

# Lab06-Graph Exploration

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2019.

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1. **Solution.** The following is the process of solution.

(a). Answer is 6 and the ccs.cpp program file is in the txf\_homework\_06 file.

(b). The ccs.gephi file is in the txf\_homework\_06 file.

□

2. **Proof.** The following is the process of proof.

If we use dfs to traverse a graph, that means we visit the vertex by using a stack.

Proof by contradiction:

Assume: if two intervals are not disjoint or one is contained within the other. and we visit  $u$  before  $v$ . we know :  $PRE[u] < PRE[v]$  and  $POST[u] < POST[v]$  but  $POST[u] > PRE[v]$  and  $POST[v] > POST[u]$  According to the rule of stack. when we visited the  $u$  at first time, that means I should push the  $PRE[u]$  into the stack and if there no other vertex connect to  $u$ , then we call  $POST[u]$  when we visit  $u$ , then we pop vertex  $u$  out of stack. but in the case, we push  $PRE[v]$  into stack before we call  $POST[u]$ ; Then the stack exist  $PRE[u]$  and  $PRE[v]$ , meanwhile  $PRE[u]$  is on  $PRE[v]$ . so  $PRE[u]$  must waiting for  $PRE[v]$  being popped out of stack, so we call the  $POST[v]$  before  $POST[u]$ . so  $POST[v] < POST[u]$ , this is contradictory to  $POST[v] > POST[u]$ .

So,  $\forall u, v \in V$ , intervals  $[PRE(u), POST(u)]$ ,  $[PRE(v), POST(v)]$  are either disjoint or one is contained within the other.

□

3. **Solution.** The following is the process of solution.

In the case. we can change this question to the shortest path problem, and two computers communicate with a constant time  $t$ . So, we can use the BFS to solve this problem. The following is the pseudo code.

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**Algorithm 1:** The shortest path.

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**Input:** computer  $s$  and computer  $t$ , and an undirected Graph  $G = (V, E)$  represent the relation of computer (connected(1) and unconnected(0)). vertex  $s \in V$ .

**Output:** The shortest time needed to send message between two computers.

```
1 for each  $u \in V$  do
2    $DIST(u) = \infty$ ;
3    $visited[i] = false$ ;
4  $DIST(s) = 0$ ;
5  $visited[s] = true$ ; (Mean the point is visited.)
6  $Q.PUSH(s)$ ; (Using a queue  $Q$  to do BFS.)
7 while  $Q$  is not empty do
8    $u = Q.POP()$ ;
9   for each  $(u, v) \in E$  do
10    if  $DIST(v) = \infty$  and  $visited = false$  then
11       $Q.PUSH(v)$ ;
12       $visited[s] = true$ ;
13       $DIST[v] = DIST(u) + t$ ;
14    if  $v = t$  then
15      return  $DIST[v]$ 
16 return unconnected
```

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**Time complexity:**

- (1) Best case: if  $\mathbf{s}$  connected with  $\mathbf{v}$  or connected by a constant  $k$  vertex, then time complexity is  $o(1)$ .
- (2) Worst case: if there is no path from  $\mathbf{s}$  to  $\mathbf{t}$ , then we must traverse all vertex and edges. then time complexity is  $o(V+E)$ .
- (3) if two computers connect by  $k$  vertex,  $k=0,1,2,3 \dots n-2$ ; and probability is  $\frac{1}{n-1}$ . then time complexity is also  $o(V+E)$ .

□

**Remark:** You need to include your .pdf and .tex files in your uploaded .rar or .zip file.