Consider the problem row by row and suppose we are considering the i-th row.

If the i-th row is not connected, we could divide it into several segments, consider them individually and merge them in the end, so let us assume that we are consider a subproblem $F(L, R, i)$ from L-th column to R-th column where the i-th row is connected in this region, and $\underline{F(L, R, i+1)}$ has been solved.

We divide the columns into 3 different types:

1. Contains at least a rook.
2. Does not contain a rook, but all cells in the column could be attacked by some rooks.
3. Does not contain a rook, and at least one cell in the column could not be attacked by any rook.

Our DP state for $F(L, R, i)$ is $f_{x,y}$, where x is the number of columns of type 1, and y is the number of columns of type 2, then the number of columns of type 3 is $(R - L + 1) - x - y$. Then merging is simply $f_{x,y} = \sum_{i=0}^{x} \sum_{j=0}^{y} f^{left}_{i,j} f^{right}_{x-i,y-j}$.

To modify the state from f^{old} to f, there are several cases to consider:

- No rook in the new line: $f^{old}_{x,y} \to f_{x,0}$
- Only put rooks in type 1 column: $f^{old}_{x,y} \times (2^x - 1) \to f_{x,y+C}$
- Put i rooks in type 2 column and j rooks in type 3 column: $f^{old}_{x,y} \times 2^x \times \binom{y+C}{i} \binom{R-L+1-x-y-C}{j} \to f_{x+i+j,y+C-i}$

Here C is the number of new columns added in the i-th row.

The number of states is $O(N^2)$, to optimize it to $O(N)$, consider the following two cases:

- In the $i - 1$ rows below, each row has at least one rook. Then the type 2 column is equivalent to the type 1 column.
- In the $i - 1$ rows below, there is at least one row that does not have any rook. The the type 2 column is equivalent to the type 3 column.

Thus, we could change the state from $f_{x,y}$ into $f_{x,0}$ (assume there is a row below which does not have a rook) and $f_{x+y,1}$ (assume there is no row below which does not have a rook). Separately handle transition of $f_{\ldots,0}$ and $f_{\ldots,1}$. The only new transition is $f^{old}_{x,0} \to f_{x,1}$, which represents that the i-th row is the last row without a rook.

The final answer is $f_{N,1}$ for row 1.