

Intuitions and Perceptual Constraints on Causal Learning from Dynamics

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Abstract

To all practical intents, many of the real world phenomena that cognizers must grapple with are continuous, not only in the values they can take, but also in how these values change over time. The mind must somehow abstract from these inputs to extract discrete useful concepts such as objects, events and causal relationships. We investigate several factors that affect basic inferences about causal relationships between continuous variables based on observations in continuous time. In a novel experiment, we explore the ways in which causal judgments are sensitive to factors that relate to causal inductive biases (e.g. causal lags, the direction of variation) and causal perception (e.g. the range and rapidity of variation). We argue standard statistical timeseries models have limited utility in accounting for human sensitivity to these factors. We suggest future work is needed to fully understand the cognitive processes that underlie causal induction from timeseries information.

Keywords: causal learning, continuous time, continuous variables, time-series data, dynamics.

Introduction

From the waxing and waning of daylight in our eyes, to the ebb and flow of the neighborhood hubbub in our ears, analogue inputs, tied to real world causal dynamics, flow in through our senses continuously. As such, causal inference requires cognizers to first be “digitizers”, extracting discrete, useful entities like objects, events and causal relationships from “the blooming buzzing confusion” of sensory inputs (James, 1890). We here investigate several factors that might influence this foundational aspect of causal model based reasoning. In particular, we test the idea that reasoning about causal relationships in dynamic systems depends on two interacting dimensions: (1) an ability to perceive and extract causal signals from background noise and (2) intuitive causal theories about the nature of causal relations and their functional relationships.

Following Davis, Bramley, and Rehder (2020a), we adapt the Ornstein-Uhlenbeck (OU) process (Uhlenbeck & Ornstein, 1930) and use it as a generative model that can simulate a variety of naturalistic continuous causal dynamics. OU networks combine a causal graph with functions expressing continuous influence of causal variables on effect variables by way of an augmented OU processes—a form of mean-regressive brownian motion. Unlike standard causal graphical models (Pearl, 2000), OU networks model causal dynamics in continuous time, with effects continually noisily regressing toward a moving target defined as a function of earlier values of their cause(s). Critically, this behavior is governed by parameters that allow us to manipulate properties of the resultant causal dynamics. For example, we can manipulate the lag, i.e. how long influences from cause X

take to arrive at Y , but also the strength or “rigidity” of the influence—how rapidly changes in Y approach a whatever moving target is caused by earlier changes in the values of X (see Figure 1). We use this as a test-bed to explore human causal judgments.

Although studies into causal cognition have long focused on simple contingency information such as covariation of binary variables across independent samples (Cheng, 1997; Griffiths & Tenenbaum, 2009), recently several studies have investigated more naturalistic situations that involve continuous variables and time-series data (Soo & Rottman, 2018, 2020; Zhang & Rottman, 2021) or dynamic causal systems (Davis et al., 2020a; Davis, Bramley, & Rehder, 2020b; Bramley, Gerstenberg, Mayrhofer, & Lagnado, 2019). These studies find that people can leverage moment-by-moment transitions (i.e. changes in the values of two variables) to successfully identify their relationship (Soo & Rottman, 2018). People also can identify the structure of dynamic causal systems that involve three continuous variables (Davis et al., 2020a, 2020b) by freely intervening on the variables in real time. Participants in these tasks appeared to follow an intervention strategy of creating occasional dramatic and rapid changes in variables and monitoring the behavior of other variables shortly afterward. Participants were able to learn better when the variables changed rapidly and affected one another rigidly rather than slowly or gradually (Figure 1; Davis et al., 2020b).

This paper aims to extend this previous work by considering factors that are particularly interesting for continuous-time causal systems with continuous variables. The novel empirical results will help future work identify and build computational models to describe the underlying cognitive process, which we will discuss towards the end of the paper. We mainly focus on four factors as listed below:

Causal lag

In reasoning about causal relationships between *events*, people are clearly sensitive to temporal delays. They generally make stronger causal attributions when a putative cause is followed by a putative effect after a short delay than after a long delay, if no specific mechanistic information is conveyed (Buehner & McGregor, 2006; Shanks, Pearson, & Dickinson, 1989; Lagnado & Speekenbrink, 2010). In other words, when a cause event occurs, people expect its effects to happen sooner rather than later. It is unclear whether people also hold similar intuitions about “lags” when judging influences between continuous variables. For example, would people

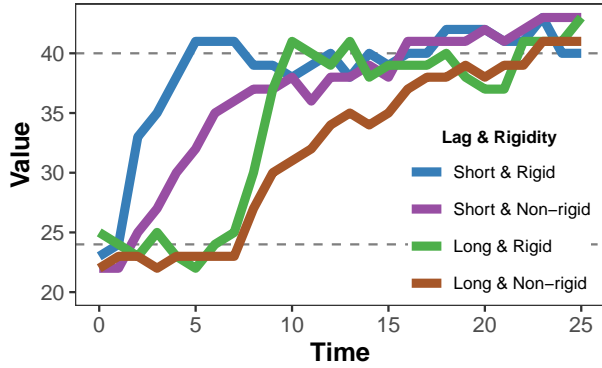


Figure 1: Examples of the effect’s OU process under different causal lags ($t=2$ vs. $t=8$) and degrees of rigidity. Dashed lines indicate the mean values before and after receiving the cause’s influence.

be more likely to make causal attribution when the effect Y ’s value at t is influenced by cause X ’s value at $t - 1$ seconds compared to when it follows X at $t - 10$ seconds. Previous studies tend to fix the causal lag (usually as $t - 1$) and have not compared different intervals (Soo & Rottman, 2018; Davis et al., 2020b). Davis et al. (2020b) manipulated a related but distinct quantity: varying the refresh rate of their causal systems between 100ms to 300ms in different conditions but scaling the causal dynamics to be otherwise identical (with causal influences still lagging exactly one refresh).

Granger causality (Granger, 1969) is an established statistical technique designed to identify potential causality in timeseries data, with a mechanism to accommodate causal lag. To assess if one variable “Granger causes” another, one searches across a range of fixed lags deemed to be mechanistically plausible, e.g. $X_{t-1} \dots X_{t-m}$, and tests whether inclusion of any of these terms statistically improve prediction of Y_t over and above its own lagged autocorrelation (modeled by including $Y_{t-1} \dots Y_{t-m}$ as a covariates). If a statistical relation is found for one or more of these lags the causal influence is deemed to be supported. As such, Granger causality does not inherently privilege longer or shorter lags. This lag-indifference may be appropriate for minimizing bias when modeling domains that are poorly understood but may not reflect human expectations. We will investigate whether laypeople are similarly indifferent, or if their causal judgments decay with lag as what they do with discrete cases.

Rigidity

For a given change in a cause, a *rigid* causal relationship would lead to the effect asymptoting to its new value sooner than a *non-rigid* relationship (see Figure 1). Davis et al. (2020b) found that people are more likely to infer a causal link between two variables if the effect responds rigidly to the cause’s change, essentially overwhelming the variables’ random motion more dramatically or saliently. However, they tested this in an active learning setting where participants’ actions complicated interpretation of accuracy patterns. In par-

ticular, in the non-rigid condition, participants faced a trade off between either not waiting long enough between their actions to observe full influence of genuine causal effects, or else spending a long time waiting for outcomes to manifest, so performing fewer interventions. We thus test rigidity in a passive learning setting to make sure that observed rigid vs. non-rigid stimuli only differ in their detailed temporal dynamics but not the final magnitude of the change (Figure 2).

The direction of change

Continuous variables can increase or decrease over time. Previous research provides people with time-series data where variables both increase and decreases within the same episode (Soo & Rottman, 2018; Davis et al., 2020a). In this study, we are more curious whether people are equally sensitive to increases and decreases and to matched or inverted changes of cause relative to effect. People frequently found to have a preference for positive over equally informative negative evidence (Newman, Wolff, & Hearst, 1980) and a prior expectation of positive linear relationships (Sanborn, Griffiths, & Shiffrin, 2010). People tend to test positive examples of a hypothesis (Coenen, Rehder, & Gureckis, 2015), and focus more on the positive aspects of their observations (e.g. the “A-cell bias”, Kao & Wasserman, 1993). As an intuitive example in daily life, positive association are far more naturally described as Y increases as X increases, rather than Y decreases as X decreases. We will test whether people are influenced by the direction of change by providing four kinds of information of the combination of X ’s increase or decrease with Y ’s increase or decrease.

The perceived magnitude of change

In situations where people experience real-time continuous dynamics, we have to also consider the role of low-level information processing, i.e. *perception*. That is, in order to discover relationships between variables, learners must first perceived that the variables have changed meaningfully—e.g. surprisingly under its baseline behavior of random fluctuations or drift.

Sensitivity to continuous change is a domain in which it seems likely that people will deviate dramatically from naive idealized observer accounts, that are typically presumed to have perfect numerical precision and computing resources. For example, it would likely be easier for us to notice a change in the quantity of green algae in a fish tank when it accumulates from 0% (an almost clean tank) to 10% than accumulating from a higher baseline (e.g. 50%-60%). It would also be easier to detect the change from 90% and 100% (as the observer can focus on the change of clean space). This is known as the boundary effect in numerical estimation that includes both lower and upper bounds (Kim & Opfer, 2017; Thompson & Opfer, 2008). In contrast, any model based in linear regression would treat these changes as the same (differing only in their intercept). Therefore, we plan to compare four ranges of Y ’s change that differ in how close they are to the limits of the range of the variables in question (Fig-

Table 1: Parameter Settings of OU Processes.

Condition	Level
<i>Lag</i>	short: $k = 2$ vs. long: $k = 8$
<i>Rigidity</i>	rigid: $\omega = 0.8$ vs. non-rigid: $\omega = 0.2$
<i>Slope</i>	positive: $\beta = 1$ vs. negative: $\beta = -1$
<i>Range_Y</i>	boundary vs. middle, controlled by α

ure 2c). We hypothesize that people are more likely to detect the causal relationships when the effect in question starts or ends nearer to its boundary than when it varies far from its boundary (Figure 2c).

OU process

As sketched at the start, we adapt OU processes to generate the stimuli in our experiment (Davis et al., 2020a; Uhlenbeck & Ornstein, 1930). A standard OU process models mean-regressive Brownian motion of the sort one might recognize in a coke can bouncing around the seat well of a car in motion. By combining a causal graph (here just $X \rightarrow Y$) with OU-processes, and replacing the fixed mean of these with a function of cause X , we can model how an effect Y changes and fluctuates in continuous time while being causally influenced by its parent X . OU processes provide a mathematically straightforward formula for producing rich continuous dynamics. Compared to using simple regressions between Y_t and X_{t-k} (Zhang & Rottman, 2021), the OU process can easily capture the “stickiness” feature where Y is “dragged” to the expected value gradually with noise.

$$P(\Delta v_y^t | v^t, \omega, k, \beta, \alpha, \sigma) = \omega[(\beta \cdot v_x^{t-k} + \alpha) - v_y^t] + N(0, \sigma) \quad (1)$$

Eq.1 formalizes the extended OU process we use such that Δv_y^t —the change in Y from t to $t + 1$ —noisily depends on the difference between the target Y value and current Y value v_y^t . We further assume the target Y value is determined by a linear function of its cause X at time $t - k$, where k is captures the time lag and the slope β and the intercept α potentially rescale and offset Y relative to X . ω then controls how “rigidly” the change in the effect occurs – for example, $\omega = 0.8$ would mean that Y moves $80\% \pm \sigma$ of the way toward its target value every time interval while $\omega = 0.2$ means moves only $20\% \pm \sigma$ of the way and so adjusts to a change in its target value more gradually. Critically, the variables additionally fluctuate according to random noise here drawn from Gaussian distribution with a mean of zero and a standard deviation of σ .

We fixed $\sigma = 1$ and varied all other parameters systematically to create our set of experimental conditions. As shown in Table 1, we consider different levels of *Lag* (k), *Rigidity* (ω), *Slope* (β) and *Range_Y* (i.e. the intercept, α). Additionally, we vary the behavior of the cause X (*Direction_X*) such that it either increases or decreases abruptly shortly after the beginning of each trial (Figure 2b). The X ’s behavior is also controlled by an OU process but since it has no parent we introduce it as being manually controlled (i.e. through external intervention) and have it approach a basin point μ

that changes either increases or decreases abruptly few seconds into the trial with $\omega = 0.6$ and $\sigma = 1$ (see Figure 2b). *Direction_Y* (whether Y ultimately increases or decreases) is thus jointly determined by *Direction_X* and the sign of *Slope* β . Critically, under our current parametric set, differences in *Lag* and *Rigidity* do not affect the final value that Y arrives at by the end of a trial (Figure 2c).

Since causal *Slope* is either $\beta = 1$ or $\beta = -1$, and X ’s change is always of the same magnitude, the causal influence of X on Y would seem naively to be equivalently detectable in all conditions with the absence of additional inductive biases or perceptual constraints.

Experiment

Methods

Participants 100 participants (47 female, 52 male, 1 non-binary, aged 42 ± 12) were recruited via Prolific Academic and were paid £1.20. The task took around 10 minutes. The anonymized pre-registration (<https://bit.ly/3fB5zSP>), data and analysis code (<https://bit.ly/3tBQ1GB>), and the experiment demo (<https://bit.ly/3tDr4uz>) are available online.

Design & Procedure Each stimulus contained 32 frames visualizing X and Y ’s values from $t = 1$ to 32. In each case, cause X starts initially follows an OU process that reverts to basin point μ_1 , and switches to a new basin point μ_2 at $t = 7$ (Figure 2). The effect Y responds according to Eq.1. Each frame lasts 750ms.

For the cover story, participants were asked to imagine playing the role of a “forestry manager” who needs to identify the causal relationship between different pairs of Plant A and Plant B following observations in which Plant A’s quantity is manipulated artificially (i.e. by the forestry commission). The quantity of each kind of plant varies across the trial. Each pair of Plant A and B had unique colors to ensure participants understand that plants from different trials are not related.

Plant A is displayed in a rectangular “forest” in which the leaves accumulate from the bottom to the top (Figure 2a). We did this to make this quantity perceptually similar to the vertical sliders used in past research (Soo & Rottman, 2018; Davis et al., 2020a; Zhang & Rottman, 2021) where the quantity and range can be perceived by simply considering the height and boundaries. Plant B (the potential effect) is displayed in a circular birds-eye view of a “forest” in which the leaves appear in random positions (Figure 2a). The quantity needs to be told by estimating the leaf number (or the area). We used slightly different presentations for Plant A and B to aid separability (Soo & Rottman, 2018).

Participants answered “What is the relationship between Plant A and B” by choosing one of the three radio buttons labeled: “Positive (regular)”, “Negative (inverse)”, and “No relationship”. The order of presentation of the radio buttons was randomized between participants. Each stimulus could be viewed only once and lasted a total of 24 seconds.

Each participants went through 16 trials, facing 16 stimuli representing all combinations of *Lag*, *Rigidity*, *Range_Y*, and

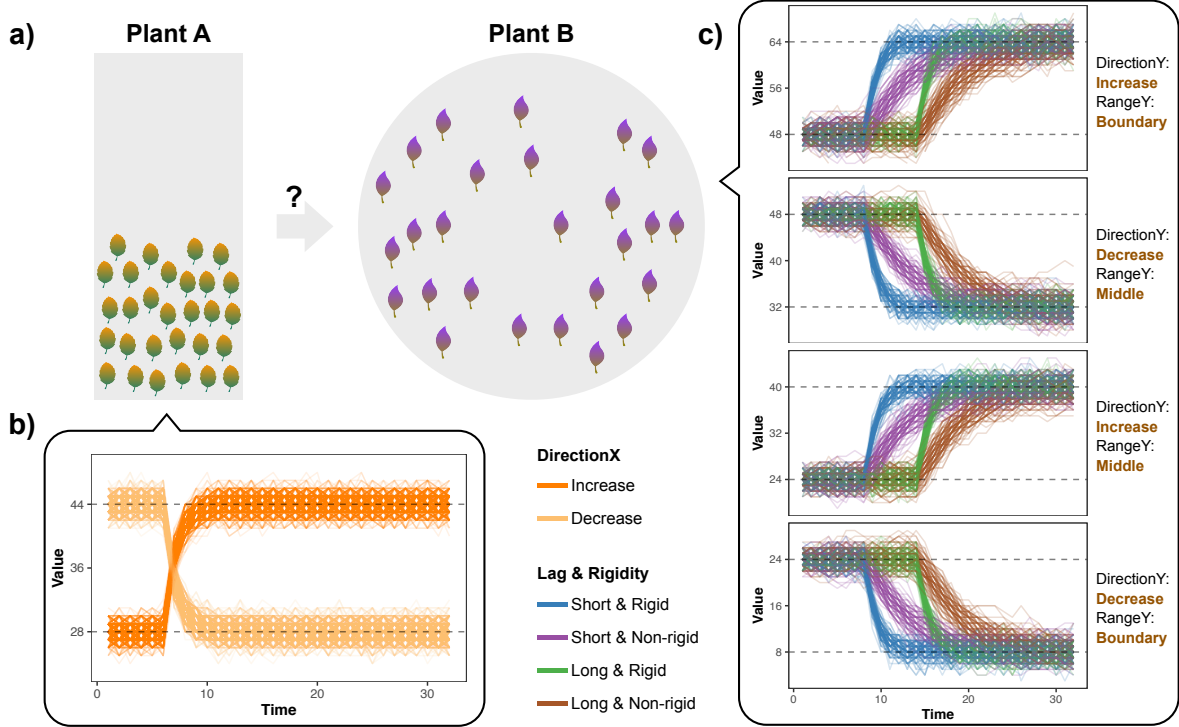


Figure 2: The cover story and experimental stimuli. a) Visualization of task interface. b) The actual runs of how X behaved in the experiment. c) How Y behaved in different conditions. Stimuli were generated individually for each trial.

Direction_Y. The *Direction_X* and hence *Slope* was randomly selected for each trial. Each trial involved a unique sequence generated by the OU process (Figure 2b and c). The order of trials was randomized independently for each participant.

Before starting the task, participants were instructed that a positive (regular) relationship is one in which an increase in the quantity of Plant A causes an increase in the quantity of Plant B, or equally that a decrease in the quantity of Plant A causes a decrease in the quantity of Plant B. A negative (inverse) relationship was described as one such that an increase in the quantity of Plant A causes a decrease in the quantity of Plant B, or equally that a decrease in the quantity of Plant A causes an increase in the quantity of Plant B. We added “(regular)” and “(inverse)” after the word positive or negative in order to emphasize that the question is about the relationship itself not about the input or the the outcome of the causal influence in the scenario (cf. Davis et al., 2020a). It was also emphasized figuring out the relationship would require a learner to observe both Plant A and B and that it would be impossible to work out the relationship by focusing on just one kind of plant. Participants had to pass comprehension check questions before starting the task.

Results

Since all stimuli in fact display a causal relationship, we code answers aligned with the underlying OU processes ($\beta = 1$ for positive and $\beta = -1$ for negative) as correct (1), and others including judgments of no connection, as incorrect (0). This accuracy serves as the primary index in this paper.

To examine the manipulated factors, we fit mixed-effects

logistic regression models using `lmerTest` in R, with Subject ID as a random effect. Under simple regressions with one predictor each, lower *Lag* ($z = 4.65$, $p < .001$), higher *Rigidity* ($z = 3.41$, $p < .001$), increasing *Direction_X* ($z = 3.94$, $p < .001$), increasing *Direction_Y* ($z = 4.43$, $p < .001$) and higher (closer to the boundaries) *Range_Y* ($z = 5.91$, $p < .001$) all relate to higher accuracy (Figure 3), while there was no evidence that *Slope* (positive or negative) made any difference to accuracy ($z = 0.93$, $p = .35$). This suggests there is no general difference between identifying positive and negative relationships (Figure 3). Since *Slope* could be seen as an interaction effect of *Direction_X* and *Direction_Y*, we later only focus on the five factors (*Lag*, *Rigidity*, *Direction_X*, *Direction_Y*, *Range_Y*) to test whether there are interactions between conditions.¹

Quantity changes We first focus on the two- and three-way interactions of three of the factors—*Direction_X* * *Direction_Y* * *Range_Y*—that determined the start and end points of X and Y (Figure 4). There was an interaction between *Direction_Y* and *Range_Y* ($z = 2.15$, $p = .03$), such that the boundary effects were stronger between Y ’s decreasing from around 24 to 8 vs. from 48 to 32, than between increasing from 48 to 64 vs. from 24 to 40 (Figure 2). Interestingly, there was also an interaction between *Direction_X* and *Range_Y* such that the boundary effect was stronger when the cause increased than decreased ($z = 5.32$, $p < .001$). This might indicate that participants’

¹ All main effects remained significant when we tested interactions so we did not report them again here.

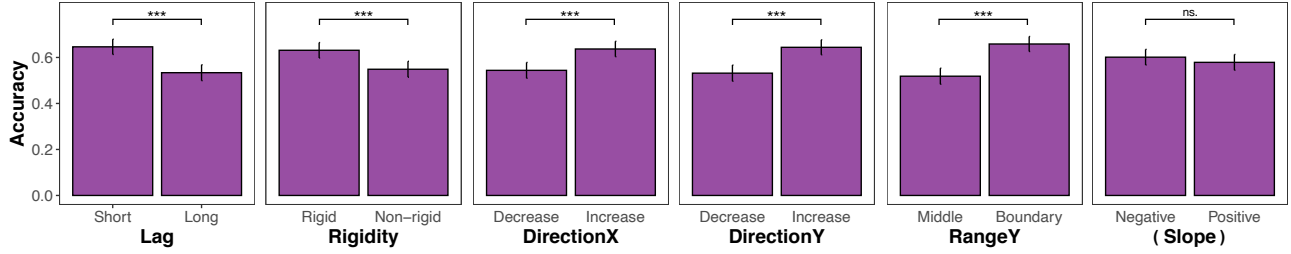


Figure 3: Accuracy separated by levels of different factors.

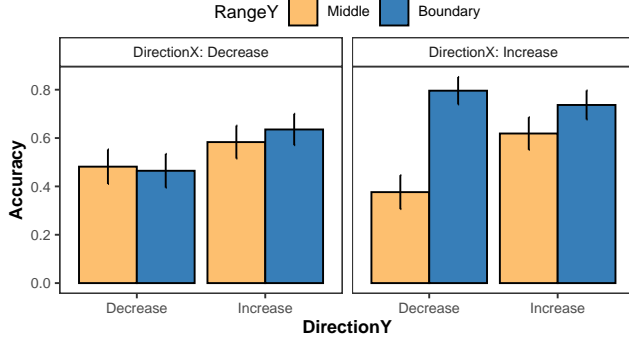


Figure 4: The interaction effect between $Direction_X$, $Direction_Y$, and $Range_Y$

processing of X and Y happened sequentially rather than in parallel, where the deficit of reasoning through X would influence the reasoning of Y . Finally there is a three-way interaction such that the boundary effect was largest when X increased and when Y decreased from 24 to 8 vs. from 48 to 32 (Figure 4).

Detailed dynamics We now investigate factors that influenced the trajectory of how Y arrived at its final value: $Lag * Rigidity$. There was no interaction between Lag and $Rigidity$ ($z = 1.12, p = .26$).

As for whether Lag or $Rigidity$ interact with $Direction_X$ or $Direction_Y$, we found no two or three-way interactions, but a four-way interaction ($z = 2.08, p = .04$). As shown in Figure 5, rigidity did not always affect learning. When both X and Y increase, there is an interaction between Lag and $Rigidity$ ($z = 2.21, p = .03$) that $Rigidity$ did not impact accuracy when the lag was short. We take this to suggest that there is unlikely to be a independent processing of rigidity. Rather, it seems to interact with other factors to influence how salient the Y 's change is. The interaction between Lag and $Rigidity$ was insignificant under other combinations of $Direction_X$ and $Direction_Y$.

$Range_Y$ did not interact with Lag or $Rigidity$ in any way, likewise there was no support for any four- or five-ways that involved $Range_Y$.

Choice frequency We finally checked the frequency of three choices (positive, negative, no relationship) under different conditions. Since the relationship cannot be determined by any single factor apart from $Slope$, we normatively would not expect any other single factor to predict

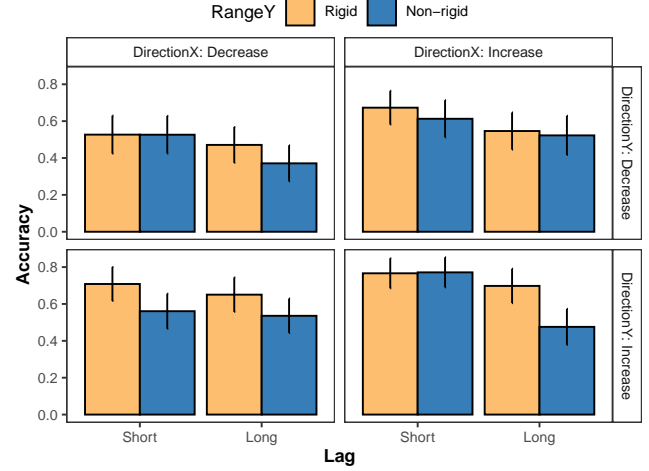


Figure 5: The interaction effect between $Rigidity$, Lag , $Direction_X$, and $Direction_Y$

the frequency of positive vs. negative choice. As shown in Figure 6, the relative frequency of positive vs. negative did not vary much depending on Lag (multinomial regression: $z = 1.44, p = .15$) or $Rigidity$ ($z = 0.35, p = .73$). The only difference is that people chose more “no relationship” when the causal lag was long or when Y 's change was non-rigid. However, the frequency of positive vs. negative did vary given $Direction_X$ or $Direction_Y$. Participants were more inclined to choose positive relationships when X increased, and choose negative relationships when X decreased ($z = 5.45, p < .001$). Similarly, they chose more positive relationships when Y increased, and more negative relationships when Y decreased ($z = 3.11, p = .002$). It means that they were influenced by the change direction of a single variable even when asked to focus on the relationship between two variables. Finally, the frequency difference between positive vs. negative was larger in the boundary than the central range condition ($z = 3.78, p < .001$). This is aligned with the finding that they performed best with the stimuli in which Y decreased to the lower bound as X increased.

Discussion

Everyday experience is one of endless small changes that occur from moment to moment. Making useful causal discoveries under these continuous dynamics may involve complex and hierarchical cognitive processes that cognitive science is just beginning to explain. In this paper we described a

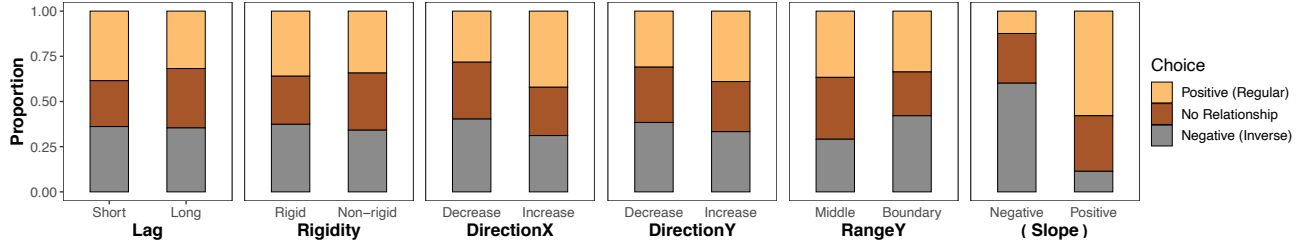


Figure 6: Choice frequency separated by levels of different factors

study that systematically investigates how several elemental aspects of continuous dynamics combine in shaping real-time dynamic causal inferences.

As has been found with discrete variables (Buehner & McGregor, 2006; Shanks et al., 1989; Lagnado & Speekenbrink, 2010), people more reliably identified a relationship when its causal lag was short than long. This is already a departure from standard non-cognitive models such as Granger causality that do not inherently favor any particular lag (Granger, 1969). Since the trajectory of Y 's change was identical for short- and long-lags except for the onset time, it is unlikely that Y 's changes under the long lag were harder to perceive. This could be more indicative of an inductive bias, or expectation, that causal lags are likely to be short, making causal attributions less certain under longer lag times. It is also possible that people had concluded no relationship within a interval and pay little attention to what happen afterward.

Accuracy was higher when the effect changed rigidly. This replicated the finding in Davis et al. (2020b) but in a cleaner passive learning setting where we controlled the total amount of change. One possibility is that people have an independent representation of rigidity and follow their causal intuitions by expecting effects to respond rigidly, especially when the cause changed rigidly, as in the current study. However, the result that rigidity interacted with lag and change direction implies a more complex mechanism. For example, rigidity may work interdependently with other factors to influence how perceptually salient the change is.

Judgments were influenced by the direction of change of both variables, with a general increase in accuracy when both were positive. Again, a purely statistical model such as a linear regression would not differentiate between increases and decreases. Participants were more accurate when they observed that cause increased or the effect increased, in alignment with the sensitivity to positive evidence in general (Kao & Wasserman, 1993; Newman et al., 1980). We also see hints of use of a heuristic of using the increase or decrease of one variable as a stand in for the direction of the relationship. It could be that people may have prototypes for how positive or negative relationships typically look in time-series data. This finding is of practical relevance to issues of creating effective data visualizations to communicate causal relationships (Soo & Rottman, 2020). One may want to transform variables so as to visualize them in ways that make use of inductive biases rather than require overriding of them.

Non-cognitive statistical models of continuous variables put little emphasis on the intercept when explaining associations. Indeed standard regression models treat is as linearly separable from other coefficients, we found participants performed better when Y 's change was easier to detect, i.e. closer to lower or upper bound of the range (Kim & Opfer, 2017; Thompson & Opfer, 2008).

Our results highlight several ways that people deviate from a simple idea of an idealized statistical timeseries learner. Future work we plan to explore a wider range of time dynamics and contrast models that represent causality either continuously or discretely. It is possible that people still use continuous representations (i.e. as something like an OU process in the head) but make different functional or parameterization assumptions that lead them to deviate from the directly reverse engineering the generative model for our stimuli. It is also possible that people abstract away to a higher level and more discrete representation, such as point events connected by parametric causal delays (Davis et al., 2020b; Gong & Bramley, 2020). For example, they may simplify the complex dynamics by thinking about them as events (X increased at $t = 1$) triggering another (Y increased/decreased at $t = 5$). This would unlock reasoning at the level of causal events, i.e. judging how likely two events separated by 4 seconds are to be causally related (Davis et al., 2020b; Gong & Bramley, 2020). In doing this, we hope these empirical findings can speak to the large topic of how bounded human learners (Simon, 1982), succeed in identifying useful causal representations of a continuous dynamic world.

In sum, we systematically tested factors that seemed likely to influence judgments about the nature of a single causal relationship. We found that, indeed, judgment patterns reflected plausible inductive biases—preferences for shorter lags, and positive changes—but also more perceptual factors—sensitivity to range of change relative to the variable's bounds, and the rigidity with which the effect adjusts to changes in its cause. The insensitivity of standard timeseries models to these factors suggests they have limited utility in accounting for human causal induction. We argue that careful work reverse engineering how human inductive biases and perceptual constraints combine in shaping causal representation is necessary for understanding the cognitive process of causal induction in a continuous world.

References

Bramley, N. R., Gerstenberg, T., Mayrhofer, R., & Lagnado,

- D. A. (2019). Intervening in time. In S. Kleinberg (Ed.), *Time and causality across the sciences* (pp. 86–115). Cambridge University Press.
- Buehner, M. J., & McGregor, S. (2006). Temporal delays can facilitate causal attribution: Towards a general timeframe bias in causal induction. *Thinking & Reasoning*, 12(4), 353–378.
- Cheng, P. W. (1997). From covariation to causation: A causal power theory. *Psychological Review*, 104(2), 367.
- Coenen, A., Rehder, B., & Gureckis, T. M. (2015). Strategies to intervene on causal systems are adaptively selected. *Cognitive Psychology*, 79, 102–133.
- Davis, Z., Bramley, N. R., & Rehder, B. (2020a). Causal structure learning in continuous systems. *Frontiers in Psychology*, 11, 244.
- Davis, Z., Bramley, N. R., & Rehder, B. (2020b). The paradox of time in dynamic causal systems. In S. Denison, M. Mack, Y. Xu, & B. Armstrong (Eds.), *Proceedings of the 42th annual conference of the cognitive science society* (pp. 808–814).
- Gong, T., & Bramley, N. R. (2020). What you didn't see: Prevention and generation in continuous time causal induction. In S. Denison, M. Mack, Y. Xu, & B. Armstrong (Eds.), *Proceedings of the 42th annual conference of the cognitive science society* (pp. 2908–2914).
- Granger, C. W. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica: Journal of the Econometric Society*, 424–438.
- Griffiths, T. L., & Tenenbaum, J. B. (2009). Theory-based causal induction. *Psychological Review*, 116(4), 661–716.
- James, W. (1890). *The principles of psychology*. Dover (2000 reprint).
- Kao, S.-F., & Wasserman, E. A. (1993). Assessment of an information integration account of contingency judgment with examination of subjective cell importance and method of information presentation. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 19(6), 1363–1386.
- Kim, D., & Opfer, J. E. (2017). A unified framework for bounded and unbounded numerical estimation. *Developmental Psychology*, 53(6), 1088–1097.
- Lagnado, D. A., & Speekenbrink, M. (2010). The influence of delays in real-time causal learning. *The Open Psychology Journal*, 3(1), 184–195.
- Newman, J. P., Wolff, W. T., & Hearst, E. (1980). The feature-positive effect in adult human subjects. *Journal of Experimental Psychology: Human Learning and Memory*, 6(5), 630–650.
- Pearl, J. (2000). *Causality*. New York: Cambridge University Press (2009 reprint).
- Sanborn, A. N., Griffiths, T. L., & Shiffrin, R. M. (2010). Uncovering mental representations with markov chain monte carlo. *Cognitive Psychology*, 60(2), 63–106.
- Shanks, D. R., Pearson, S. M., & Dickinson, A. (1989). Temporal contiguity and the judgement of causality by human subjects. *The Quarterly Journal of Experimental Psychology*, 41(2), 139–159.
- Simon, H. A. (1982). *Models of bounded rationality: Empirically grounded economic reason*. MIT press.
- Soo, K. W., & Rottman, B. M. (2018). Causal strength induction from time series data. *Journal of Experimental Psychology: General*, 147(4), 485–513.
- Soo, K. W., & Rottman, B. M. (2020). Distinguishing causation and correlation: Causal learning from time-series graphs with trends. *Cognition*, 195, 104079.
- Thompson, C. A., & Opfer, J. E. (2008). Costs and benefits of representational change: Effects of context on age and sex differences in symbolic magnitude estimation. *Journal of Experimental Child Psychology*, 101(1), 20–51.
- Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823.
- Zhang, Y., & Rottman, B. (2021). Causal learning with interrupted time series. In T. Fitch, C. Lamm, H. Leder, & K. Teßmar-Raible (Eds.), *Proceedings of the 43th annual conference of the cognitive science society* (pp. 1333–1339).