Lab7 Solution

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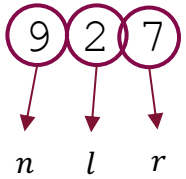
Lab7.A Modify

- Recently, Andrea got a list with only one element n. Since she doesn't like anything other than 0 and 1, she performed some operations on this list. In each operation, she removed every element x, such that $x > 1$ from the list. Then, in the same position, she added $\left\lfloor \frac{x}{2} \right\rfloor, x \bmod 2, \left\lfloor \frac{x}{2} \right\rfloor$ into the list. She stopped the operations until the list contained only 0 or 1.
- Now, she wants to know the sum of the elements whose indexes are in the range $[l, r]$. Given n, l, r, please tell her the answer.

Input:



[9]

9 → 4, 1, 4

[4, 1,4]

4 → 2, 0, 2

[2, 0, 2, 1, 2, 0, 2]

2 → 1, 0, 1

[1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1]

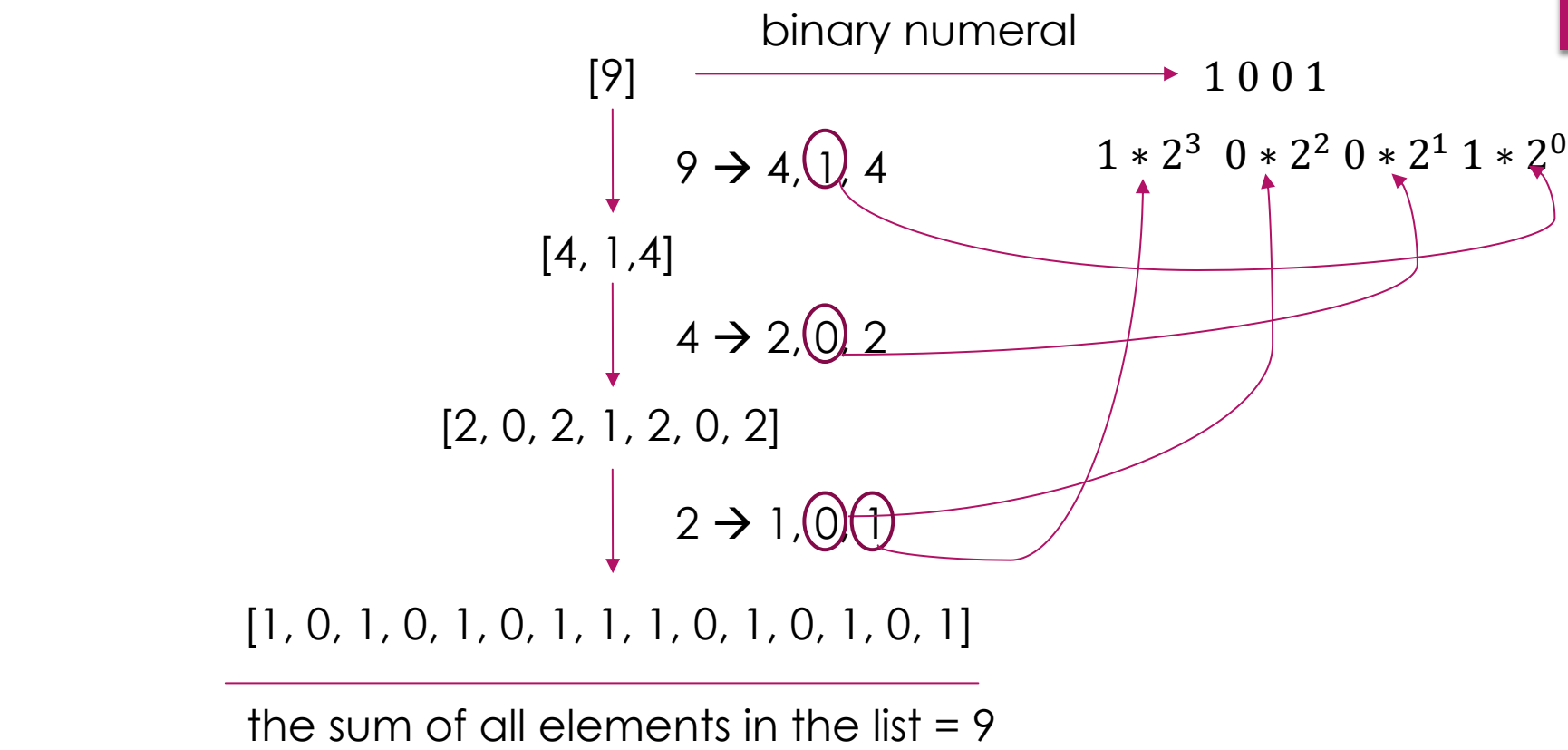
index: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

l r

sum = 3

Output:

3



$2^3 \leq 9 < 2^4$

the length of the final list = $2^3 + 2^2 + 2^1 + 2^0 = 2^4 - 1$

let $sum([l,r])$ = the sum of the elements whose indexes are in the range $[l,r]$
$sum([l,r]) = sum([1,r]) - sum([1,l-1])$

$sum([1,15]) = 9$

[1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1]

index: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$sum([1,7]) = \frac{sum([1,15])}{2} = 4$

$sum([1,1]) = ?$

[1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1]

index: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$sum([1,3]) = sum([1,7])\ /2 = 2$

[1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1]

index: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

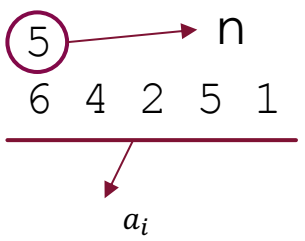
$sum([1,1]) = sum([1,3])\ /2 = 1$

$sum([1,13]) =?$

Lab7.B The Best Way to Wipe out a Friendship

- Andrea is wise, rich, noble, famous, sacred, sociable, powerful, diligent and intelligent, so she has a lot of friends. However, she has gotten bored and she wants to reduce the number of her friends.
- Andrea has listed the names of n friends of hers on a piece of paper and assigned a number a_i ($i \in [1, n]$) for each friend. She is going to keep an interval and abandon other friends. Every friend is also a believer in Andrea. The remaining interval $[\boldsymbol{l}, \boldsymbol{r}]$ is religious, if and only if there is some integer $\boldsymbol{m} \geq \boldsymbol{2}$, such that $\boldsymbol{a_l \bmod m = a_{l+1} mode m = \ldots = a_r \bmod m}$.
- Can you find out **the maximal length of a religious interval**?

Sample Input 1：



6 4 2 5 1

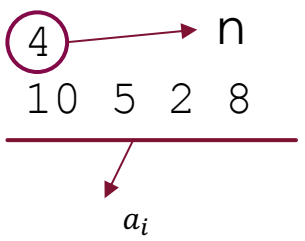
6 % 2 = 4 % 2 = 2 % 2 = 0

the maximal length

Sample Output 1：

3

Sample Input 2:



10 5 2 8

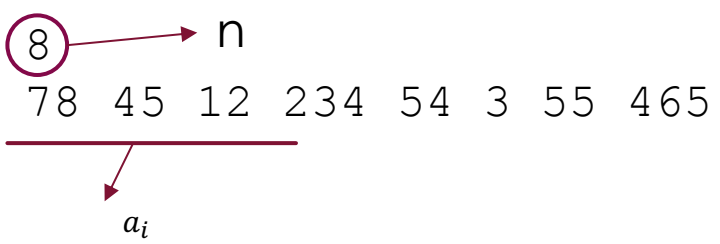
5%3 = 2%3 = 8%3 = 2

the maximal length

Sample Output 2:

3

Sample Input 3:



78 45 12 234 54 3 55 465

%3 = 0

the maximal length

Sample Output 3:

6

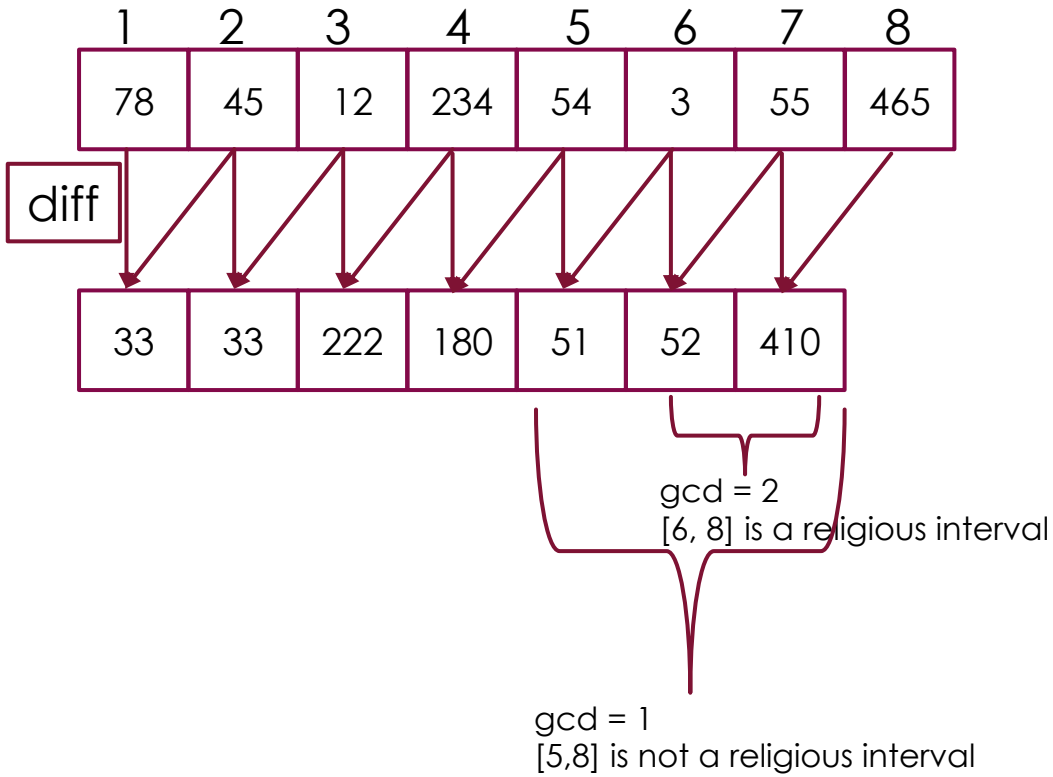
How to find the integer $m \geq 2$, such that $a_l \bmod m = a_{l+1} \bmod m = \ldots = a_r \bmod m$?

When $r - l > 1$

$$a_l \bmod m = a_{l+1} \bmod m = \ldots = a_r \bmod m$$

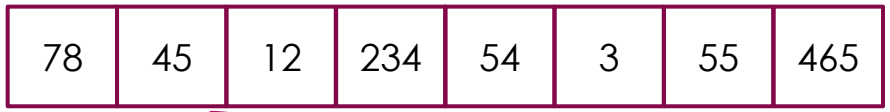
$$|a_l - a_{l+1}| \bmod m = |a_{l+1} - a_{l+2}| \bmod m = \ldots = |a_{r-1} - a_r| \bmod m = 0$$

$$m = gcd(|a_l - a_{l+1}|, \quad |a_{l+1} - a_{l+2}|, \quad \ldots |a_{r-1} - a_r|)$$



How to find out the maximal length of a religious interval?

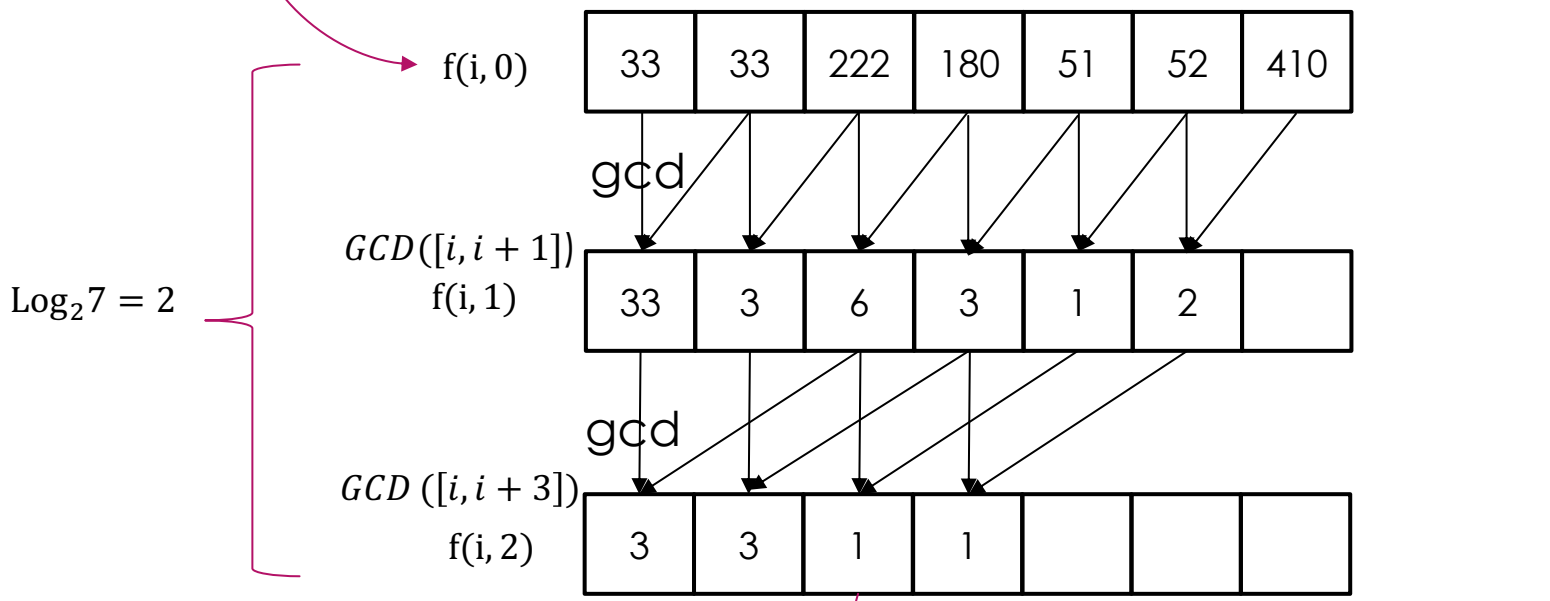
ST table : Creat



diff

Creating ST table:

$let\ f(i,j) represent\ the\ \gcd\ of\ inteval\ [i, i+2^j-1] while\ j>0$



1 is the gcd of inteval [3, 6] which contains {222, 180, 51, 52}

https://oi-wiki.org/ds/sparse-table/

ST table: Query

78	45	12	234	54	3	55	465

diff

ST table:

$let\ f(i,j) represent\ the \gcd of\ inteval\ [i, i+2^j-1] while\ j>0$

f(i, 0)

33	33	222	180	51	52	410

gcd

$GCD([i, i+1])$
f(i, 1)

33	3	6	3	1	2	

gcd

$GCD\ ([i, i+3])$
f(i, 2)

3	3	1	1			

$GCD\ Query$:

$Inteval: [l, r]$

$[l, l+2^s-1] \quad [r-2^s+1, r]$

$s = \lfloor \log_2(r-l+1) \rfloor$

$GCD_l = the \gcd of\ inteval\ [l, l+2^s-1]$
$= f(l, s)$
$GCD_r = the \gcd of\ inteval\ [r-2^s+1, r]$
$= f(r-2^s+1, s)$

$GCD = \gcd(GCD_l, GCD_r)$ $\quad \boldsymbol{O(1)}$

ST table: Query example

78	45	12	234	54	3	55	465

diff

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gcd

$GCD\ ([i, i+1])$
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gcd

$GCD\ ([i, i+3])$
f(i, 2)

3	3	1	1			

$GCD\ Query$:

$Inteval$: [1,7] $s = \lfloor \log_2(7-1+1) \rfloor = 2$

$[1, 1+2^2-1=4]$ $[7-2^2+1=4, 7]$

$GCD_l = the\ \gcd\ of\ inteval\ [1,4]$
$= f(1,2) = 3$
$GCD_r = the\ \gcd\ of\ inteval\ [4,7]$
$= f(4,2) = 1$

$GCD([1,7]) = \gcd(3,1) = 1$ $\quad \boldsymbol{O(1)}$

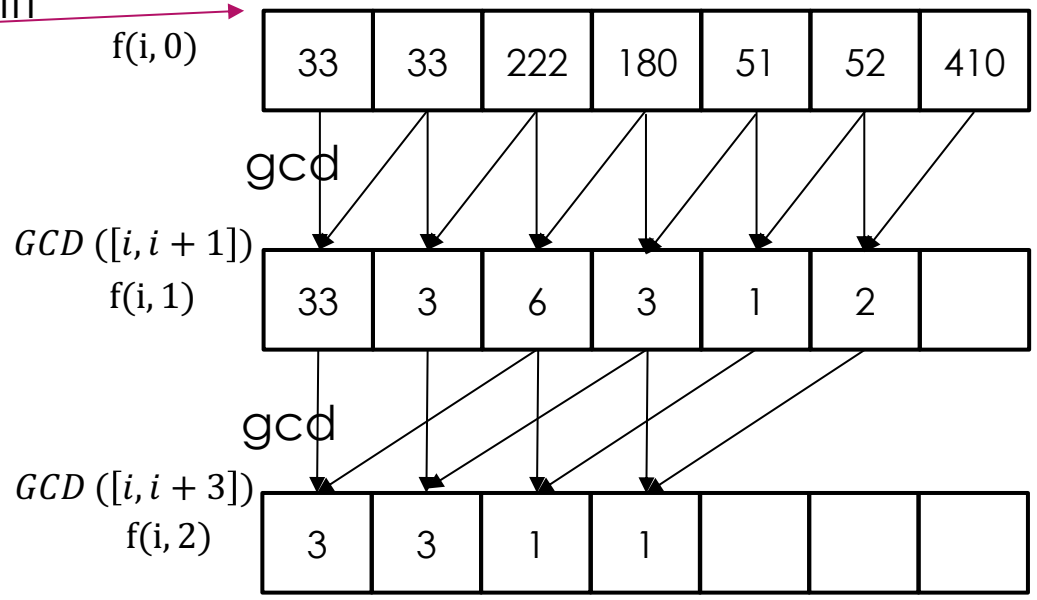
ST table: Query example

78	45	12	234	54	3	55	465

diff

ST table:

$let\ f(i,j) represent\ the\ \gcd of\ inteval\ [i, i+2^j-1] while\ j>0$



First, we can query the half interval, for example, [1, 4]
we know GCD([1,4]) = 3, which means the interval may enlarge, then can try the ¾ interval [1, 6]
Now GCD([1,6]) = 1, now the interval should be reduced.
Try GCD([1,5]) , GCD([1, 5])= 3
Hint: binary search

While fixing 1 to be the left point of the interval, the maximum length is 6.

Enumerate all indexes to be the left point of the interval, and get the maximum length of all cases.