Optimal Fuel Feed Strategy for Asymmetric Arranged Multi-tanks Aircraft

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***Abstract*—Aircraft fuel system, which provides a continuous source of fuel to the engine, is an important component of the aircraft. Although the sequential fuel feed strategy for the symmetric arranged multi-tanks aircraft is widely used in today's aircraft, the fuel feed strategy of the asymmetric arranged multi-tanks is still a challenge. In this article, a two layer offline approach is developed to obtain the fuel feed strategy to minimize the difference between the actual center of gravity (CG) and the desired CG. The performance of the proposed approach is tested in a case study based on the test data of two cases, level flight and pitch. The result indicates that the proposed approach solves the problem with an offline manner from the optimization perspective.**

***Index Terms*—asymmetric arranged multi-tanks, center of gravity (CG), fuel feed strategy, offline optimization**

I. Introduction

The aircraft fuel system is fundamental to provide a continuous source of fuel to the engine to maintain thrust and powered flight [1]. Since the system carries a large amount of aviation fuel, it has a strong effect on the aircraft weight and balance. This effect is valued both in the aircraft design phase and operation phase [2]. Because there exists a strong functional interaction between the CG and the aircraft flight performance [3].

During the operation phase, it is vital for the aircraft to comply with the weight and balance limits. There are strict fore and aft limits beyond which the CG should not be located during the flight [4]. If the CG moves aft, the aircraft becomes less controllable. Stall and spin recovery would be impossible if the CG beyond the aft limit. On the other hand, moving the CG forward will result in the aircraft nose-heavy, which means the elevator may be unable to hold up the nose, especially at low airspeeds. Hence to keep the aircraft CG within the limited range is necessary, otherwise it will lead to serious safety accidents.

The aircraft fuel system has a huge effect on the location of aircraft in-flight CG, however from the other perspective, the fuel system could be used to active CG control. In fact, aircraft fuel system has been used for active CG control for a long history. The active CG control system was firstly implemented on the wide body aircraft by Airbus Industries in the 1980s [5]-[6]. The installation of the Center of Gravity Control System (CGCS) by fuel transfer results in fuel benefit and loading benefit [6]. According to Langton [1], the fuel feed strategy adopted by many aircraft, like A340-600, is an sequential strategy. The center tank is consumed first by transferring fuel into each feed tank as fuel is burned.

In recent years, Tudosie propose a meshing method to calculate the fuel CG of each tank [7]. The European research project, SmartFuel, which includes six industry partners and three universities, focused on the aircraft distributed fuel management system [8]. Jimenez [9] builds a two layer simulation model includes process layer and sequential control layer with the MATLAB and Simulink for a symmetric arranged seven tank and two engine aircraft. And they make a simulation and visualization about the fuel feed process and center of gravity evolution process. Yan designs an aircraft CG control law based on sliding mode control [10].

However, most of the above mentioned study focus on the symmetric arranged multi-tanks aircraft. Obviously, the sequential fuel feed strategy provided by the fuel management system of the symmetric arranged multi-tanks aircraft is not suitable for the asymmetric arranged ones. The later needs a more flexible and intelligent fuel feed strategy.

In this paper, we propose a novel two layer offline approach of the fuel feed strategy optimization for asymmetric arranged multi-tanks aircraft.

The remainder of this article is organized as follows. In section II, the system model is introduced. The problem formulation is developed in Section III. In Section IV, the two-layer fuel management approach is presented. The performance of the proposed approach is evaluated in Section V based on simulation results. In section VI, we draw the conclusion and discuss future research direction.

II. System Model

Before we introduce the aircraft CG model, we describe the fuel tanks aboard. The fuel tanks can be divided into two classes: main tanks and backup tanks. Note that the main tanks directly connect to the engine. We use $\mathcal{M}$, $\mathcal{M}_m$ and $\mathcal{M}_b$ to denote the fuel tanks set, main tanks and backup tanks respectively. Hence

$$n = n_m + n_b \tag{1}$$

where $n = |\mathcal{M}|$,$n_m = |\mathcal{M}_m|$, $n_b = |\mathcal{M}_b|$, and $|\cdot|$ represents the cardinality of the set. In addition, we just consider single engine aircraft in this paper.

A. CG Model

Now we introduce the aircraft CG model. When the aircraft doesn't carry any kerosene, the aircraft is periodically weighed to calibrate the empty weight CG (EWCG) location [11]. Along with the fuel injection and fuel consumption, the actual CG varies from EWCG and forms a CG trajectory. Both intuitively and conventionally, we build the coordinate system with the origin at calibrated EWCG.

We assume that the aircraft is only equipped with rigid fuel tanks, and no bladder tanks or integral tanks. Then the cuboid could be used to model the fuel tank, where the length, width and height of each tank are represented by a_i, b_i, c_i respectively. The geometric center of each tank is $\mathbf{P}^i = [x_p^i, y_p^i, z_p^i]^T$ under the EWCG coordinate system. The weight of the fuel in the i-th tank at time slot t is $m_i(t)$, and the weight of the fuel fed by the i-th tank at time t is $y_i(t)$. We assume the tanks have no partitions and the coordinate of CG-of-fuel in each tank is $\mathbf{co}^i = [x_{co}^i, y_{co}^i, z_{co}^i]^T$. According to the CG computing formula, CG = total moment / total weight, the aircraft CG at time slot t is $\mathbf{c}_1(t) = [x_c(t), y_c(t), z_c(t)]^T$, where

$$\begin{cases} x_c(t) = \dfrac{\mathbf{m}(t)^T \mathbf{x}_{co}(t)}{M + \mathbf{1}^T \mathbf{m}(t)} \\ y_c(t) = \dfrac{\mathbf{m}(t)^T \mathbf{y}_{co}(t)}{M + \mathbf{1}^T \mathbf{m}(t)} \\ z_c(t) = \dfrac{\mathbf{m}(t)^T \mathbf{z}_{co}(t)}{M + \mathbf{1}^T \mathbf{m}(t)} \end{cases} \tag{2}$$

where M is the manufacture's empty weight (MEW) of the aircraft and $\mathbf{m}(t) = [m_1(t) \cdots m_n(t)]^T$ is the weight vector of the fuels in the tanks. The weight of the pilot and passengers are not considered here, actually this part of weight can be add to the MEW of the aircraft.

According to (2), $\mathbf{co}^i(t)$ needs to be calculated to determine the aircraft CG. Obviously, it is affected by both weight of the fuel in the tank and the attitude of the aircraft. As a result, it is necessary to construct the CG-of-fuel computing model. For simplification, we just consider the aircraft only with pitch movement.

We construct the Cartesian coordinate system with the origin at the geometric center of each tank, and the direction of the axis coincide with the aircraft coordinate system. The CG of the fuel in the i-th tank under the tank coordinate system is $\mathbf{co}_{tc}^i = [x_{tc}^i, y_{tc}^i, z_{tc}^i]^T$. We assume the fuel level in the tank is flat at any time. Combining with the above assumption, we can get $y_{tc}^i = 0$. Hence, the calculation of the CG-of-fuel is equivalent to the calculation of the face centroid.

As Fig.1 shows, the pitch angle is θ, the coordinate of A and B in the tank coordinate system is $[x_A, 0, 0.5c_i]^T$ and $[0.5a_i \text{sign } \theta, 0, z_B]$ respectively. The center of gravity of the blank part is

$$\mathbf{co}_{tc}^{M_i - m_i} = [\frac{x_A + a_i \text{sign } \theta}{3}, 0, \frac{z_B + c_i}{3}]^T \tag{3}$$

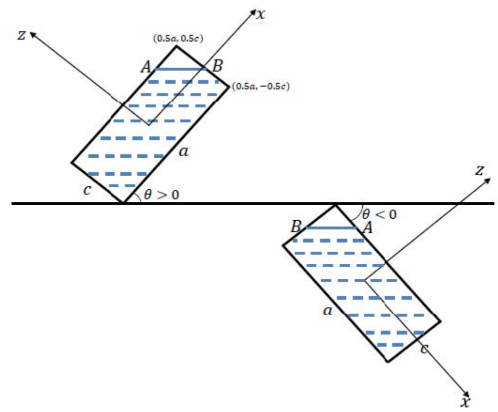

Fig. 1. Schematic diagram of aviation fuel in the fuel tank

where sign$(\cdot)$ is the sign function and M_i is the weight of the fuel that full filled with the i-th tank, then

$$m_i \mathbf{co}_{tc}^i + (M_i - m_i)\mathbf{co}_{tc}^{M_i - m_i} = 0 \tag{4}$$

from geometry, there is

$$\begin{gathered} x_A = \frac{z_B}{\tan\theta} + \frac{a_i \tan\theta \text{sign } \theta - c_i}{2\tan\theta}, \\ z_B \in (-\frac{c_i}{2}, \frac{c_i}{2}), x_A \in (-\frac{a_i}{2}, \frac{a_i}{2}) \end{gathered} \tag{5}$$

$$\frac{M_i - m_i}{m_i} = \frac{0.5(0.5a_i - x_A \text{sign } \theta)(0.5c_i - z_B)}{a_i c_i} \tag{6}$$

x_A and z_B can be solved through (5) and (6), then $\mathbf{co}_{tc}^i$ can be achieved by (4), after that the coordinate of the CG of the fuel in the tank under the aircraft coordinate system can be attained by adding a coordinate translation to it.

There is a special case in aircraft pitch movement, i.e. the pitch angle $\theta = 0°$. In this case, $\mathbf{x}_{co}(t)$ and $\mathbf{y}_{co}(t)$ coincide with $\mathbf{x}_p(t)$ and $\mathbf{y}_p(t)$ respectively, and $\mathbf{z}_{co}(t)$ can be calculated by

$$z_{co}^i(t) = z_p^i - \frac{c_i}{2} + \frac{m_i(t)}{2a_i b_i \rho}, \ \forall i \in \mathcal{M} \tag{7}$$

where ρ is the density of the fuel.

B. Fuel Feed Model

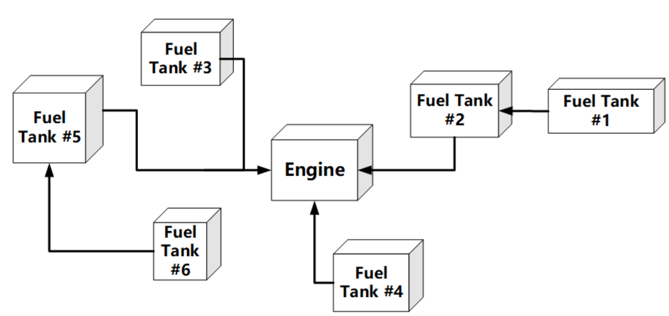

Fig. 2. Schematic diagram of a certain aircraft fuel tank distribution

As Fig.2 shows, the topology of the fuel tanks and the engine can be described through the schematic diagram. According to the topology, we can build the dynamics of the weight of the fuels in the connected fuel tanks. Hence

$$\begin{aligned} \mathbf{m}(t+1) &= f(\mathbf{m}(t), \mathbf{y}(t)) \\ \mathbf{m}(0) &= \mathbf{m}_0 \end{aligned} \tag{8}$$

where $\mathbf{y}(t) = [y_1(t) \cdots y_n(t)]^T$ is the the weight vector of the supplied fuels by the tanks and $\mathbf{m}_0$ is the initial fuel weight in the tanks. There is a maximum of the $y(t)$ in any time slot, i.e.

$$0 \leq \mathbf{y}(t) \leq \bar{\mathbf{y}}, \ \forall t \tag{9}$$

Actually (8) is a group of linear equations. For example, the dynamics in the Fig.2 is

$$\begin{cases} m_1(t+1) = m_1(t) - y_1(t) \\ m_2(t+1) = m_2(t) - y_2(t) + y_1(t) \\ m_3(t+1) = m_3(t) - y_3(t) \\ m_4(t+1) = m_4(t) - y_4(t) \\ m_5(t+1) = m_5(t) - y_5(t) + y_6(t) \\ m_6(t+1) = m_6(t) - y_6(t) \end{cases} \tag{10}$$

The volume of the fuel in the tank is $V_i(t) = m_i(t)/\rho$. And $m_i(t)$ should be restricted

$$0 \leq m_i(t) \leq a_i b_i c_i \rho, \ \forall i \in \mathcal{M}, \forall t \tag{11}$$

We use binary variable $x_i(t)$ to represent whether the i-th tank feeds the fuel at time slot t, where $x_i(t) = 1$ means the i-th tank feeds the fuel at t. Due to the limitations of the aircraft structure, there exists a set of constraints to the numbers of simutaneously feeding tanks. During the flying process, the aggregation of the fuel fed by the tanks should satisfy the engine consumption demand,

$$\sum_{i \in \mathcal{M}_m} x_i(t) y_i(t) \geq h(t), \forall t \in \mathcal{T} \tag{12}$$

where $h(t)$ is the engine consumption demand at t. Observed that the constraint includes bilinear terms, which is not easy to handle with. So we use Big-M method to transform the constraints to linear ones.

$$-M_1 x_i(t) \leq y_i(t) \leq M_1 x_i(t), \forall t \tag{13}$$

where M_1 is a big number, then (12) can be transformed to

$$\sum_{i \in \mathcal{M}_m} y_i(t) \geq h(t), \forall t \in \mathcal{T} \tag{14}$$

Considering the lifetime of the equipment, the valve of the fuel tank should not be opened and closed too frequently. So we assume once the fuel tank is open, it must last for more than L time slots. The relationships of $x_i(t)$ and $x_i(t+1)$ are shown in Table I, then we can get

$$[x_i(t+1) - x_i(t)] \times \left[\sum_{j=1}^{L} (x_i(t+j)) - L \right] \geq 0 \tag{15}$$

If $x_i(t+1) = x_i(t)$, then (15) becomes redundant. If $x_i(t+1) - x_i(t) = -1$, i.e. the i-th tank ends feeding fuel, this constraint is also redundant since $\sum_{j=1}^{L}(x_i(t+j))$ is strictly less than L. If $x_i(t+1) - x_i(t) = 1$, i.e. the i-th tank starts to feed fuel, then $\sum_{j=1}^{L}(x_i(t+j)) - L$ must be great than or equal to 0 to satisfy the assumption.

This constraint can also be linearized using Big-M method.

$$\begin{aligned} -M_1 \{1 - [x_i(t+1) - x_i(t)]\} \leq \sum_{j=1}^{L} (x_i(t+j)) - L \\ \leq M_1 \{1 - [x_i(t+1) - x_i(t)]\} \end{aligned} \tag{16}$$

TABLE I
THE RELATIONSHIP OF $x_i(t)$ AND $x_i(t+1)$

$x_i(t)$	$x_i(t+1)$	State
0	0	Not Feed
0	1	Start to Feed
1	0	End Feeding
1	1	Continue Feeding

III. PROBLEM FORMULATION

During the flying process, the aircraft control system will provide a desired aircraft CG curve, $\mathbf{c}_2(t)$. The objective of the fuel management system (FMS) is to choose a fuel feed strategy such that the actual CG is close to the desired one. Therefore we can formulate the problem as

$$\begin{aligned} (\mathbf{P}) \min_{\mathbf{x},\mathbf{y}} \quad & \max_{t} \quad \|\mathbf{c}_1(t) - \mathbf{c}_2(t)\|_2 \\ s.t. \quad & (2)(8)(9)(11)(13)(14)(16). \end{aligned} \tag{17}$$

The (17) is a Mixed Integer Nonlinear Programming Problem (MINLP) and it is NP-hard even when the aircraft is under the level flight. Since the original optimization problem contains nonlinear constraints (2), it is difficult to solve. Linearization technique is used here to transform the nonlinear constraints to linear ones. According to (2) and (7),

$$z_c(t) = \frac{\sum_{i=1}^{n} [m_i(t)(z_p^i - \frac{c_i}{2} + \frac{m_i(t)}{2a_i b_i \rho})]}{M + \mathbf{1}^T \mathbf{m}(t)} \tag{18}$$

We use $\hat{M}(t)$ to approximate the aircraft gross weight (AUW), i.e.

$$M + \mathbf{1}^T m(t) \approx \hat{M}(t) \tag{19}$$

where

$$\begin{cases} \hat{M}(t) = \hat{M}(t-1) - h(t-1) \\ \hat{M}(0) = M + \mathbf{1}^T \mathbf{m}_0 \end{cases} \tag{20}$$

Here the engine fuel consumption demand data is utilized. Though this approximation brings accumulated error, it provides a well upper bound estimation of the AUW. Similarly, $\hat{m}_i(t)$ is adopted to approximate the real fuel weight $m_i(t)$ while $\frac{m_i(0)}{\mathbf{1}^T \mathbf{m}_0}$ is the coefficient represents the component of the initial fuel weight.

$$m_i(t) \approx \hat{m}_i(t) \tag{21}$$

where

$$\begin{cases} \hat{m}_i(t) = \hat{m}_i(t-1) - \dfrac{m_i(0)}{\mathbf{1}^T \mathbf{m}_0} h(t) \\ \hat{m}_i(0) = m_i(0) \end{cases} \tag{22}$$

Hence the (18) could be rewritten as

$$z_c(t) = \frac{\sum_{i=1}^{n} [m_i(t)(z_p^i - \frac{c_i}{2} + \frac{\hat{m}_i(t)}{2a_i b_i \rho})]}{\hat{M}(t)} \tag{23}$$

this equation doesn't has quadratic term of $m_i(t)$ and is linear with $m_i(t)$. Through the linearization, (17) is transformed to a Mixed Integer Linear Programming Problem (MILP). Then we proposed a two stage algorithm to solve this problem in a fast manner.

IV. Two Layer Offline Algorithm

The requirement that the fuel feeding lasts for at least L time slots for each tank, represented by inequality (16), significantly increases the time for directly solving the problem . Motivated by the idea of divide and conquer, we decompose the original problem to a master problem (MP) and a subproblem (SP). Here we assume $\mathcal{T}$ can be divided into continuous time blocks. Each time block has L time slots. New decision variables $x_i^{mp}(j)$ are introduced in the MP to describe whether the i-th tank feeds fuel in the j-th time block. Similarly, $y_i^{mp}(j)$ and $h^{mp}(j)$ are the expansion of $y_i(t)$ and $h(t)$ under the time block setting.

Constraints (2), (8), (9), (11), (13), (14) are updated to the new form accordingly.

$$\begin{cases} x_c(j) = \dfrac{\mathbf{m}(j)^T \mathbf{x}_{co}(j)}{M + \mathbf{1}^T \mathbf{m}(j)} \\ y_c(j) = \dfrac{\mathbf{m}(j)^T \mathbf{y}_{co}(j)}{M + \mathbf{1}^T \mathbf{m}(j)} \\ z_c(j) = \dfrac{\mathbf{m}(j)^T \mathbf{z}_{co}(j)}{M + \mathbf{1}^T \mathbf{m}(j)} \end{cases} \tag{24}$$

$$\begin{aligned} \mathbf{m}(j+1) &= f(\mathbf{m}(j), \mathbf{y}^{mp}(j)) \\ \mathbf{m}(0) &= \mathbf{m}_0 \end{aligned} \tag{25}$$

$$0 \leq \mathbf{y}^{mp}(j) \leq L\bar{\mathbf{y}}, \ \forall j \tag{26}$$

$$0 \leq m_i(j) \leq a_i b_i c_i \rho, \ \forall i \in \mathcal{M}, \forall j \tag{27}$$

$$-M_1 x_i^{mp}(j) \leq y_i^{mp}(j) \leq M_1 x_i^{mp}(j), \forall j \tag{28}$$

$$\sum_{i \in \mathcal{M}_{DC}} y_i^{mp}(j) \geq h^{mp}(j), \forall j \tag{29}$$

As for the constraint (16), since the big time block setting ensures the valve is open if $x_i^{mp} = 1$, this constraint could be relaxed. Although this relaxation makes the original problem easier to handle with, it restricts the state switch time to be integer multiples of the least continuing time L, which will affect the optimality of the solution.

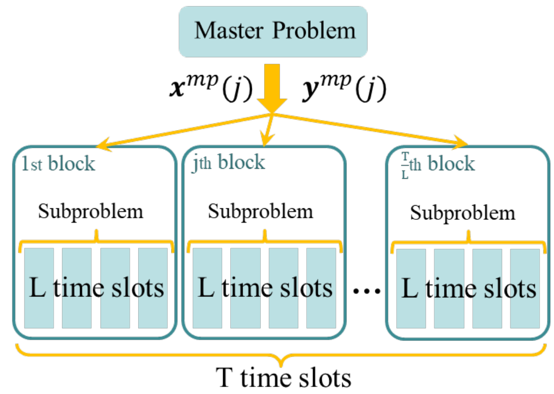

Fig. 3. The Time Block Setting

Then the master problem is shown as below.

$$\begin{aligned} (\mathbf{MP}) \min_{\mathbf{x}^{mp}, \mathbf{y}^{mp}} \quad & \max_t \quad \|\mathbf{c}_1(t) - \mathbf{c}_2(t)\|_2 \\ s.t. \quad & (24)(25)(26)(27)(28)(29). \end{aligned} \tag{30}$$

Solving the master problem, the optimal $\mathbf{x}^{mp*}$ could be attained, hence whether the i-th tank feed fuel or not in the j-th time block is known. The $\mathbf{y}^{mp*}$ is also known, however, in our algorithm, the value of the $\mathbf{y}^{mp*}$ is a reference to the subproblem. If we restrict the solution of the subproblem completely meets $\mathbf{y}^{mp*}$, then the subproblem may be infeasible. As a result, we introduce the new constraint

$$\begin{cases} \displaystyle\sum_{t=(j-1)L+1}^{jL} y_i(t) \geq y_i^{mp}(j) \quad x_i^{mp}(j) = 1 \\ \displaystyle\sum_{t=(j-1)L+1}^{jL} y_i(t) = 0 \quad x_i^{mp}(j) = 0 \end{cases} \tag{31}$$

So the subproblem is

$$\begin{aligned} (\mathbf{SP}) \min_{\mathbf{x}} \quad & \|\mathbf{c}_1(t) - \mathbf{c}_2(t)\|_2 \\ s.t. \quad & (2)(8)(9)(11)(13)(14)(31). \end{aligned} \tag{32}$$

Based on the above analysis, we propose a two layer fuel feeding optimization algorithm.

Algorithm 1 Two Layer Fuel Feeding Optimization Algorithm

solve the master problem (MP) with big time block
get the value of decision variable $x_i^{mp}(j)$
for $j \in [1, \frac{T}{L}]$ **do**
  if $x_i^{mp}(j) = 1$, then $x_i(t) = 1, \forall t \in [L(j-1)+1, Lj]$,
  solve the subproblem (SP) of each time block
  get the fuel feed weight $y_i(t)$
end for

V. Case Study

In the case study, we consider the example in the Fig.2. $\#2, \#3, \#4, \#5$ are main tanks, $\#1, \#6$ are backup tanks. The geometric center coordinates of the fuel tanks are given, as the Table II shows. Due to the limitations of the aircraft structure, at most 2 main tanks can feed the engine simultaneously, and totally no more than 3 tanks can feed simultaneously. Our case study is performed in MATLAB 2018b on a desktop computer with Intel® Core™ i7-8750H CPU @2.20GHz. The master problem (MP) and subproblem (SP) are solved with MATLAB 2018b and YALMIP toolbox [12] and gurobi.

TABLE II
GEOMETRIC CENTER COORDINATES OF FUEL TANKS

Fuel Tank	**Geometric Center Coordinates (Unit:m)**		
	x	***y***	***z***
#1	8.913043	1.20652174	0.61669004
#2	6.91304348	-1.39347826	0.21669004
#3	-1.68695652	1.20652174	-0.28330996
#4	3.11304348	0.60652174	-0.18330996
#5	-5.28695652	-0.29347826	0.41669004
#6	-2.08695652	-1.49347826	0.21669004

The jet fuel density ρ is 850 kg/m^3, the initial fuel $\mathbf{m}_0 = [344.25, 1645.6, 2019.6, 2254.2, 2448, 1020]^T$. The manufacture's empty weight (MEW) of the aircraft $M = 3000$ kg. The time length of the test data is 7200 seconds. The duration for each fuel tank to keep open is at least 60 seconds.

A. Pitch angle $= 0°$

The actual engine fuel consumption rate obtained from our fuel feed strategy is compared with the desired engine fuel consumption rate as shown in Fig. 5. The total fuel feed of all the main tanks is 6783.2 kg, while the desired total fuel consumption is 6441.5 kg. The fuel feed rate of each tank is shown as Fig. 6. The comparison of the actual CG location (represented by solid line) and the desired CG location (represented by dotted line) is shown as Fig. 4. The maximum of the difference distance is 0.1533 m.

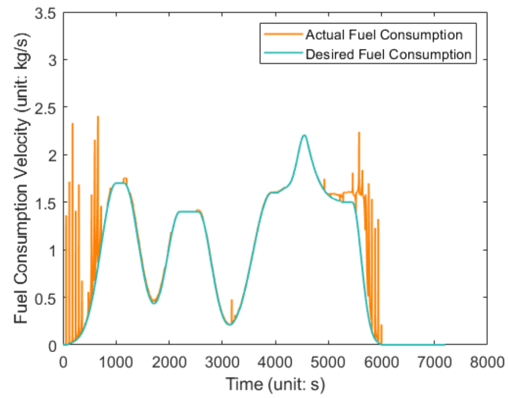

Fig. 5. Comparison of actual fuel consumption and desired fuel consumption in level flight

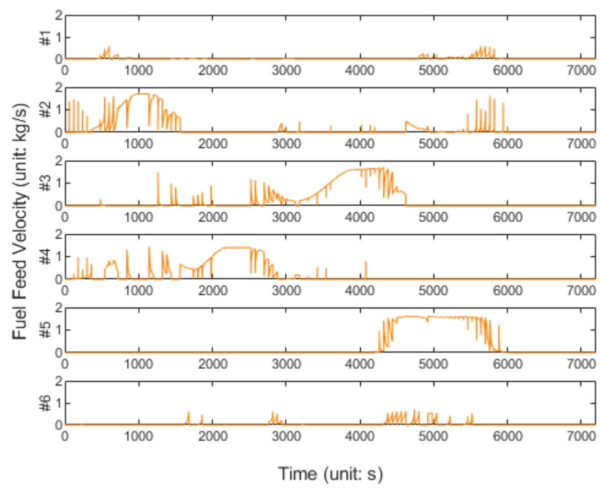

Fig. 6. Fuel consumption of each tank in level flight

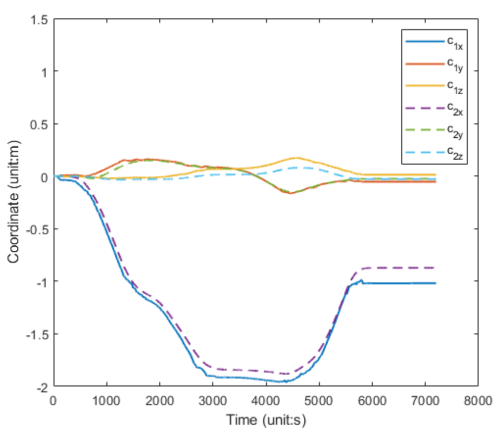

Fig. 4. Comparison of actual CG position and desired CG position in level flight

B. Pitch angle $\neq 0°$

When the pitch angle θ is not $0°$, the pitch angle variation data used in the test case is shown in Fig. 7. The actual engine fuel consumption rate obtained from our fuel feed strategy is compared with the desired engine fuel consumption rate as shown in Fig. 8. The total fuel feed of all the main tanks is 7477.3 kg, while the desired total fuel consumption is 7033.5.5 kg. The fuel feed rate of each tank is shown as Fig. 9. The maximum of the difference distance is 0.1870 m.

VI. CONCLUSION AND FUTURE WORK

In this article, a two layer fuel feed strategy optimization approach is proposed for the multi-tanks aircraft. In order to analyse the effect of fuel in the tanks to the aircraft CG, a CG model is built considering pitch movement. Then aircraft fuel feeding is formulated as an MINLP based on the CG model and the fuel feeding dynamics. Through the linearization, a fast and efficient two layer algorithm is developed. The performance of the proposed approach is evaluated via the case study. In this work, the fuel feed time of each tank is limited to an integer multiple of the smallest duration since the two layer time block setting, hence a more flexible algorithm will be designed in the future work. The online optimization approach to the fuel feed strategy will also be studied in the future.

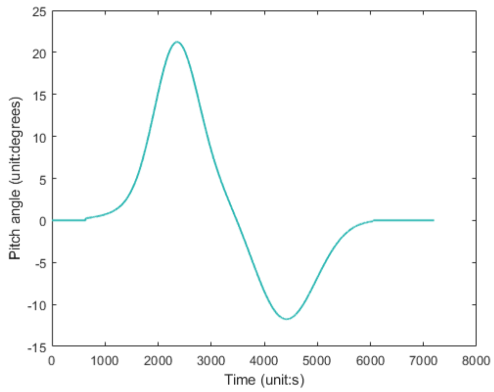

Fig. 7. Pitch angle variation data in the test case

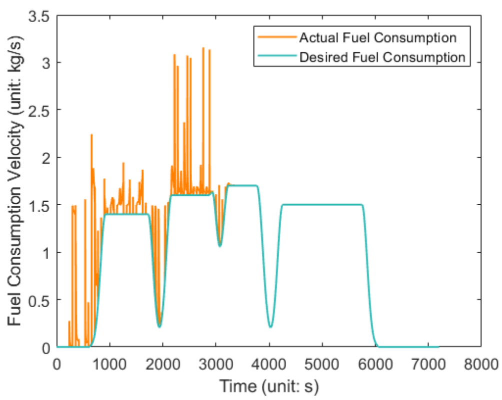

Fig. 8. Comparison of actual fuel consumption and desired fuel consumption in pitch

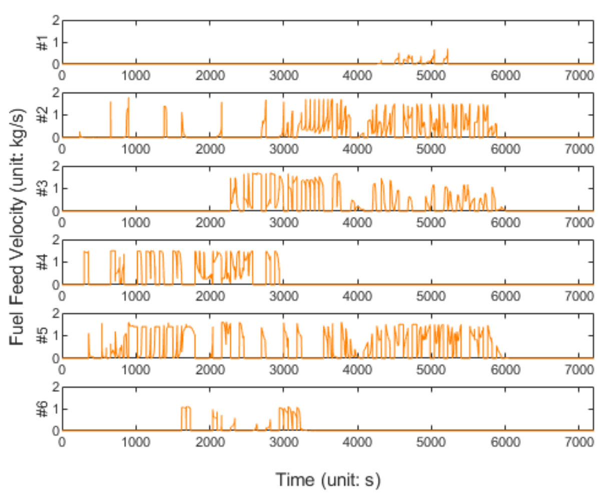

Fig. 9. Fuel consumption of each tank in pitch

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