A rational model of spatial neglect

Anonymous CogSci submission

Abstract

Spatial neglect has been a phenomenon of interest for perceptual and neuropsychological researchers for decades. However, the underlying cognitive processes remain unclear. We provide a Bayesian framework for the classic line bisection task in spatial neglect, regarding as rational inferences in the face of uncertain information. A Bayesian observer perceives the left and right endpoints of a line with uncertainty, and leverages prior expectations about line lengths to compensate for this uncertainty. This Bayesian model provides a basis for characterizing different patterns of behavior. Our model also captures the paradoxical cross-over effect observed in earlier studies as a natural outcome when uncertainty is high and the observer falls back on priors. It provides measures that correlate well with measures from other neglect tests, and can accurately distinguish stroke patients from healthy controls. It has the potential to facilitate spatial neglect studies and inform clinical decisions.

Keywords: Spatial neglect, Visual neglect, Line bisection, Attention, Perception, Bayes.

Introduction

Spatial neglect, a pronounced asymmetry of attention and behavior away from one side of space and towards the other, is a behavior disorder that typically results from right hemispheric brain damage. It has attracted considerable attention from researchers seeking to understand the psychological or neurological mechanisms of awareness, attention, and spatial cognition (Corbetta & Shulman, 2011). It can be measured via different tasks, such as the cancellation tasks, where patients often fail to 'cancel' or strike out items on the left side, or the copying tasks, where patients' copying of items on the left side is not as complete as for items on the right side. Among those standard tasks, the line bisection task, given its simple materials and easy administration, has seen especially widespread use (Schenkenberg, Bradford, & Ajax, 1980; Sperber & Karnath, 2016). Participants are required to mark the midpoint of a presented line (Figure 1a). Patients with spatial neglect often mark the answer to the right of the true midpoint, which is conventionally explained as a distortion or compression of the left space (Bisiach, Bulgarelli, Sterzi, & Vallar, 1983; Milner, Harvey, Roberts, & Forster, 1993).¹

Despite being one of the most well-established tasks to diagnose neglect, some debates remain about how we should interpret the line bisection task. The first challenge lies in the

relatively low correlation between bisection errors and other tasks (Sperber & Karnath, 2016; McIntosh, Ietswaart, & Milner, 2017), which has been taken to suggest there are multiple, dissociable mechanisms underlying spatial neglect. The second challenge is a paradoxical cross-over effect, where neglect patients sometimes mark the midpoint as left rather than right of the true midpoint (Marshall & Halligan, 1988; Mennemeier, Rapcsak, Pierce, & Vezey, 2001; McIntosh, Schindler, Birchall, & Milner, 2005). McIntosh et al. (2005) proposed that rather than underestimating the leftward extent of the line, patients in the line bisection task may lack a clear idea of where the left endpoint is. It was found that when manipulating the left and right endpoints independently, patients' answers were much more responsive to the right endpoint than the left endpoint (McIntosh et al., 2005, 2017; McIntosh, 2018). A measure of the difference in responsiveness to the two endpoints, the "endpoint weightings bias", tends to be highly sensitive to neglect, and to correlate better with other tasks (McIntosh et al., 2005, 2017).

We show that a Bayesian framework provides a way to expand and generalize McIntosh's idea that spatial neglect patients have unbalanced left- vs. right-side sensitivity, by understanding the bisection problem as one of the rational inferences in the face of uncertain or unreliable information. This perspective acknowledges perception as involving both bottom-up input constraints and top-down generative expectations (De Lange, Heilbron, & Kok, 2018; Clark, 2013), and takes the root cause of line bisection errors to be a loss of perceptual precision in the affected hemifield. A rational Bayesian observer should balance between prior expectations and evidence optimally, relying more on the prior when the evidence is ambiguous, but more on the evidence when the evidence is precise (De Lange et al., 2018). This framework has been applied successfully to a variety of phenomena and illusions in human static and motion perception (see De Lange et al., 2018; Seriès & Seitz, 2013, for review), as well as visual phenomena of people with neurological disorders (Valton et al., 2019).

In this paper, we model the line bisection task under the Bayesian framework. Our model provides a unified account for the behavior of both patients with right hemispheric brain damage and healthy controls. It captures the paradoxical cross-over effect observed in earlier studies, demonstrates the connections with other spatial neglect tasks, and potentially provides measures of the task that can later be used for better diagnosis.

¹Spatial neglect typically refers to left neglect, following right hemisphere damage, which is more common, severe and persistent than left neglect following right hemisphere damage (Mesulam, 1981).

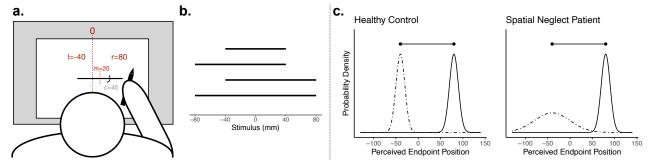


Figure 1: The line bisection task: a) Participants are asked to mark the midpoint of the line presented in front of them. Left endpoint (l), right endpoint (r), true midpoint (m), and response (c) are not marked on the stimulus sheet itself, but are only for the figure. The spatial neglect patients are often found to provide a rightward answer. b) Four types of experimental stimuli in the current dataset. c) Spatial neglect patients may have lower left-side perceptual precision.

Methods

Dataset

We used the dataset presented in (McIntosh et al., 2017). McIntosh et al. (2017) administered a series of unilateral neglect tests to patients diagnosed with unilateral right hemisphere stroke (N = 42; 12 female, 30 male; 68.64 years \pm 9.76), including the line bisection task, the line and star cancellation tasks, the copying task, the drawing task, and the multiple line bisection task, which involves multiple lines presented at once. A healthy control group (N = 30; 18 female, 12 male; 71.27 years \pm 9.12) was recruited for the line bisection task (data original from McIntosh et al., 2005).

The line bisection task

In the line bisection task, each line stimulus is printed individually in black on a white A4 paper in landscape orientation. Each sheet is positioned directly in front of the participant, and the page is aligned centrally with the body midline. Participants are instructed to mark the midpoint of the line with a pen held in their right hand. Before each trial, they are asked to remove their hand off the table to avoid the invariable reaction.

McIntosh et al. (2017) manipulated left and right endpoints independently. Four types of lines were created by combining endpoint locations with two different distances from the midpoint of the page (see Figure 1b): [-40, 40], [-80, 40], [-40, 80], and [-80, 80] mm (we standardized the data by treating 80 mm as 1 unit in later analysis, McElreath, 2020).

The Bayesian neglect model

A rational Bayesian observer would integrate the evidence, i.e. the perceived endpoints, with their prior expectations. There are different ways to represent prior expectations, but we adopt a simple prior distribution with a small number of interpretable parameters (see below).

We provide an illustration in Figure 1c, where the observer's perceived left and right endpoints are assumed to follow Gaussian distributions with a mean at the actual endpoint and a standard deviation parameter (σ_L on the left side and σ_R on the right side). Patients might exhibit high uncertainty on the left side, and hence, a large σ_L (Figure 1c).

A more technical illustration is shown in Figure 2. Specifically, three key principles are assumed to guide the solution of line bisection.

- 1. The midpoint response c should have the same distance from the perceived left P_L and right P_R endpoints, i.e. $P_L = c \frac{ll}{2}$, and $P_R = c + \frac{ll}{2}$, where ll represents the expected line length.
- 2. There is uncertainty when one perceives the left endpoint (σ_L) and the right endpoint (σ_R) , i.e. $l \sim N(P_L, \sigma_L)$, $r \sim N(P_R, \sigma_R)$, where l and r represent the left and right endpoints in the stimulus.
- 3. One would utilize the expectation to compensate their perceptual uncertainty of the left and right endpoints. This includes the expectation of the line length, drawn from a gamma distribution: $ll \sim \Gamma(\alpha, \beta)$, and the expectation that the midpoint should be on the middle of the page: $c \sim N(0, \sigma_C)$.

One key advantage of the Bayesian framework is that it considers the role of expectations, in ways that can accommodate (and reduce variance due to) order effects in stimuli, and offer new predictions. For example, we assume that the learner has a priori expectations about line length, which follows a Gamma distribution with parameters α and β . The expectation for a given trial could be based on general beliefs that precede the task as well as the line lengths perceived in earlier line bisection trials. If a learner has a precise sense of where the endpoints are, this expectation should reflect the distribution of actual lengths of previous trials. To simplify

²We later used the same criteria for these other tasks as in McIntosh et al. (2017). Eight patients from the original sample of 50 were excluded from the analysis because the raw line bisection data were missing.

³Gamma distributions are widely used to model probability over non-negative quantities, like length, but unlike the simpler exponential distribution, can decouple expectations about central tendency and variability.

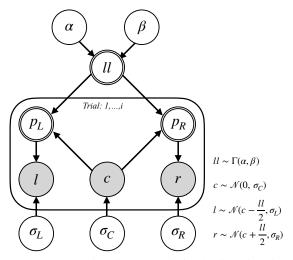


Figure 2: The Bayesian diagram to infer the line bisection parameters for individuals. The model infers parameters given the bisection answer (c) and the stimulus's the actual left and right endpoint position (l and r).

our analyses, we leave effects of sequence order for future work, and fit a single distribution to approximate expectations across individual trials.

Using our model, we can estimate individual differences given judgments from different people. Specifically, we can model each individual's data and obtain five parameters (σ_L , σ_R , α , β , σ_C) for each person; we will later examine whether all of them are necessary. These parameter values provide insight into the causes of patients' and healthy controls' judgments, and we will show that they are more informative than existing measures in predicting whether a patient has had a stroke. The model fitting was implemented in Stan via the RStan package (Stan Development Team, 2022).

Alternative measures of performance

Directional bisection error The conventional way to measure line bisection performance is with the directional bisection error, which calculates the average difference between human answers (c) and true midpoints $\frac{l+r}{2}$ across trials:

$$DBE = \frac{1}{i} \sum_{trial=1}^{i} (c - \frac{l+r}{2})$$
 (1)

Endpoint weightings The endpoint weightings bias measures calculate the difference between participants' sensitivity to the left and right endpoint manipulations. It depends on the linear slopes when using the left and right endpoint (d_{PL} and d_{PR}) to predict the midpoint answers:

$$c = (d_{PL} \cdot l) + (d_{PR} \cdot r) + k \tag{2}$$

$$EWB = d_{PR} - d_{PL} \tag{3}$$

Results

Parameter values

Main measures By fitting individual data to different models, we can get different measures for the line bisection task. We first visualize those parameters in Figure 3. Different measures demonstrated varying degrees of discrimination between stroke patients and healthy controls. In the endpoint weightings model, the left endpoint weight (d_{PL}) achieved an accuracy of 0.847 (Figure 3a). The endpoint weightings bias, which further calculates the difference between left and right weights, has a slightly higher accuracy at 0.861 (Figure 3b). In contrast, the directional bisection error has a max accuracy of 0.778 (Figure 3b).

For the Bayesian neglect model, we consider two measures: the left-side uncertainty (i.e. σ_L) and the difference between left-side and right-side uncertainty (i.e. $\sigma_L - \sigma_R$). The former emphasizes the individual difference on the leftside uncertainty, while the latter further uses the individual's right-side uncertainty as a baseline, similar to the endpoint weightings bias. As shown in Figure 3c, stroke patients had higher σ_L than the healthy controls (t(70) = 4.69, p < .001). σ_L distinguished stroke patients and healthy controls with a max accuracy of 0.917. Meanwhile, the difference between left-side and right-side uncertainty (i.e. $\sigma_L - \sigma_R$) was larger in the stroke patients than in the healthy controls (t(70) = 3.80, p < .001). It had a max accuracy of 0.875 in distinguishing two groups. It highly correlated with the endpoint weightings bias (r = 0.923, Figure 3d), which is consistent with the theoretical assumption that both of them measure the relative sensitivity on left and right sides.

Expectation parameters The Bayesian neglect model provided two parameters α and β that can help examine the line length expectations (mean and variance) participants had. We visualized each participant's prior distribution in Figure 4a. In the healthy control group, participants' line expectations on average were 1.48 units, which was around the average line length across trials (1.5 units). The stroke patients' line expectation was 1.38 units on average, which was shorter than that of the healthy controls (t(70) = 2.32, p = .02). As for the variance of line length expectation, participants from the healthy control group and 50% of participants from the patient group showed a middle level of variance in line length expectation (Figure 4a). Another 50% of patients demonstrated a very strong line prior (with a variance smaller than 10^{-3}), which were shown as sharp probability distributions in Figure 4a. The expectation variance was smaller in the patient group than in the healthy control group (t(70) = 6.45,p < .001), and this tendency remained even after removing patients with very strong prior (t(49) = 3.63, p < .001, Fig-

We visualized several representative individual cases to further address the relationship between line length expecta-

⁴All analysis code could be found via an anonymous link: https://anonymous.4open.science/r/bayesNeg-87B3/

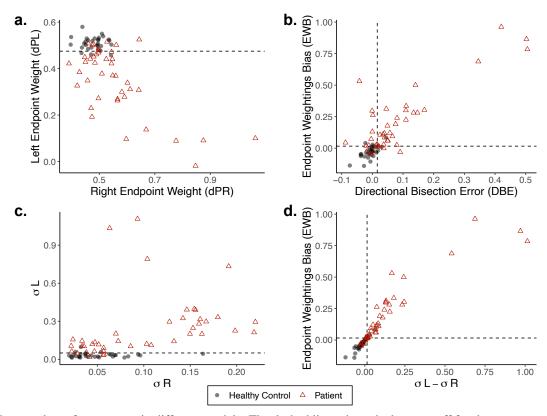


Figure 3: Scatter plots of parameters in different models. The dashed lines show the best cut-off for the corresponding measure under the current dataset.

tion and the answers (Figure 5). Given that patients had difficulty perceiving the left side, the expectation was assumed to play an important role in their judgments (De Lange et al., 2018). For example, the Bayesian model diagnosed that Patient16 had a strong expectation that the line was short. Accordingly, most of their answers were close to the right endpoint. Patient 46 demonstrated a strong expectation over the true line length, which indicated they could perceive the line length during some trials but may have difficulty during other trials. Accordingly, their answers showed a fixed distance from the right endpoint. Importantly, more of their answers were on the left side of the true midpoint, which was called the paradoxical cross-over effect under the directional bisection measure. This indicated that the earlier found cross-over effect could simply result from patients having a strong expectation over a certain line length and the uncertainty about the true endpoint positions. The σ_I negatively correlated with the line expectation variance (r = -0.73). Compared to the two aforementioned patients, Patient35 and HealthyControl30 were better at detecting the left endpoints, and hence showed less reliance on the expectation.

For the center prior parameter σ_C , as shown in Figure 4c, four stroke patients had a σ_C that was much higher than others, which indicated they may have highly variable performance that cannot be clearly captured by any other parameters . There was no difference between the patient and

healthy control groups after excluding these four data points (t(66) = 1.79, p = .08).

Correlation with other tasks

One of the main puzzles of the directional bisection error is the low levels of correlation with other tasks. Here we examined the relationships between new Bayesian indices and other tasks.

Table 1 showed the correlation between line bisection measures and other tasks. We used Steiger's Z (Hoerger, 2013) to compare the correlation coefficients. The directional bisection error had relatively low correlations compared to the endpoint weighting bias for all tasks (zs > 2.22, ps < 0.03) but the line cancellation (z = 1.87, p = 0.06). The correlations of the Bayesian σ_L were slightly lower than those of the endpoint weighting bias, while all the differences were not significant (zs < 1.78, ps > 0.08). Similarly, the correlation of the $\sigma_L - \sigma_R$ was slightly lower than those of the endpoint weighting bias, while they were not significant except for the copying task (z = 2.45, p = 0.01).

Classification

We have earlier shown that different indices had a certain ability to distinguish between the stroke patients and the healthy controls with a single threshold. We here perform more formal classification tests to test how indices can clas-

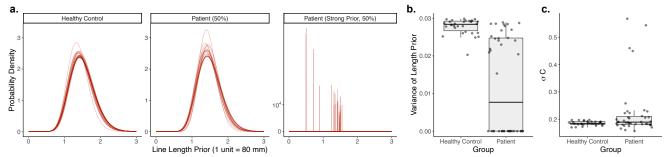


Figure 4: a) The probability distributions of the line length prior in the healthy control, 50% of patients with a weak prior, and another 50% of patients with a strong prior. b) The fitted line length prior variance in two groups. c) The fitted center prior parameter σ_c in two groups.

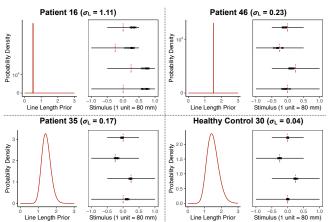


Figure 5: The line bisection answers and fitted line length priors of four participants.

Table 1: Correlations between line bisection indices and other tasks.

	LINES	STARS	COPY	DRAW	MULTI
DBE	0.47	0.28	0.27	0.21	0.61
EWB	0.61	0.52	0.49	0.40	0.77
σ_L	0.59	0.42	0.40	0.37	0.75
$\sigma_L - \sigma_R$	0.61	0.40	0.34	0.37	0.73

sify the participants and whether we can use the index combinations to improve the accuracy.

We applied one linear algorithm (logistic regression) and one non-linear algorithm (decision tree), and used the leave-one-participant-out cross-validation as the procedure. We used the accuracy (the proportion of true positive plus true negative) and F1-score (the harmonic mean of precision and sensitivity). The results were shown in Table 2. Predictors from the Bayesian neglect models, especially the left-side uncertainty σ_L , performed better than the directional bisection error or parameters from the endpoint weightings model $(d_{PL}$ and $d_{PR})$. Under the decision tree algorithm, the single σ_L can achieve 0.917 accuracy and 0.925 F1-score. Combining σ_L with d_{PL} from the endpoint weightings model may

Table 2: Cross-validation classification results.

	Logistic Regression		Decision Tree	
	ACC	F1	ACC	F1
DBE	0.764	0.785	0.639	0.667
d_{PL}	0.847	0.864	0.764	0.785
d_{PL}, d_{PR}	0.847	0.867	0.819	0.854
σ_L	0.903	0.914	0.917	0.925
σ_L , σ_R	0.903	0.914	0.917	0.925
σ_L , σ_R , α , β	0.875	0.892	0.847	0.871
σ_L , σ_R , σ_C	0.903	0.914	0.861	0.881
σ_L , σ_R , α , β , σ_C	0.889	0.902	0.847	0.876
σ_L, d_{PL}	0.917	0.927	0.917	0.925
σ_L , σ_R , d_{PL} , d_{PR}	0.903	0.914	0.903	0.918

Note: Classification decisions based on linear combinations of d_{PL} and d_{PR} correspond to EWB. The α and β parameters were transferred into the line length expectation and variance before fitting to ensure different predictors were on relatively similar scales.

slightly improve the performance under the linear algorithm, indicating the possibility of integrating parameters from different models in future practice.

Ablation study

To better understand the importance of each component of our model, we also compared several ablations of the full model:

- No line expectation: Replace $ll \sim \Gamma(\alpha, \beta)$ (Figure 2) with the true line length. It assumes no prior line length expectation, and relies solely on the other three parameters σ_C , σ_L and σ_R to make decisions.
- No lateral difference: Use the same parameter σ for σ_L and σ_R . It tests the importance of allowing unbalanced uncertainty between two sides.
- No center expectation: Removing $c \sim N(0, \sigma_c)$ (Figure 2). It tests the contribution of σ_C .

Figure 6 summarizes model performances in terms of the average mean squared errors. All three ablation models were

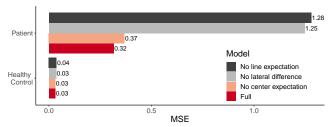


Figure 6: MSE of model fits for the full and ablation models.

much worse than the full model, and most reduction came from fitting the patients. Dropping center uncertainty σ_C resulted in the least model fit reduction, showing that line expectation and uncertainty asymmetricity are the dominant factors in the full model.

Discussion

In this paper, we provide a new model to describe the perceptual process in the line bisection task. Our model is based on the Bayesian perception hypothesis that people use Bayesian inference to integrate signal input and prior knowledge to produce conscious perception (Clark, 2013; Seriès & Seitz, 2013). Consistent with McIntosh et al. (2005), we showed that the previous suspicions about the validity of the line bisection task is mainly due to the problem of the conventional measure, rather than the task itself. The new Bayesian measure successfully addressed those challenges. Along with the simple and clean task structure, the line bisection task has the potential to be one of the most efficient tools to diagnose spatial neglect.

The advantages of the current Bayesian neglect model are multifaceted. Firstly, it captures a general and intuitive representation of the perceptual processes in the line bisection task, i.e. the uncertainty in perceiving the left and right endpoints. Spatial neglect could be naturally explained as greater uncertainty on the left side, and reduced to parameters, σ_L or $\sigma_L - \sigma_R$, in our model. We demonstrated that both measures performed well in distinguishing stroke patients from healthy controls and outperformed measures from previous studies. They also correlated better with other spatial neglect tasks than the conventional directional bisection error, indicating that, rather than relying on dissociated underlying functions, various spatial neglect tasks may share a common underlying mechanism (McIntosh et al., 2005). As such, the Bayesian measure could provide useful information to diagnose spatial neglect in the future.

Secondly, the Bayesian neglect model naturally integrates the role of expectation, which can explain the variety of behavior patterns in different patients. Previous studies have shown that humans can learn an expectation very quickly (Seriès & Seitz, 2013; De Lange et al., 2018). The stimuli that participants have gone through could serve as a resource for the prior for future trials. It explains why most of the healthy controls fit a distribution similar to the true stimulus distribution. In contrast, the patients' expectations are less adap-

tive and flexible compared to those of the healthy controls. Specifically, some patients demonstrated a very strong prior distribution that could provide a guideline for decision when the left-side perception is very uncertain. It also clarified a long-standing confusion in the literature that some patients showed mixed leftward and rightward errors, or even primarily leftward errors: If the patient has a strong line length expectation and a poor ability to perceive the left side, they would frequently mark answers that have the same distance from the right endpoint. These answers could be either rightward or leftward from the true midpoint, given different stimulus lengths. We thus showed that the cross-over effect is not a problem of the task, but rather reflects how patients rely on their expectations to compensate for perceptual difficulties.

The Bayesian neglect model presented in this paper could open several directions for future work. From the practical perspective, the Bayesian neglect model not only describes a possible process of spatial neglect but also demonstrates a high accuracy in identifying stroke patients. It indicates the Bayesian measures could be related to particular brain damage or brain functions. With suitable datasets, future work could directly explore how Bayesian measures could predict neuroimaging differences in individuals (Halligan, Fink. Marshall, & Vallar, 2003). One relevant debate is whether the individual's right-side uncertainty should be used as a baseline when measuring spatial neglect. Although the asymmetry between left and right sides of space (i.e. $\sigma_L - \sigma_R$) may better reflect the definition of spatial neglect (McIntosh et al., 2005), the left-side uncertainty alone (σ_L) also performs well in our analysis. In fact, the two measures are highly correlated in the current dataset (r = 0.97). With brain data, future studies should further explore how these two measures correspond to neuroimaging differences.

Meanwhile, the Bayesian neglect model provides a generative process of line bisection solutions, which could be very useful if researchers want to simulate the response under different situations. It could be useful for making a range of new predictions to be tested in future studies. It also could be combined with the modern optimal experimental design tool (Valentin, Kleinegesse, Bramley, Gutmann, & Lucas, 2021) to provide guidelines for task administration, such as what types of stimuli are required and how many trials are required to provide a valid diagnosis.

From the theoretical perspective, the asymmetrical perceptual uncertainty and the prior expectation are not exclusive for line bisection tasks, but for various types of spatial neglect tasks (Sperber & Karnath, 2016). Future work could use the same framework to explain behavior patterns such as omission and distortion in other spatial neglect tasks, building toward a general model for spatial neglect.

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