Lab5 Solution

YAO ZHAO

Lab5.A Shopping

- Festival is coming and Andrea decides to buy n gifts for her friends. However, n gifts are sold on different days. So Andrea will buy them **in order**. For the i^{th} gift, the price is c_i.
- In her city, there are only 1 dollar coins and 100 dollar notes. Before shopping, Andrea prepares m coins and plenty of notes (you can assume that she always has enough notes to buy gifts).
- There is only one cashier in the store and his efficiency depends on the gift he sells. For the i^{th} gift, his efficiency is w_i and he needs $x * w_i$ seconds to check out where x is the total number of the notes and coins he needs to give Andrea. The cashier always minimizes x.
- Andrea wants to pay in such a way that the total time she needs to check out should be as small as possible.
- Please help her to find out the minimum time she needs!

Input:

5 34 — the number of gifts; the number of coins

15 760 755 411 950 → price

2 2 2 2 2 → efficiency

price	15	760	755	411	950
weight	2	2	2	2	2

34 -15→ 19 +40→ 59 -55→ 4 +89→ 93 -50→ 43

total time:
40*2+89*2 = 258

price	15	760	755	411	950
weight	2	2	2	2	2

34 -15→ 19 +40→ 59 +45→ 104 -11→ 93 -50→ 43

total time:
40*2+45*2 = 170

Output:

170

Input:

```
5 193
178 887 466 475 10
816 136 880 340 800
```

price	178	887	466	475	10
weight	816	136	880	340	800

total time：

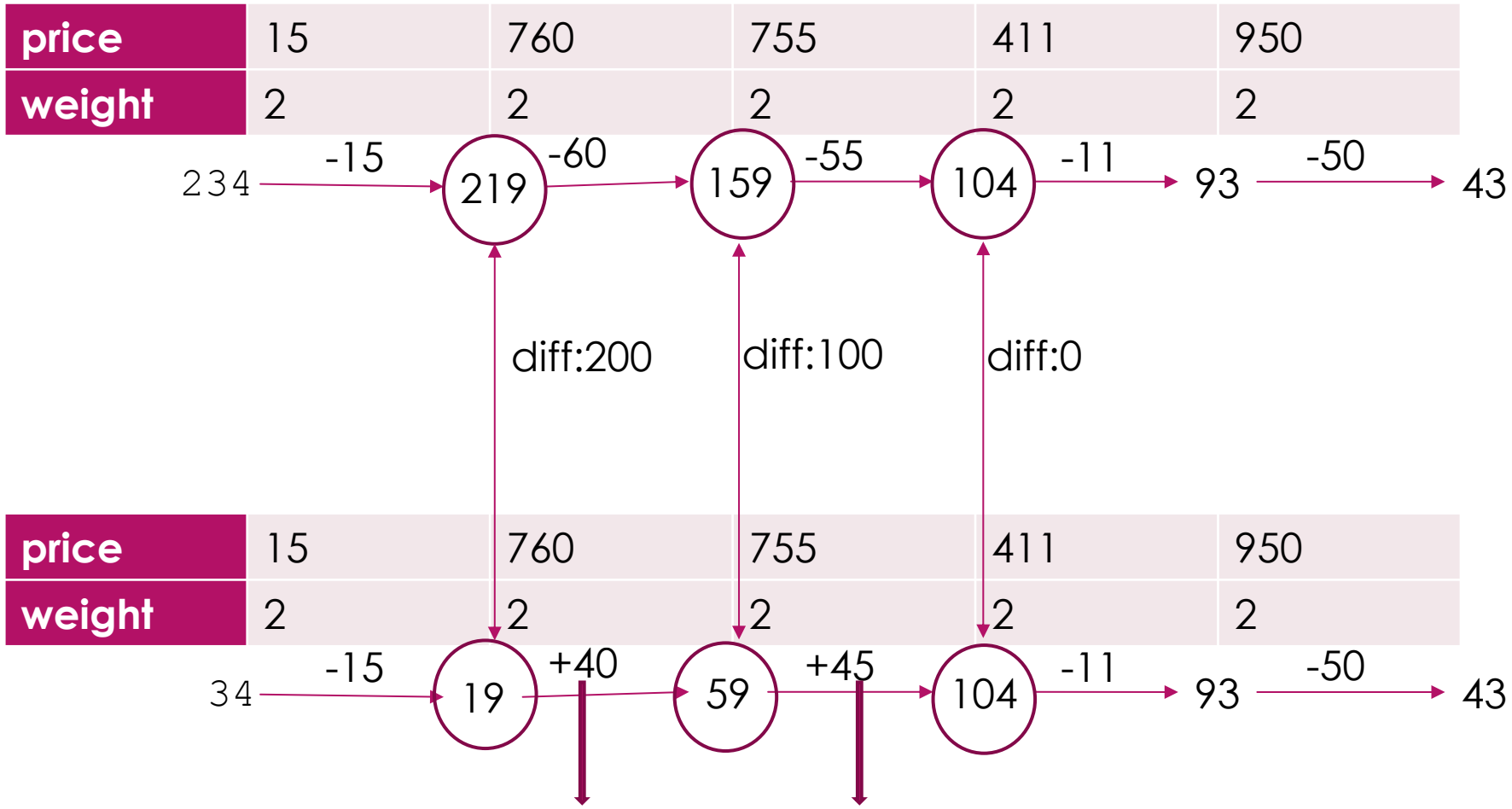
13*136+25*340=10268

Output:

```
10268
```

for testcase 1

assume the initial coins is 234



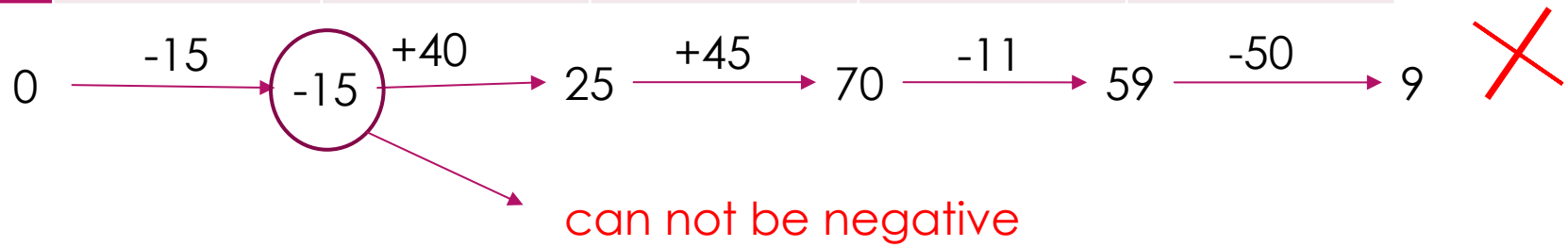
each time the cashier the coin, means the coin increase 100

Here, the remaining coins equal the case of initial coins = 234

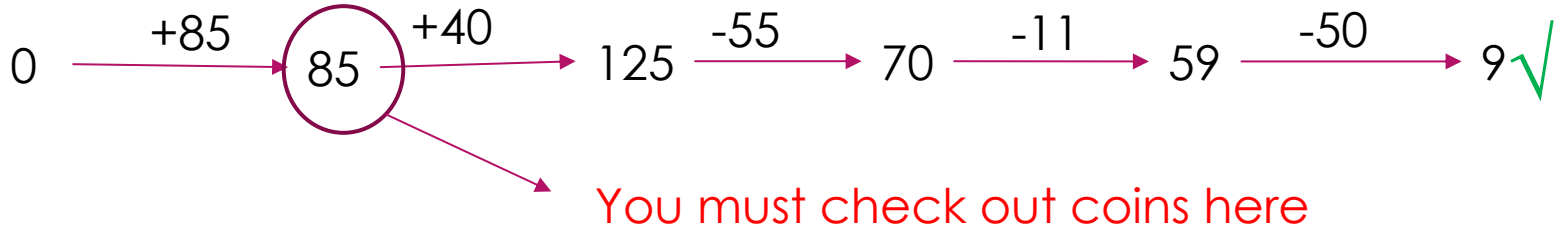
If you want to minimize the total time spent checking out, choose the gifts that take the least time to check out.

Always choose the smallest costs?

price	15	760	755	411	950
weight	2	2	2	2	2

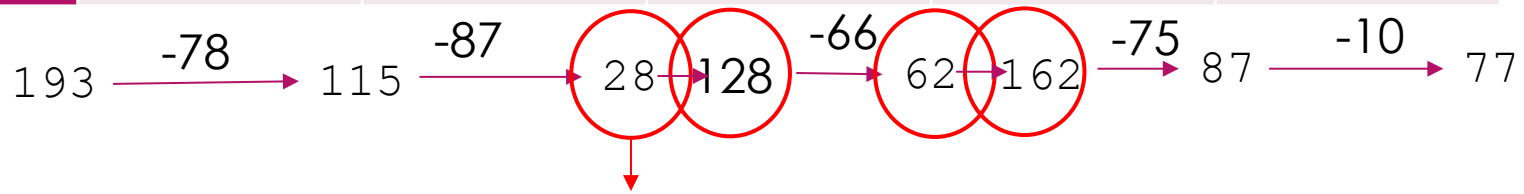


price	15	760	755	411	950
weight	2	2	2	2	2



You can trace the remaining coins:

price	178	887	466	475	10
weight	816	136	880	340	800



Here, the remaining coins are not enough. 28 < 66
Choose the check out with the smallest cost in the historical transaction. 13*136 < 22*816 and 34*880

total cost += 13*136
the remaining coins +=100

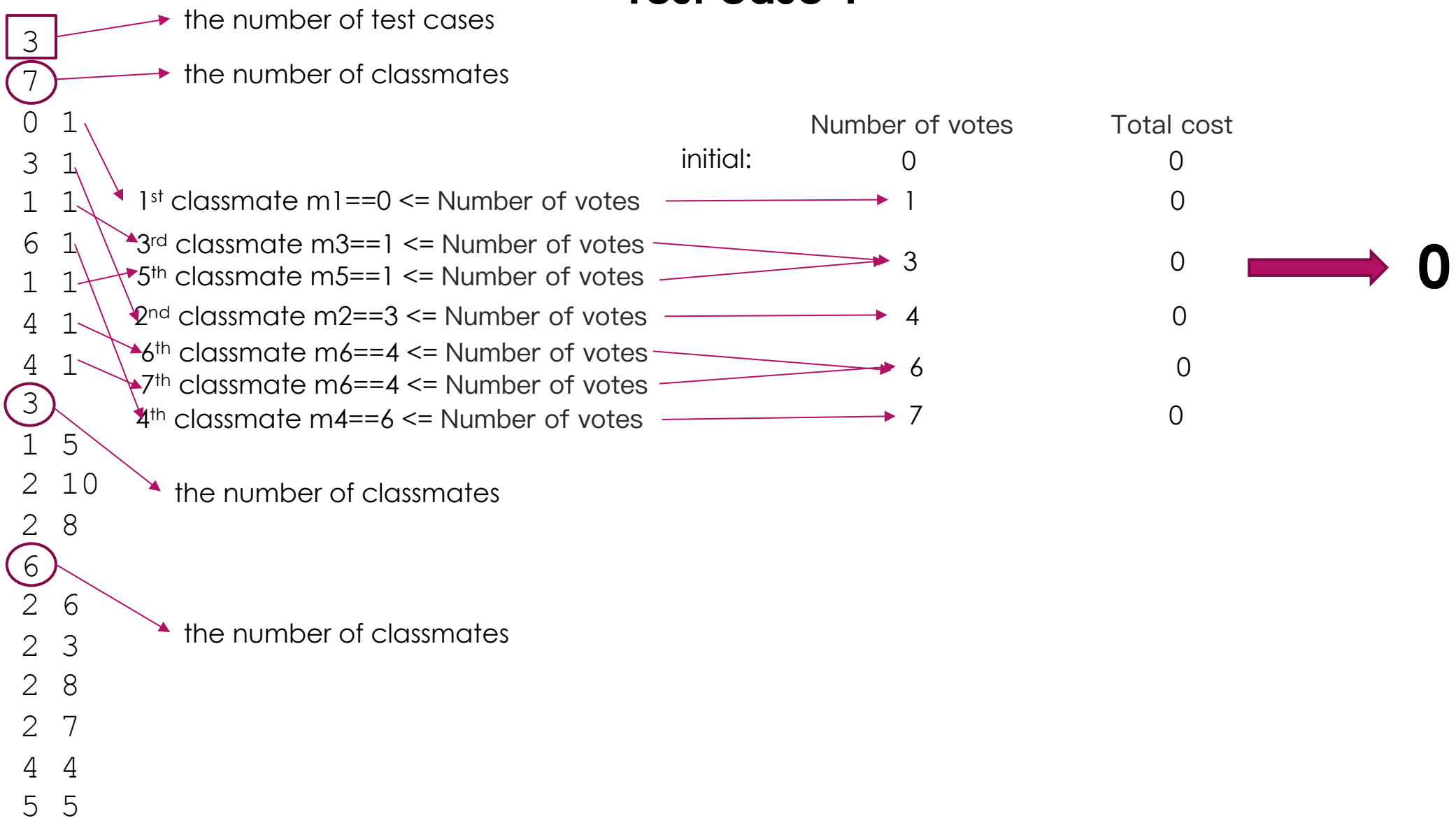
Here, the remaining coins are not enough. 62 < 75
Choose the check out with the smallest cost in the historical transaction. 25*340 < 22*816 and 34*880

total cost += 25*340
the remaining coins +=100

Lab5.B Voting

- A monitor election is going to be held in your class. There are $\$(n+1)\$$ students in your class. And you want all your $\$n\$$ classmates to vote for you.
- There exist two ways to convince each of your classmates to vote for you. The first way to convince your i^{th} classmate is to pay him/her c_i coins. The other way is to make m_i other classmates vote for you, and the i^{th} classmate will vote for you for free.
- It should be noticed that the voting takes places in several steps. For example, if you have four classmates with $m_1 = 1, m_2 = 2, m_3 = m_4 = 3, m_5 = 5$, then if you buy the vote of the 5^{th} classmate, then all your classmates will vote for you. And the set of classmates vote for you changes as: $\{5\} \rightarrow \{5,1\} \rightarrow \{5,1,2\} \rightarrow \{5,1,2,3,4\}$.
- Please calculate the minimum coins you need to spend so that all your classmates will vote for you.

Test case 1


3
the number of test cases
7
the number of classmates
0 1
3 1
1 1
6 1
1 1
4 1
4 1
3
1 5
2 10
2 8
6
2 6
2 3
2 8
2 7
4 4
5 5
the number of classmates
the number of classmates
Number of votes
Total cost
initial:
0
0
1st classmate m1==0 <= Number of votes
1
0
3rd classmate m3==1 <= Number of votes
5th classmate m5==1 <= Number of votes
3
0
2nd classmate m2==3 <= Number of votes
4
0
6th classmate m6==4 <= Number of votes
7th classmate m6==4 <= Number of votes
6
0
4th classmate m4==6 <= Number of votes
7
0
0

Test case 2

```
3
7
0 1
3 1
1 1
6 1
1 1
4 1
4 1
3
1 5
2 10
2 8
6
2 6
2 3
2 8
2 7
4 4
5 5
```

the number of classmates

	Number of votes	Total cost
initial:	0	0
1st classmate cost the least →	1	5
then 3rd classmate cost the least →	2	13
2nd classmate m2==2 <= Number of votes →	3	13 ✗

Not the minimum cost

	Number of votes	Total cost
initial:	0	0
3rd classmate →	1	8
1st classmate m1==1 <= Number of votes →	2	8
2nd classmate m2==2 <= Number of votes →	3	8 √

Test case 3

```
3
7
0 1
3 1
1 1
6 1
1 1
4 1
4 1
3
1 5
2 10
2 8
6
2 6
2 3
2 8
2 7
4 4
5 5
```

the number of classmates

	Number of votes	Total cost
initial:	0	0
6th classmate has the largest m	1	5
then 5th classmate	2	9
m1,m2,m3,m4==2 <= Number of votes	6	9 ✗

Not the minimum cost

	Number of votes	Total cost
initial:	0	0
2nd classmate, cost 3	1	3
5th classmate, cost 4	2	7
m1,m3,m4==2 <= Number of votes	5	7
m5==5 <= Number of votes	6	7 ✓

Give priority to m? Give priority to c?

Consider both.

How to implement greedy strategy?

Lazy to make choice, buy a vote when you have to, and buy the smallest c in your choice.

6
2 6
2 3
2 8
2 7
4 4
5 5

Order m from largest to smallest

5 5
4 4
2 6
2 3
2 8
2 7

large

small

First, consider (5, 5), Let's start with the optimistic hypothesis, if s/he doesn't vote, the rest of classmates will vote, putting (5, 5) to a set S.
Now the size of S is 1, and the total number of classmates is 6. If everyone else is going to vote except for the classmates in S, then the number of votes will be 6 − the size of S = 5, equal to m = 5. So you don't have to buy votes.
Continue to consider (4,4), put (4, 4) to S
6 − 2 = 4, equal to m = 4. So you don't have to buy votes n yet.
Then put (2 6) (2 3) (2 8) (2 7) to S together, because all the classmates have the same m value.
Now, 6 – 6 = 0， but m = 2, 0 < 2, you have to buy 2 votes. you can buy the 2 votes with the least cost in the S.