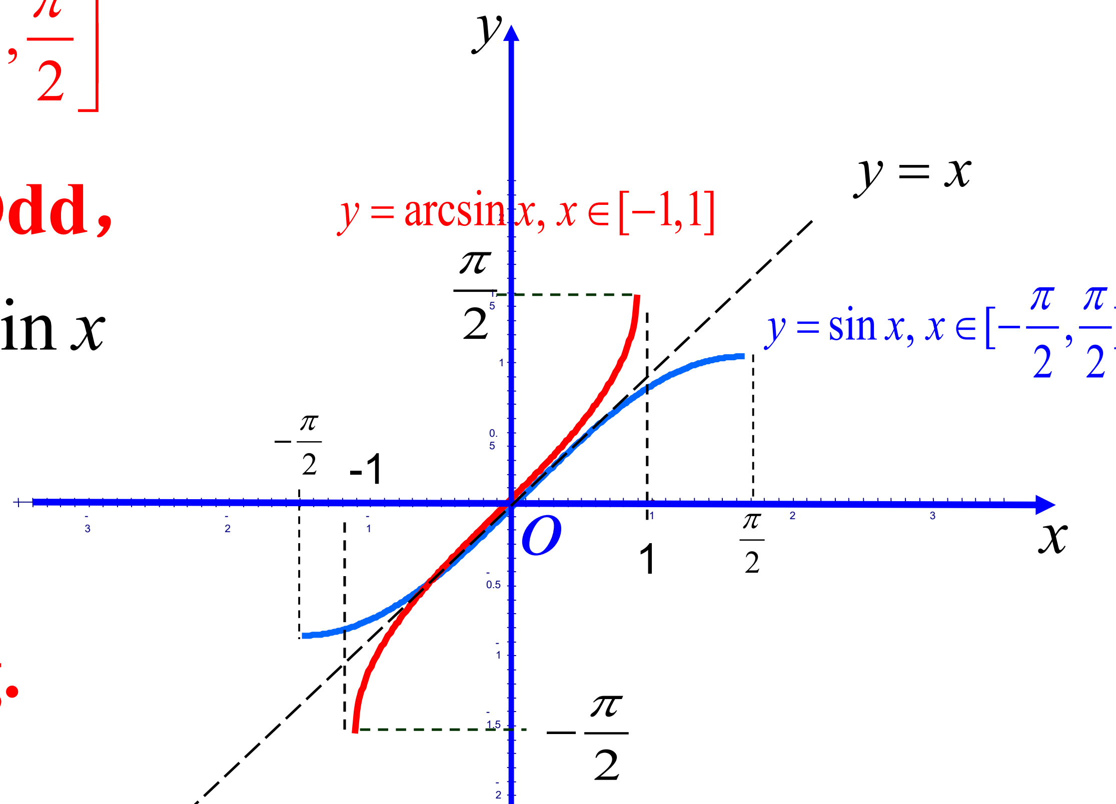
(3) Odd/even: **Odd,**

$\arcsin(-x) = -\arcsin x$

$x \in [-1, 1]$.

(4) Monotonicity :

Increasing.



$\sin^{-1} x$ 表示一个$[-\dfrac{\pi}{2}, \dfrac{\pi}{2}]$内的角 y，这个角的正弦等于x.

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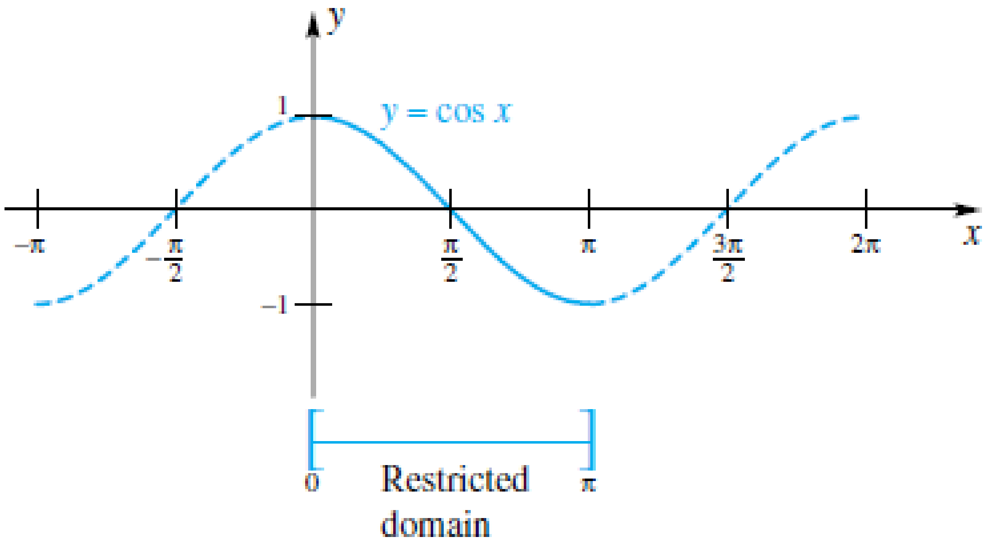
To obtain inverses for cosine, it is necessary to restrict its domains to $[0, \pi]$.

The function $y = \cos x \ \ (x \in [0, \pi])$ has an inverse

$$x = \cos^{-1} y \ \ (y \in [-1, 1]).$$



$$x = \cos^{-1} y \ \Leftrightarrow \ y = \cos x, \ 0 \le x \le \pi$$

2. Inverse for cosine: $y=\cos^{-1}x \ \ (x\in[-1,1])$

or $y=\arccos x \ \ (x\in[-1,1])$

Warning

$\cos^{-1}x\neq(\cos x)^{-1}$, $\cos^{-1}x\neq\dfrac{1}{\cos x}$

(1) Domain: $[-1,\ 1]$

(2) Range: $[0,\pi]$

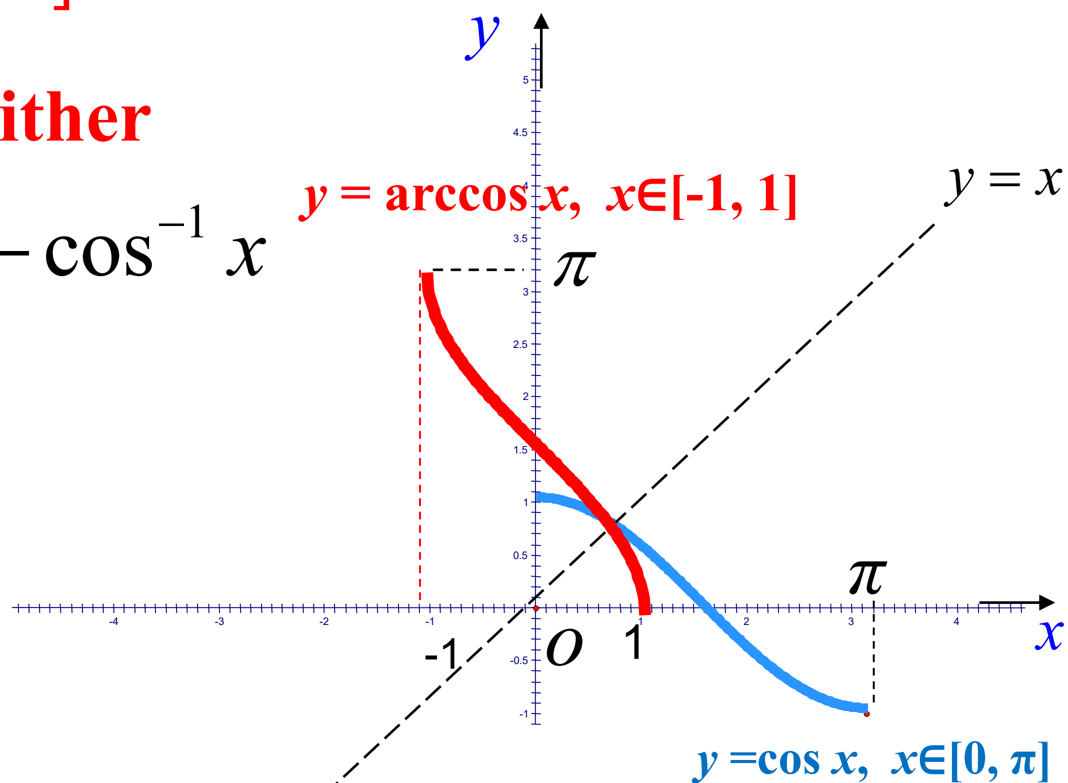
(3) Odd/even: **Neither**

$$\cos^{-1}(-x)=\pi-\cos^{-1}x$$

$$x\in[-1,1].$$

(4) Monotonicity :

Decreasing.



$\cos^{-1}x$ 表示一个$[0,\pi]$内的角y，这个角的余弦等于x.

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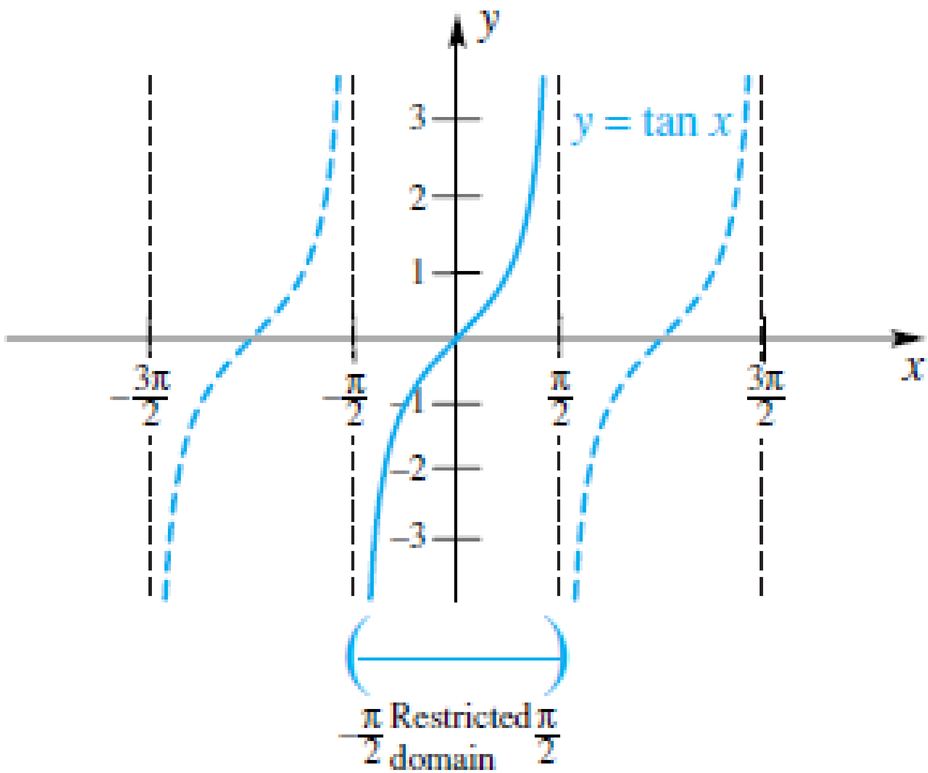
To obtain inverses for tangent, it is necessary to restrict its domains to $(-\frac{\pi}{2},\frac{\pi}{2})$.

The function $y=\tan x\ \ (x\in(-\frac{\pi}{2},\frac{\pi}{2}))$ has an inverse

$$x=\tan^{-1}y\ \ (y\in(-\infty,\infty))\quad \text{or}\quad y=\tan^{-1}x\ \ (x\in(-\infty,\infty)).$$



$$x=\tan^{-1}y \Leftrightarrow$$

$$y=\tan x,\ -\frac{\pi}{2}<x<\frac{\pi}{2}$$

3. Inverse for tangent

$$y=\tan^{-1}x \quad x\in R$$

$$(\text{or, } y=\arctan x\ )$$

Warning

$$\tan^{-1}x\neq(\tan x)^{-1},\quad \tan^{-1}x\neq\frac{1}{\tan x}$$

(1) Domain: ***R***

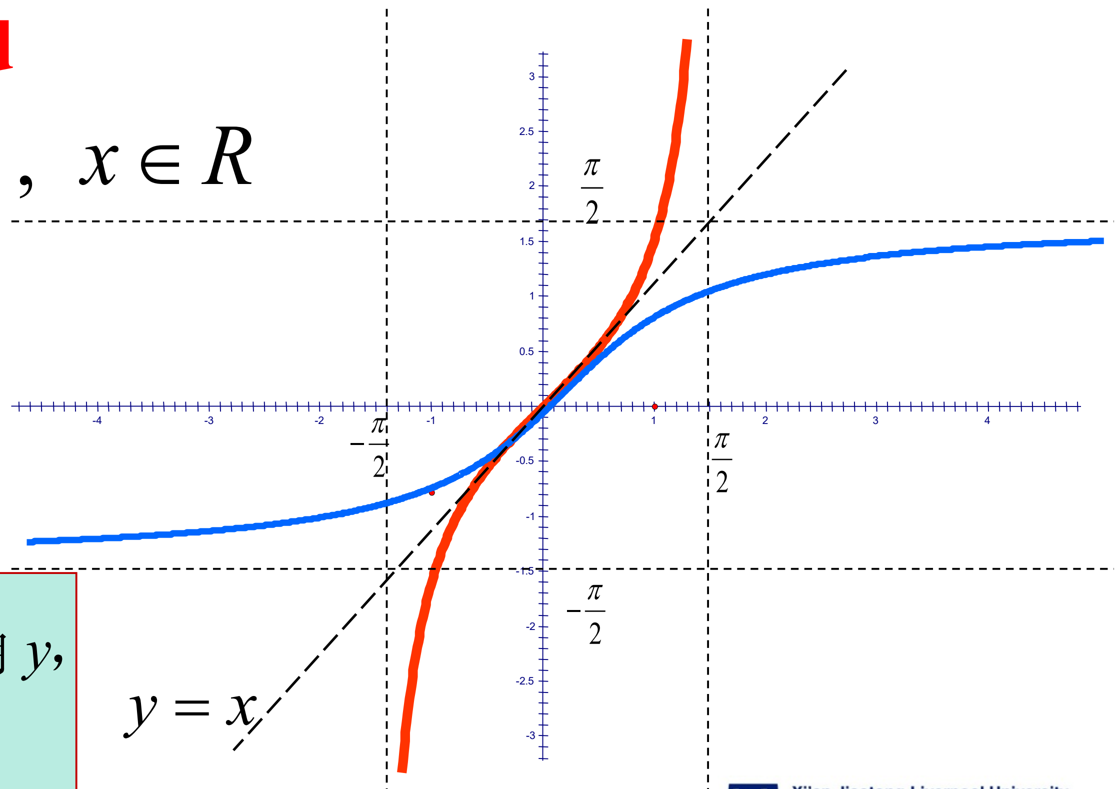
(2) Range: $(-\frac{\pi}{2},\frac{\pi}{2})$

(3) Odd/even: **Odd**

$$\tan^{-1}(-x)=-\tan^{-1}x\,,\quad x\in R$$

(4) Monotonicity：

Increasing.



$\tan^{-1}x$ 表示一个$(-\frac{\pi}{2},\frac{\pi}{2})$内的角 y,

这个角的正切等于x.

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Theorem A

(1) $\sin\left(\cos^{-1} x\right) = \sqrt{1 - x^2}$

(2) $\cos\left(\sin^{-1} x\right) = \sqrt{1 - x^2}$

(3) $\sec\left(\tan^{-1} x\right) = \sqrt{1 + x^2}$

Partial Catalog of Functions to Be Used in Calculus

Constant function: $f(x)=k$, where k is a constant(real number)

常数函数

Identity function: $f(x)=x$,

恒等函数

$(+,-,\times)$

Polynomial function: $f(x)=a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

多项式函数

(In particular, linear function: $f(x)=ax+b$

quadratic function: $f(x)=ax^2+bx+c$)

线性函数

二次函数

$(\div)$

Rational function: $f(x)=\dfrac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$

有理函数

$(+,-,\times,\div,\sqrt[n]{\ })$

Algebraic function, such as $f(x)=[(x+2)\sqrt{x}]/(x^3+\sqrt[3]{x^2-1})$

代数函数

Non-algebraic functions are called transcendental functions, including trigonometric, inverse trigonometric, exponential, logarithmic functions.

超越函数

Elementary functions 初等函数

Basic elementary functions:

Constant function: $f(x)=k$, where k is a constant (real number)

Power function: $f(x)=x^r$

Exponential function: $f(x)=a^x,\ \ a>0, a\neq 1$

Logarithmic function: $f(x)=\log_a x,\ \ a>0, a\neq 1$

Trigonometric function: $f(x)=\sin x, \cos x, \tan x, \cot x, \sec x, \csc x$

Inverse Trigonometric function:

$$f(x)=\arcsin x, \arccos x, \arctan x$$

Elementary functions: formed through $+, -, \bullet, \div, \circ$ of basic elementary functions

Three special functions:

1. Absolute value function: $|x| = \begin{cases} x\,, & x \ge 0 \\ -x\,, & x < 0 \end{cases}$

绝对值函数

2. Sign function: $\operatorname{sgn} x = \begin{cases} 1\,, & x > 0 \\ 0\,, & x = 0 \\ -1\,, & x < 0 \end{cases}$

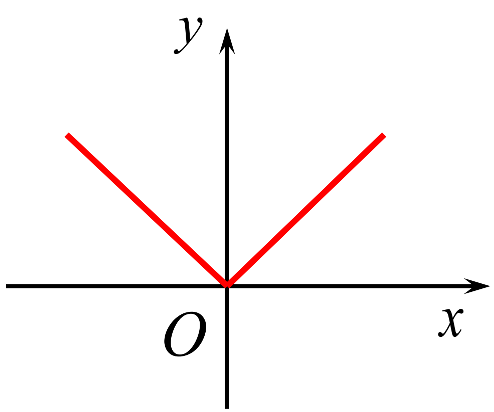
符号函数

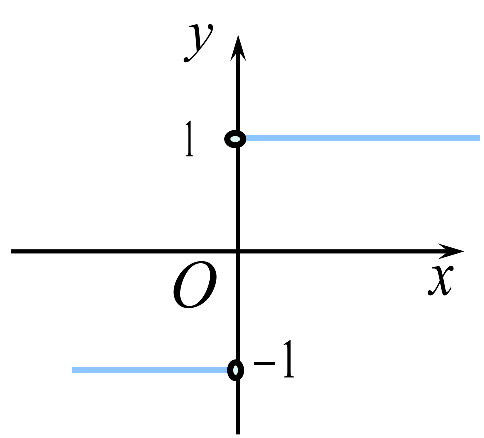
The two functions here are examples of **piecewise defined function (分段函数):** the function that has different expressions in different parts of the domain.

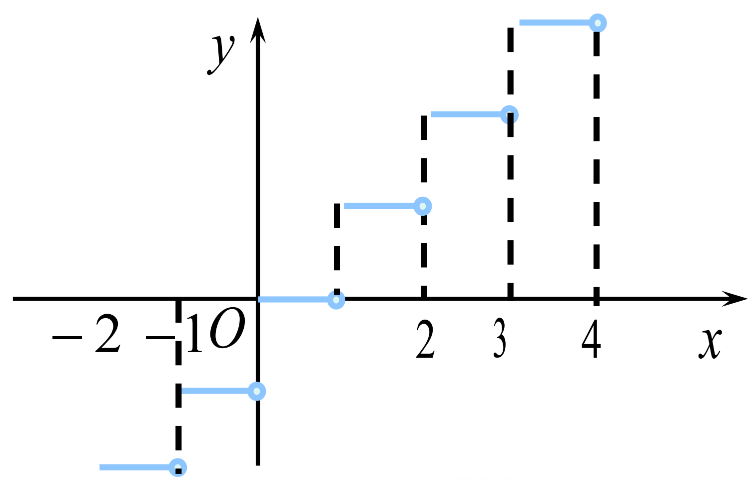
3. Greatest integer function:

最大整数部分，或 取整函数

$[x] =$ the greatest integer less than or equal to x.







Example 1 Find the natural domain for the following

(a) $f(x)=\sqrt{-(x^2+4x+3)}$, (b) $f(x)=\sqrt{1-2x}+\arcsin\dfrac{3x-1}{2}$.

Example 2 If $f(x)=\sqrt{x^2-1}$ and $g(x)=\dfrac{2}{x}$, find formulas for the following and state their domains.

(a) $(f\circ g)(x)$, (b) $(g\circ f)(x)$.

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Example 3 Find the inverse of $y = 2^{x-1}$.

Example 4 Find the exact value of the following without using a calculator.

(a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, (b) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$, (c) $\arctan\left(-\frac{\sqrt{3}}{3}\right)$.

Example 5 Find the exact value of the following.

(a) $\cos[2\sin^{-1}(-\frac{2}{3})]$, (b) $\tan\left[2\tan^{-1}\left(\frac{1}{3}\right)\right]$, (c) $\sec(\arctan 2)$.

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Thank You !