Surface Plasmon Resonance and Measurement of Dispersion Relation

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Abstract

In this experiment we studied the surface plasmons resonance and measured its dispersion relations. We measured the index of refraction of the prism by minimum angle deviation method and calculated the critical angle for different wavelengths. We made a silver film with thickness of 25nm in evaporator, measured the incident angle and relative intensity of the reflective light at resonance. We measured the index of refraction of prism to be 1.5165 ± 0.0002, critical angle to be $41°16.188' \pm 0.007'$ and angle of resonance to be $42°30.989' \pm 0.005'$ at 650nm. We plotted the dispersion relation and calculated the electron density to be $5.151 \pm 0.507 \times 10^{22}\text{cm}^{-3}$, which is 12% off the typical value $5.86 \times 10^{22}\text{cm}^{-3}$. The relative intensity of reflective light is plotted as a function of incidence angle. We observe a wavelength dependence of the position and the width of the resonance peak.

1 Introduction and Theory

1.1 Drude Theory of Free Electron Gas

Drude theory proposed in 1900 explained the electrical conduction mechanism in metals. It assumes that gas of electrons are free(no electron-ion interaction) and independent(no electron-electron interaction) particles except when they collide, and collisions of electrons occur instantaneously, with probability per unit time $1/\tau$. [1] As a result of Drude theory and Ohm's law, the conductivity of a metal in an AC electric field is:

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}, \sigma_0 = \frac{ne^2\tau}{m} \tag{1}$$

where ω is the frequency of the AC field, n is the density of electron per volume, m is the electron mass. And when the wavelength of the field is large compared to the electron mean free path l, we can assume the electric field does not vary appreciably in space, then Ohm's law can be written as:

$$\boldsymbol{j}(\boldsymbol{r}, \omega) = \sigma(\omega)\boldsymbol{E}(\boldsymbol{r}, \omega). \tag{2}$$

Now consider in the presence of a specified current density $\boldsymbol{j}$ and no bound charge $\rho = 0$, we may write Maxwell's equations as:

$$\nabla \cdot \boldsymbol{E} = 0 \tag{3}$$

$$\nabla \cdot \boldsymbol{H} = 0 \tag{4}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} \tag{5}$$

$$\nabla \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \tag{6}$$

We can organize these equations and rewrite in a form of usual wave equation

$$-\nabla^2 \boldsymbol{E} = \frac{\omega^2}{c^2}\varepsilon(\omega)\boldsymbol{E} \tag{7}$$

with a complex dielectric constant given by

$$\varepsilon(\omega) = 1 + \frac{4\pi i \sigma}{\omega} \tag{8}$$

Take the limit $\omega\tau \gg 1$ and insert Eq.1, to the first approximation we get

$$\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega^2}\frac{i\tau\omega}{1 - i\tau\omega} \approx 1 - \frac{\omega_p^2}{\omega^2} \tag{9}$$

where plasma frequency ω_p is given by

$$\omega_p^2 = \frac{4\pi n e^2}{m^*}(\text{CGS}) \tag{10}$$

$$\omega_p^2 = \frac{n e^2}{\varepsilon_0 m^*}(\text{SI}) \tag{11}$$

1.2 Surface Plasmons and Dispersion Relation

Surface plasmons are defined as coherent charge density oscillations at the interface of two materials. And the surface charge oscillations are naturally coupled to the electromagnetic waves which explains their designation as polaritons. Now let's consider two materials with interface is plane $z = 0$ of a Cartesian coordinate system as shown in Fig.1. Material 1(metal in our case) is characterized by a complex frequency dependent dielectric function $\varepsilon_1(\omega)$ whereas material 2(air in our case) has a real dielectric function $\varepsilon_2(\omega)$. We are looking for homogeneous solutions of Maxwell's equations that are localized at the interface. That is, solution must satisfy the wave equation

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}, \omega) - \frac{\omega^2}{c^2}\varepsilon(\omega)\boldsymbol{E}(\boldsymbol{r}, \omega) = 0 \tag{12}$$

with $\varepsilon(\omega) = \varepsilon_1(\omega)$ for $z < 0$ and $\varepsilon(\omega) = \varepsilon_2(\omega)$ for $z > 0$. The localization at the interface requires exponentially decaying EM fields. So it is sufficient to consider TM(p-polarized)

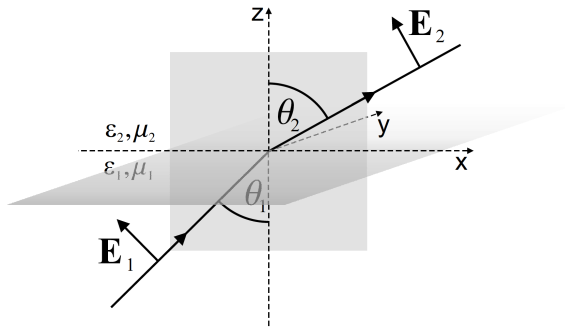


Figure 1: Interface between two media 1 and 2 with dielectric function ε_1 and ε_2.

waves in both $z < 0$ and $z > 0$ region because no solutions exist for the case of TE(s-polarized) waves. Thus the solution is of the form [2]

$$\boldsymbol{E} = \begin{pmatrix} E_{j,x} \\ 0 \\ E_{j,z} \end{pmatrix} e^{ik_x x} e^{ik_{j,z} z} . j = 1, 2 \tag{13}$$

First it must satisfy the wave equation as shown in Eq.(12) and we get the following relations:

$$k_x^2 + k_{j,z}^2 = \varepsilon_j k_0^2, j = 1, 2 \tag{14}$$

where $k_0 = 2\pi/\lambda = \omega/c$ is vacuum wave number. Second electric displacement field must be source free, i.e $\nabla \times \boldsymbol{D} = 0$, this gives:

$$k_x E_{j,x} + k_{j,z} E_{j,z} = 0, j = 1, 2 \tag{15}$$

Besides, we need to impose the boundary conditions at the interface by requiring the continuity of the parallel component of $\boldsymbol{E}$ and the perpendicular component of $\boldsymbol{D}$ and we get:

$$E_{1,x} - E_{2,x} = 0 \tag{16}$$

$$\varepsilon_1 E_{1,z} - \varepsilon_2 E_{2,z} = 0 \tag{17}$$

Eq.(15), (16), (17) form four homogeneous equations for four field components. The existence of solution requires the parameter determinant to vanish, i.e.

$$\varepsilon_1 k_{2,z} - \varepsilon_2 k_{1,z} = 0 \tag{18}$$

Together with Eq.(14), we finally get the expression of the dispersion relation:

$$k_x^2 = \frac{\varepsilon_1(\omega)\varepsilon_2}{\varepsilon_1(\omega) + \varepsilon_2} k_0^2 \tag{19}$$

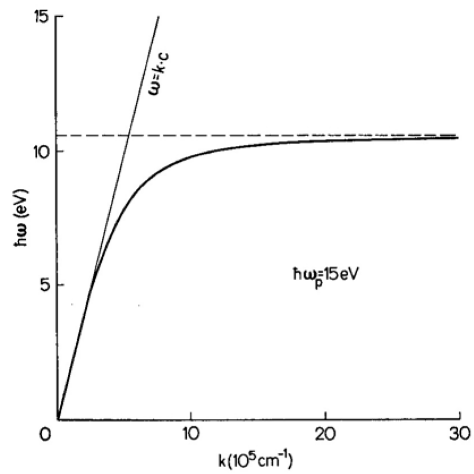


Figure 2: Dispersion relation $\omega(k)$ of nonradiative surface plasmon wave at the surface of a free electron gas($\varepsilon = 1 - \omega_p^2/\omega^2, \hbar\omega_p = 15\text{eV}$) with vacuum. The straight line is the dispersion of plane electromagnetic waves in vacuum. This figure is adapted from Otto [3]

1.3 Excitation of Surface Plasmons

Since the surface plasmons have a nonlinear dispersion relation with wavevectors greater than the incoming photons, they can not be excited by directly shining light onto a metal surface. Thus we need a greater wavevector hence a larger incident phonon momentum to generate the surface plasmons. The most simple way is to excite surface plasmons by means of evanescent waves created at the interface between a medium with refractive index $n > 1$. [2] This is first developed by Otto [3] in 1968, who showed that when light going through a prism undergoes total internal reflection. The wavevector of the photons along the base of the prism would be $k = k_0 n sin\alpha$, where n is the refractive index of the prism and α is the angle of incidence from vertical. Under total internal reflection, no light propagates into the air but the continuity of the fields at the interface makes the fields extend a short distance into the air with an exponentially decay, which is known as the evanescent wave.

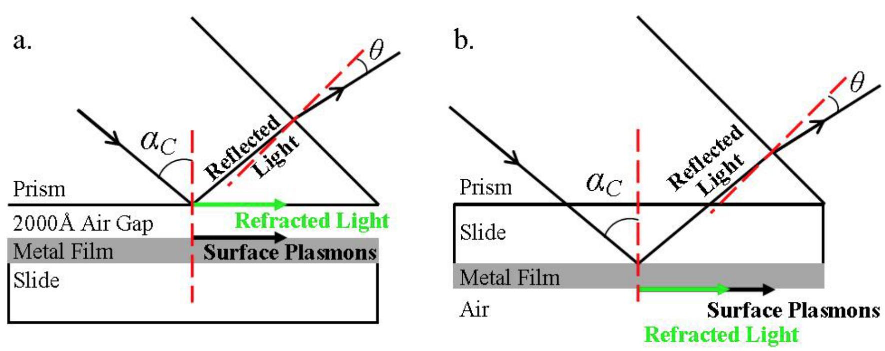


Figure 3: The Otto (a) and our variation of the Kretschmann (b) configurations for generating surface plasmons in the metal films. Kretschmann deposited the metal film directly on the prism, while we deposit it on a glass slide. This figure is adapted form the lab manual [4]

In the Otto configuration, the tail of the evanescent wave at the glass-air interface is brought into contact with a metal-air interface that generates surface plasmons. For a sufficiently large separation between the two interfaces the evanescent wave has a minimum spatial effect on the metal. By tuning the angle of incidence of the totally reflected light inside the prism, the resonance condition for excitation of surface plasmons can be achieved. In this resonance condition, we will observe a clear absorption of the reflected light.

Due to the rapid decay of the fields far from the photons, the air separation must be maintained at a distance of approximately 100nm. This tiny air gap is very difficult to control in reality, which makes Otto's configuration experimentally inconvenient. In 1971, Kretschmann came up with an alternative configuration that solved this problem. He deposited a thin metal film on top of the prism and the evanescent wave was developed at the interface of glass and metal. In order to excite surface plasmons at the metal-air interface, the evanescent wave have to tunnel through the thin film to reach air on the other side. This phenomenon is known as frustrated total internal reflection. Two different configurations are shown in Fig.3.

2 Setup and Procedures

2.1 Apparatus and Setup

The schematic of the experimental setup is shown in Fig.4. In our experiment, the light source is provided by a 12V, 100W tungsten bulb powered by a large, highly regulated DC supply. Then we select light with particular wavelength by the monochromator which contains two 1180 grooves/mm high dispersion gratings blazed to 300nm and 600nm, respectively. One adjustable polarizer is placed after the monochromator to control the direction of polarization of the incident light. The heart of this experiment lies in the spectrometer, which allows us to precisely do the aligning and calibrating the optics. A photometer is used when measuring the intensity of the reflected light. This photometer mainly gives the counts of photons of the coming light.

2.2 Procedures

We measure and calculate the index of refraction by the minimum angle deviation method. We record the reading of the spectrometer when collimator and telescope is at 180°. After cleaning the prism, we put it on the spectrometer table. When light with different wavelength is incident into the prism, we rotate the telescope to find the refracted image of the light slit and record the reading. By dividing the two angles we get the minimum deviation angle δ_m. As shown in Fig.5, the deviation angle δ_m is minimized when the light beam entering and exiting are symmetric. By simple geometry we get the relation of the angles:

$$2\theta_2 = \alpha \tag{20}$$

$$\delta_m/2 + \theta_2 = \theta_1 \tag{21}$$

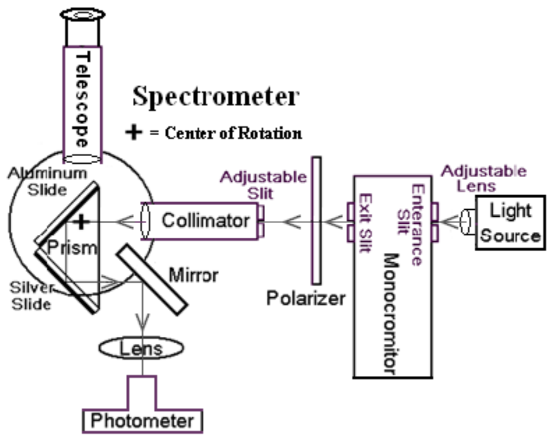


Figure 4: Schematic of the experimental setup used for excitation and measurement of surface plasmons

Together with $\alpha = \pi/4$, substitute into Snell's law $n_{prism}/n_{air} = sin\theta_1/sin\theta_2$ and we get the index of refraction of the prism:

$$n_{prism} = n_{air}\frac{sin(\frac{\delta_m}{2} + \frac{\pi}{8})}{sin(\frac{\pi}{8})} \tag{22}$$

The alignment of optics is quite import because this ensures that the image we observe is optimized and the data we take is with respect to the calibration position. We rotate the telescope precisely 90° CW from the 180° position and observe the reflected slit image falling precisely on the intersection of the cross hairs. Now in this position we record the reading and take it as calibration angle θ_0.

In order to excite surface plasmons, we have to let the incoming light at an angle $\alpha \geq \alpha_c$ where α_c is the critical angle. With the calculated refractive index and Snell's law, we can then calculate the critical angle at different wavelength:

$$\alpha_c = arcsin(\frac{n_{air}}{n_{prism}}) \tag{23}$$

Then we rotate the prism to approximately the critical angle where the total internal reflection begins. The position where we should rotate the telescope to is calculated as follows. By simple geometry, we can calculate the relation between prism incident angle θ and SP incident angle α in Fig.6. (Let θ' to be the refracted angle of the prism).

$$\theta' = |\alpha - \frac{\pi}{4}| \tag{24}$$

$$\frac{sin\theta}{sin\theta'} = \frac{n_{prism}}{n_{air}} \tag{25}$$

Then we get the value of θ:

$$\theta = arcsin(\frac{n_{prism}}{n_{air}}sin|\frac{\pi}{4} - \alpha|) \tag{26}$$

and we are to set the reading of the divided circle to be $\theta_0 - 45° - \theta$. At this position, we make the surface plasmons incident angle to be α. Now we observe from the photometer to do the alignment of the rest part of the optics. In the rest part of the experiment, the prism can be considered as a mirror because the incident and reflected light beam is always parallel. When aligning the mirror, lens and the photometer, we must ensure that the parallel light is reflected, converged and detected by the photometer. If we observe a bright and clear slit image in the circle of the photometer's eyepiece, the alignment of optics is well done.

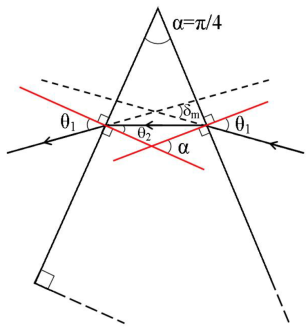


Figure 5: Measurement of refractive index by minimum deviation angle method

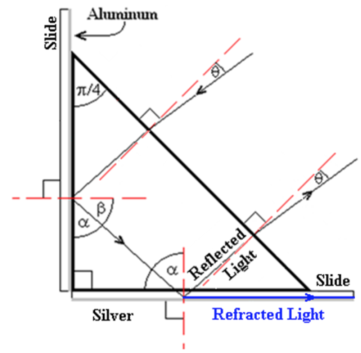


Figure 6: Path of light through the prism

We then deposited silver and aluminum films in the evaporator. The aluminum film is used to reflect the light to keep it inside the prism instead of refracting out in the air. The silver film is the source of the surface plasmons. So the thickness of the silver film should be thin enough to allow penetration of the evanescent wave. In this experiment we prepared a 25nm thick silver film and 92.5nm thick aluminum film.

After the silver film is prepared, we need to take the measurement of surface plasmons dispersion relation immediately due to the high oxidation rate of silver. We first calibrate the photometer that when no light is incident, the reading is 5 counts with Gain and Integration time set to 1. Then we block the light source and take the background signal, the counts read 329 with Gain and Integration time set to 1 and 10, respectively. This background signal will be removed from the latter measurement. Using the technique described in the manual ????, we record the angle and photometer counts when absorption occurs under the condition of TM, TE polarization and different wavelengths. We only take data in wavelength 650nm, 600nm, 550nm, 500nm and 450nm because we can not see the absorption dark line when incident light is 400nm.

3 Data Analysis

3.1 Measuring the Index of Refraction of Prism

Using the minimum angle deviation method, we calculate the index of refraction of the prism according to Eq.22. The error is estimated using error propagation equation:

$$\sigma_n = \frac{n_a ir}{2sin(\pi/8)} cos(\frac{\delta_m}{2} + \frac{\pi}{8})\sigma_{\delta_m} = 0.0002 \tag{27}$$

where we take the error in measurement of angle to be 0.5'.

Wavelength(nm)	Minimum deviation angle δ_m	Refractive index of prism n_{prism}
650	$21°19.5'$±0.5'	1.5165±0.0002
600	$21°14.5'$±0.5'	1.5180±0.0002
550	$21°7.5'$±0.5'	1.5202±0.0002
500	$20°59'$±0.5'	1.5228±0.0002
450	$20°46.5'$±0.5'	1.5267±0.0002

Table 1: Refractive index of the prism at different wavelength

From calculation we see that the index of refraction of our prism is $\sim$ 1.52 for visible light. This is consistent with the value of crown glass which has the index of refraction 1.52 at 293K and 589nm. [5]

3.2 Dispersion Relation and Electron Density

Using the calculated index of refraction, we first calculated the critical angle to get a sense of to what angle we should rotate the prism to detect the resonance. In the following table, we present the calculation result, where α_c is critical angle and θ is the incident angle of prism shown in Fig.6.

Wavelength(nm)	Critical angle α_c	Incident angle of prism θ
650	$41°16.188'$±0.007'	5° 39.6'
600	$41°13.113'$±0.007'	5° 44.7'
550	$41°8.826'$±0.007'	5° 51.7'
500	$41°3.646'$±0.007'	6° 0.2'
450	$40°56.082'$±0.007'	6° 12.7'

Table 2: Critical angle and incident angle of prism at different wavelength

The error of the critical angle is calculated by error propagation [6]:

$$\sigma_{\alpha_c} = \frac{n_{air}}{n_{prism}\sqrt{n_{prism}^2 - n_{air}^2}}\sigma_n = 0.007' \tag{28}$$

Here we do not calculate the error of incident angle θ is because this just provides us an approximately position and we still need to adjust the prism to look for the absorption. Now we are going to calculate the resonance angle α_r. It is worth noticing that when we find the minimum intensity position and take data for a few angles near this position, these data do not directly correspond to the angles near resonance, i.e. near the peak. This is because the function of α with respect to angle θ is not a monotone function, for $\theta < 0$(which is the case shown in Fig.6, light incident from the right side of vertical)

$$\alpha = \frac{\pi}{4} + arcsin(\frac{n_{air}}{n_{prism}}sin\theta) \tag{29}$$

But for $\theta > 0$, light incident from the other side of vertical, we have:

$$\alpha = \frac{\pi}{4} - arcsin(\frac{n_{air}}{n_{prism}} sin\theta) \tag{30}$$

According to the theoretical analysis described in Sec.1.3, surface plasmon resonance absorption occurs where a dark slit is observed, which means the counts we read from the photometer will show a sharp drop. That is to say, the position with minimum counts is considered to be the resonance angle. The result is shown in Table.3.

Wavelength(nm)	Resonance angle α_r	Electron density n(cm^{-3})
650	$42°30.989'$±0.005'	5.782×10^{22}
600	$42°48.255'$±0.005'	5.467×10^{22}
550	$43°11.783'$±0.005'	5.195×10^{22}
500	$43°53.334'$±0.005'	4.792×10^{22}
450	$44°54.103'$±0.005'	4.519×10^{22}

Table 3: Resonance angle and electron density at different wavelength

The error of the resonance angle is calculated by propagation:

$$\sigma_{\alpha_r} = \sqrt{\frac{sin^2\theta}{n^2(n^2 - sin^2\theta)}\sigma_n^2 + \frac{cos^2\theta}{n^2 - sin^2\theta}\sigma_\theta^2} = 0.005' \tag{31}$$

where we take $\sigma_\theta = 0.5' = 1.45 \times 10^{-4}$ rad

Otto showed that the surface plasmon wavevector is defined as:

$$k_{sp} = k_0 n sin\alpha_r = \frac{\omega}{c} n_{prism} sin\alpha_r \tag{32}$$

and thus the error is given by:

$$\sigma_k = \frac{\omega}{c}\sqrt{sin^2_\alpha \sigma_n^2 + cos\alpha^2 n^2 \sigma_\alpha^2} \tag{33}$$

Thus we get 5 data points corresponding to 5 wavelengths in visible light region. The plot of dispersion relation is shown in Fig.7. In this figure we also showed the theoretical result of Drude model described by Eq.19. If we set $\varepsilon_2 = 1.00059 \approx 1$ and $\varepsilon_1 \approx 1 - \omega_p^2/\omega^2$ for simplicity, the dispersion relation can be written as:

$$k = \frac{\omega}{c}\sqrt{\frac{\omega^2 - \omega_p^2}{2\omega^1 - \omega_p^2}} \tag{34}$$

One important feature of surface plasmons is that for a given energy $\hbar\omega$ the wave vector k is always larger than the wave vector of light in free space. This is easily observed from Fig.7, where the line $k = \omega/c$ is plotted as solid black line. We can also see that for small energies, the SPs dispersion relation asymptotically approaches this black line. We assume that this is due to the strong coupling between light and surface charges for increased momentum of SPs, with an effect that the electrons are tend to stay along metal surface. This means that

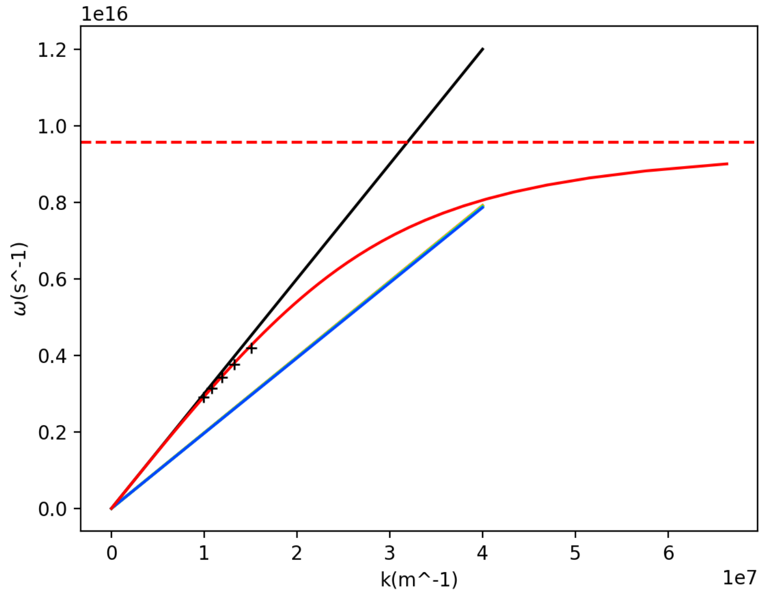


Figure 7: Dispersion relation of surface plasmons. Black line is $k_{air} = \frac{\omega}{c} n_{air}$. Blue lines are $k_{prism} = \frac{\omega}{c} n_{prism}$. Black cross points are our experiment data of surface plasmons for 5 wavelengths. Red solid line is simulation result of SPs dispersion relation with $\hbar\omega_p = 8.9$eV for silver. Red dashed line is the asymptotic line $\omega = \omega_p/\sqrt{2}$

SPs at the interface can not be excited by light directly propagates in free space, and they can only be generated by the light with increased wavevector, which the blue lines shown in Fig.7.

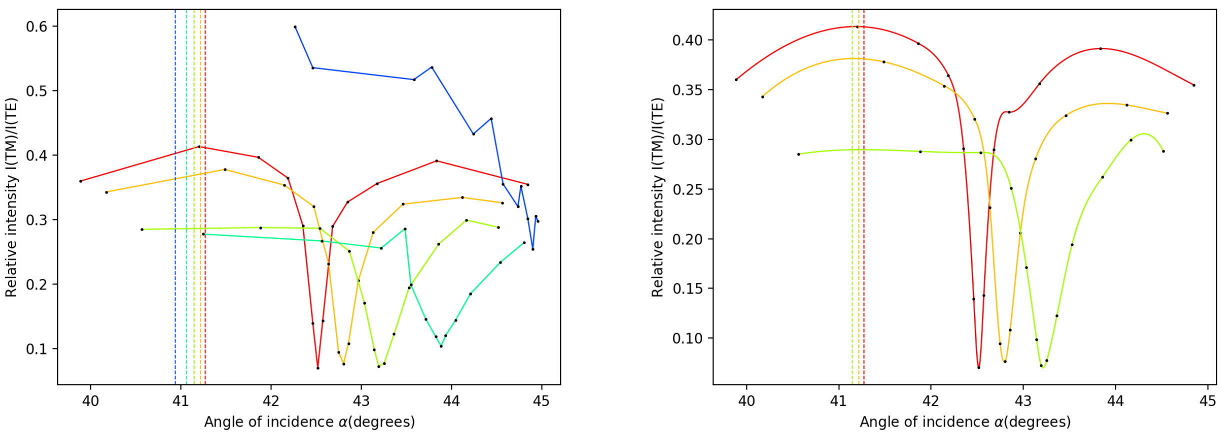
We can also calculate the electron density of silver with the data we measured. Combining Eq.9, Eq.11 and Eq. 19, we can get the expression of the electron density n:

$$n = \omega^2 \left(\frac{1}{n_{prism}^2 sin^2\alpha - 1} + 2\right) \frac{\varepsilon_0 m^*}{e^2} \tag{35}$$

where dielectric constant of air $\varepsilon_2 = 1.00059 \approx 1$, $k_0 = \omega/c$ is free space wavevector and $\varepsilon_0 = 8.85 \times 10^{-12}$F/m is dielectric constant of vacuum. For now we assume the effective mass in silver is simply the mass of an electron in free space. To calculate the electron density, we simply calculate n using Eq.35 for 5 wavelengths, with result shown in Table.3. Take the average and the standard deviation of 5 data points to be the experiment result and error and we finally get the result of $n = 5.151 \pm 0.507 \times 10^{22}\text{cm}^{-3}$. Compared with typical value of silver electron density $\sim 5.9 \times 10^{22}\text{cm}^{-3}$, our calculated value is 12% off, which is slightly out of the 1σ region. We attribute this inconsistency to our calculation method of averaging discrete data points, because the approximation of assuming effective mass as electron mass should not bring so much effect on the electron density. This result shows a clear dependence on wavelength and longer wavelength tends to be more accurate than others, which is consistent with the goodness of fit in Fig.7.

3.3 Surface Plasmon Resonance Spectrum

As described in Sec.1.1, surface plasmons can only be excited by TM polarized waves, and can not be generated by TE polarized waves. As a consequence, we see that the counts we read from the photometer have a sharp drop along incident angles of TM waves while the counts stay almost the same with change of incident angles of TE waves. This is plotted in Fig.8. First we plot the data points and connect them by lines, we can see that our



(a) **(b)**

Figure 8: Excitation of surface plasmons. The relative intensity of reflected light is plotted as a function of incident angle α for different wavelengths. Red, yellow, green, indigo and blue line graph corresponds to 650nm, 600nm, 550nm, 500nm, 450nm . (a) is just simple connecting of discrete data points and (b) is the fitted curve to data points by the method of spline interpolation with smoothing factor set to 0. Critical angles are shown in dashed lines.

data do not cover the both sides of the resonance peak for 450nm wavelength. This is consistent with what we described in Sec.3.2 because the function of α is not monotone with prism incident angle θ and we only take data near resonance θ value rather than α. From this figure, we notice (1) the minimum intensity occurs at angle larger than the critical angle, (2) the resonance angle and the width of the resonance peak increased with increased with decreased wavelength(increased incident energy). This can be understood as follows. The excitation of surface plasmons is actually a momentum matching process. The surface plasmons momentum is dependent on the energy of incoming light from Drude model, also dependent on the index of refraction of prism and the incident angle from Otto experiment configuration. Thus, the matching of momentum must bring the wavelength dependence of the incident angle. In analogy to NMR spectrum, here we can understand the width of the resonance peak as the effect of damping. Broader width means higher damping rate so this figure shows the damping of SP waves is smaller if SPs are excited by larger wavelengths.

4 Conclusion and Discussion

In this experiment, we measured the index of refraction of prism to be 1.5165 ± 0.0002, critical angle to be $41°16.188' \pm 0.007'$ and angle of resonance to be $42°30.989' \pm 0.005'$ at 650nm.

From the plotted dispersion relation, we calculated the electron density to be $5.151 \pm 0.507 \times 10^{22}\text{cm}^{-3}$, which is 12% off the typical value $5.86 \times 10^{22}\text{cm}^{-3}$. We think this inconsistency might arise from the calculation method we use, which is averaging the value calculated from discrete data points. The proper way of doing this is to fit the experiment data points to the theoretical dispersion relation curve, described by Eq.19. But here for better visualization we need to plot frequency vs wavevector, that is to say, we are fitting the data to the inverse function of Eq.19, given as follows:

$$\omega = \sqrt{\frac{-\sqrt{4c^4k^4 + \omega_p^4} + 2c^2k^2 + \omega_p^2}{2}} \tag{36}$$

But we have met problems when doing the curve fit using python and due to the limit of time we used the method described above to calculate the electron density, though it is clearly not a good method.

The lab manual suggested us to examine 6 wavelength in the range of 650nm to 400nm. In the experiment we found that at the 400nm wavelength, the light slit is extremely difficult to see from the telescope and even harder to see the absorption dark line from the photometer. Consequently, we just got 5 data points in the dispersion relation section. In order to get more data points in the visible light region, we suggest that 6 wavelengths could be examined in the range of 650nm to 450nm.

Besides, if possible, we could make silver slides with different thickness and do the same measurement to see the shape of the resonance peak. Thinner silver film enables the evanescent wave to penetrate through more easily but the surface plasmon waves may be strongly damped into the glass. In this condition, we may expect to see a less sharp peak in the resonance spectrum.

References

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Appendix

A.1 Python codes of calculation and plotting

```
from math import *
import xlrd
import matplotlib.pyplot as plt
import numpy as np
from scipy import interpolate
from scipy.optimize import curve_fit

# Import data from xlsx file

book1 = xlrd.open_workbook('Refraction2.xlsx')
sheet1 = book1.sheet_by_index(0)

book2 = xlrd.open_workbook('Data2.xlsx')
sheet2 = book2.sheet_by_index(0)

lambda_legend_list = sheet1.col_values(0, 3, 8)
lambda_list = [650, 600, 550, 500, 450]
color_list = ((1, 0, 0), (1, 0.75, 0), (0.64, 1, 0), (0, 1, 0.57), (0, 0.27, 1))

# Calculate index of refraction
delta_m = sheet1.col_values(4, 3, 8)
sd_angle = 1.45e-4          # Error of angle measured in rad
n_air = 1.00028             # Index of refraction of air in STP
n_prism = [sin(x/2+pi/8)*n_air/sin(pi/8) for x in delta_m]
sd_n_prism = [n_air*cos(x/2+pi/8)/2/sin(pi/8)*sd_angle for x in delta_m]
print(r'Refractive index is', n_prism)
print(r'Refractive index error is', sd_n_prism)


# Calculate critical angle and needed angle of divided circle


def r2dm(x):    # Rad to Degree Minute transform
    return [int(x/pi*180), (x/pi*180-int(x/pi*180))*60]


alpha_c = [asin(n_air/x) for x in n_prism]
alpha_c_dm = [r2dm(x) for x in alpha_c]
sd_alpha_c = [n_air/x/sqrt(x**2-n_air**2)*0.0002*180/pi for x in n_prism]


def cal_theta(x1, x2):
    return asin(x1/n_air*sin(pi/4-x2))


theta_theory = [cal_theta(x1, x2) for (x1, x2) in zip(n_prism, alpha_c)]
theta_theory_dm = [r2dm(x) for x in theta_theory]
print(theta_theory_dm)


def cal_exp_alpha(t, n):
    if t > 0:
        return pi / 4 - asin(n_air * sin(t) / n)
    else:
        return pi / 4 + asin(n_air * sin(t) / n)
```

```python
# Plot dispersion relation


c = 3e8                     # Speed of light (m/s)
h_bar = 6.58212e-16         # Planck constant (eV*s)
omega_p_theory = 8.9        # eV/h_bar

epsilon_d = 1.00059       # Dielectric const for air
epsilon_0 = 8.85419e-12 # Vacuum permittivity (F/m)
q_e = 1.60218e-19         # Electron charge (C)
m_eff = 9.10938e-31       # Effective mass of electron in metal (assume = electron mass)


omega = [2*pi*c/x/10**-9 for x in lambda_list]

exp_theta_r = sheet1.col_values(11, 3, 8)
exp_alpha = [cal_exp_alpha(t, n) for (t, n) in zip(exp_theta_r, n_prism)]
exp_alpha_dm = [r2dm(x) for x in exp_alpha]
sd_exp_alpha = [sqrt((sin(t)**2*0.0002**2/n**2/(n**2-sin(t)**2))+
                     (cos(t)**2*sd_angle**2/(n**2-sin(t)**2)))
                for (t, n) in zip(exp_theta_r, n_prism)]
print(r'Critical angle is', alpha_c_dm)
print(r'Critical angle error (in minute) is', sd_alpha_c)
print(r'Resonance angle is', exp_alpha_dm)
print(r'Resonance angle error is', sd_exp_alpha)


k_space = np.linspace(0, 4e7)
omega_space = np.linspace(0, 9e15)
omega_air = k_space*c/n_air
omega_prism = [k_space*c/x for x in n_prism]
k_air = [x/c*n_air for x in omega]
k_sp = [x1/c*x2*sin(x3) for (x1, x2, x3) in zip(omega, n_prism, exp_alpha)]
k_theory = [i/c*sqrt((i**2-(omega_p_theory/h_bar)**2)/
                     (2*i**2-(omega_p_theory/h_bar)**2)) for i in omega_space]
omega_asp = omega_p_theory/h_bar/sqrt(2)

sd_k_sp = [x1/c*sqrt((sin(x3)*0.0002)**2+(cos(x3)*x2*1e-4)**2)
           for (x1, x2, x3) in zip(omega, n_prism, exp_alpha)]
print(r'k_sp error is', sd_k_sp)

# Curve-fit of dispersion relation


def disp(x, a):
    return np.sqrt((2*(c*x)**2 - np.sqrt(4*(c*x)**4 + a**4) + a**2) / 2 )


popt, pcov = curve_fit(disp, np.array(k_sp), np.array(omega))
perr = np.sqrt(np.diag(pcov))
print(popt, perr)

plt.figure(1)
plt.plot(k_space, omega_air, 'k')
for i in range(0, 5):
    plt.plot(k_space, omega_prism[i], color=color_list[i])
plt.plot(k_theory, omega_space, 'r')
plt.plot(k_sp, omega, 'k+')
# plt.errorbar(x=k_sp, y=omega, xerr=np.array(sd_k_sp))
plt.axhline(y=omega_asp, color='r', linestyle='--')
```

```python
plt.xlabel(r'k(m^-1)')
plt.ylabel(r'$\omega$(s^-1)')


# Calculate electron density of silver


def cal_n(npr, alpha, omega):
    return (1 / (npr**2 * sin(alpha)**2 - 1) + 2) * omega**2 \
           * (epsilon_0 * m_eff / q_e**2) / 1e6


density = [cal_n(x1, x2, x3) for (x1, x2, x3) in zip(n_prism, exp_alpha, omega)]
print(r'Electron density of silver is', density)

# Plot absorption spectrum near resonance
for i in range(0,3):
    # Import raw data from book2.sheet2
    angle_raw = sheet2.col_values(0+5*i, 4, 17)
    TM_raw = sheet2.col_values(3+5*i, 4, 17)
    TE_raw = sheet2.col_values(4+5*i, 4, 17)

    # Prepare data for plot
    # 12669 is calibration angle in minutes
    # Calculate angle alpha with function cal_exp_alpha
    # Remove background signal (32.9 counts) and calibration (5 counts)
    #   from raw data
    theta = [(12669 - x - 45 * 60) / 60 / 180 * pi for x in angle_raw]      # in rad
    alpha = [cal_exp_alpha(t, n_prism[0]) / pi * 180 for t in theta]    # in degree
    TM = [x - 32.9 for x in TM_raw]
    TE = [x - 32.9 for x in TE_raw]
    Intensity = [x1 / x2 for (x1, x2) in zip(TM, TE)]

    # Fitting to data points using spline interpolation
    # Set smoothing factor to 0
    spl = interpolate.UnivariateSpline(alpha, Intensity)
    alpha_space = np.linspace(min(alpha), max(alpha), 1000)
    spl.set_smoothing_factor(0)
    Intensity_spline = spl(alpha_space)

    plt.figure(2)
    #plt.plot(alpha, Intensity, color=color_list[i], linewidth=0.9)
    plt.plot(alpha_space, Intensity_spline, color=color_list[i], linewidth=0.9)
    plt.axvline(x=alpha_c[i]/pi*180, color=color_list[i], linestyle='--', linewidth=0.7)
    plt.plot(alpha, Intensity, 'k.', markersize=2)

plt.xlabel(r'Angle of incidence $\alpha$(degrees)')
plt.ylabel(r'Relative intensity I(TM)/I(TE)')
# plt.legend(lambda_legend_list, loc='best')
plt.show()
```

A.2 Inventory Sheet

Surface Plasmons, B&H Room ~~211a~~ 209
Inventory Sheet
10/24/17

Start Up	#	Inventory	Close Out
✓	1	Spectrograph	✓
✓	1	Monochromator	✓
✓	1	Tungsten-Light Source	✓
✓	1	Evaporative Coating System with accessories	✓
✓	1	Solid-State Photometer with manual	✓
✓		Acetone, Alcohol and Distilled Water with beakers	✓
✓		Assortment of Optical Holders	✓
✓	1	Convex Lens	✓
✓	1	Flashlight	✓
✓		Mineral Oil	✓
✓	1	Mirror	✓
✓	1	Optical ~~Bench~~ (Long) Rail	✓
✓	1	Optical ~~Bench~~ (Short) Rail	✓
✓	1	Polarizer	✓
✓	1	Prism (90^{o})	✓
✓	2	Pair rubber gloves and Pair Safety Glasses	✓
✓	1	Table Lamp	✓
✓	1	**Safety Manual in Room 203**	✓

Comments:

AT TIME OF CHECKOUT THIS AREA WAS NEAT AND ORDERLY AND THE FOLLOWING ITEMS WERE DISCUSSED: __ THE LAB DOOR, ✓ NO FOOD OR DRINK IN LAB, ✓ USE AND DISPOSAL OF CHEMICALS, ✓ DETAILED EVAPORATOR INSTRUCTIONS, AND ✓ LAB SAFETY. Fan Instruction.

Tianyi Zhou 10/25/17 — Fan [illegible] 10/25/17
(Student signature) (Date) (Student signature) (Date)

AT TIME OF START UP/CHECK OUT THIS AREA WAS NEAT AND ORDERLY AND ALL INVENTORY ITEMS WERE ACCOUNTED FOR.

Start Up: [illegible] 10/25/17
(Staff signature) (Date)

Check Out: [illegible] 11/19/17
(Staff signature) (Date)