Lab10-Approximation & Randomized Algorithm

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1. **Solution.** The following is the process of solution.

a. For each $x_i, x_i \in \{0,1\}$, and for the convenience of using formula by integer programming, we use $\overline{x_i} = 1 - x_i$.

$$\begin{cases} min \sum_1^n x_i \\ \sum \overline{x_j} + \sum x_i \geq 1; x_i, x_j \in clause_1 \\ \sum \overline{x_j} + \sum x_i \geq 1; x_i, x_j \in clause_2 \\ \vdots \\ \sum \overline{x_j} + \sum x_i \geq 1; x_i, x_j \in clause_m \\ x_1, x_2, \dots x_n \in \{0,1\} \\ \overline{x_1}, \overline{x_2}, \dots \overline{x_n} \in \{0,1\} \end{cases}$$

Then we can change the formula to:

$$\begin{cases} min \sum_1^n x_i \\ \sum (1 - x_j) + \sum x_i \geq 1; x_i, x_j \in clause_1 \quad i, j \in 1, 2 \dots n \\ \sum (1 - x_j) + \sum x_i \geq 1; x_i, x_j \in clause_2 \quad i, j \in 1, 2 \dots n \\ \vdots \\ \sum (1 - x_j) + \sum x_i \geq 1; x_i, x_j \in clause_m \quad i, j \in 1, 2 \dots n \\ x_1, x_2, \dots x_n \in \{0,1\} \end{cases}$$

b.LP-relaxation.

Algorithm 1: Approximation algorithm.

Input: A CNF Φ with n boolean variables $\{x_i\}_{i=1}^n$ and m clauses with each clause consisting of 3 boolean variables.

Output: Feasible satisfiable with fewest true boolean variables.

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Find an optimal algorithm solution to the LP-relaxation.
count = 0;
/Fori to n  if x_i ≥ 1/3 then
    roundx_i = 1;
    count = count + 1;
else
    roundx_i = 0;
return count
```

□

$OPT_{LP} \leq 3 * OPL_{ILP} \leq 3 * OPL$

So, approximation algorithm ratio is 3.

2. (a) **Solution.** Random choose a position in $[l, r]$, and choose to color in this position.Then we check the every position and every color for query. in this case,the pre-processing complexity is $O(n)$.

We assume: the number of position of i_{th} color pearl are k_i; Repeat the pre-processing 10 times. For query, it will be execute $10(r-l)$ times, since r-l can achieve n-1,so time complexity per query is $O(n)$.No extra space needed.Then, prove accuracy will be better than 99.9%.

proof(two cases):

• For all color in $[l, r]$,k_i is smaller than $\frac{1}{2}(r-l)$. after 10 operations, we can not find any color which satisfies the condition. So,the answer is there does not exist this color. This answer is true.Accuracy is 100%.

• For all color in $[l, r]$, if there exist one color: $k_i \geq \frac{1}{2}(r-l)+1$; when we choose a color in $[l, r]$ and repeat 10 times, the probability of i_th color not being chosen is $(1-\frac{k_i}{m})^{10}$. and $p = (1-\frac{k_i}{m})^{10} \leq (1-\frac{1}{2})^{10} \leq \frac{1}{2^{10}} \leq 0.1\%$.then.the probability that the answer will go wrong is not smaller than 0.1%, So ,Accuracy is 99.9%.

So, the accuracy of this random algorithm will be better than 99.9%.

□

Remark: You need to include your .pdf and .tex files in your uploaded .rar or .zip file.