Session-3

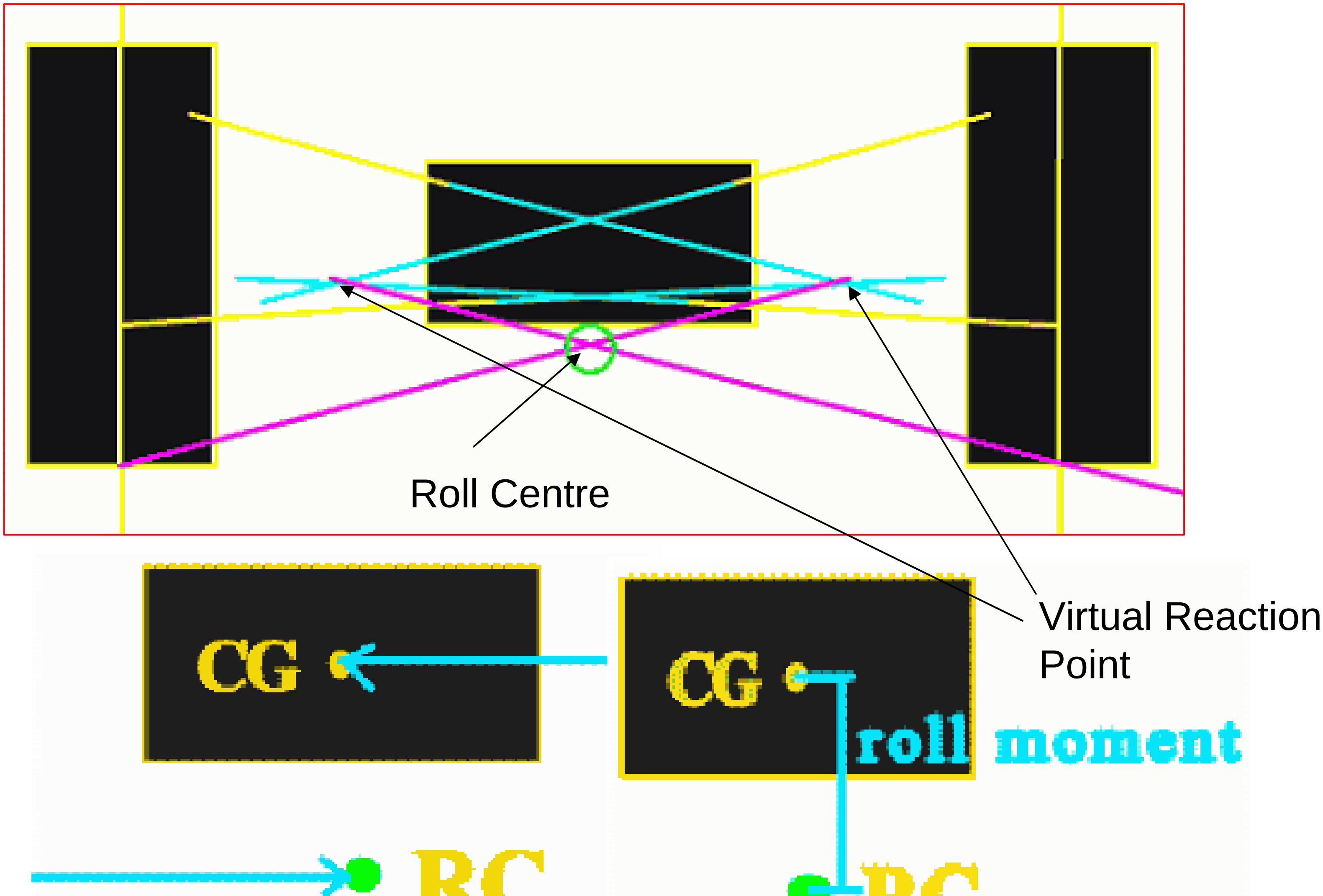
4.00 PM-5.30 PM

Vehicle Suspension-Roll Centre, Squat and Dive

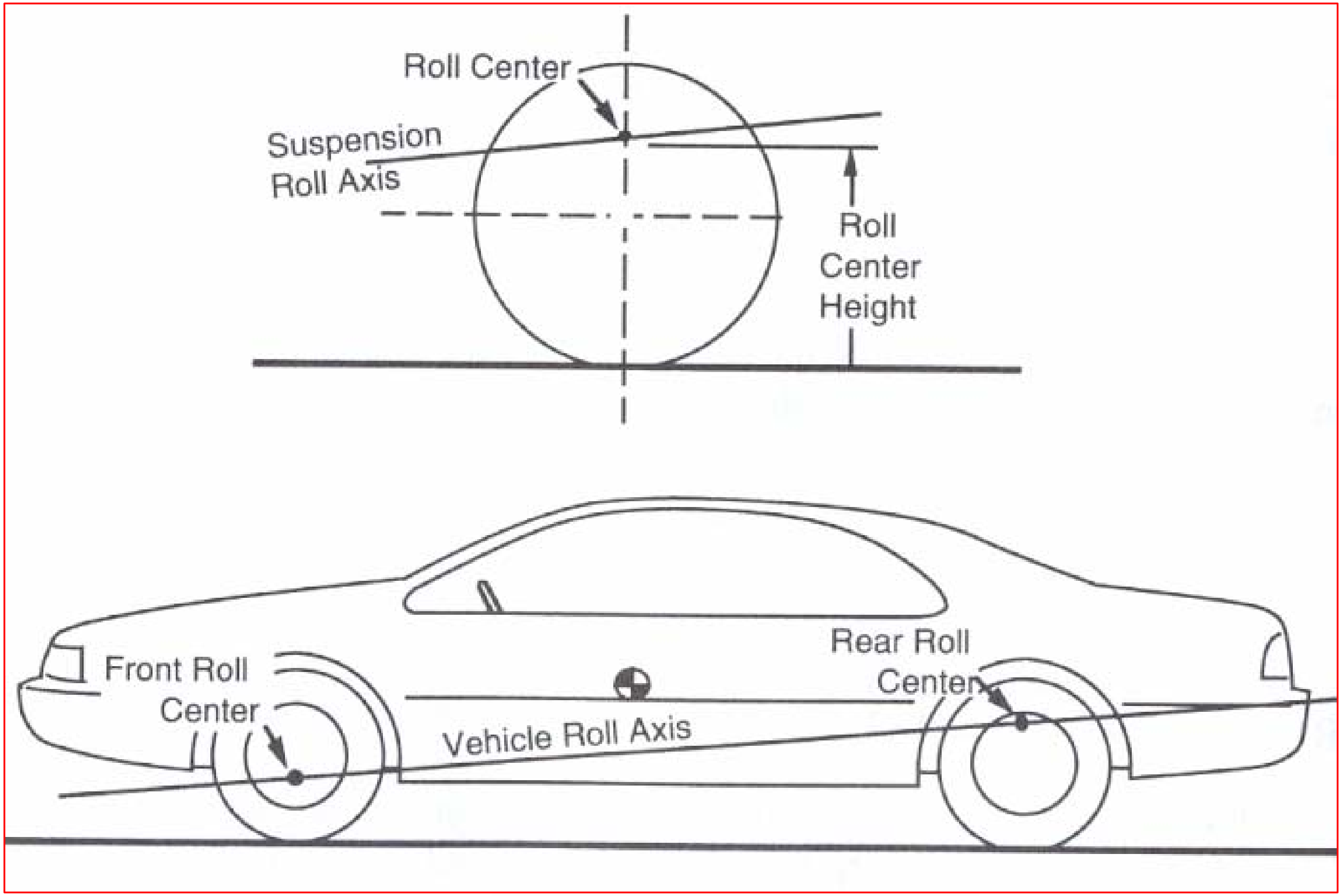
- Roll Centers, Roll Axis
- Squat and Dive

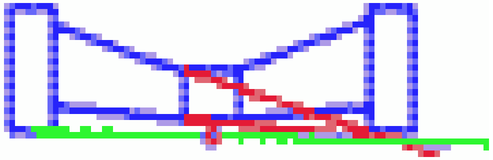
Roll Centre and Roll Axis

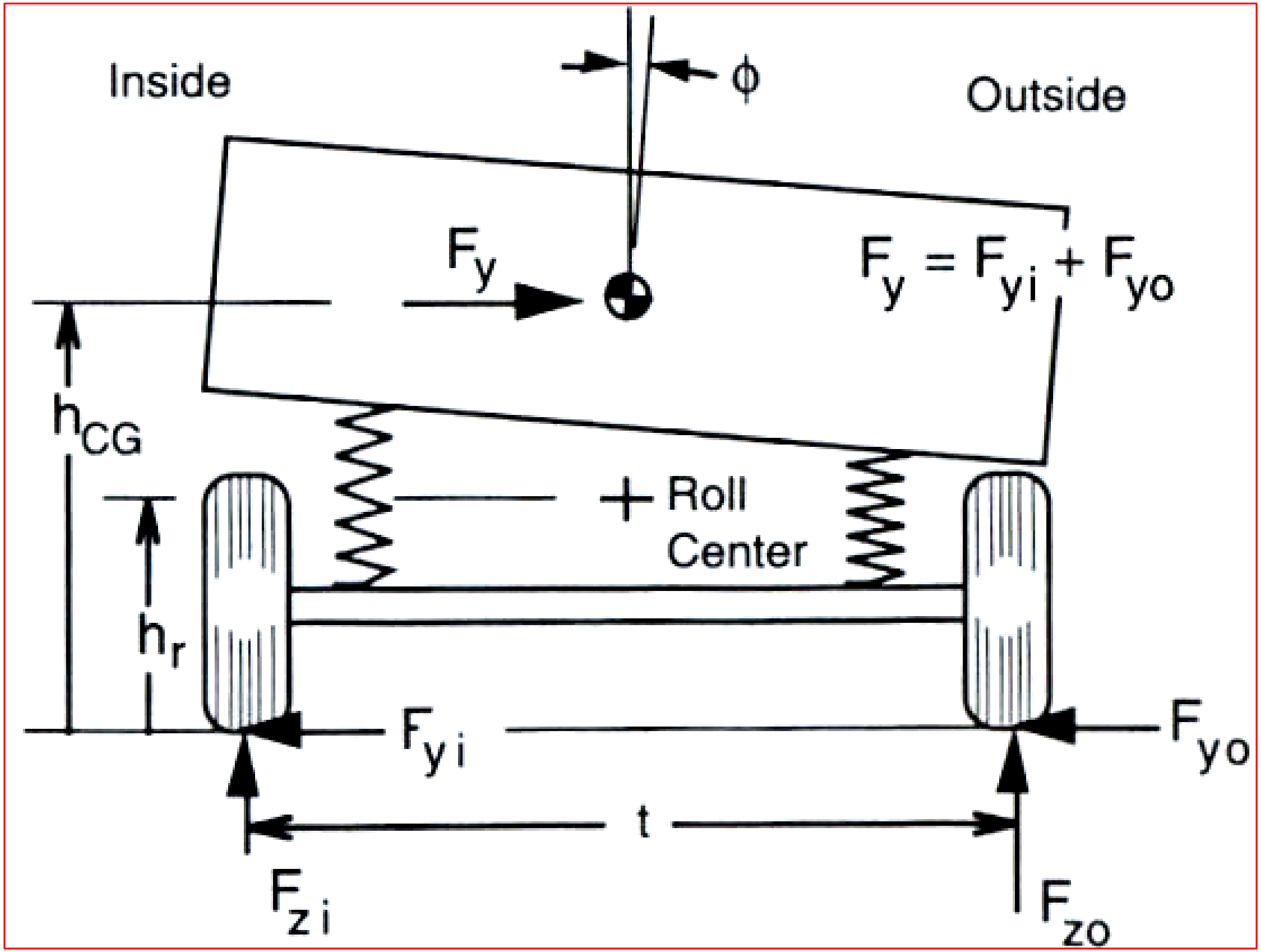
- Roll Centre and Roll Axis
  - The suspension is characterized by a Roll Centre
  - Roll Centre is a point at which the lateral forces are transferred from the axle to the sprung mass
  - Roll centre is a point on the body at which a lateral force application will produce no roll angle
  - It is the point around which the axle rolls when subjected to a pure roll moment
  - Roll Axis; It is the line connecting the roll centres of front and rear suspensions
  - In the median plane of the car, a transverse plane in which horizontal lateral forces applied to the rolling mass of the car will move the car sideways without causing it to roll
  - It follows that forces applied above the roll axis as at the CG will cause roll. Also that forces, if any applied below the roll axis will cause "Banking"



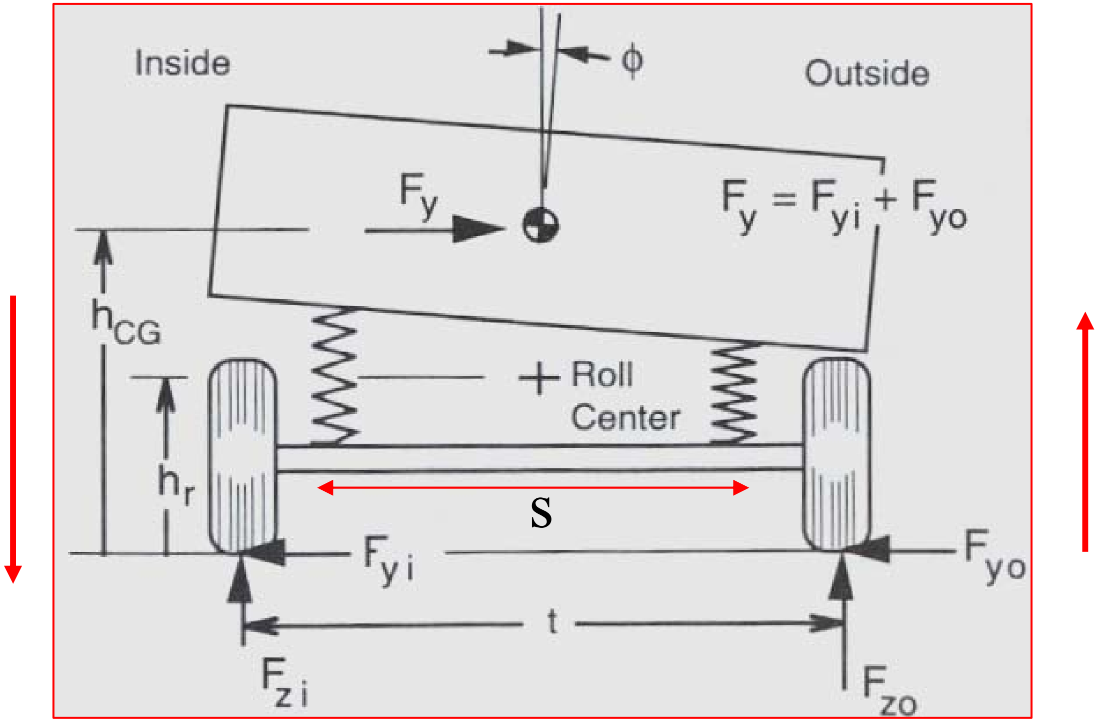
- Physically, the virtual reaction point is the intersection of the axes of any pair of suspension control arms.
- Mechanistically, it is the point where the compression/tension forces in the control arms can be resolved into a single lateral force


Roll Center
Suspension
Roll Axis
Roll
Center
Height
Front Roll
Center
Rear Roll
Center
Vehicle Roll Axis




Inside
ϕ
Outside
F_y
$F_y = F_{yi} + F_{yo}$
h_{CG}
Roll Center
h_r
F_{yi}
F_{yo}
t
F_{zi}
F_{zo}

Suspension Roll Stiffness

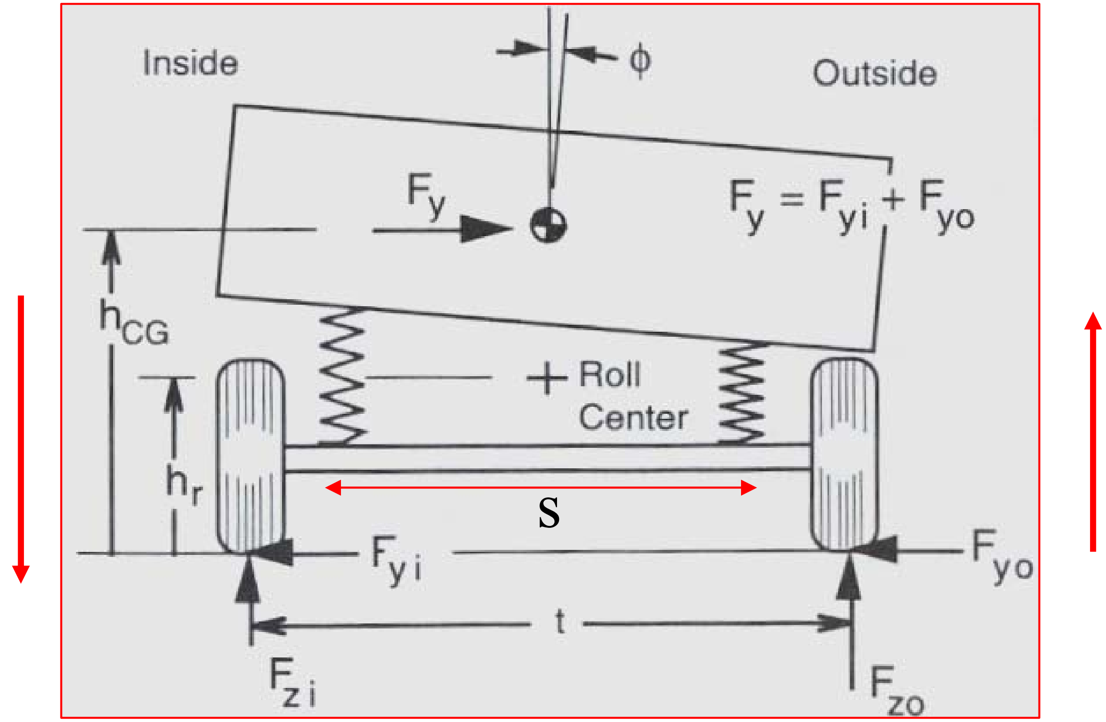


K_ϕ : Roll stiffness of the suspension

K_s : Vertical Spring rate of the left and right springs

s: Lateral separation between the springs

ϕ: Roll angle of the body



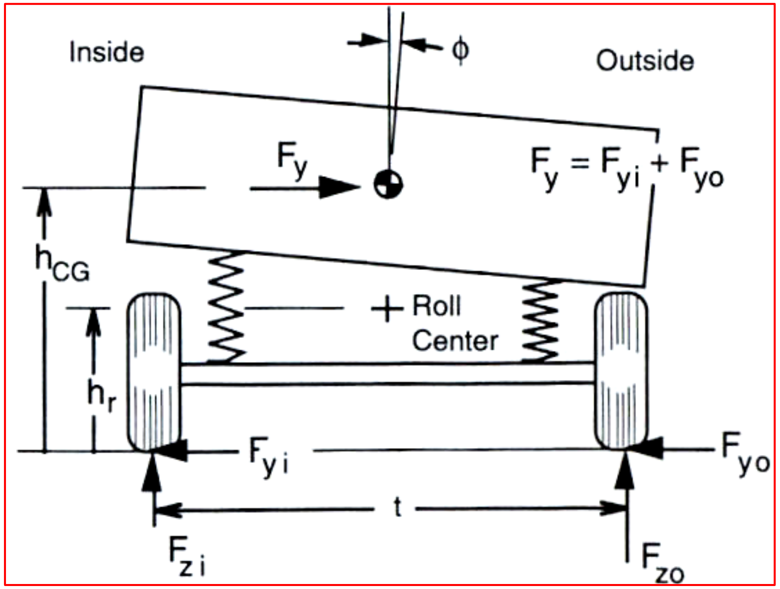
$$\sum M_{cG} = \left(K_s\left(\frac{s}{2}\right)\phi\right)\frac{s}{2} + \left(K_s\left(\frac{s}{2}\right)\phi\right)\frac{s}{2} = \frac{1}{2}K_s s^2\phi = K_\phi \phi$$

If a roll bar is included then

$$\sum M_{cG} = \frac{1}{2}K_s s^2\phi + K_r\phi \;= (K_\phi + K_r)\phi$$

$$\mathrm{K}_\varphi = \text{Roll stiffness of the suspension} = 0.5\mathrm{K}_s s^2$$

Lateral Load Transfer

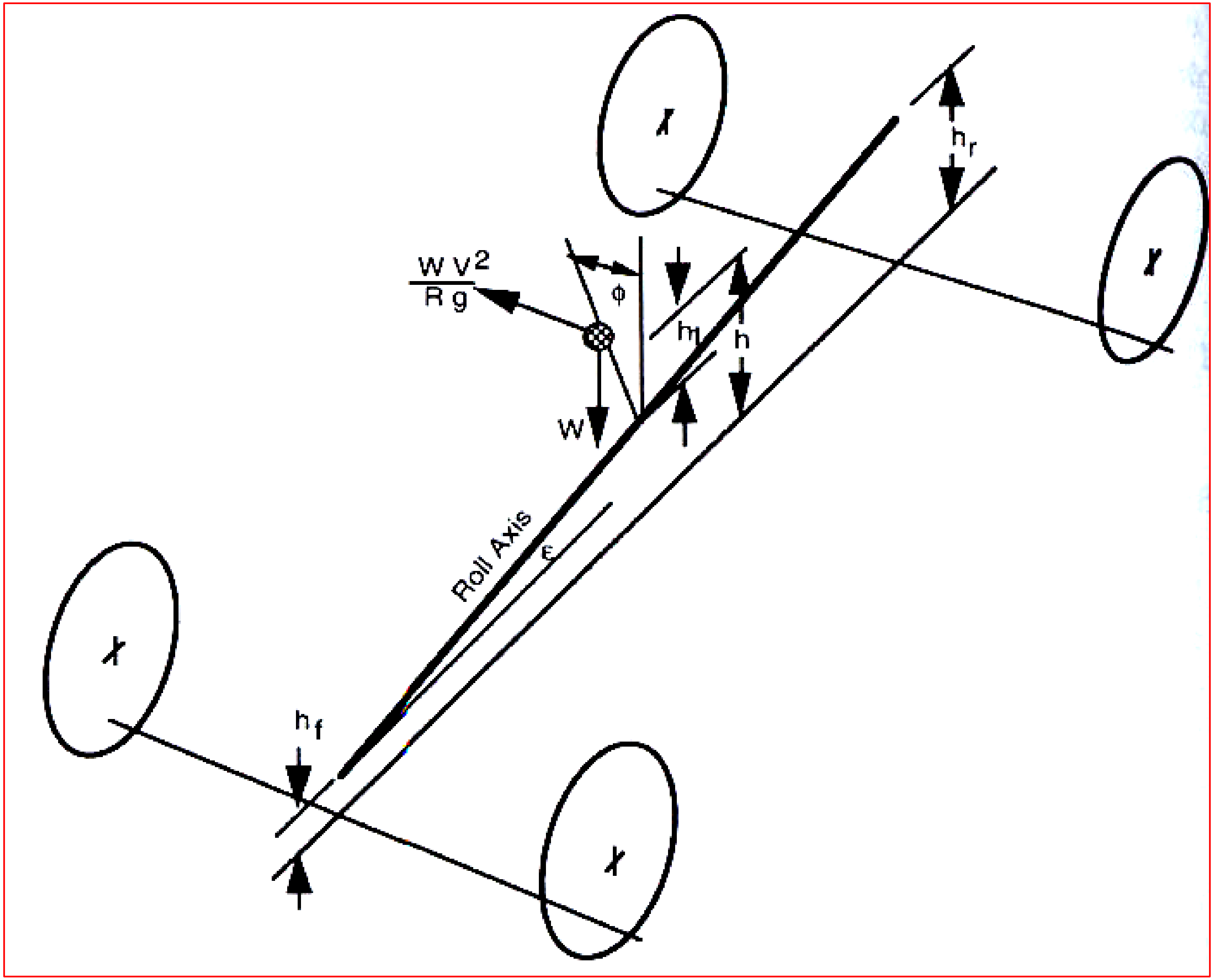


$$F_{z0} - F_{zi} = \frac{2F_y h_r}{t} + \frac{2K_\phi \phi}{t}$$

$$\text{Lateral load transfer due to cornering force} = \frac{2F_y h_r}{t}$$

$$\text{Lateral load transfer due to vehicle roll} = \frac{2K_\varphi \varphi}{t}$$

Roll Angle and Roll Rate



Roll Angle and Roll Rate

$$Roll\ angle = \phi = \frac{Wh_1V^2/(Rg)}{K_{\phi f} + K_{\phi r} - Wh_1}$$

$$\phi = \frac{Wh_1 a_y}{K_{\phi f} + K_{\phi r} - Wh_1}$$

$$Roll\ \ Rate = \frac{d\phi}{da_y} = \frac{Wh_1}{K_{\phi f} + K_{\phi r} - Wh_1}$$

The roll rate is usually K in the range of 3 to 7 degrees/g on typical passenger cars

Roll Moment

Roll Moments

$$M'_{\phi f} = K_{\phi f}\frac{Wh_1V^2/(Rg)}{K_{\phi f}+K_{\phi r}-Wh_1} + W_f h_f \frac{V^2}{Rg} = \Delta F_{zf} t_f$$

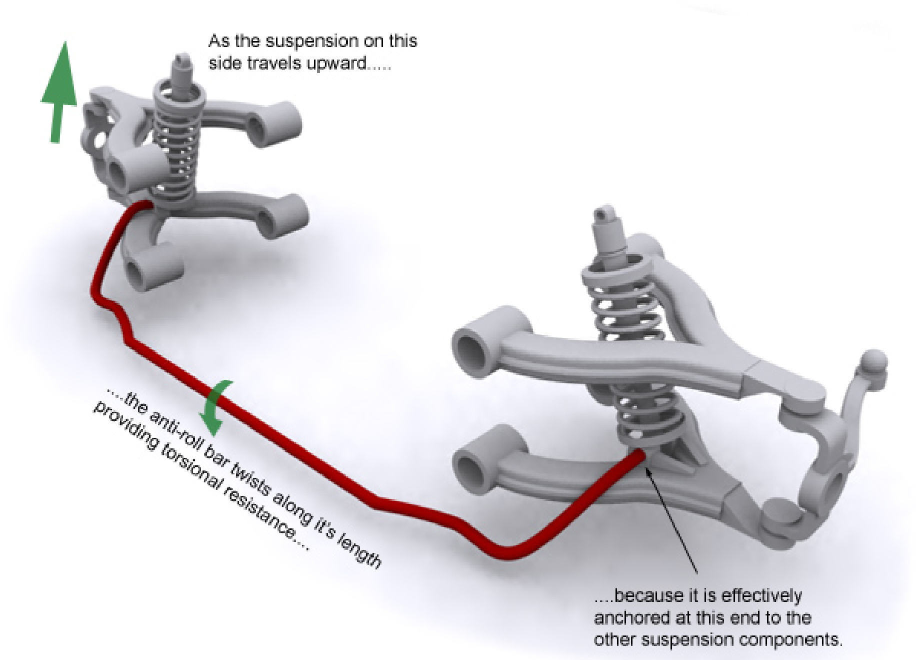
$$M'_{\phi r} = K_{\phi r}\frac{Wh_1V^2/(Rg)}{K_{\phi f}+K_{\phi r}-Wh_1} + W_r h_r \frac{V^2}{Rg} = \Delta F_{zr} t_r$$

The Roll moments magnitude depend on $K_{\Phi f}$ and $K_{\Phi t}$ which in turn depend on suspension stiffness

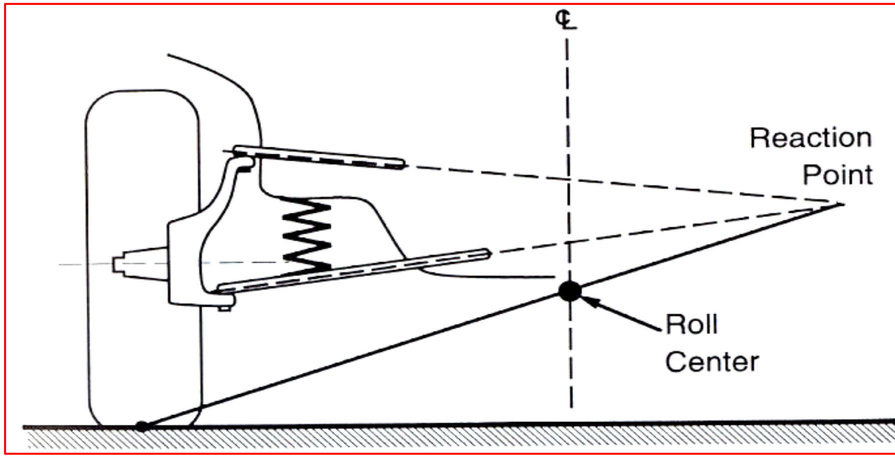
1. In general, the roll moment distribution on vehicles tends to be biased toward the front wheels due to a number of factors:
2. Relative to load, the front spring rate is usually slightly lower than that at the rear (for flat ride), which produces a bias toward higher roll stiffness at the rear. However, independent front suspensions used on virtually all cars enhance front roll stiffness because of the effectively greater spread on the front suspension springs.
3. Designers usually strive for higher front roll stiffness to ensure under-steer in the limit of cornering.

DIMENSIONS	
Overall Length (mm)	3565
Overall Width (mm)	1595
Overall Height (mm)	1550
Wheelbase (mm)	2380
Ground Clearance (mm)	165
Front Track (mm)	1400
Rear Track (mm)	1385
Fuel Tank capacity (l)	35

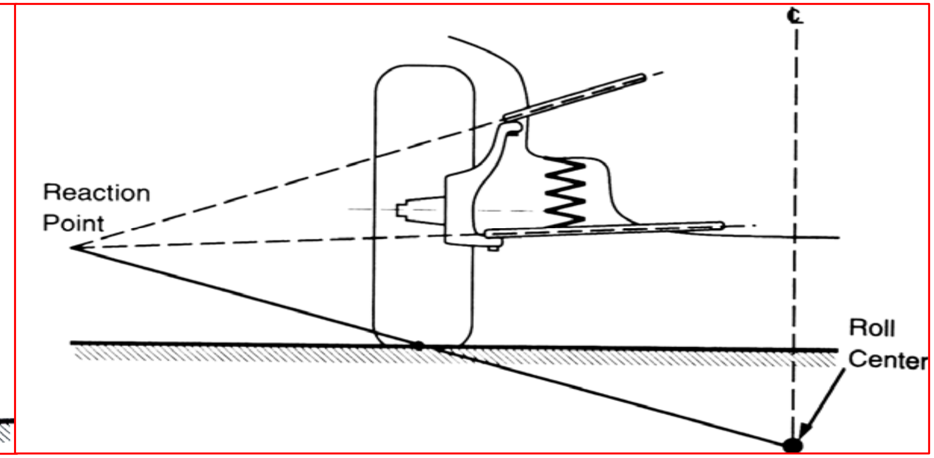
4. Stabilizer bars are often used on the front axle to obtain higher front roll stiffness.
5. If stabilizer bars are needed to reduce body lean, they may be installed on the front or the front and rear. Caution should be used when adding a stabilizer bar only to the rear because of the potential to induce unwanted oversteer.



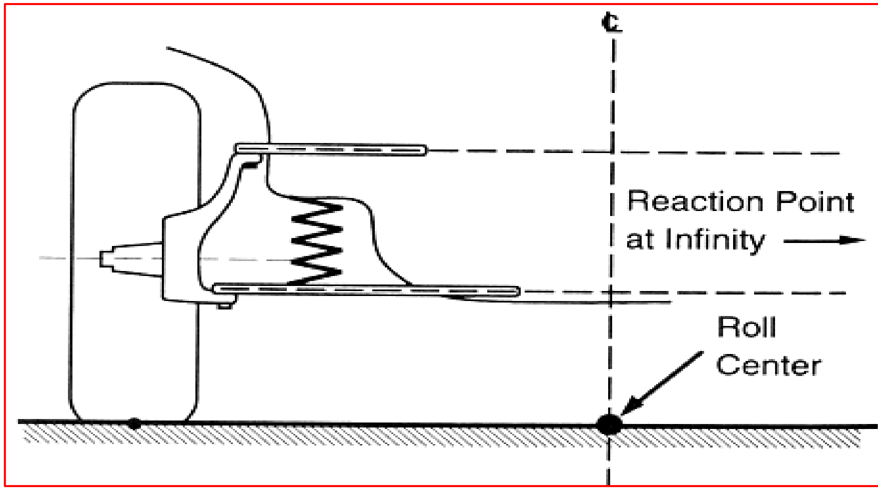
Independent Suspension Roll Centers



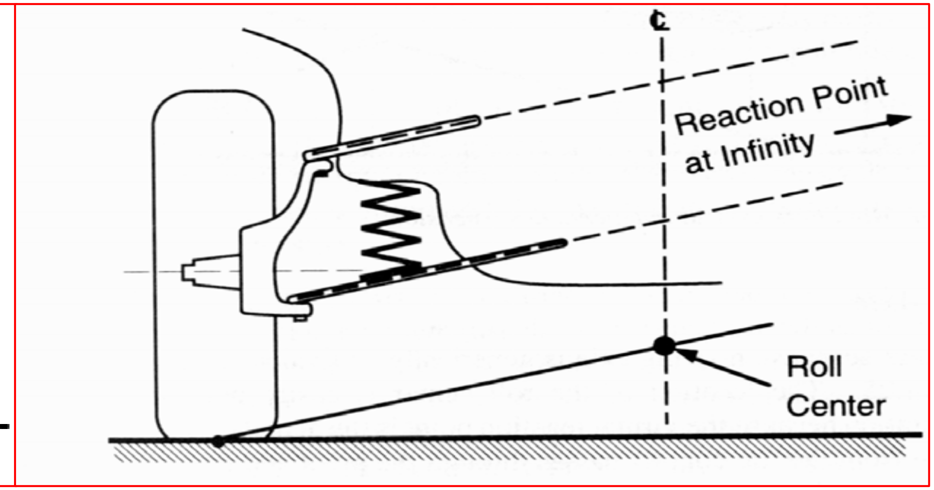
Positive swing arm independent suspension



Negative swing arm independent suspension

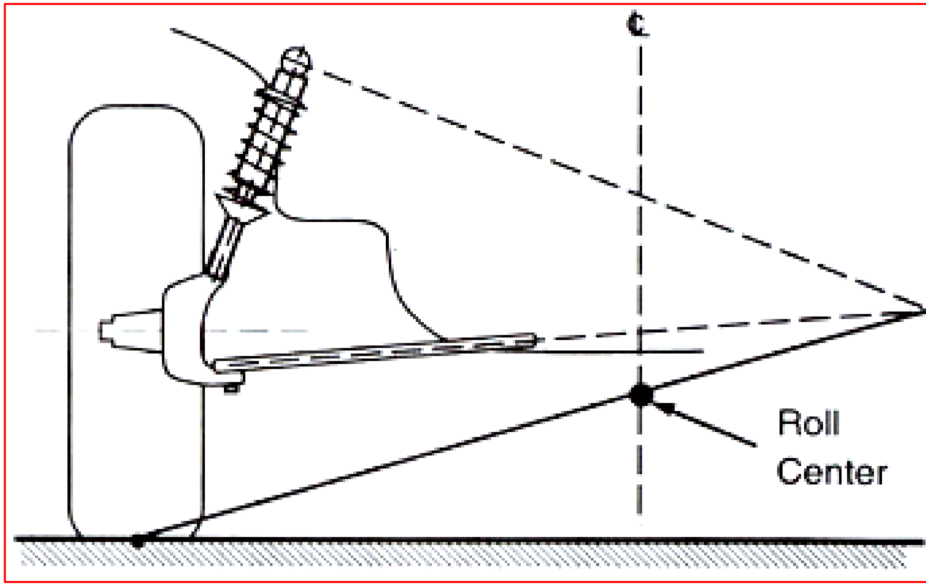


Parallel horizontal link independent suspension

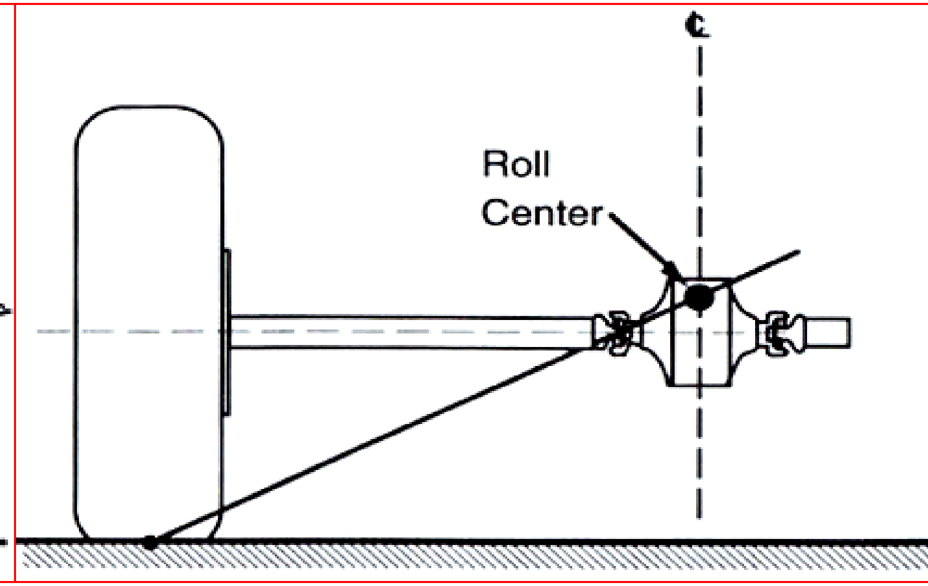


Inclined parallel link independent suspension

Independent Suspension Roll Centers

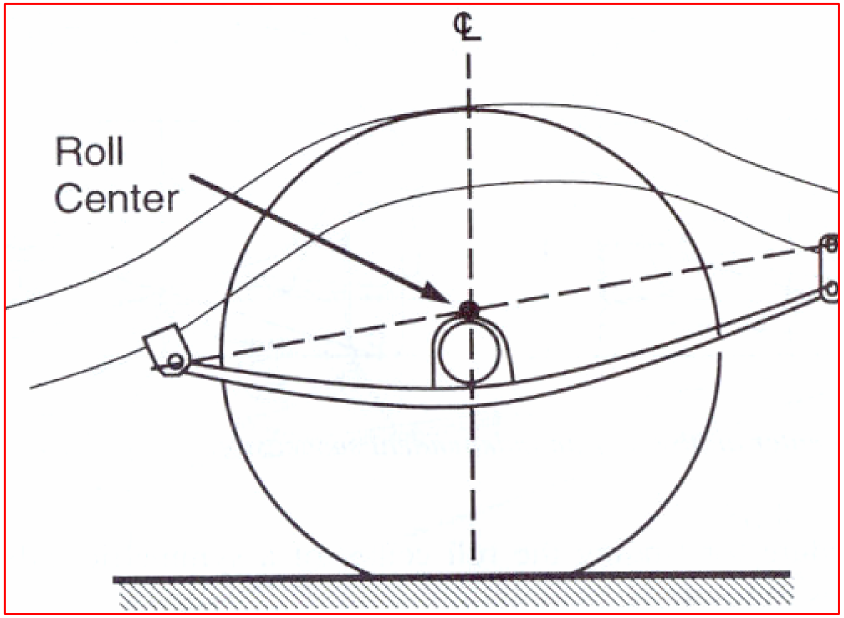


Macpherson strut independent suspension

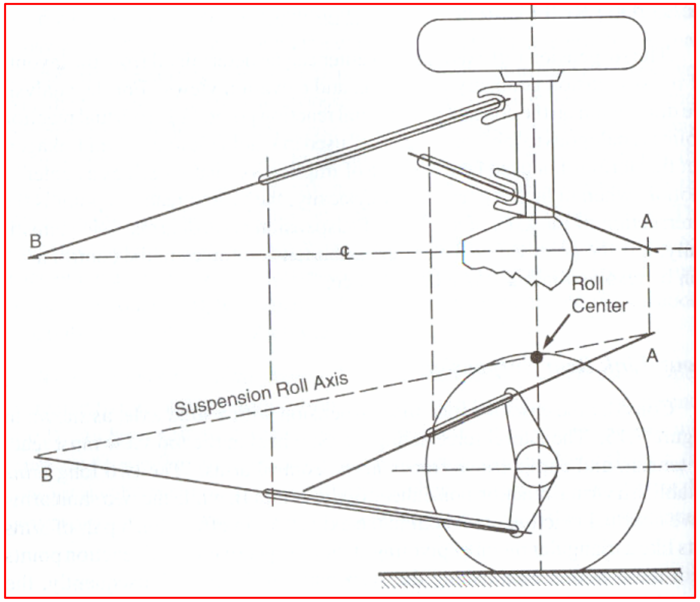


Swing axle independent suspension

Roll Centers of Dependent Suspension

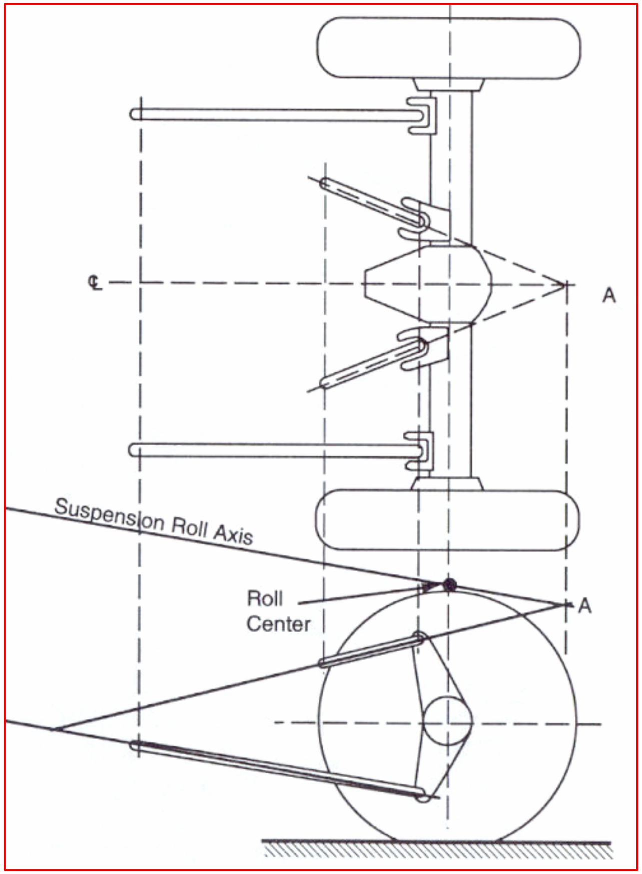


Hotchkiss Suspension

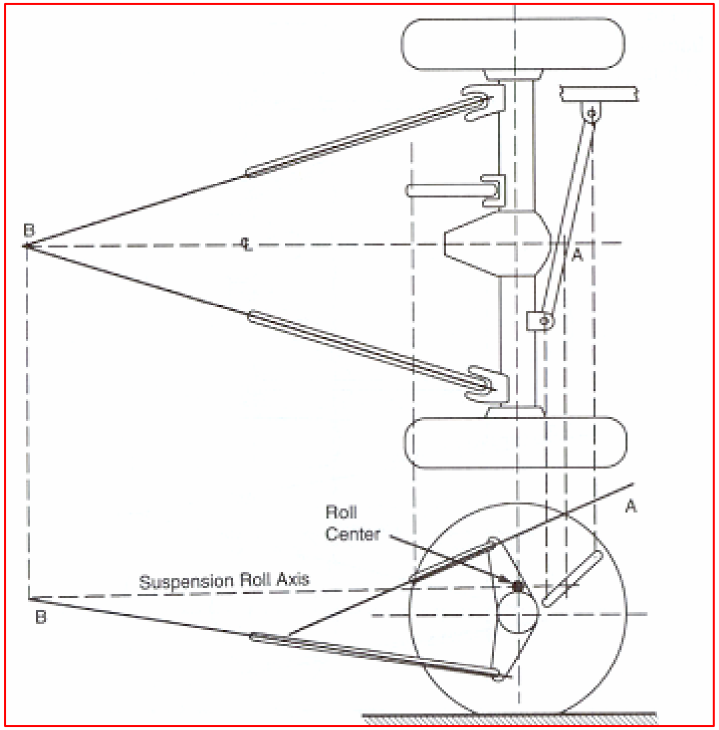


Four Link Rear Suspension

Roll Centers of Dependent Suspension



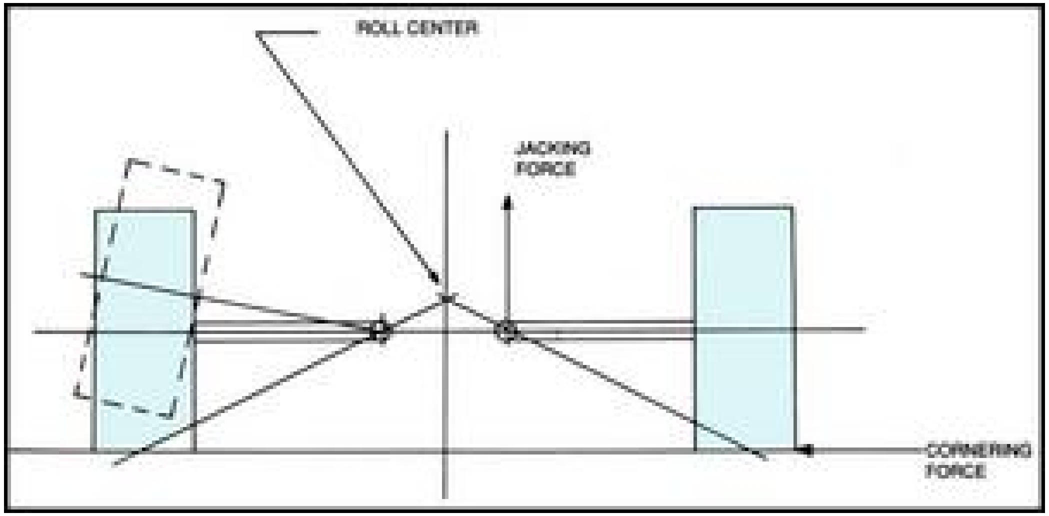
Four Link with Parallel Arms



Three Link Rear Suspension

What is the Effect if Roll Centre Located on Ground or Located Below the Ground?

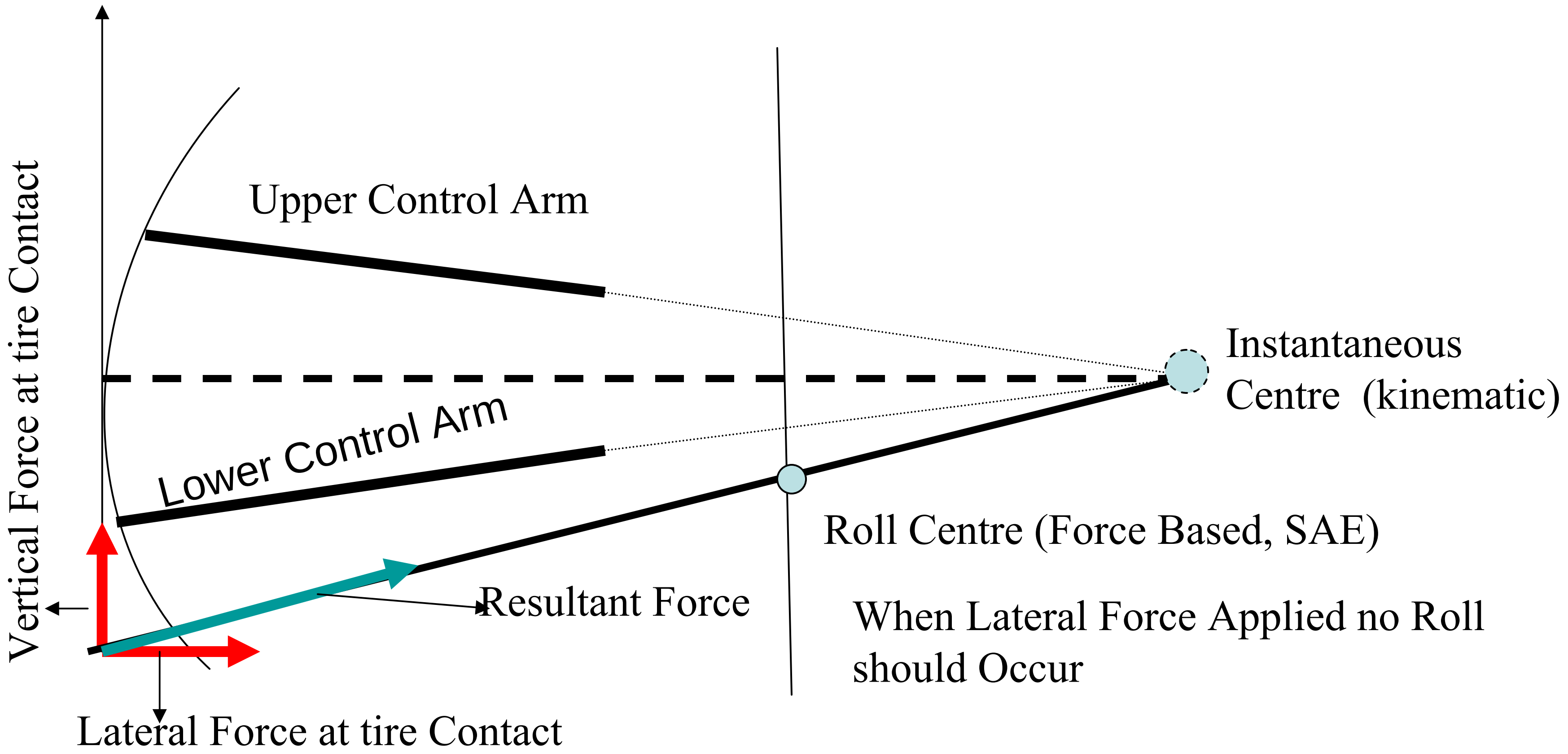
- Jacking forces are the sum of the vertical force components experienced by the suspension links. The resultant force acts to lift the sprung mass if the roll centre is above ground, or compress it if underground. Generally, the higher the Roll Centre, the more Jacking force is experienced



Instant Centre

- Due to the fact that the wheel and tire's motion is constrained by the suspension links on the vehicle, the motion of the wheel package in the front view will scribe an imaginary arc in space with an “instantaneous center" of rotation at any given point along its path. The instant center for any wheel package can be found by following imaginary lines drawn through the suspension links to their intersection point.
- A component of the tire's force vector points from the contact patch of the tire through instant center. The larger this component is, the less suspension motion will occur. Theoretically if the resultant of the vertical load on the tire and the lateral force generated by it points directly into the instant center, the suspension links will not move. In this case all weight transfer at that end of the vehicle will be geometric in nature. This is key information used in finding the force-based roll center as well.
- In this respect the instant centers are more important to the handling of the vehicle than the kinematic roll center alone, in that the ratio of geometric to elastic weight transfer is determined by the forces at the tires and their directions in relation to the position of their respective instant centers.

Kinematic and Force Roll Centre



If the above condition is met the suspension will not move, otherwise suspension moves

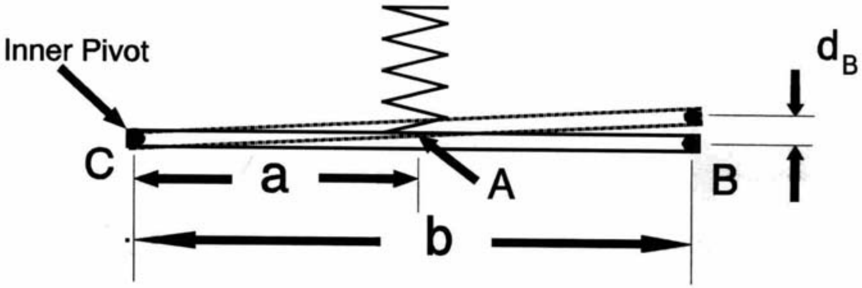
Spring and Wheel Rates

- Spring rates
  - It is the ratio of vertical load on spring to its displacement (N/mm)
- Wheel rates
  - It is the ratio of load acting on the wheel/rise of the wheel
- Motion ratios
  - The displacement relationship between the spring and the wheel determines the actual rate the wheel works against for any spring rate. This displacement relationship may be defined as a motion ratio. The rate at the wheel is defined as the wheel rate (K_w). The rate of the spring itself is called the spring rate (K_s). The displacement relationship is a function of both spring position on the load carrying member and the angular orientation of the spring to that member.

Motion Ratio Analysis

- From the simple lever system a number of relationships can be drawn.

$$F_B = F_A\left[\frac{a}{b}\right]$$

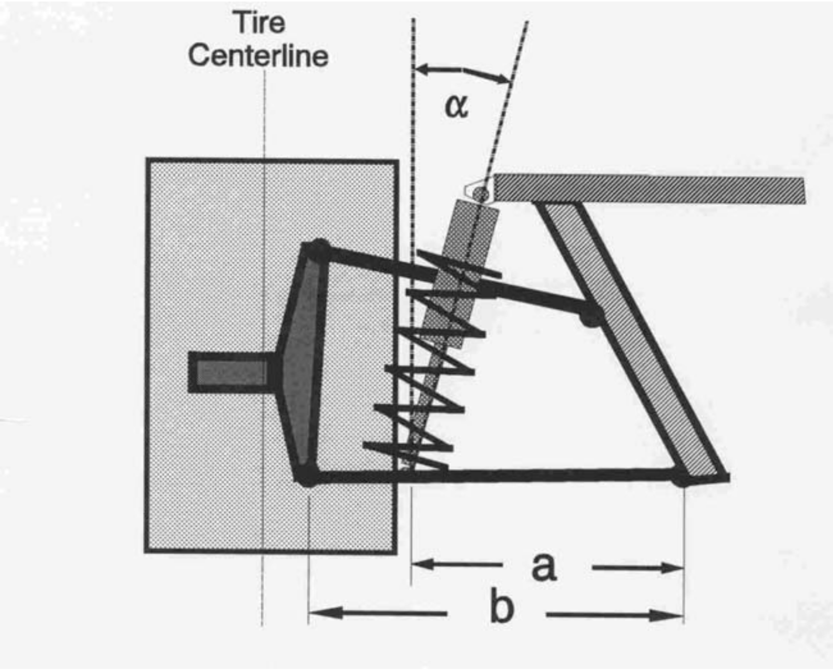


$$d_B = d_A\left[\frac{b}{a}\right]$$

$$\frac{F_B}{d_B} = k_B = \frac{F_A\left(\frac{a}{b}\right)}{d_A\left(\frac{b}{a}\right)} = k_A\left(\frac{a}{b}\right)^2$$

Motion Ratio Analysis

- Motion Ratio in the Road Vehicle.
  - The motion ratio describes the displacement ratio between the spring and the centerline of the wheel. The motion ratio squared times the spring rate gives the wheel rate.



Motion Ratio Analysis

- Using the previous analysis and Figure , the following apply.

$$K_w = K_s \left(\frac{a}{b}\right)^2 \text{Cos}^2 \alpha$$

  – The above analysis assumes minimal camber change at the wheel.

  – The motion ratio can be determined experimentally and the measured distance ratio squared for an accurate value.

$$K_w = K_s \left(\frac{\textit{travel along spring axis}}{\textit{vertical travel of wheel centerline}}\right)^2$$

Squat and Dive

- **Squat** - Squat is the dipping of a car's rear end that occurs during hard acceleration.
- Squat is caused by a load transfer from the front to the rear suspension during acceleration.
- **Dive**- Dive is the dipping of a car’s front end that occurs during braking
- Dive is caused by a load transfer from the rear to the front suspension during braking

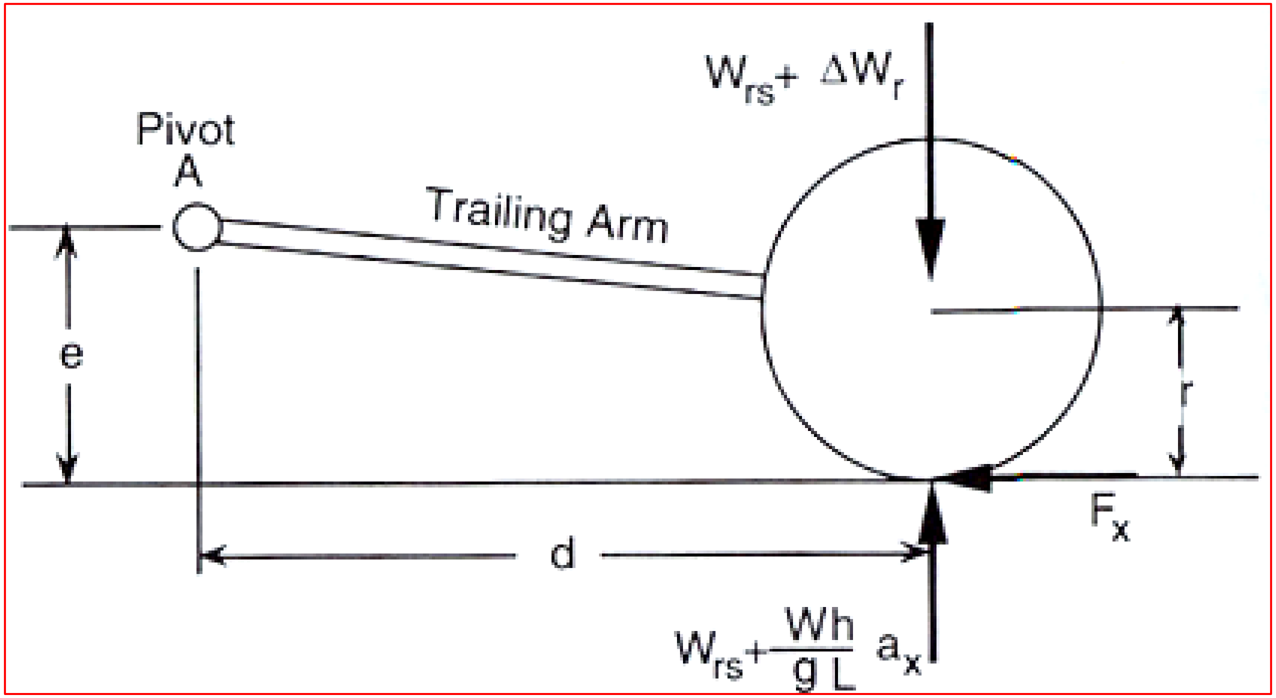
Anti-Squat and Anti-Pitch Suspension Geometry

- During acceleration the load on the rear wheels increases due to longitudinal weight transfer. The load on the rear axle is:

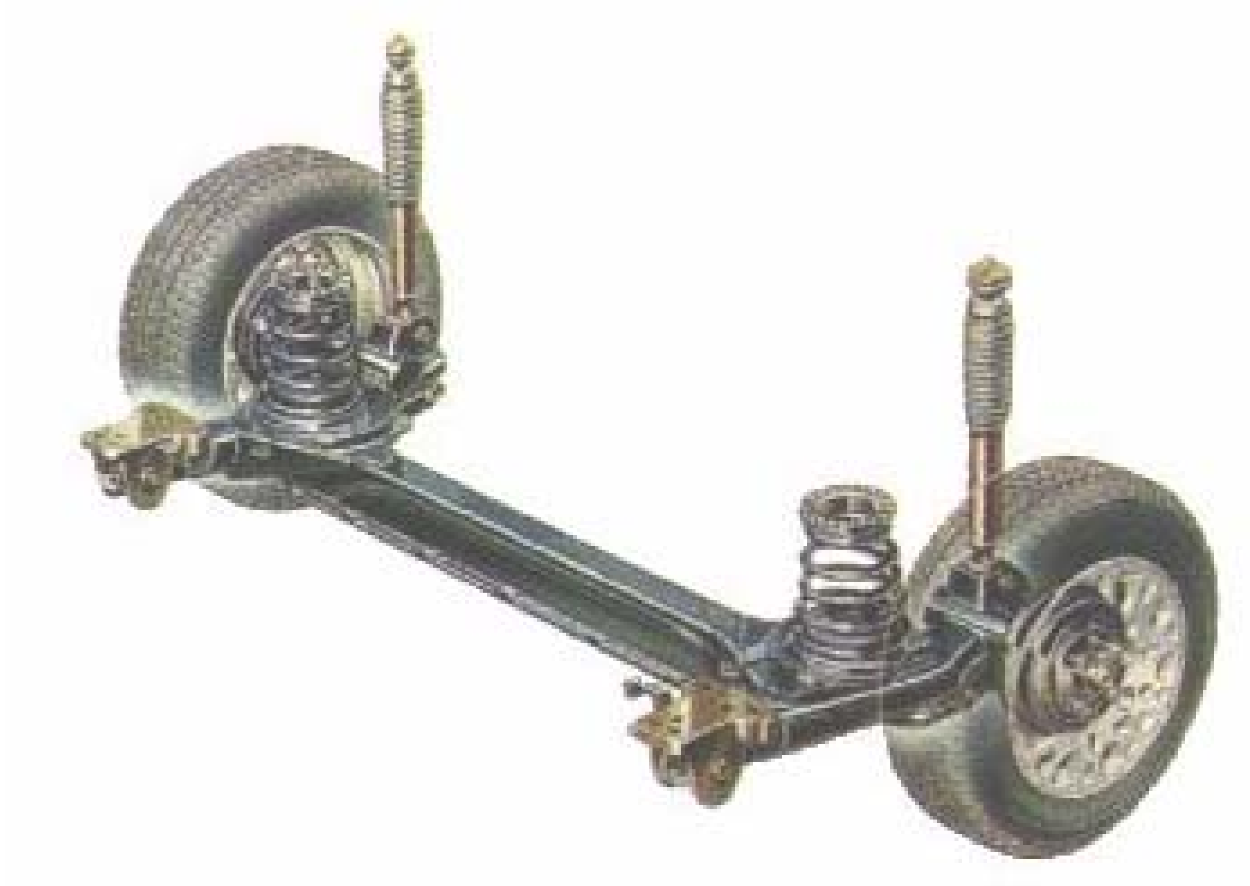
$$W_r = W\left(\frac{b}{L} + \frac{a_x}{g}\frac{h}{L}\right)$$

- The second term on the right side of this equation is the weight transfer effect.
- The weight is transferred to the axle and wheels principally through the suspension. Therefore, there is an implied compression in the rear suspension which, in the case of rear-drive vehicles, has been called "Power Squat."
- Concurrently, there is an associated rebound in the front suspension. The combination of rear jounce and front rebound deflections produces vehicle pitch. Suspension systems may be designed to counteract the weight transfer and minimize squat and pitch.

Trailing Arm Analysis-Rear Solid Drive Axle



- **Trailing arm** - A suspension element consisting of a longitudinal member that pivots from the body at its forward end and has a wheel hub rigidly attached to its trailing end.



Twist Beam Suspension

Rear Solid Drive Axle

- The system is analyzed by applying NSL for the torques around the pivot point "A." The sum of these torques must be zero when the system is in equilibrium.

$$\Sigma M_A = W_{rs} d + \frac{W}{g}\frac{h}{L} a_x d - W_{rs} d - \Delta W_r d - F_x e = 0$$

Where,

W_{rs} = Static load on the axle = Static load in the suspension

ΔW_r = Change in the suspension load under acceleration

This equation can be solved for the change in rear suspension load.

$$\Delta W_r = \frac{W}{g}\frac{h}{L}a_x - F_x\frac{e}{d} = K_r\delta_r$$

Where:

Kr = Rear suspension spring rate

δr = Rear suspension deflection (positive in jounce)

The front suspension is undergoing a rebound deflection because of the longitudinal load transfer, and has a magnitude of :

$$\Delta W_f = -\frac{W}{g}\frac{h}{L}a_x = K_f\delta_f$$

The pitch angle of the vehicle, θ_p, during acceleration is simply the sum of the suspension deflections divided by the wheelbase. Thus we can write:

$$\theta_p = \frac{\delta_r - \delta_f}{L} = \frac{1}{L}\frac{W}{g}\frac{h}{L}\frac{a_x}{K_r} - \frac{1}{L}\frac{F_x}{K_r}\frac{e}{d} + \frac{1}{L}\frac{W}{g}\frac{h}{L}\frac{a_x}{K_f}$$

Since F_x is simplify the mass times the acceleration, $(W/g)a_x$, the equation can be written :

$$\theta_p = \frac{1}{L}\frac{W}{g}\frac{h}{L}\frac{a_x}{K_r} - \frac{1}{L}\frac{W}{g}\frac{a_x}{K_r}\frac{e}{d} + \frac{1}{L}\frac{W}{g}\frac{h}{L}\frac{a_x}{K_f}$$

$$\theta_p = \frac{1}{L}\frac{W}{g}a_x\left(\frac{1}{K_r}\frac{h}{L} - \frac{1}{K_r}\frac{e}{d} + \frac{1}{K_f}\frac{h}{L}\right)$$

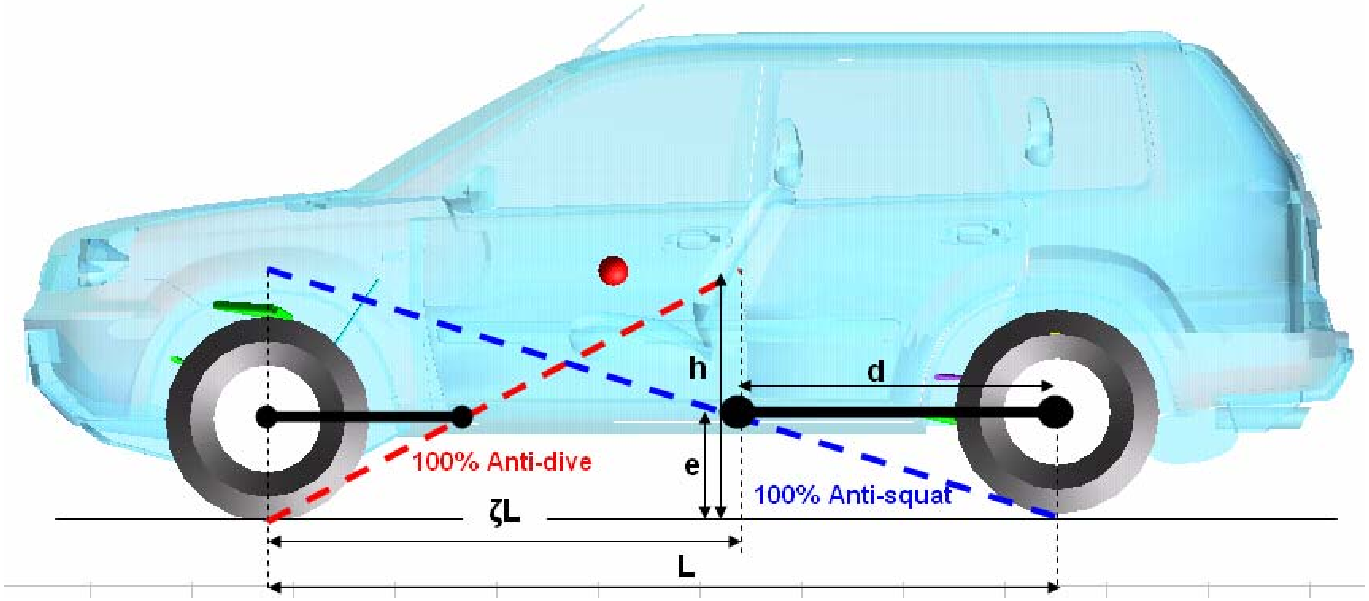
From this equation it is easy to show that zero pitch angle is achieved when the following condition is satisfied :

$$\frac{e}{d} = \frac{h}{L} + \frac{h}{L}\frac{K_r}{K_f}$$

Condition for anti-squat

Rear suspension lift to compensate rebound of front suspension

- The first term on the right-hand side corresponds to the condition by which anti-squat is achieved on the rear suspension.
- That is, if $e/d = h/L$, the rear suspension will not deflect (jounce) during acceleration. The degree to which this is achieved is described as the percent anti-squat.
- For example, if $e/d = 0.5\ h/L$, the suspension is said to be 50% anti-squat. Since h/L is in the vicinity of 0.2 for most passenger cars, full anti-squat generally requires an effective trailing arm length of about five times the elevation of "e."
- The anti-squat equation ($e/d = h/L$) defines a locus of points extending from the tire contact point on the ground to the height of the CG over the front axle.


h
d
100% Anti-dive
e
100% Anti-squat
ζL
L

- Locating the trailing arm pivot at any point on this line will provide 100% anti-squat.
- Satisfying the equation with inclusion of the second term implies that the rear suspension will lift to compensate for rebound of the front suspension, thereby keeping the vehicle level.
- The complete equation may be interpreted as the full anti-pitch relationship. Because the ratio of suspension stiffnesses is nominally 1, the anti-pitch condition is approximately:

$$\frac{e}{d} \approx \frac{h}{L} + \frac{h}{L} = 2\,\frac{h}{L} \qquad \text{(Full anti-pitch)}$$

- The locus of points for anti-pitch extends from the tire contact point on the ground to the height of the CG at the mid-wheelbase position.
- Anti-pitch is achieved when the trailing arm pivot is located on the line from the center of tire contact on the ground to the CG of the vehicle.
- Normally some degree of squat and pitch is expected during vehicle acceleration, so full compensation is unusual.
- Anti-squat performance cannot be designed without considering other performance modes of the vehicle as well. When the trailing arm is short, the rear axle may experience "power hop" during acceleration near the traction limit.
- The goals for anti-squat may conflict with those for braking or handling. In this latter case, placing the pivot center above the wheel center can produce roll oversteer.

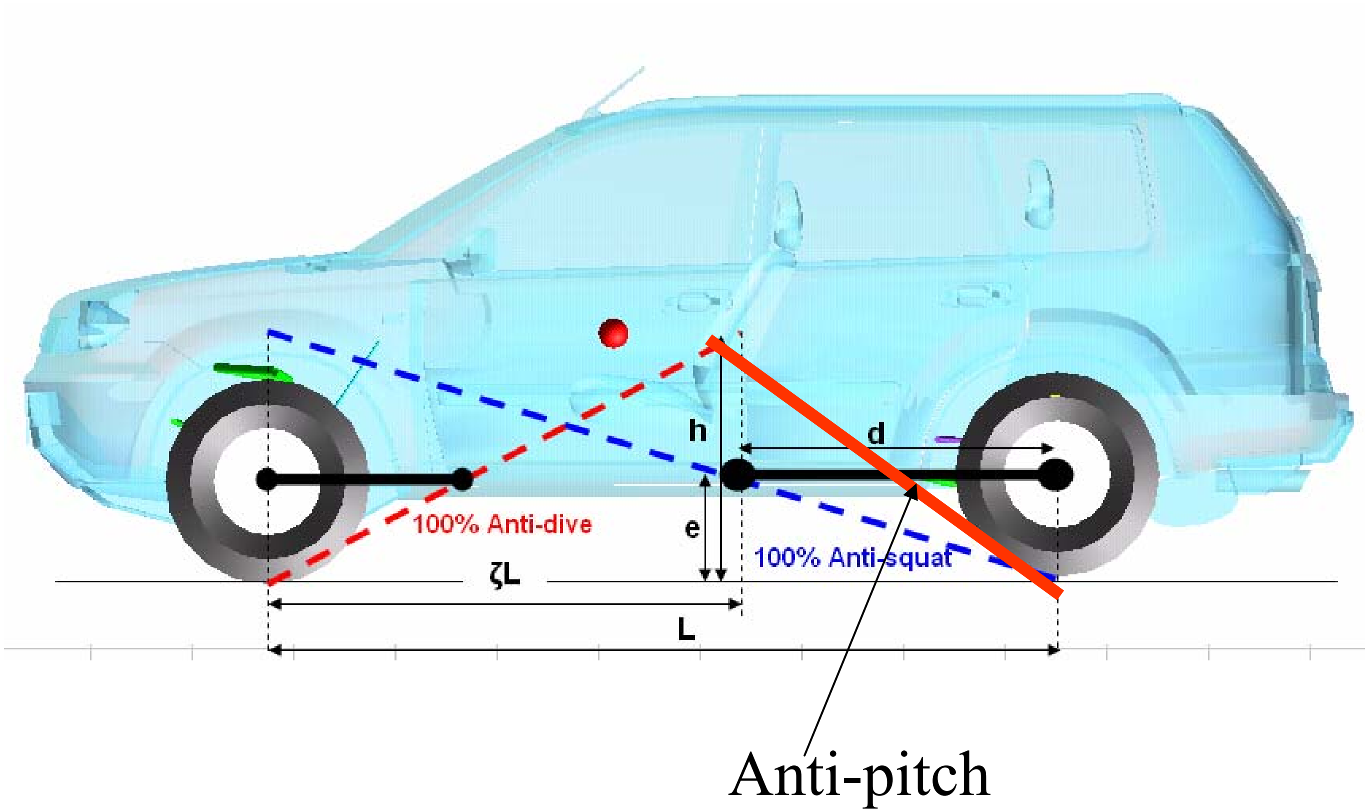
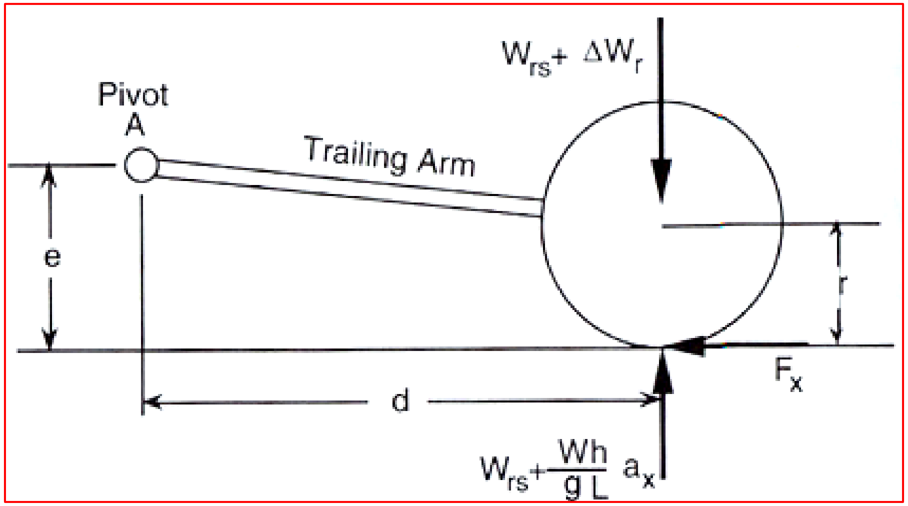


Fig: 16.2b

Independent Rear Drive



$$\frac{e-r}{d}=\frac{h}{L}+\frac{h}{L}\frac{K_r}{K_f}$$

Front Solid Drive Axle

- For a front-drive axle, conditions for anti squat and anti pitch are:

$$\theta_p = \frac{1}{L}\frac{W}{g}a_x\left(\frac{1}{K_r}\frac{h}{L} + \frac{1}{K_f}\frac{h}{L} + \frac{1}{K_f}\frac{e}{d}\right)$$

$$\frac{e}{d} = -\frac{h}{L} - \frac{h}{L}\frac{K_f}{K_r}$$

Independent Front-Drive Axle

- The comparable equations for an independent front-drive axle, as is common on most front-drive cars today, are:

$$\theta_p = \frac{1}{L}\frac{W}{g}a_x\left(\frac{1}{K_r}\frac{h}{L} + \frac{1}{K_f}\frac{h}{L} + \frac{1}{K_f}\frac{e-r}{d}\right)$$

and $$\frac{e-r}{d} = -\frac{h}{L} - \frac{h}{L}\frac{K_f}{K_r}$$

Anti-dive Suspension Geometry

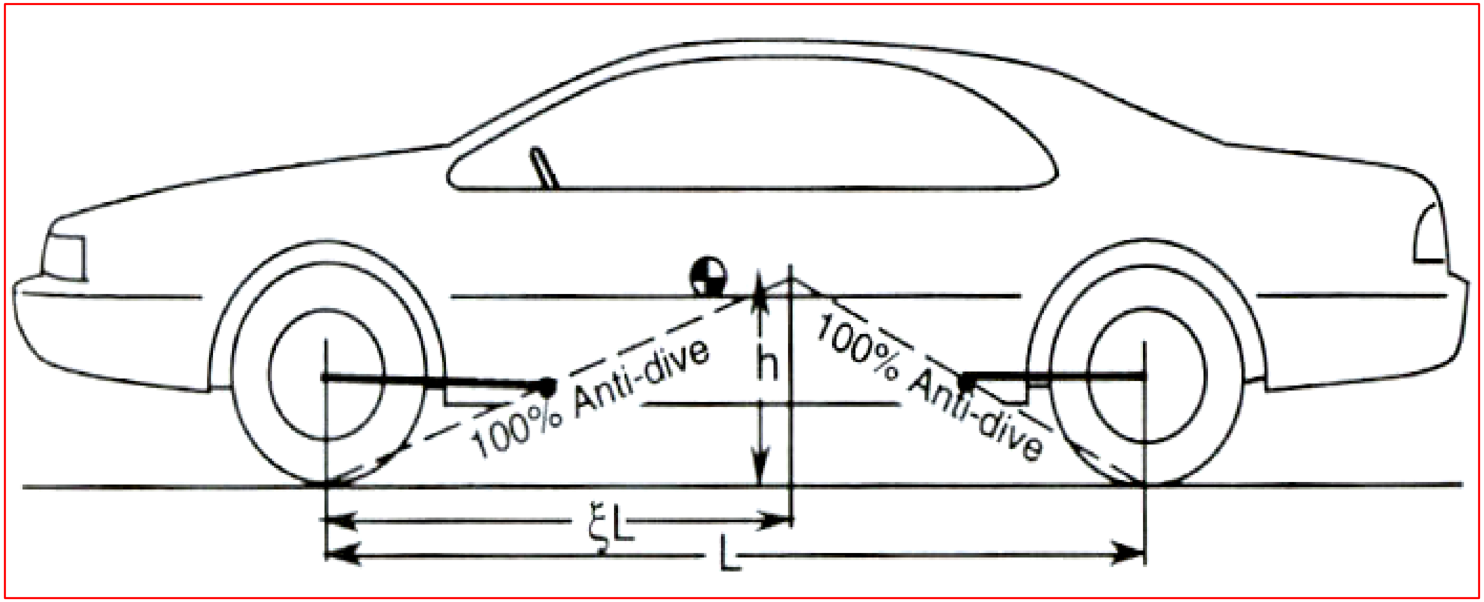
- The longitudinal load transfer incidental to braking acts to pitch the vehicle forward producing "brake dive."
- Just as a suspension can be designed to resist acceleration squat, the same principles apply to generation of anti-dive forces during braking.
- Because virtually all brakes are mounted on the suspended wheel (the only exception is in-board brakes on independent suspensions), the brake torque acts on the suspension and by proper design can create forces which resist dive.
- Using an analysis similar to that developed for the four-wheel-drive anti-squat example given previously, it can be shown that the anti-dive is accomplished when the following relationships hold:

Front suspension: $$\frac{e_f}{d_f} = \tan \beta_f = -\frac{h}{\xi L}$$

Rear suspension: $$\frac{e_r}{d_r} = \tan \beta_r = \frac{h}{(1-\xi)L}$$

Where:

ξ = Fraction of the brake force developed on the front axle eg: 60: 40.



- To obtain 100% anti-dive on the front and 100% anti-lift on the rear, the pivot for the effective trailing arm must fall on the locus of points defined by the defined ratios.
- If the pivots are located below the locus, less than 100% anti-dive will be obtained; if above the locus the front will lift and the rear will squat during braking.
- In practice, 100% anti-dive is rarely used. The maximum anti-dive seldom exceeds 50%.
- Full anti-dive requires that the pivot be located above the point required for full anti-squat. Thus acceleration lift would be produced on solid drive axles.

- Flat stops are subjectively undesirable.
- With full anti-dive, front suspension caster angle changes may increase the steering effort substantially during braking.
- The required steering system geometry may be quite complex.
- Excessive variation in rotational speed can occur in the drivetrain as the wheels move in jounce and rebound causing rattling and noise in the drive gears.
- In the rear suspension, oversteer problems may be created by the high location of the pivot.
- Brake hop may be induced if the effective trailing arm is too short. The propensity for brake hop is reduced by a suspension design with a long effective arm.
- NVH performance may be compromised.

Leaf Spring Windup

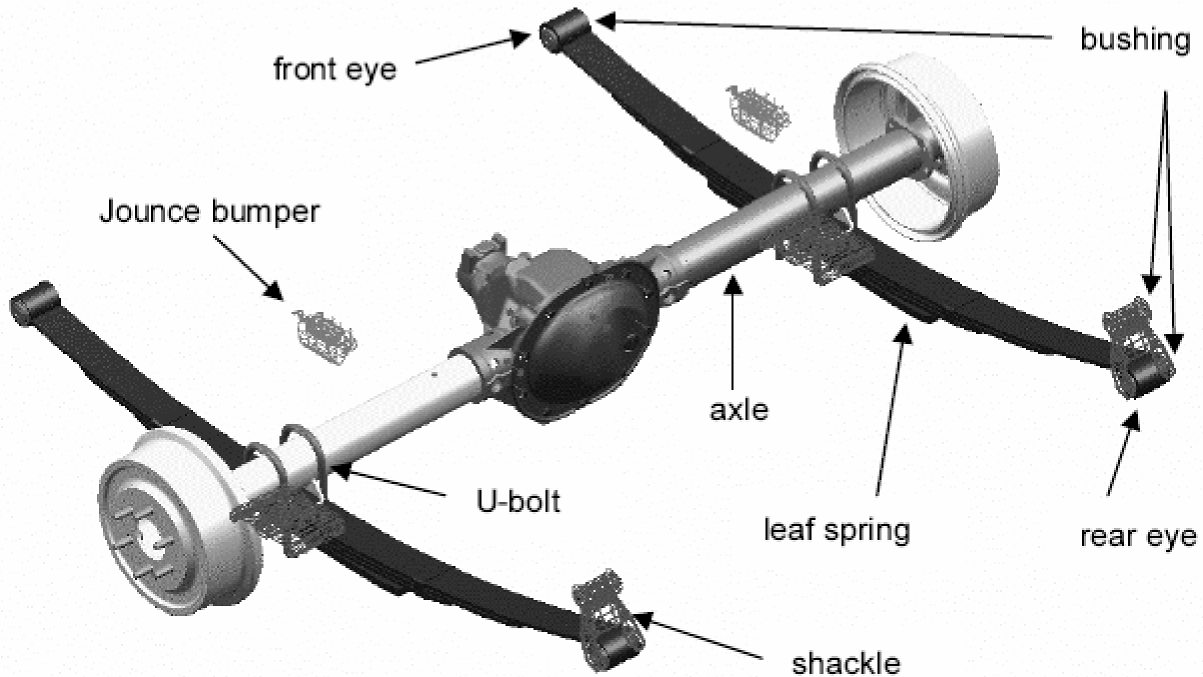
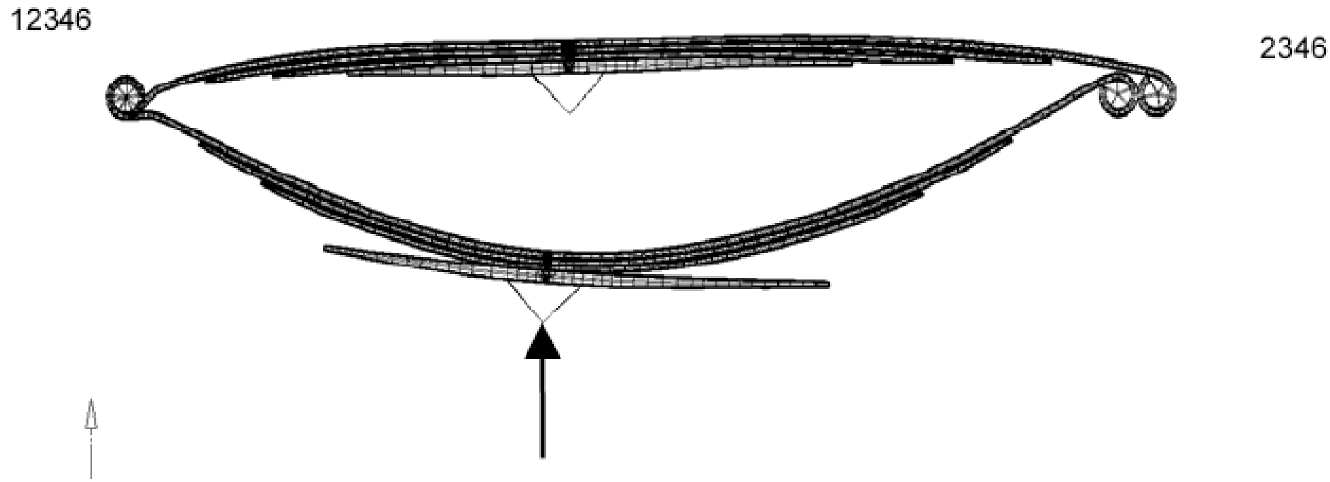


Figure 1. A Hotchkiss suspension.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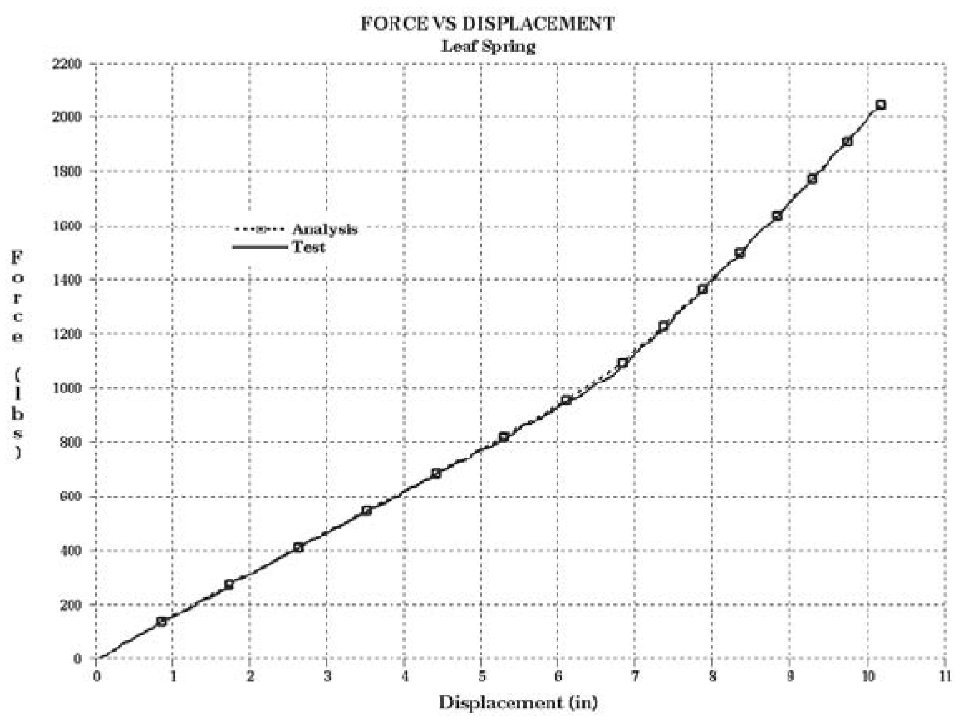
FORCE VS DISPLACEMENT
Leaf Spring
Analysis
Test
Force (lbs)
Displacement (in)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
1
2
3
4
5
6
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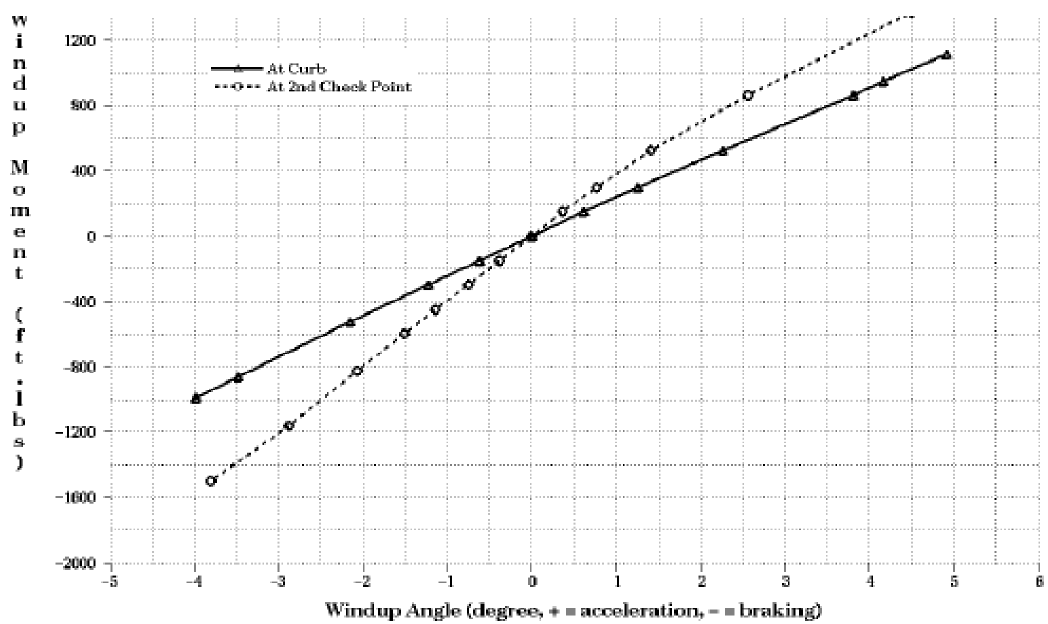
(e) Braking at 2^{nd} checkpoint

(f) Acceleration at 2^{nd} checkpoint



2^{nd} Check Point- Laden Condition