A Simulation of the Effect of Temperature and Salinity on Ice Melting

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1 Introduction

1.1 Motivation

This study aims to study the effect of temperature and salinity on ice melting through simulation. Specifically, the main goal is to establish a relationship between the ice melting rate and the initial conditions including temperature and salinity.

Initiated by Professor Nicolas Grisouard and Erica Rosenblum, the study originated from the observation of divergent behaviors in ice melting in pure water and salty water.

In the current field, one of the recent studies of ice melting simulation was conducted by Hester et al. (2021)[2]. It focused on the effect of aspect ratios and fluid velocity on ice melt. However, its numerical model can also change the initial conditions of temperature and salinity to generate different simulations. Therefore, I utilized the numerical model and focused on the investigation of how temperature and salinity influence ice melt.

1.2 Background Theory

1.2.1 Equations in the Simulation

The equations involved in the simulation are shown as following. They are moving boundary conditions in Equation 1, convection-diffusion equations of temperature and salinity in Equation 2 3, incompressible Navier-Stokes equation with Boussinesq Approximation in Equation 4, and the incompressible fluid assumption in Equation 5. Equation 3 is from the paper *Improved phase-field models of melting and dissolution in multi-component flows*[1] while the others are from the paper *Aspect Ratio Affects Iceberg Melting*[2]:

$$\varepsilon \frac{5}{6} \frac{\Lambda}{C_p \kappa} \partial_t \Phi - \gamma \nabla^2 \Phi + \frac{1}{\varepsilon^2} \Phi(1-\Phi)[\gamma(1-2\Phi) + \varepsilon(T + \lambda C)] = 0, \tag{1}$$

$$\partial_t T + \nabla \cdot [(1-\Phi)uT - \kappa \nabla T] = \frac{\Lambda}{C_p} \partial_t \Phi, \tag{2}$$

$$\partial_t C + u \cdot \nabla C - \mu \nabla^2 C = \frac{1}{1 - \Phi + \delta}(C \partial_t \Phi - \mu \nabla \Phi \cdot \nabla C), \tag{3}$$

$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla})\vec{u} - \nu \nabla^2 \vec{u} + \vec{\nabla} p + g \frac{\rho(T, C) - \rho_0}{\rho_0} \hat{z} = -\frac{\nu}{\eta} \Phi \vec{u}, \tag{4}$$

$$\vec{\nabla} \cdot \vec{u} = 0. \tag{5}$$

This simulation focuses on the phase Φ, temperature T, salinity C, velocity $\vec{u}$ and pressure p of the ice cube and the salt water. With only 2 dimensions, $\vec{u}$ has only x and z components. Phase Φ equals 1 for solid and 0 for liquid, with the interface expressed using a Sigmoid function. The parameters κ, μ, ν, are the thermal, salt, and momentum diffusivity, C_p is the heat capacity and Λ is the latent heat, η is the damping time, γ is the coefficient that shows curvature dependence of the melting temperature, δ regularizes the salinity equation within the ice, η, γ, $\delta \ll 1$, all of the three coefficients are small values.[2]

1.2.2 Convection-diffusion equations of temperature and salinity

This simulation used the convection-diffusion equations to consider the change of temperature and salinity field in the container. The general form of convection-diffusion equation is Equation 6.[5]

$$\frac{\partial F}{\partial t} = \vec{\nabla} \cdot (D \vec{\nabla} F) - \vec{\nabla} \cdot (\vec{u} F) + R \tag{6}$$

It indicates the convection and diffusion effect of the variable F in the field. The left hand side term $\frac{\partial F}{\partial_t}$ is the local time derivative of F. On the right hand side, $\vec{\nabla} \cdot (D \vec{\nabla} F)$ is the diffusion of F where D is the diffusivity. $\vec{\nabla} \cdot (\vec{u} F)$ shows the convection of F, which is also the change of F due to fluid flow. R is the source or sink of F.

In the simulation, the temperature convection-diffusion equation is:

$$\partial_t T + \nabla \cdot [(1 - \Phi)uT - \kappa \nabla T] = \frac{\Lambda}{C_p} \partial_t \Phi \tag{7}$$

$\partial_t T$, $\nabla \cdot \kappa \nabla T$, and $\nabla \cdot (1-\Phi)uT$ are local derivative, diffusion, and convection of temperature. The function of $(1-\Phi)$ in the convection term is to make sure there is no convection inside the ice cube. $\frac{\Lambda}{C_p}\partial_t \Phi$ represents the heat source in the system, which is the heat transfer in ice melting.

In the simulation, the salinity convection-diffusion equation is:

$$\partial_t C + u \cdot \nabla C - \mu \nabla^2 C = \frac{1}{1 - \Phi + \delta}(C \partial_t \Phi - \mu \nabla \Phi \cdot \nabla C) \tag{8}$$

Similarly, $\partial_t C$, $u \cdot \nabla C$, and $\mu \nabla^2 C$ are local derivative, diffusion, and convection of salinity. There is no salinity source in the system. Equation 3 produces the normal salinity convection-diffusion equation in the fluid[1].

1.2.3 Incompressible Navier-Stokes Equations with Boussinesq Approximation

In this simulation, I used incompressible Navier-Stokes equations to simulate the motion of fluid in the system. In this way, the salty water was treated as viscous incompressible fluid. Besides, the incompressible fluid assumption indicated zero divergence of velocity in the fluid in Equation 5.

In addition, the Boussinesq approximation was applied to the system. It assumed that all the fluid density was constant in all the terms except for the buoyancy term. This is because the difference in the inertia of fluid can be neglected compared to the effect of buoyancy forces[4]. The equations implemented in the simulation is:

$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla})\vec{u} - \nu \nabla^2 \vec{u} + \vec{\nabla} p + g \frac{\rho(T, C) - \rho_0}{\rho_0} \hat{z} = -\frac{\nu}{\eta} \Phi \vec{u} \tag{9}$$

$\partial_t u + u \cdot \nabla u$ is the material derivative of fluid velocity u. $\nu \nabla^2 u$ is the viscosity term, with ν being momentum diffusivity. ∇P is pressure gradient. $g\frac{\rho(T,C)-\rho_0}{\rho_0}\hat{z}$ is the buoyancy term, with ρ_0 being the constant density in other terms and $\rho(T, C)$ being the changing density in the buoyancy term. g is the gravitational acceleration. The term $\frac{\nu}{\eta}\Phi\vec{u}$ suppresses the convection in the solid. The damping time $\eta \ll 1$ makes this term dominantly large in the solid ($\Phi = 1$) compared to the other terms. In order to ensure the equation still holds, the $\vec{u}$ in the $\frac{\nu}{\eta}\Phi\vec{u}$ need to be sufficiently small. In this way, the velocity in the ice cube is forced to be nearly zero.

1.2.4 Moving Boundary condition

Equation 1 represents the moving boundary condition of ice melting.[1] It simulates the detailed temperature and salinity change at the ice-water interface.

1.2.5 Equation of State

The density of salty water depends on temperature, salinity, and pressure. According to incompressible fluid assumption, the effect of pressure on density can be neglected. Therefore, the density is a function of temperature and salinity $\rho(T, C)$ as is indicated in the buoyancy term in Equation 4.[3]

The dependence of density on temperature and salinity can be approximated by the highly simplified equation of state[3]:

$$\rho = \rho_0 + \rho_{ref}(-\alpha_T[T - T_0] + \beta_C[C - C_0]) \tag{10}$$

where the dependence of density on temperature and salinity is approximated to be linear. ρ_0 is the density of fluid at temperature T_0 and salinity C_0. ρ_{ref} is the reference density $1000 kg \cdot m^{-3}$. The linear coefficients α_T and β_C are defined as below[3]:

$$\alpha_T = -\frac{1}{\rho_{ref}}\frac{\partial \rho}{\partial T} \tag{11}$$

$$\beta_C = \frac{1}{\rho_{ref}}\frac{\partial \rho}{\partial C} \tag{12}$$

α_T has a typical value of 1×10^{-4}. However, it is significantly larger at higher temperatures. β_C has a typical value of $7.6 \times 10^{-4}(g/kg)^{-1}$. This indicates that at low temperatures the effect of temperature and salinity on density competes. In the open ocean temperature and salinity range ($T = 0\ 30°C$, $C = 33\ 36 g/kg$), temperature influences density more significantly than salinity does[3].

2 Methods and Findings

2.1 Simulation Design

The simulation describes a 2D melting process where a 2cm×2cm ice cube (in 2D context, ice rectangle) floats in the middle of a 10cm×5cm rectangular container filled with salty water. The initial state of the ice cube is as shown in Figure 1.

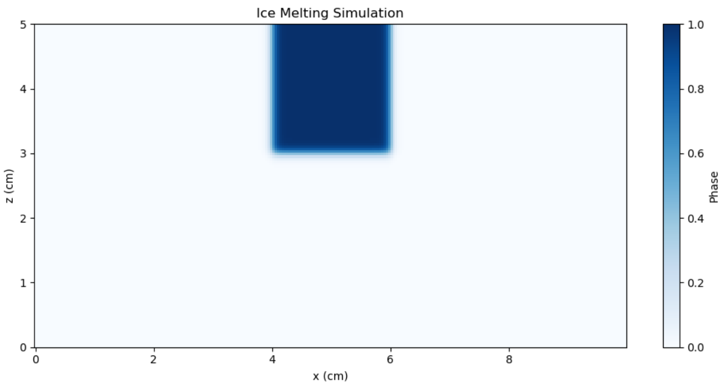

Figure 1: The initial state of the ice cube. The colorbar indicates phase with blue (1.0) representing ice and white (0.0) representing liquid. Any color between them represents a transitional state between solid and liquid

2.2 Simulation Goals and Methods

The aim of this experiment is to investigate the dependence of ice melting rate on initial temperature and salinity conditions. First, I compared the average melting rate at different initial conditions. I used the remaining ice volume after 25 seconds as the indicator of the average melting rate in the first 25 seconds. This is analyzed in the Preliminary Analysis section. Next, I studied the detail of the melting process case by case. I simulated near-total melt where 90% of the total ice volume was melt. Besides, I plotted the temperature and velocity field in the container. In this way, the effect of temperature and salinity on ice melting can be analyzed visually. The near-total melt analysis is in the section Near-total Melt Analysis.

The simulations I ran for the preliminary analysis are presented in the following table:

	0 g/kg	...	30 g/kg	...	200 g/kg
2 $°C$					
...					
20 $°C$					

where each column represents a list of simulations with the same initial salinity condition and the initial temperature conditions changing gradually from $T_0 = 2°C$ to $T_0 = 20°C$. Each row represents a list of simulations with the same initial temperature condition and the initial salinity conditions changing gradually from $C_0 = 0g/kg$ to $C_0 = 200g/kg$.

The values of the initial temperature and salinity were chosen with the following reasons. $C_0 = 0g/kg$ represents the ice melt in fresh water. $C_0 = 30g/kg$ represents the ice melt in an environment similar to sea water. (open ocean salinity is about $33-36g/kg$ [3], which is close to $30g/kg$ here) $C_0 = 200g/kg$ represents the ice melt in extremely salty environment. The saturated salinity at $T = 20°C$ is $263g/kg$[6]. The reason why I only set the upper limit of salinity to be $200g/kg$ was because the salinity above this value harmed the stability of the simulation. $T_0 = 2°C$ represents the ice melt in polar regions. Similarly, an initial temperature below $2°C$ harmed the stability of the simulation. $T_0 = 20°C$ represents an initial temperature close to the global average temperature of the open ocean.

In the section Near-total Melt Analysis, I investigated the 3 melting process with the initial salinity of $C_0 = 0g/kg$, $C_0 = 200g/kg$, and $C_0 = 30g/kg$. The initial temperature conditions are all $T_0 = 20°C$. Due to the limit of time and the limited calculating ability of my computer, I only ran these three near-total simulations.

2.3 Preliminary Analysis

2.3.1 Effect of Temperature on Ice Melting

First, I compared the remaining ice volume after 25 seconds at different initial temperature conditions. The result is plotted in Figure 2.

As is demonstrated in Figure 2, no matter at which initial salinity condition, the remaining ice volume after 25 seconds decreases as initial temperature rises, which means that the average melting rate grows. This is up to the expectation that the melting rate is proportional to the temperature difference between the ice cube and the environment temperature.

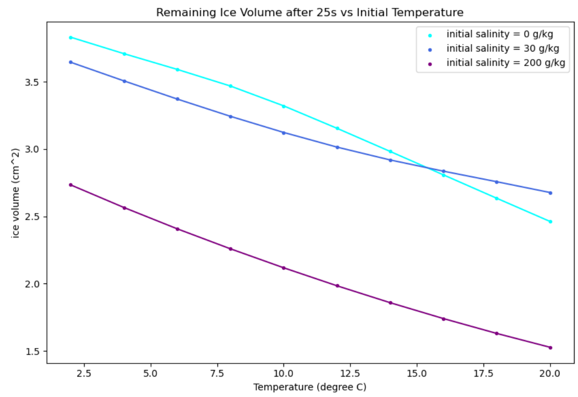

Figure 2: y axis is the remaining ice volume after 25 seconds and x axis is the initial temperature in the fluid. Cyan line represents initial salinity equal to 0 g/kg; blue line represents initial salinity equal to 30 g/kg; purple line represents initial salinity equal to 200 g/kg.

2.3.2 Effect of Salinity on Ice Melting

I compared the remaining ice volume after 25 seconds at different initial salinity conditions. The result is plotted in Figure 3.

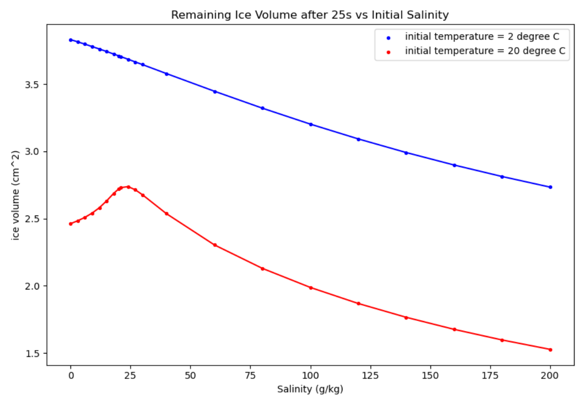

Figure 3: y axis is the remaining ice volume after 25 seconds and x axis is the initial salinity in the fluid. The blue line represents initial temperature equal to 2 $°C$; the red line represents initial temperature equal to 20 $°C$.

According to Figure 3, the remaining ice volume after 25 seconds decreases with increasing initial salinity when the initial temperature equals to 2 $°C$. However, the trend is different at initial temperature equal to 20 $°C$. The remaining ice volume first rises as the initial salinity rises, peaks at salinity equals to around 25 g/kg, and then decreases afterwards. This indicates a minimum average melting rate at initial salinity 25 g/kg. Therefore I decided to further inspect the effect of salinity on ice melting by observing the near-total melting process and analyzing the detailed change in temperature and fluid velocity.

2.4 Near-total Melt Analysis

2.4.1 The Melting Process at $C_0 = 0g/kg$, $T_0 = 20°C$

The velocity and temperature field of the system is plotted in Figure 4.

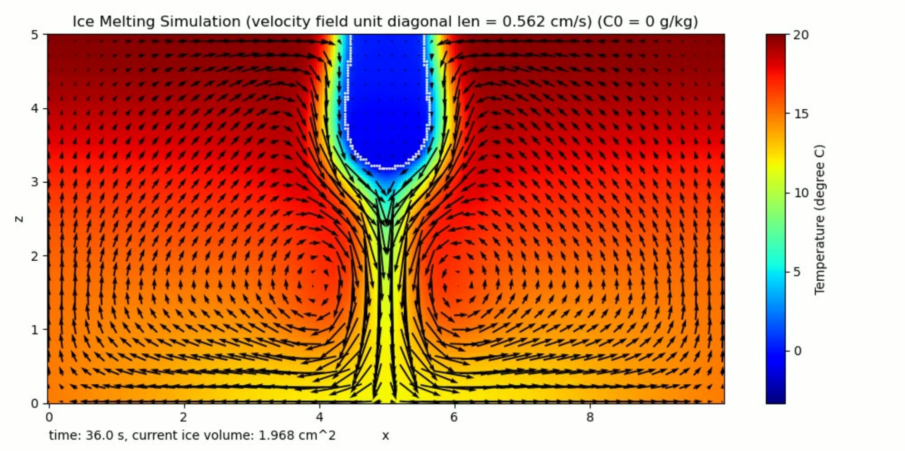

Figure 4: The velocity and temperature field of the system with $T_0 = 20°C$ and $C_0 = 0g/kg$ at time $36.0s$. x and y axes represent horizontal and vertical dimensions of the container. The color represents the local temperature, with warmer colors representing higher temperatures. The arrows represents the velocity field. The arrow with a length equal to the diagonal length of the velocity field grid represents a velocity with magnitude 0.562 cm/s. The white dots in the blue area represents the interface between water and ice.

I observed the change of the velocity and temperature field as time evolved and found that the field plot is similar throughout the melting process.

At the beginning of the melt, the melt water went straight down to the bottom of the container. The melt water occupied the space on the bottom and pushed the warm water upwards. This process formed a symmetrical pair of vortices. The circulations sustained till the end. Therefore, the ice cube was surrounded by warm water throughout the process.

The downwards tendency of melt water can be explained through density. Since the ice cube melt in pure water, the density of the fluid only depended on temperature. According to the equation of states, melt water is much denser than the water at room temperature. Therefore, buoyancy force drove melt water to flow downwards.

Due to the warm temperature surrounding the ice cube, the ice melt at a steady and relatively high rate. The total time it took for the ice to melt 90% of its initial volume was 83.0 seconds.

2.4.2 The Melting Process at $C_0 = 200g/kg$, $T_0 = 20°C$

The velocity and temperature field of the system is plotted in Figure 5 and 6.

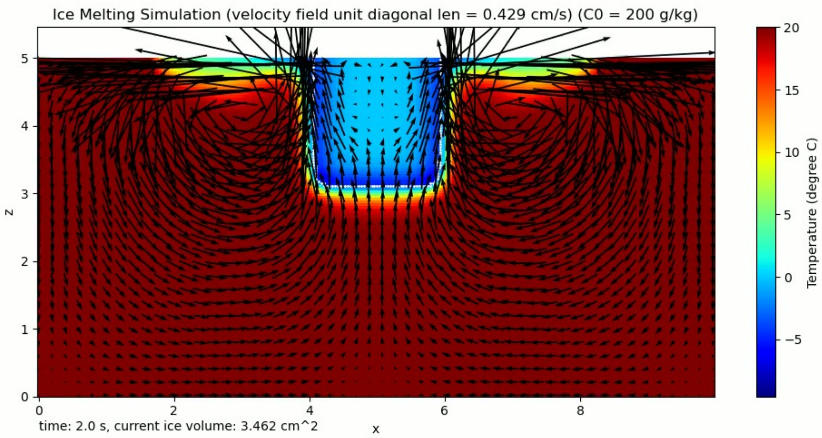

Figure 5: The velocity and temperature field of the system with $T_0 = 20°C$ and $C_0 = 200g/kg$ at time $2.0s$. The diagonal length of velocity field grid represent a velocity magnitude of $0.429cm/s$.

At the beginning of the melt in Figure 5, the ice cube melt fast. The melt water had upward initial velocities. The fresh melt water quickly occupied the surface regions and drove the salty warm water downwards. This process formed a symmetrical pair of vortices that had significantly large velocities. The ice cube was surrounded by quickly swirling flows on the early stage.

The upwards tendency of melt water can also be explained through density. According to the theory in section Equation of State, α_T becomes large at high temperatures, leading to a significant effect of temperature on density. However, the salinity was extrenely high in this situation, making the effect of salinity overtake that of temmperature. Consequently, the fresh melt water was much lighter than the salty warm water. The large difference in density led to the large buoyancy force that pushed the fresh melt water to the surface. This formed the the quickly swirling flow around the ice cube at the beginning.

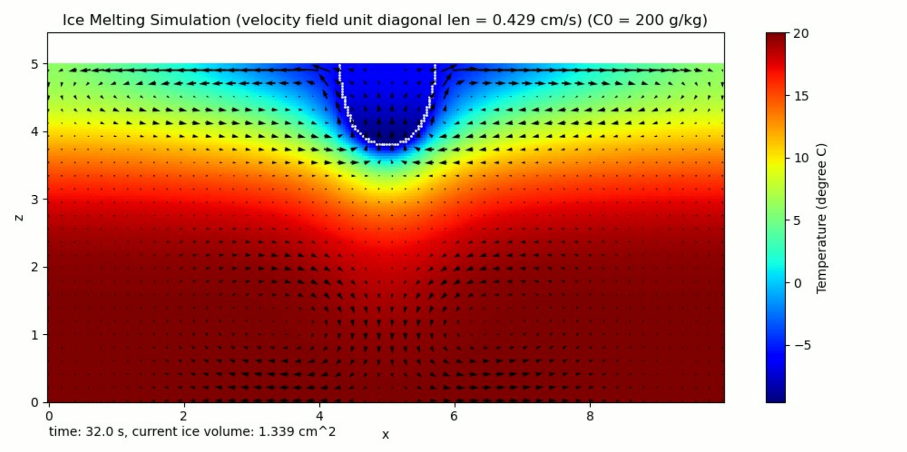

Figure 6: The velocity and temperature field of the system with $T_0 = 20°C$ and $C_0 = 200g/kg$ at time $12.0s$. The diagonal length of velocity field grid represent a velocity magnitude of $0.429cm/s$.

As time evolved in Figure 6, the vortices died out. Cold water completely wrapped around the ice cube, making the ice melt very slowly. The flow around the ice cube was also very slow.

The fresh melt water was lighter than the salty warm water. Therefore, the fresh melt stayed on the surface and the salty warm water stayed on the bottom, which greatly slowed down the convection.

The high-velocity fluid around the ice cube at the beginning made the ice cube melt fast initially. However, as time evolved, the cold temperature and the slow flow around the ice cube made it melt slowly. Due to the joint effect, the total time it took for the ice to melt 90% of its initial volume was 100.0 seconds.

2.4.3 The Melting Process at $C_0 = 30g/kg$, $T_0 = 20°C$

The velocity and temperature field is plotted in Figure 7.

The velocity and temperature field of the system was similar throughout the process. The melt water had small upward initial velocities and slowly occupied the surface. Soon after the beginning, as is indicated in Figure 7, the ice cube was surrounded by cold water and slow flows. The ice cube melt at a steady but slow rate all the time.

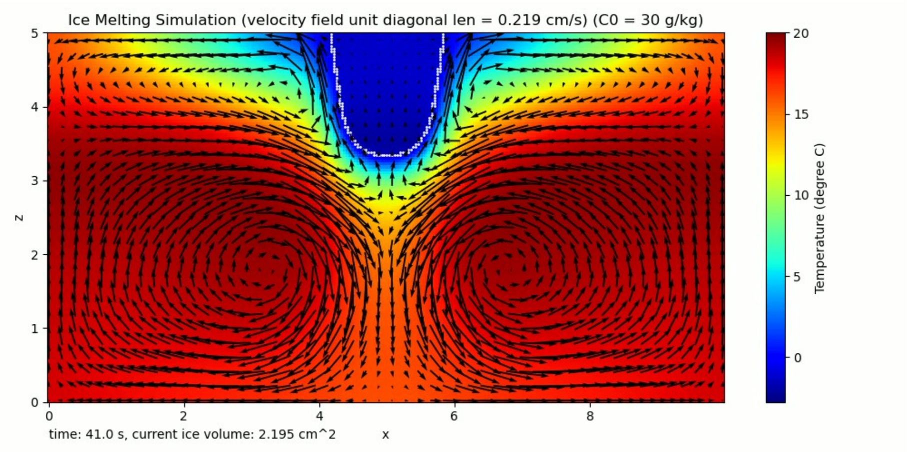

Figure 7: The velocity and temperature field of the system with $T_0 = 20°C$ and $C_0 = 30g/kg$ at time $41.0s$. The diagonal length of velocity field grid represent a velocity magnitude of $0.219cm/s$.

According to the simplified equation of state in Equation 10, temperature and salinity compete to influence density. I ran the Python seawater module that was used in the simulation to compare the density of fresh melt water ($C = 0g/kg, T = 0°C$) and salty warm water ($C = 30g/kg, T = 20°C$). The former was $999.84kg{\cdot}m^{-3}$ and the latter was $1020.95kg{\cdot}m^{-3}$. This shows that the warm salty water was slightly denser than the fresh melt water. This explained the small buoyancy forces that drove the fresh melt water to the surface.

Due to the cold temperature and the slow flow around the ice cube, it took 176.0 seconds to melt 90% of its initial volume.

2.4.4 Comparison of the three nearly-total melt

I compared the three nearly-total melt in more details. I plotted the change of the ice volume and melting rate in the three initial conditions in Figure 8 and 9.

In Figure 8, the green line declined quickly, indicating a high melting rate of ice cube in pure water. The relatively high position the green line in the melting rate plot also proved this. In Figure 8, the blue line had a large slope magnitude at the beginning. However, it gradually became flatter as time evolved. The blue line in the melting rate plot was the lowest among the three after around 20 seconds. This demonstrated the initially high and subsequently low melting rate pattern of the ice cube in extremely salty environment. The orange line in Figure 8 indicated a steady

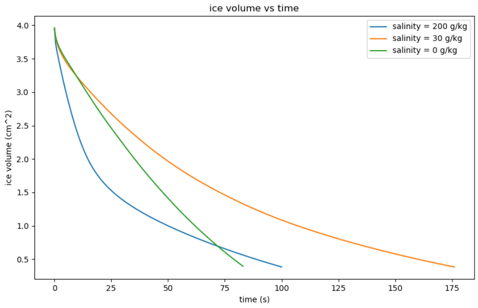

Figure 8: Ice volume (cm^2) vs Time (s). Green, blue, and orange curves represent $C_0 = 0, 200, 30 g/kg$ respectively. The initial temperatures are all $T_0 = 20°C$.

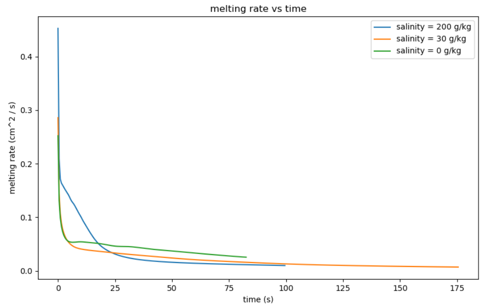

Figure 9: Ice melting rate (cm^2/s) vs Time (s). It is the inverse of the derivative of the curves in Figure 8.

but slow melting rate of the ice cube in initial salinity $C_0 = 30g/kg$.

3 Discussion and conclusions

In this study, I investigated the effect of temperature and salinity on ice melting through simulation. I analyzed the simulation and drew several conclusions, which also gave rise to potential future work.

3.1 The Effect of Temperature on Ice Melt

It was found in section Preliminary Analysis that as the initial temperature rose from $T_0 = 2°C$ to $T_0 = 20°C$, the average melting rate in the first 25 seconds rose. This trend was consistent for the three salinity conditions in the simulation ($C_0 = 0, 30, 200g/kg$). This is up to the expectation, as warmer environmental temperature usually means higher melting rate.

3.2 The Effect of Salinity on Ice Melt

The effect of salinity on ice melt was analyzed in the section Near-total Melt Analysis, which turned out to be complicated. The near-total melt is defined as the process that 90% of the volume of an ice cube is melt. I compared the three near-total ice melt at initial salinity $C_0 = 0, 30, 200g/kg$. The initial temperatures were all $T_0 = 20°C$. When the ice cube melt in fresh water, it took the least time ($83.0s$) to complete the near-total melt. This was because the ice cube was surrounded by warm water all the time. When the ice cube melt in extremely salty water ($C_0 = 200g/kg$), it took $100.0s$ to complete the near-total melt. I observed that the initially fast water flow around the ice cube led to a high melting rate at the beginning. As time evolved, the ice cube was surrounded by cold and slow flow, which made it melt very slowly. The joint effect led to its medium average melting rate. When the ice cube melt in a fluid with initial salinity $C_0 = 30g/kg$, it melt the most slowly (near-total melting time was $176.0s$). This was because it was surrounded by cold and slow fluid throughout the process.

3.3 Future Work

Although there are many conclusions drawn in the study, more detailed investigations need to be conducted. Firstly, when studying the effect of temperature, I only considered the melting at the first 25 seconds, which was not enough. As is demonstrated in Figure 8 and 9, the melting rate could also change in a longer period. Therefore, it is necessary to extend the simulation time. Moreover, when studying the effect of salinity, I only considered the initial temperature at $T_0 = 20°C$. However, the theory in section Equation of State indicates that the dependence of water density on temperature is different at different temperatures. Therefore, more simulations on different temperatures need to be conducted.

References

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