# Fuel Feed Control for Asymmetric Arranged Multi-tanks Aircraft: A Hybrid MPC Approach

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**Abstract:** Aircraft fuel system, which provides a continuous source of fuel to the engine, is an important component of the aircraft. Although the sequential fuel feed strategy for the symmetric arranged multi-tanks aircraft is widely used in today's aircraft, the fuel feed strategy of the asymmetric arranged multi-tanks is still a challenge. In this article, a two layer offline approach is developed to obtain the fuel feed strategy to minimize the difference between the actual center of gravity (CG) and the desired CG. The performance of the proposed approach is tested in a case study based on the test data of aircraft pitch movement. The result indicates that the proposed approach solves the problem with an offline manner from the optimization perspective.

**Key Words:** asymmetric arranged multi-tanks, fuel feed control, hybrid MPC, warm start

## 1 Introduction

To be completed.

## 2 Related Work

To be completed.

## 3 System Model and Problem Formulation

The details of the model is refered to [1].

### 3.1 System Model

$$\begin{cases} x_c(t) = \dfrac{\mathbf{m}(t)^T \mathbf{x}_{co}(t)}{M + \mathbf{1}^T \mathbf{m}(t)} \\ y_c(t) = \dfrac{\mathbf{m}(t)^T \mathbf{y}_{co}(t)}{M + \mathbf{1}^T \mathbf{m}(t)} \\ z_c(t) = \dfrac{\mathbf{m}(t)^T \mathbf{z}_{co}(t)}{M + \mathbf{1}^T \mathbf{m}(t)} \end{cases} \tag{1}$$

$$\mathbf{u}_{t|\tau}^T = \left[\mathbf{x}_{t|\tau}^T, \mathbf{y}_{t|\tau}^T\right]^T \in \mathbb{R}^{2n} \tag{2}$$

$[T] = 0, 1, \cdots, T$
Constraints:

$$\begin{aligned} &\mathbf{m}_{0|\tau} = \mathbf{m}_\tau \\ &\mathbf{m}_{t+1|\tau} = A\mathbf{m}_{t|\tau} + B\mathbf{u}_{t|\tau}, t \in [T] \setminus T \end{aligned} \tag{3}$$

$$\mathbf{0} \leq \mathbf{m}_{t|\tau} \leq \bar{\mathbf{m}}, \ \forall t \in [T] \tag{4}$$

$$\mathbf{0} \leq \mathbf{y}_{t|\tau} \leq \min\{K\mathbf{x}_{t|\tau}, \bar{\mathbf{y}}\}, \ \forall t \in [T] \setminus T \tag{5}$$

The least fuel consumption constraint:

$$\mathbf{1}_{\mathcal{M}}^T \mathbf{y}(t) \geq h(t), \ \forall t \tag{6}$$

The minimum duration constraint.

$$[x_i(t+1) - x_i(t)] \times \left[\sum_{j=1}^{L} (x_i(t+j)) - L\right] \geq 0 \tag{7}$$

$$\begin{aligned} -M_1 \{1 - [x_i(t+1) - x_i(t)]\} &\leq \sum_{j=1}^{L} (x_i(t+j)) - L \\ &\leq M_1 \{1 - [x_i(t+1) - x_i(t)]\} \end{aligned} \tag{8}$$

### 3.2 Problem Formulation

In the time slot $\tau$, the optimization problem to be solved is

$$\begin{aligned} \min_{\mathbf{u}_{t|\tau}} \quad & \|\mathbf{c}_1(t) - \mathbf{c}_2(t)\|_2 \\ s.t. \quad & \mathbf{m}_{0|\tau} = \mathbf{m}_\tau, \\ & \mathbf{m}_{t+1|\tau} = A\mathbf{m}_{t|\tau} + B\mathbf{u}_{t|\tau}, t \in [T] \setminus T, \\ & \mathbf{0} \leq \mathbf{m}_{t|\tau} \leq \bar{\mathbf{m}}, \ \forall t \in [T], \\ & \mathbf{0} \leq V_y \mathbf{u}_{t|\tau} \leq \min\{K \cdot V_x \mathbf{u}_{t|\tau}, \bar{\mathbf{y}}\}, \ \forall t \in [T] \setminus T, \\ & \mathbf{1}_{\mathcal{M}}^T V_y \mathbf{u}_{t|\tau} \geq h_{t|\tau}, \ \forall t \in [T] \setminus T, \\ & V_x \mathbf{u}_{t|\tau} \in \{0, 1\}^n, \ \forall t \in [T] \setminus T, \\ & V_y \mathbf{u}_{t|\tau} \in \mathbb{R}_+^n, \ \forall t \in [T] \setminus T, \\ & \text{constraints determined by counter.} \end{aligned} \tag{9}$$

## 4 Hybrid Model Predictive Control

**Algorithm 1** Hybrid Model Predictive Control

Initialize model parameters, prediction horizon $T$
Initialize the timer $\mathbf{t}_{dura}$
Initialize current state $\mathbf{m}_0$
**while** not at the end of the time horizon **do**
  Measure current state $\mathbf{m}_\tau$
  Compute optimal control sequence $\mathbf{u}_{t|\tau}^* = \text{HMPC}(\mathbf{m}_\tau)$
  Apply the first control action $\mathbf{u} = \mathbf{u}_{0|\tau}^*$
  Simulate system dynamics using $\mathbf{u}$
  $\tau = \tau + 1$
**end while**

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## 5 Numerical Examples

$$\begin{cases} m_{1,t+1|\tau} = m_{1,t|\tau} - y_{1,t|\tau} \\ m_{2,t+1|\tau} = m_{2,t|\tau} - y_{2,t|\tau} + y_{1,t|\tau} \\ m_{3,t+1|\tau} = m_{3,t|\tau} - y_{3,t|\tau} \\ m_{4,t+1|\tau} = m_{4,t|\tau} - y_{4,t|\tau} \\ m_{5,t+1|\tau} = m_{5,t|\tau} - y_{5,t|\tau} + y_{6,t|\tau} \\ m_{6,t+1|\tau} = m_{6,t|\tau} - y_{6,t|\tau} \end{cases} \tag{10}$$

A is an identity matrix.

$$B = \left[\begin{array}{c|cccccc} & -1 & 0 & 0 & 0 & 0 & 0 \\ & 1 & -1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & -1 & 0 & 0 & 0 \\ \mathbf{0} & 0 & 0 & 0 & -1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & -1 & 1 \\ & 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right] \tag{11}$$

Table 1: Geometric Center Coordinates of Fuel Tanks

| Fuel Tank | Geometric Center Coordinates (Unit:m) | | |
|---|---|---|---|
| | ***x*** | ***y*** | ***z*** |
| #1 | 8.913043 | 1.20652174 | 0.61669004 |
| #2 | 6.91304348 | -1.39347826 | 0.21669004 |
| #3 | -1.68695652 | 1.20652174 | -0.28330996 |
| #4 | 3.11304348 | 0.60652174 | -0.18330996 |
| #5 | -5.28695652 | -0.29347826 | 0.41669004 |
| #6 | -2.08695652 | -1.49347826 | 0.21669004 |

## 6 Conclusion

## References


[1] Haoyu Miao, Zikai Ouyang, Shunpeng Yang, Weichao Yan, Mengfan Cao, Shibo Chen, and Zaiyue Yang. Optimal fuel feed strategy for asymmetric arranged multi-tanks aircraft. In *2021 3rd International Conference on Industrial Artificial Intelligence (IAI)*, pages 1–6. IEEE, 2021.