DICE 2023:
Introduction and User's Manual

By

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I. *Preface* [1]

The present manual combines a discussion of the subject of integrated assessment models (IAMs) of climate-change economics, a detailed description of the DICE model as an example of an IAM, and the results of the latest projections and analysis using the DICE-2023 model.

The main focus here is an introduction to the DICE-2023 model (which is an acronym for the Dynamic Integrated model of Climate and the Economy). The 2023 version is a major update from the last fully documented version, which was the DICE-2016 model (Nordhaus 2017, 2017a, 2018, 2019). The purpose of this manual is to explain in a self-contained publication the structure, calculations, algorithmics, and results of the current version. Some of the materials has been published in earlier documents, but this manual attempts to combine the earlier materials in a convenient fashion.

The author would like to thank the many co-authors and collaborators who have contributed to this project over the many decades of its development. Most important is Professor Lint Barrage (Zurich ETH), who is co-author of the DICE-2023 model and has contributed several chapters in the current handbook. Additionally, Zili Yang has contributed a review of the RICE model, while Paul Storcz prepared the earlier version of the appendix.

More than any single person, my colleague and co-author Tjalling Koopmans was an intellectual and personal inspiration for this line of research. I will mention particularly his emphatic recommendation for using mathematical programming rather than econometric modeling for energy and environmental economics.

Other important contributors have been George Akerlof, Lint Barrage, Scott Barrett, Joseph Boyer, William Brainard, William Cline, Jae Edmonds, Ken Gillingham, William Hogan, Charles Kolstad, Tom Lovejoy, Alan Manne, Robert Mendelsohn, Nebojsa Nakicenovic, William Pizer, David Popp, John Reilly, Richard Richels, John Roemer, Tom Rutherford, Jeffrey Sachs, Leo Schrattenholzer, Herbert Scarf, Robert Stavins, Nick Stern, Richard Tol, David Victor, Martin Weitzman, John

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Weyant, Zili Yang, Janet Yellen, and Gary Yohe, as well as many anonymous referees and reviewers.

Some of the background material including the appendix draws heavily on earlier versions of this handbook or other literature. Those who would like access to the model and material can find it at a box site at https://bit.ly/3TwJ5nO.

II. *Background on Integrated Assessment Models*

Many areas of the natural and social sciences involve complex systems that link together multiple physical or social networks. This is particularly true for environmental problems, which are intrinsically ones having strong roots in the natural sciences and require social and policy sciences to solve in an effective and efficient manner. A good example is climate change science and policy, which involve a wide variety of sciences such as atmospheric chemistry and climate dynamics, ecology, economics, political science, game theory, and international law.

As understanding progresses across the different fronts, it is increasingly necessary to link together the different areas to develop effective understanding and efficient policies. In this role, integrated assessment analysis and models play a key role. *Integrated assessment models (IAMs) can be defined as approaches that integrate knowledge from two or more domains into a single framework.* These are sometimes theoretical but are increasingly computerized dynamic models of varying levels of complexity.

A. Emerging problems of climate change

Before getting into modeling details, it will be useful to sketch the scientific basis for concerns about global warming, as reviewed by the Intergovernmental Panel on Climate Change (IPCC)'s Sixth Assessment Report (IPCC, 2013) with updates from other sources. As a result of the buildup of atmospheric greenhouse gases, it is expected that significant climate changes will occur in the coming decades and beyond. The major industrial greenhouse gases are carbon dioxide (CO2), methane, ozone, nitrous oxides and Fluorinated gases (F-gases such as CFCs). The most important greenhouse gas is CO2, whose emissions have risen rapidly in recent decades.

The atmospheric concentration of CO2 of 427 parts per million (ppm) in 2024 far exceeds the range over the last 650,000 years, estimated to be between 180 and

300 ppm (current estimates of CO_2 concentrations at Mauna Loa are available at https://gml.noaa.gov/ccgg/trends/weekly.html. Current calculations from climate models are that doubling the amount of CO_2 or the equivalent in the atmosphere compared with preindustrial levels will, in equilibrium, lead to an increase in the global surface temperature of about 3 °C.

The suite of models and emissions scenarios used by the IPCC produces a range of temperature change over the twenty-first century of between 2.0 and 4.4 °C (for intermediate and high scenarios). Other projected effects are increases in precipitation and evaporation, an increase in extreme events such as hurricanes, and a rise in sea level of 0.3 to 1.1 meters over this century. Some models also predict regional shifts, such as hotter and drier climates in mid-continental regions, including the US Midwest. Climate monitoring indicates that actual global warming is occurring in line with scientific predictions.

The agreed framework for all international climate change deliberations is the UN Framework Convention on Climate Change, which took force in 1994. That document stated, "The UNFCCC's ultimate objective is to stabilize greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic (human induced) interference with the climate system." (United Nations, 2009). The Framework Convention was implemented in the Kyoto Protocol in 1997, in which both high-income countries and countries in transition from central planning agreed to binding emissions limits for the 2008-2012 period. While the Kyoto Protocol expired in 2013, the framework for implementing the Protocol was institutionalized in the EU's Emissions Trading System (ETS), which covers almost half of Europe's CO_2 emissions.

B. Climate change as a global public good

Climate change is a polar case of economic phenomena known as global public goods (Samuelson, 1954). Public goods are activities for which the cost of extending the service to an additional person is zero and for which it is impossible or expensive to exclude individuals from enjoying. Global public goods are ones whose influences are felt around the world rather than in one nation, town, or family. What makes global public goods different from normal economic activities is that there are at best weak economic and political mechanisms for resolving these issues efficiently and effectively.

C. Economic modeling of climate change

Most economic studies of climate change, including most IAMs, integrate geophysical stocks and flows with economic stocks and flows. The major difference between IAMs and geophysical models is that economic measures include not only quantities but also valuations, which for market or near-market transactions are prices. The essence of an economic analysis is to convert or translate all economic activities into monetized values using a common unit of account and then to compare different approaches by their impact on total values or a suite of values. While this approach is often found objectionable, it is necessary to convert many different quantities (perhaps even thousands of goods and time period) into a simpler metric.

There are different ways of creating a standardized unit of account. The most satisfactory is to use a common "purchasing power parity" (PPP) exchange rate across different regions (see Nordhaus 2007 for a discussion in the IAM context).

To illustrate the economic approach, suppose that an economy produces only corn. We might decide to reduce corn consumption today and store it for the future to offset the damages from climate change on future corn production. In weighing this policy, we consider the economic value of corn both today and in the future in order to decide how much corn to store and how much to consume today. In a complete economic account, "corn" would represent all economic consumption. It would include all market goods and services as well as the value of non-market and environmental goods and services. That is, economic welfare properly measured should include everything that is of value to people, even if those things are not included in the marketplace.

The central questions posed by economic approaches to climate change are the following: how sharply should countries reduce CO2 and other greenhouse gas emissions? What should be the time profile of emissions reductions? How should the reductions be distributed across industries and countries?

There are also important and politically divisive issues about the instruments that should be used to impose cuts on consumers and businesses. Should there be a system of emissions limits imposed on firms, industries, and nations? Or should emissions reductions be primarily induced through taxes on greenhouse gases? Should we subsidize green industries? What should be the relative contributions of rich and poor households or nations? Are regulations an effective substitute for fiscal instruments?

In practice, an economic analysis of climate change weighs the costs of slowing climate change against the damages of more rapid climate change. On the side of the costs of slowing climate change, this means that countries must consider whether, and by how much, to reduce or offset their greenhouse gas emissions. Reducing greenhouse gases, particularly deep reductions, will require taking costly steps to reduce CO2 emissions. Some steps involve reducing the use of fossil fuels; others involve using different production techniques or alternative fuels and energy sources. Societies have considerable experience in employing different approaches to changing energy production and use patterns. Economic history and analysis indicate that it will be most effective to use market incentives, primarily higher prices on carbon fuels, to give signals and provide incentives for consumers and firms to change their energy use and reduce their carbon emissions. In the longer run, higher carbon prices will also provide incentives for firms to develop new technologies to ease the transition to a low-carbon future.

On the side of climate damages, our knowledge is very meager. For most of the time span of human civilizations, global climatic patterns have stayed within a very narrow range, varying at most a few tenths of a degree Centigrade from century to century. Human settlements, along with their ecosystems, pets, and pests, have generally adapted to the climates and geophysical features they have grown up with. Economic studies suggest that those parts of the economy that are insulated from climate, such as air-conditioned houses and most manufacturing operations, will be less affected directly by climate change over the next century or so (see by reference IPCC, 2007b).

However, those human and natural systems that are "unmanaged," such as rain-fed agriculture, seasonal snowpacks and river runoffs, and most natural ecosystems, may be significantly affected. While economic studies in this area are subject to large uncertainties, recent surveys of the literature on damages from future climate change indicate that the economic damages from climate change with no interventions will be in the order of 3 - 5% of world output per year by the end of the twenty-first century (for a recent review of damage estimates (see the Background Paper on Damages in Appendix A). The damages are likely to be most heavily concentrated in low-income and tropical regions such as tropical Africa and India. While some countries may benefit from climate change, or at least for small warming, there is likely to be significant disruption in any area that is closely tied to climate-sensitive physical systems, whether through rivers, ports, hurricanes, monsoons, permafrost, pests, diseases, frosts, or droughts. Moreover, damage estimates cannot reliably include estimates of the costs of ecological impacts such as ocean acidification, species extinction, ecosystem disruption, or of the dangers posed by tipping points in the earth systems.

III. DICE and RICE Models as Integrated Assessment Models

The DICE model views climate change in the framework of economic growth theory. In a standard neoclassical optimal growth model known as the Ramsey-Solow-Koopmans model, society invests in capital, thereby reducing consumption today, in order to increase consumption in the future. The DICE model augments the standard Ramsey model to include climate investments, which are analogous to capital investments in the standard model. The augmented model contains all elements of the process from economic activity and emissions through climate change to damages and policy in a manner that represents simplified best practice in each area.

A. ***Introduction to the models***

The DICE model (Dynamic Integrated model of Climate and the Economy) is a simplified analytical and empirical model that represents the economics, policy, and scientific aspects of climate change. Along with its more detailed regional version, the RICE model (Regional Integrated model of Climate and the Economy), the models have gone through several revisions since their first development around 1990.

The prior fully documented versions are the RICE-2010 and DICE-2016 model. The present version, DICE-2023, is an update of those earlier models, and this study describes the revision of DICE, whereas an updated version of RICE (joint with Zili Yang) will be available shortly. We begin with a description of the DICE-2023 model, after which we provide the detailed equations. This section draws heavily on earlier expositions Nordhaus 2017, 2018, 2018a, Yang 2020, Nordhaus and Sztorc 2013,Yang (1996) and Nordhaus and Boyer (2000) as well as the published version of DICE-2023 (Barrage and Nordhaus 2023 with online materials).

The DICE-2023 model is a globally aggregated model. The RICE-2024 model, currently in final development, is essentially the same except that output, population, emissions, damages, and abatement have regional structures for 13 regions. The discussion in this manual will focus on the DICE model, and the analysis applies equally to the RICE model for most modules. The differences will be described later along with a short discussion of the RICE-2024 model.

The DICE model views the economics of climate change from the perspective of neoclassical economic growth theory (see particularly Solow 1970). In this approach, economies make investments in capital, education, and technologies, thereby reducing consumption today, in order to increase consumption in the future. The DICE model extends this approach by including the "natural capital" of the climate system. In other words, it views concentrations of GHGs as negative natural capital, and emissions as disinvestment that lowers the quantity of natural capital. By devoting output to emissions reductions, economies reduce consumption today but prevent economically harmful climate change and thereby increase consumption possibilities in the future.

Figure 1 shows a schematic flow chart of the major modules and logical structure of the DICE and RICE models.

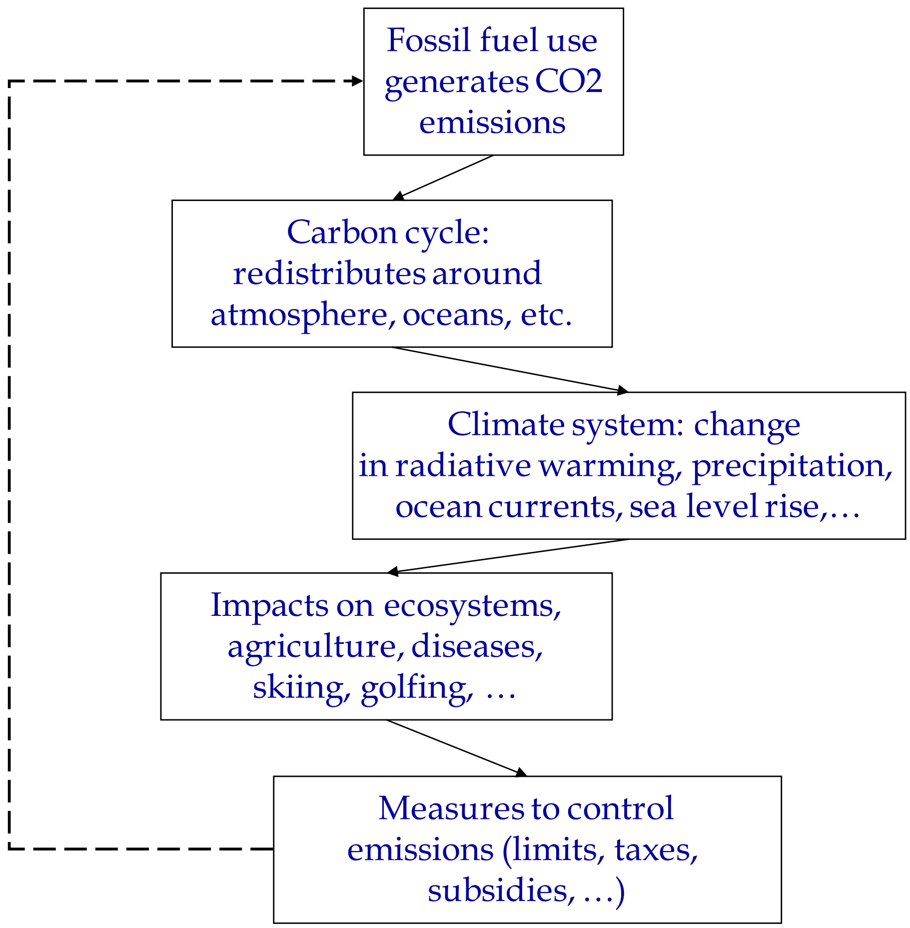

Figure 1. Schematic flow chart of a full integrated assessment model for climate change science, economics, and policy

B. Objectives of Integrated Assessment Models (IAMs)

IAMs can be divided into two general classes – policy optimization and policy evaluation models (this distinction was emphasized in an excellent chapter of the IPCC report by Weyant et al. (1996). Policy evaluation models generally are recursive or equilibrium models that generate paths of important variables but do not optimize an economic or environmental outcome.

Policy optimization models have an objective function or welfare function that is maximized and can be used to evaluate alternative paths or policies. In models that have an economic structure, the objective function is generally a measure of economic welfare. This would typically be a set of utility functions in general equilibrium models or consumer and producer surplus in partial equilibrium models.

These two approaches are not as different as might be supposed, as policy optimization models can be run in a non-policy mode, while policy evaluation models can compare different policies. However, there are often differences in the solution algorithms as recursive models are often much simpler to solve computationally than are optimization models.

The DICE/RICE models are primarily designed as policy optimization models, although they can be run as simple projection models as well. In both modes, the approach is to maximize an economic objective function. The objective function represents the goal implicit in the problem. For the DICE/RICE models, the objective function refers to the economic well-being (or utility) associated with a path of consumption.

As will be emphasized below, the use of optimization can be interpreted in two ways: First, from a positive point of view, optimization is a means of simulating the behavior of a system of competitive markets; and second, from a normative point of view, it is a possible approach to comparing the impact of alternative paths or policies on economic welfare. The models are available online at https://bit.ly/3TwJ5nO.

IV. Detailed Equations of the DICE-2023 Model

A. Preferences and the Objective Function

In the DICE and RICE models, the world or individual regions are assumed to have well-defined preferences, represented by a social welfare function, which ranks different paths of consumption. The social welfare function is increasing in the number of people and in the per capita consumption of each generation, with diminishing marginal utility of consumption.

The importance of a generation's per capita consumption depends on the size of the population. The relative importance of different generations is affected by two central normative parameters, the pure rate of social time preference ("generational discounting") and the elasticity of the marginal utility of consumption (the "consumption elasticity"). In the simplest approach, these two parameters interact to determine the discount rate on goods, dimensionally the same as a real interest rate, which is critical for intertemporal economic choices. In the modeling, we set the preference parameters to be consistent with observed economic outcomes as reflected by market interest rates, risks, and rates of return on capital, a choice that will be central to the results and is further discussed in the section on discounting below.

Note as well a major change in the current version of DICE and RICE with respect to discounting. While the underlying preference structure is as described above, the current approach corrects for uncertainty about the growth of consumption as well as for the systematic risk of investments in climate mitigation. These additions will lead to a discounting structure that differs significantly from earlier treatments and from most other current IAMs.

The DICE model assumes that economic and climate policies should be designed to optimize the flow of consumption over time. It is important to emphasize that consumption should be interpreted as "generalized consumption," which includes not only traditional market goods and services like food and shelter but also non-market items such as leisure, health status, and environmental services. It might (but currently does not) include non-human factors such as the welfare of other species or ecosystems. The inclusion of non-market values will affect the level of total income, but because of the normalization of market output, it will not have a significant effect on the results.

We add a note of interpretation of the equilibrium in the DICE model. We have specified the baseline case so that, from a conceptual point of view, it represents the outcome of market and policy factors as they currently exist. In other words, the baseline model is an attempt to project, from a descriptive perspective, the levels and growth of major economic and environmental variables as would occur with existing climate-change policies. The baseline is dated as of 2021 and excludes, for example, the US Inflation Reduction Act or other policies adopted after that time. The baseline is distinguished from a "no controls" policy, such as might have existed in the 1960s, where there were no policies targeted to slow climate change. Similarly, the baseline does not include countries' announced or aspirational policies.

Finally, we emphasize that the approach takes as a baseline the existing distribution of incomes and wealth over space or time. The approach of "equity weighting," however appealing, is not implemented in the current models.

B. Equations of the DICE-2023 model[2]

We next describe the equations of the model. We omit minor equations such as accounting identities.

Objectives

To begin with, we assume that policies are chosen to maximize a general concept of economic welfare. More precisely, we maximize a social welfare function, *W,* which is the discounted sum of the population-weighted utilities of per capita consumption.

Equation (1) shows the objective function, which is standard in modern theories of optimal economic growth.

$$(1) \quad W = \sum_{t=2020}^{Tmax} U[c(t)]L(t)\Pi(t) = \sum_{t=2020}^{Tmax} [c(t)^{1-\varphi}/(1-\varphi)]L(t)\Pi(t)$$

In equation (1), W is the welfare total, U is the utility function, *c(t)* is per capita consumption per period *t=2020, 2025, 2030…, L(t)* is population and labor inputs, and $\Pi(t)$ is the discount factor.

Each period's preferences are represented by the utility of consumption, with a constant elasticity of the marginal utility of consumption, φ, which represents the extent of substitutability of the consumption of different years or generations. Note, importantly, that the elasticity is distinct from personal behavioral characteristics and is not used in the DICE model to represent risk aversion.

In the new DICE specification, the discount factor $\Pi(t)$contains three elements: (i) the pure rate of social time preference, ρ, which reflects the welfare weights on the utilities of different generations, (ii) a precautionary term reflecting consumption growth uncertainty, and (iii) an adjustment for the non-diversifiable risk of climate investments. Elements (ii) and (iii) are new in DICE-2023 and are corrections to reflect uncertainty through the use of *certainty-equivalent discount rates*. This term is used to designate the single discount rate delivering the same discount factor as the expected value from the distribution of uncertain future discount rates.

[2] This section draws heavily on Barrage and Nordhaus (2024).

The precautionary effect in (ii) is associated with the uncertainty about the trend growth of per capita consumption. Based on different studies, we assume the average growth rate of per capita consumption from t = 0 to t = T is normally distributed with a standard deviation (σ_C) of 1%-point / year. The adjustment in (iii) for risky climate investments is based on the concept of the climate beta, β^{CLIM} (Dietz et al. 2018). The climate beta measures the extent to which climate investments (such as renewable power) share the non-diversifiable risk characteristics of economy-wide investments. When $\beta^{CLIM} = 0$, the risks on climate investments are uncorrelated with market returns; if $\beta^{CLIM} = 1$, climate investments have risk properties of the aggregate economy. Based on our review, we assume that $\beta^{CLIM} = 0.5$, which implies an intermediate correlation with market risks. We further assume, based on historical data, a near-term risk-free real rate of return of 2%/year and the economy-wide non-diversifiable risk premium of π = 5%/year.

Using these assumptions, we substitute a time-varying risk-adjusted time preference parameter, $\rho^*(t)$, for ρ, where $\rho^*(t) = \rho - \frac{1}{2}\varphi^2{\sigma_C}^2 t + \beta^{CLIM}\pi$. When calibrated to the parameters noted above, this approach yields a near-term real rate of return of 4.5%/year, declining over time. A full discussion of the new approach is contained in Appendix A's *Background Note on Rates of Returns and Discounting*.

Population, output, and productivity

The DICE model is a standard one-sector model with output (Q) determined by a Cobb-Douglas production function in capital and labor (K and L) with growing total factor productivity (A). Output is reduced by abatement and damages as shown in equation (2), where Ω and Λ represent climate damages and abatement costs respectively and are discussed in the next section.

(2) $$Q(t) = [1-\Lambda(t)][1-\Omega(t)]A(t)K(t)^{\gamma}L(t)^{(1-\gamma)}$$

Population and the labor force are exogenous and are based on UN projections (UN 2022). Output is measured in PPP exchange rates using World Bank and IMF estimates. Future productivity growth is based on estimates from various studies on future growth (P. Christensen, K. Gillingham, W. D. Nordhaus (2018); R. Newell, W. Pizer, B. Prest (2022); U. Müller, J. Stock, M. Watson (2022); K. Rennert, et al. (2021), see Appendix A for details). Technological change is exogenous and takes two major forms: economy-wide technological change and carbon-saving technological change. Carbon-saving technological change is represented in two ways: first, as reducing the baseline ratio of CO_2 emissions to output and, second, as reducing the cost of the backstop technology.

Damages

Equation (3) represents the economic impacts or damages from climate change, which has been one of the thorniest issue in climate-change economics. Providing reliable estimates of the damages from climate change over the long run has proven extremely difficult, and we examine alternative approaches.

$$(3) \quad \Omega(t) = \psi_1 T_{AT}(t) + \psi_2 [T_{AT}(t)]^2$$

The damage function is a quadratic function of global temperature increase since pre-industrial times (1765), $T_{AT}(t)$. It is based on three key assumptions: (i) The increase in global mean surface temperature from pre-industrial levels is assumed to be a reasonable sufficient statistic for damages. This specification omits or captures only indirectly cumulative effects (such as the effects of prolonged rather than instantaneous warming on sea-level rise) and also omits effects that depend on the speed of temperature change. (ii) Damages scale proportionately with global output. (iii) Damages are quadratic in warming, in line with recent reviews (W.D. Nordhaus and A. Moffat 2017 and S. Hsiang, et al. 2017.) but with potential limitations discussed below. The estimates are based on three components.

The first component is an updated literature synthesis as described in Appendix F. DICE-2023 builds on Nordhaus and Moffat (2017) and adds studies published since that review. The update is based on a survey by Piontek, et al. (2021), which overlaps closely also with global damage studies reviewed by the IPCC's AR6 (O'Neill, et al. 2022). The updated results imply a 1.6% GDP-equivalent loss at 3 °C warming over pre-industrial temperatures, up from 1.2% in the review for DICE-2016. It is important to note that surveyed studies generally omit many climate change impact channels, such as biodiversity loss, ocean acidification, extreme events, and social unrest.

The second component, based on a comprehensive study of tipping points (Dietz et al. 2021), adds a 1% output loss at a 3 °C warming. The third component is a judgmental adjustment for excluded impacts totaling 0.5% output loss at 3 °C warming. This adjustment reflects concerns over missing sectors, climate change impacts not yet reliably quantified in the literature, uncertainty, and recent research that is not reflected in our synthesis of aggregate damage estimates.

Each of the three components is assumed to be proportional to output and quadratic in the temperature change from pre-industrial times. In total, damages are estimated to be 3.1% of output at 3°C warming and 7.0% of output at 4.5 °C warming. The resulting damage coefficient is almost twice as large as in DICE-2016, resulting in more stringent emissions reductions and a large increase in the social cost of carbon.

Note that the damage function has been calibrated for damage estimates with temperature increases up to 4 °C and is not well-suited for temperature increases above that range. The evidence is very limited for warming beyond 4 °C, and the quadratic functional form in equation (5) does not reflect potential concerns about threshold damages.

Abatement

The abatement cost equation in equation (4) is a reduced-form type model in which the ratio of the costs of emissions reductions to output, $\Lambda(t)$, is a polynomial function of the emissions-control rate, $\mu(t)$.

(4) $\Lambda(t) = \theta_1(t)\mu(t)^{\theta_2}$

The intercept, $\theta_1(t)$, represents the fraction of output that is required to reduce emissions to zero.

The DICE model includes a backstop technology, which is a set of technologies that can replace all fossil fuels, albeit at a relatively high price. These technologies might be solar or wind power, safe nuclear power, or some as-yet-undiscovered source. Conceptually, at the cost of the backstop technology, the economy achieves zero net carbon emissions.

Two revisions in the current version are noteworthy. Estimates of the cost of the backstop technology are controversial, with the DICE model having a high backstop cost relative to some estimates of the cost of renewables or carbon capture. The cost function is derived from highly detailed process models. Examining estimates of the marginal cost of scenarios with zero net emissions, we can estimate the marginal cost of the backstop technology. A statistical analysis from the results of the ENGAGE study (K. Riahi, et al. 2021 2021a) indicates a median backstop price of \$515/tCO_2 in 2019\$ in 2050, which is the earliest year that most models can reach zero net emissions. Models assume improvements over time in the technologies needed to attain zero emissions. The decline rate of the cost of the backstop technology is assumed to be 1%/year from 2020 to 2050, and then 0.1%/year after that.

The backstop technology is introduced into the model by setting the time path of the parameters in the abatement-cost Equation (4) so that the marginal cost of abatement at a control rate of 100 percent is equal to the backstop price. By construction, the cost of a zero-emissions policy is determined by the cost of the backstop technology and the emissions-output ratio. With the assumed parameters, the cost of net-zero emissions is 11% of output in 2020, declining at 1.7% per year from 2020 to 2100 to 2.7% of output in 2100.

The other revision is the inclusion of emissions other than industrial CO_2. This addition is basically a scalar increase in the abatement cost function. For further discussion, see the section on emissions below as well as Appendix A.

Emissions

DICE-2023 has a major revision in its treatment of greenhouse-gas (GHG) emissions. In earlier versions, only industrial CO_2 emissions were controllable (abatable), while other GHGs and forcings were taken to be exogenous. The current version includes all abatable emissions in the endogenous category and excludes only a small fraction of forcings as non-abatable emissions. Appendix A contains a full discussion of the methods.

The lion's share of GHG emissions is from CO_2. However, a large suite of processes and gases also contribute to radiative forcings. According to IPCC AR6, total CO_2-equivalent (CO_2-e) abatable emissions are 140% of industrial emissions in 2020, declining to 121% of industrial CO_2 emissions in 2100. This ratio indicates the increase in abatable emissions in DICE-2023 compared to DICE-2016. The cost function is drawn from studies of the abatement cost function for non-CO_2 emissions. This extension allows a larger potential abatement and the possibility of attaining more ambitious targets.

Projections of baseline emissions are a function of total output, time, a time-varying emissions-output ratio, and the emissions-control rate. The baseline emissions control rate reflects current policy, which we estimate to be about 5%, or a carbon price of about $6/t$CO_2$. There is no major change in the function form of the abatement-cost function from earlier DICE models, but the extension to non-industrial CO_2 emissions is completely new and based on studies of the abatement-cost function of non-industrial CO_2 and abatable non-CO_2 GHGs.

The final two equations in the economic block are the emissions equation for CO_2 and that for abatable non-CO_2 GHGs:

$$(5)\ \ ECO2(t) = [\sigma(t)Y(t) + ECO2_{Land}(t)]\,[1-\mu(t)]$$
$$(6)\ \ ECO2\mathrm{e}_{NonCO2GHGabate}(t) = [ECO2e_{NonCO2GHGabate_{base}}(t)]\,[1-\mu(t)]$$

Equation (5) defines total CO_2 emissions per period. The first term is industrial emissions, given by the level of no-controls carbon intensity, $\sigma(t)$, times output. $\sigma(t)$ is taken to be exogenous and declines initially at a rate of 1.5% per year. The second term is land-use emissions of CO_2, which decline by 2% per year. Actual CO_2 emissions are base emissions times (one minus the emissions-control rate) or $[1-\mu(t)]$. Equation (6) represents abatable non-CO_2 GHG emissions measured on a CO_2-equivalent (CO_2-e) basis. These emissions equal uncontrolled emissions (based on the SSP2 scenario, NAS 2017) times $[1-\mu(t)]$. Our treatment assumes the same control rate on CO_2 and non-CO_2 abatable emissions. Total abatable emissions in CO_2-e units are given by the sum of (5) and (6).

Geophysical sectors

A key feature of IAMs is the inclusion of geophysical relationships that link the economy with the different forces affecting climate change. In the DICE model, these relationships include the carbon cycle, a radiative forcing equation, and the climate-change equations. The purpose of including these is that they operate in an integrated fashion rather than taking inputs as exogenous from other models or assumptions.

This block of equations links economic activity and greenhouse-gas emissions to the carbon cycle, radiative forcings, and climate change. As with the economics, the modeling philosophy for the geophysical relationships has been to use parsimonious specifications so that the theoretical model is transparent and so that the optimization model is empirically and computationally tractable and robust.

For purposes of the carbon/forcings/climate modules, CO_2 emissions are linked to the carbon cycle and thence to forcings. The other GHGs are linked directly to forcings and short-circuit the atmospheric chemistry.

Carbon Cycle

The carbon cycle and climate model are key components of any IAM. DICE-2023 has made a major change in the treatment of these modules, particularly the carbon cycle. Earlier versions of DICE and most other IAMs used linear carbon-cycle structures. While these approaches seemed acceptable as a simplification, they did not allow for the important finding that the ability of non-atmospheric sinks to absorb CO_2 declines with higher emissions (NAS 2017, Dietz et al. 2021). The latest and most extensive multimodel carbon-cycle comparison by (Joos et al. 2013) showed that the atmospheric retention at 100 years would be 70% for a pulse of 5000 billion tons of carbon (GtC) compared to only 30% for a pulse of 100 GtC.

The major structural revision of DICE-2023 is the introduction of the DFAIR module, the DICE version of the FAIR or Finite Amplitude Impulse-Response model developed by Millar et al. (2017), which represents the dynamics of the carbon cycle. The FAIR model is based on a linear four-reservoir impulse-response model of the response of CO_2 concentrations to emissions. A key innovation is the structural parameter $\alpha(t)$, which increases the fraction of total CO_2 emissions that resides in the atmosphere as cumulative CO_2 emissions increase. While the reservoirs may have geophysical names (permanent, long, etc.), they have no physical or structural interpretation but are variables in reduced-form dynamic equations and may take negative values.

Simulations reported in Appendix A's Background Paper indicate that the DFAIR model tracks the historical emissions-concentrations paths closely, as well as small emissions pulses. However, the DFAIR atmospheric retention for very large pulses (e.g., the 5000 GtC pulse in Joos et al.) tracks the full carbon-cycle models poorly.

The DFAIR equations are the following. Equation (7) is the set of equations for the four reservoirs, whose contents are $R^i(t)$. We note that only CO_2 emissions (industrial and land-based) enter the carbon cycle, that is, CO_2-equivalent emissions from other gases are not included in the emissions term *E(t)*. Equation (8) then sums the four reservoirs to obtain atmospheric CO_2, $MAT(t)$. Equation (9) provides the equation for accumulated CO_2 in non-atmospheric sinks, defined as $Cacc(t)$. Equation (10) yields the predicted 100-year integrated impulse response function $IRF100(\mathrm{t})$ and (11) implicitly defines the saturation parameter $\alpha(t)$. All equations are straightforward to calculate except for (11).

(7) $$\Delta R^i(t+1) = \xi_i E(t) - \left[\frac{R^i(t)}{\alpha(t)\tau_i}\right], \ \mathrm{i} = 1,\ldots,4$$

(8) $$MAT(t) - MAT(1765) = \sum_{i=1}^{4} R^i(t)$$

(9) $$Cacc(t) = \sum_{v=1765}^{t} E(v) - [MAT(t) - MAT(1765)]$$

(10) $$\mathrm{iIRF100}(t) = \varsigma_0 + \varsigma_C Cacc(t) + \varsigma_T T_{AT}(t)$$

(11) $\text{iIRF100}(t) = \sum_{i-1}^{4} \alpha(t)\xi_i\tau_i\{1 - \exp[-100/(\alpha(t)\tau_i)]\}$

The variables are MAT = atmospheric concentrations, R^i = carbon content of reservoir i, E = emissions of CO_2, $iIRF100$ = 100-year integrated impulse response value, $Cacc$ = accumulated carbon stock in the land and ocean, α = scaling factor for carbon reservoirs, ξ^i = fraction of emissions entering reservoir i , and τ^i = time constant for reservoir i. Note that values of R, $Cacc$, and E are all zero in 1765. The values of the parameters are described in the Background Note on DFAIR.

Climate equations

The other equations of the climate system contain the relationships for radiative forcing and for global mean temperature. These specifications are similar to earlier versions of the DICE model but update the parameters and change the structure to parallel the treatment in Millar et al. (2017).

DICE employs a small structural model that captures the basic relationship between GHG concentrations, radiative forcing, and the dynamics of climate change. Accumulations of GHGs lead to warming at the earth's surface through increases in radiative forcing. The relationship between GHG accumulations and increased radiative forcing is derived from empirical measurements and climate models, as shown in Equation (12).

(12) $F(t) = F_{\text{CO22x}}\{ log_2[MAT(t)/MAT(1765)]\} + F_{ABATE}(t) + F_{EX}(t)$

$F(t)$ is the change in total radiative forcings of GHGs since 1765 from anthropogenic sources such as CO_2 and other GHGs. $F_{EX}(t)$ is exogenous forcings from non-abatable GHGs and other sources, and $F_{ABATE}(t)$ is the forcings resulting from abatable non-CO_2 GHGs (see equation (6) and Appendix A). The equation uses estimated carbon stocks in the year 1765 as the pre-industrial equilibrium.

The climate module in equations (13) through (15) uses a two-box model of the temperature response to radiative forcing developed by IPCC AR5 and parameterized in Millar et al. (2017). DICE-2023 further adjusts the parameters to match the equilibrium climate sensitivity (ECS) and transient climate response (TCR) to the centers of the IPCC Sixth Assessment Report (2021) with estimates of 3.0 °C for ECS and 1.8 °C for TCR. The equations are:

$$(13) \quad Tbox1(t+1) = Tbox1(t)\,exp(-5/d1) + teq1\,F(t+1)\,[1 - exp(-5/d1)]$$
$$(14) \quad Tbox2(t+1) = Tbox2(t)\,exp(-5/d2) + teq2\,F(t+1)\,[1 - exp(-5/d2)]$$
$$(15) \quad T_{AT}(t) = Tbox1(t) + Tbox2(t)$$

The increase in global mean surface temperature, $T_{AT}(t)$, is computed as the sum of a two components, *Tbox1* and *Tbox2.* These are the contributions to temperature increase due to processes of the deep and upper ocean, respectively. The FAIR model assumes a neutral biosphere and is therefore likely to overestimate atmospheric accumulation in the early years. The parameters *d1* and *d2* are time lags for the two temperature boxes (in years). The parameters *teq1* and *teq2* are the diffusion rates for the boxes (in m^2K/W). Note that the equilibrium climate sensitivity is given by $ECS = F_{CO22x}(teq1+teq2) = 3.93 \times (0.324+0.440) = 3.0$. The model's transient climate response is 1.80 °C (for the complex formula defining the value, see Millar et al. (2017), eq. 5).

This completes the description of the DICE-2023 model. A full discussion of the DFAIR module – including updates such as initial conditions relevant for 2020 – is in the Background Paper in Appendix A.

C. The RICE-2010 Model

The RICE model (Regional Integrated model of Climate and the Economy) is a regionalized version of the DICE model. It has the same basic economic and geophysical structure, but contains a regional elaboration. The last full version is described in Nordhaus (2010), with detailed in the Supplemental Information to Nordhaus (2010). A new version is under development with Zili Yang.

The general structure of the RICE model is similar to the DICE model with disaggregation into regions. However, the specification of preferences is different because it must encompass multiple agents (regions). The general preference function is a Bergson-Samuelson social welfare function over regions of the form $W = \mathscr{W}(U^1,\ldots,U^N)$, where U^I is the preference function of the I^{th} region. The model is specified using the Negishi approach in which regions are aggregated using time- and region-specific weights subject to budget constraints, yielding

$$(19) \quad W = \sum_{t=1}^{T\,max} \sum_{I=1}^{N} \psi_{I,t} U^I[c^I(t), L^I(t)]R^I(t)$$

In this specification, the $\psi_{I,t}$ are the “Negishi weights” on each region and each time period. Each region has its consumption and population. In principle, they may have different rates of time preference, although in practice the RICE model assumes that they are all equal. The Negishi algorithm in the RICE model sets each of the weights so that the marginal utility of consumption is equal in each region and each period, which ensures that the requirement for maximization as market simulation principle holds. We elaborate below on the Negishi approach, which is widely used in IAMs for climate change, in the section on “Computational and algorithmic aspects.“

The RICE-2010 model divides the world into 12 regions. These are US, EU, Japan, Russia, Eurasia (Eastern Europe and several former Soviet Republics), China, India, Middle East, Sub-Saharan Africa, Latin America, Other high income countries, and Other developing countries. Note that some of the regions are large countries such as the United States or China; others are large multi-country regions such as the European Union or Latin America.

A discussion of the RICE-2024 model will be forthcoming.

D. Interpretation of Positive and Normative Models

One of the issues that pervades the use of IAMs is whether they should be interpreted as normative or positive.[3] In other words, should they be seen as the recommendations of a central planner, a world environmental agency, or a disinterested observer incorporating a social welfare function? Or are they meant to be a description of how economies and real-world decision makers (consumers, firms, and governments) actually behave? This issue also arises in the analysis of the discount rate.

For most simulation models, such as general circulation climate models, the interpretation is clearly that these are meant to be descriptive. The interpretation of optimization models is more complex, however. In some cases, the purpose is clearly normative. For example, the *Stern Review* represented an attempt to provide normative guidance on how to cope with the dangers raised by climate change. In other cases, such as baseline projections, these are clearly meant to be descriptive.

The ambiguity arises particularly because many models use optimization as a technique for calibrating market outcomes in a positive approach. This is the interpretation of “market mechanisms as maximization or minimization devices.”

[3] This section draws heavily on Nordhaus (2012).

The question was addressed in one of the earliest energy-model comparisons, chaired by Tjalling Koopmans, "The use of optimization in these models should be seen as a means of simulating, as a first approximation, the behavior of a system of interacting competitive markets." (MRG 1978, p. 5, emphasis added.)

This point was elaborated at length in the integrated assessment study of copper by Gordon, Koopmans, Nordhaus, and Skinner (1987, with minor edits to simplify and emphasis added):

> We can apply this result to our problem of exhaustible resources as follows: if each firm is faced with the same market prices for its inputs and outputs, and if each firm chooses its activities so as to maximize the firm's discounted profits, then the outcome will be economically efficient. In more precise language, such an equilibrium will be economically efficient in the sense that (1) each firm will provide its share of the market at minimum discounted cost; and (2) the requirements of the market will be met by producers in a manner that satisfies total demand at minimum discounted total cost to society.
>
> Examining these two conditions, we see that our competitive equilibrium has indeed solved a minimization problem of sorts – it has found a way of providing the appropriate array of services at lowest possible costs. But this minimization is exactly the objective of a linear-programming problem as well. Consequently, we can mimic the outcome of the economic equilibrium by solving the LP problem that minimizes the same set of cost functions subject to the same set of technical constraints. Put differently, given the appropriate quantities of resources available and the proper demand requirements, by solving a cost-minimizing LP problem we can determine the equilibrium market prices and quantities for all future periods. We call this lucky analytical coincidence the correspondence principle: *determining the prices and quantities in a general economic equilibrium and solving the embedded cost-minimization problem by linear programming are mathematically equivalent.*

This discussion implies that we can interpret optimization models as a device for estimating the equilibrium of a market economy. As such, it does not necessarily have a normative interpretation. Rather, the maximization is an algorithm for finding the outcome of efficient competitive markets.

I. V

. Scenarios to evaluate

Integrated assessment models such as DICE have a wide variety of applications. Among the most important ones are the following:

- making consistent projections, i.e., ones that have consistent inputs and outputs of the different components of the system;
- calculating the impacts of alternative assumptions on important variables such as output, emissions, temperature change, impacts, prices, and economic growth;
- tracing through the effects of alternative policies on all variables in a consistent manner;
- estimating the costs and benefits of alternative strategies, and
- estimating the uncertainties associated with alternative variables and strategies.

The current study presents a suite of scenarios as follows.

Baseline: This scenario contains estimates of current climate policies, and the trends of current policies as of 2022 are extended indefinitely. This approach is standard for forecasting, say of government budgets, and is appropriate for a world of evolving climate policies. The baseline assumption is that the global average carbon price on CO_2 emissions is \$6/tCO_2, growing at 2.5% per year.

No controls: We sometimes will refer to a no-controls path. This is a scenario with a carbon price equal to \$0. It is for reference in calculating variables and is not used as a scenario for evaluation.

Cost-benefit optimal (C/B optimal): In this scenario, climate change policies maximize economic welfare according to the principles of cost-benefit analysis, with full participation by all nations starting in 2025. The C/B optimal scenario involves a balancing of the present values of the costs of abatement and the benefits of reduced climate damages. Although the underlying assumptions are highly optimistic, this scenario provides an efficiency benchmark against which other policies can be measured. (Note that this scenario was called "optimal" in earlier versions. The term cost-benefit was added to emphasize that it relies on monetized impacts and uses standard economic approaches to welfare maximization.)

Temperature-limited: In this scenario, the C/B optimal policies are undertaken subject to a further (precautionary) constraint that global temperature does not exceed 2 °C (or other targets) above pre-industrial levels. The temperature-limited scenarios are variants of the C/B optimal scenario that build in a precautionary temperature constraint.

Alternative discount rates. The assumptions about discounting are highly controversial and have major implications for the SCC and for policies. We consider alternatives to the standard approach discussed above by setting constant discount rates of 1%, 2%, 3%, 4%, or 5% per year.

Alternative damage function. This scenario uses an alternative damage function quantification based on Howard and Sterner (2017) and Howard (2021). The damage function has the same structure as the DICE version. While there are several potential results to choose from in Howard and Sterner, a reasonable middle ground of their preferred estimates is a 9% damage/output ratio at a 3 °C increase. This temperature-damage coefficient is 3 times larger than the one used in the current DICE model.

Paris Accord extended. The Paris Accord of 2015 codified a policy that would aim to limit climate change to 2 °C above pre-industrial levels. To achieve this goal, countries agreed to make their best efforts through nationally determined contributions. This scenario assumes that countries meet their objectives in 2030 according to their revised pledges as of summer 2022 and projects slightly less than ½ percentage point increase per year in the control rate from 2030 to 2100. This scenario further assumes that pledges are implemented through internationally harmonized carbon prices. It should be emphasized that any projections beyond 2030 are not based on country commitments and are therefore conjectural.

All scenarios have some important implementation constraints built in. One constraint is that climate policies have limits on implementation. These involve emission control rates increasing at a maximum of 12 percentage points per five-year period. Additionally, the emissions control rate is limited to 100% through 2120 and to 110% after that. The control limits are drawn from runs that stress high-resolution IAMs with extremely high carbon prices.

Finally, all scenarios assume 100% country participation with harmonized and comprehensive carbon prices. These assumptions about policy, particularly of full participation and harmonization, are highly optimistic and will lead to lower costs and better implementation of targets than scenarios where country actions and international agreements fall short of the ideal. Note as well that we do not consider solar-management geoengineering, which raises a host of other issues.

VI. Results

We now report on a set of representative results. All scenarios ran smoothly with the exception of the 1.5 °C limit, which is infeasible within the constraints of realism and of the technologies considered.[4]

A. *Emissions, Concentrations, Temperature*

For the major results, we focus on the baseline, C/B optimal, 2 °C, and Paris policies. Figure 2 and Table 1 report the results for CO_2 emissions under different scenarios. The baseline implies increasing emissions in the coming decades, in stark contrast with the declining emissions needed to achieve any of the policy objectives.

Figure 3 and Table 2 report the results for CO_2 concentrations for different scenarios. The 2 °C target will require stabilization of CO_2 concentrations at slightly more than 10% above current levels. Note that the Paris Accord will reduce concentrations about one-third of the way to the 2 °C target.

Figure 4 and Table 3 report the results for the increases in global temperature in different scenarios. The temperature change for 2100 in the baseline (current policy) run is 3.6 °C. The 2100 temperature change for the C/B optimal run is 2.6 °C. The C/B optimal temperature change is significantly above 2 °C run because damages do not have a kink at the threshold 2 °C temperature change, but the C/B optimal temperature path would depend as well on other key parameters such as abatement costs. Among the broader scenarios considered, the 2 °C target does pass the cost-benefit test in cases with sufficiently low discounting (2%) or the alternate damage function. In 2100, the Paris Accord reduces the temperature increase by one-third of the way from the base path to the 2 °C target. Note that these temperature increases are slightly above conventional measures because they use the pre-industrial (1765) baseline rather than later benchmarks.

[4] To meet the 1.5 °C target, emissions would be required to fall virtually to zero in the next five years. This would entail either a deep depression (output declining by around 75%) or an implausibly sharp increase in emissions reductions (by at least 50% within a decade). The scenario is so far from what any economic model can hope to capture realistically that it is best thought of as infeasible.

B. *Policies and Impacts on Income*

We next discuss key policy variables. Table 4 shows results for the emissions control rate across the main scenarios (with further results reported in Table 5 and Figure 5). Recall that this applies to all of the CO_2 emissions as well as abatable non-CO_2 GHGs. The emissions control rates across all scenarios start at 5% in 2020. In the base case, they remain low because of the weak level of current policy. The emissions control rates for policies in 2050 are 27%, 39%, and 55% for the Paris, C/B optimal, and 2 °C targets; and in 2100 are 57%, 84%, and 99% for the Paris, C/B optimal, and 2 °C targets. These necessary control rates are low relative to some other analyses because of the comprehensive nature of the controlled gases and because the runs assume complete efficiency and participation.

The carbon prices associated with these emissions control rates start at an estimated baseline price of \$6/tCO_2 (2019\$) for 2022. These reflect either the trading price for universal capped emissions or the harmonized level of universal carbon taxes. In order to implement the C/B optimal, Paris, and 2 °C emissions reductions targets, global carbon prices must rise to \$115/tCO_2, \$63/tCO_2, and \$200/tCO_2 by 2050, respectively (see Figure 6 for further results on carbon prices). In these calculations, the average carbon prices are modest relative to other estimates primarily because the emissions control rates are lower.

In the baseline (current policy) scenario, annual damages reach 4.4% of output by 2100. The extended Paris program improves on the baseline, with losses of 3.1% of output in 2100. The C/B optimal program reduces damages by half compared to the current policy scenario, with a damage-output ratio of 2.3% in 2100. The 2 °C limit scenario has a damage-output ratio of 1.4%.

Table 5 shows the total "wealth" in each scenario. Wealth is defined as the present value of consumption (technically, this is the present value of utility calibrated to first-period consumption). The stakes in an efficient program are clearly substantial. The C/B optimal program increases wealth by \$120 trillion. The 2 °C and Paris programs also make substantial improvements, increasing wealth by around \$107 and 85 trillion in present value, respectively.

C. *The social cost of carbon*

The most important single economic concept in the economics of climate change is the social cost of carbon (SCC). This term designates the discounted value of the change in consumption caused by an additional ton of carbon dioxide emissions or its equivalent. The SCC has become a central tool used in climate change policy, particularly in the determination of regulatory policies that involve greenhouse gas emissions. While estimates of the SCC are necessarily complex, IAMs are ideally suited to calculate them because of their comprehensive and internally consistent structure.

The definition of the SCC is the derivative of the objective function (or of the present value of consumption) with respect to CO_2 emissions in a given year. In actual calculations, the estimates are calculated as the ratio of shadow prices (which are algorithmic derivatives) in the different scenarios. [5]

Table 6 and Figure 7 show estimates of the SCC from DICE-2023. The SCC in the baseline run is \$66/tCO_2 for the 2020 period (in 2019 international \$). This is above the SCC for the C/B optimal run of \$50/tCO_2 because damages are smaller in the C/B optimum. It is far below the SCC for the 2 °C run of \$76/tCO_2. The higher SCC in the temperature-limited run reflects the economic interpretation that a tight temperature limit is equivalent to a damage function with a sharp kink at the temperature limit and therefore to a sharply higher damage function above 2 °C.

One of the most instructive findings involves the importance of discounting for the SCC and other policies. Table 6 shows alternative estimates of the SCC in the DICE-2023 scenarios and particularly emphasizes the powerful impacts of discounting and climate damages on the SCC.

Additionally, Figure 8 compares DICE estimates of the year-2020 SCC with several other current values, as explained in the legend. The surprising conclusion from Figure 8 is that the estimates from different sources are quite close conditional on the discount rate. Figure 8 highlights the importance of the discount rate in determining the SCC.

[5] As a technical note, the calculation of the SCC is not exactly equal to the carbon price in the optimal C/B scenario, which should be the case. For this run, the price is generally about 93% of the SCC. The reason is that the addition of the non-CO2 GHGs changes slightly the abatement cost function so that the price-SCC relationship is not one-to-one.

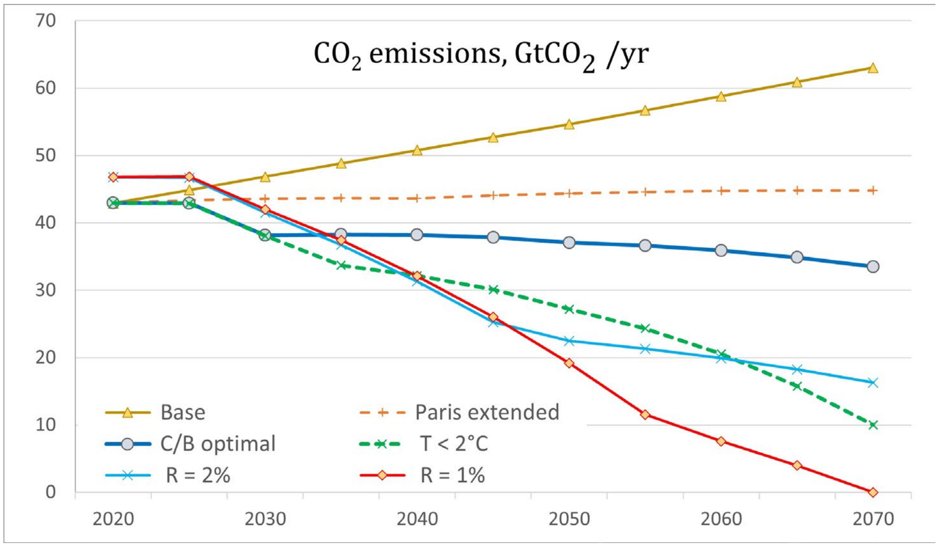

Figure 2. Results for CO_2 emissions in different scenarios

Note that emissions in low-discount scenarios are higher in early years because of higher output due to higher savings rates. In Figs 1 – 3, the label "R = X%" is scenario with a constant discount rate of X% per year.

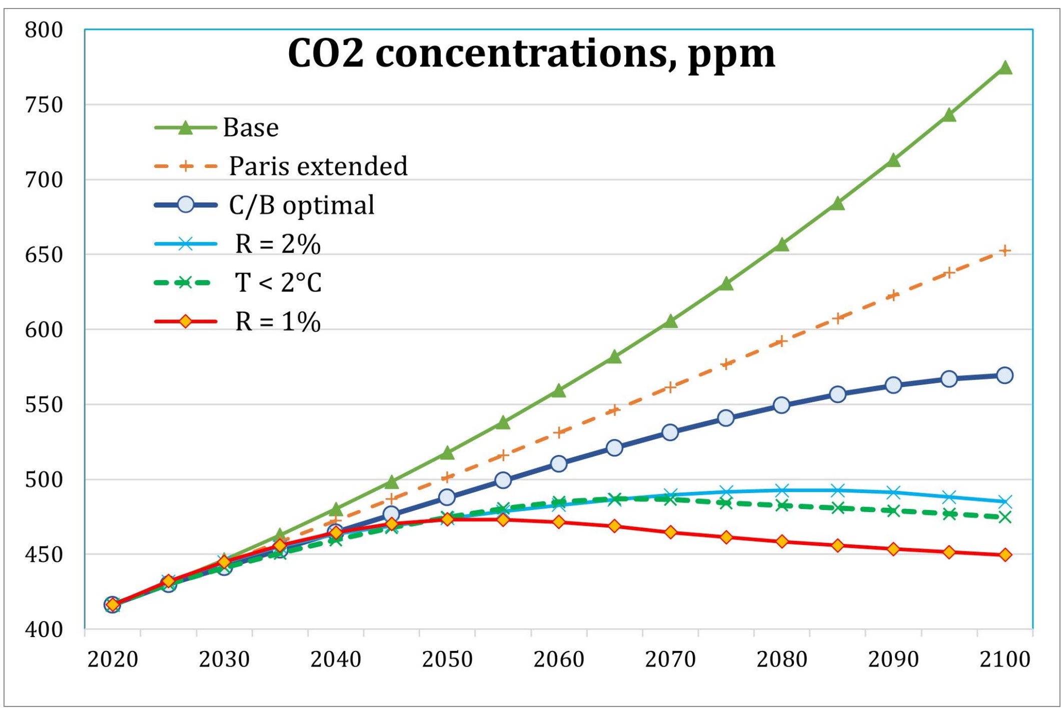

Figure 3. CO_2 concentrations in different scenario

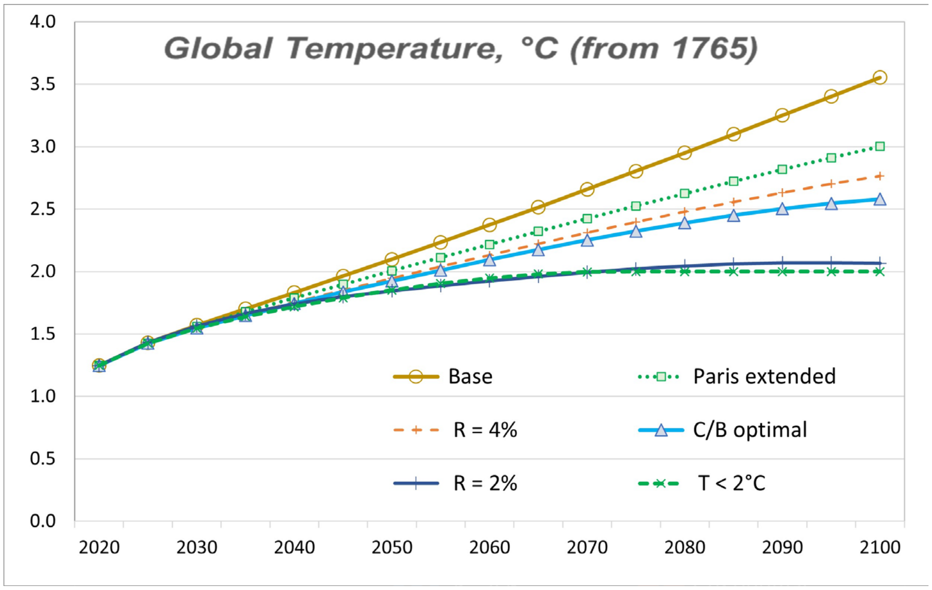

Figure 4. Global temperature increases in different scenarios

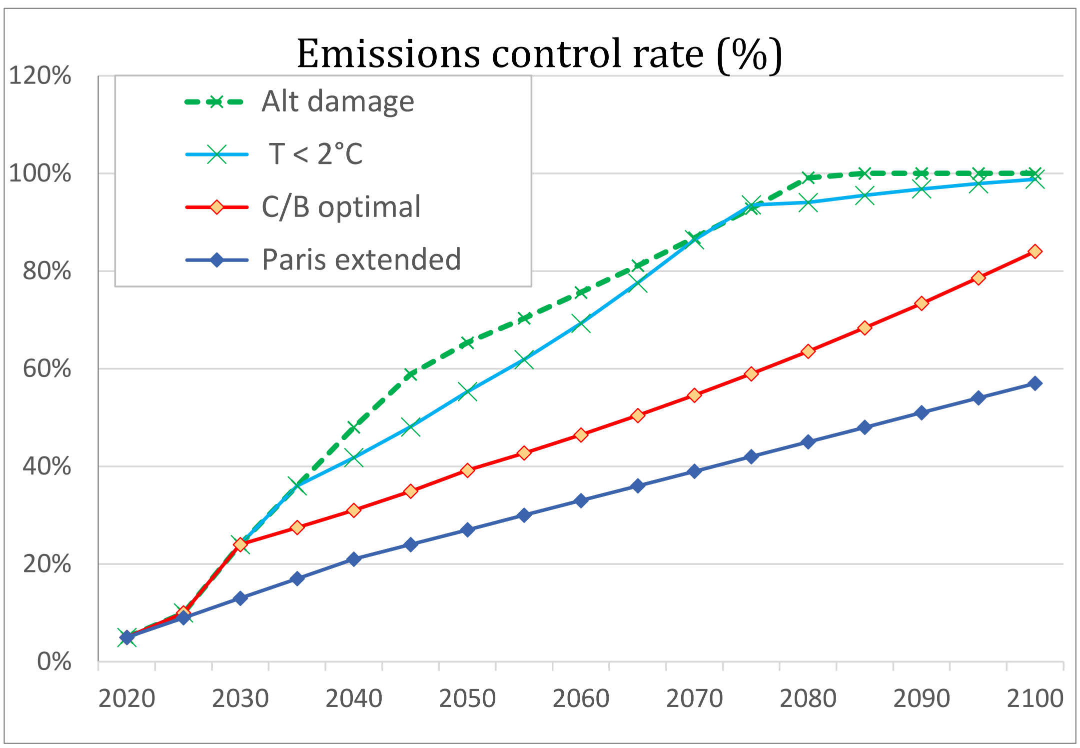

Figure 5. Emissions control rate for CO_2 and abatable GHGs (percent of no control)

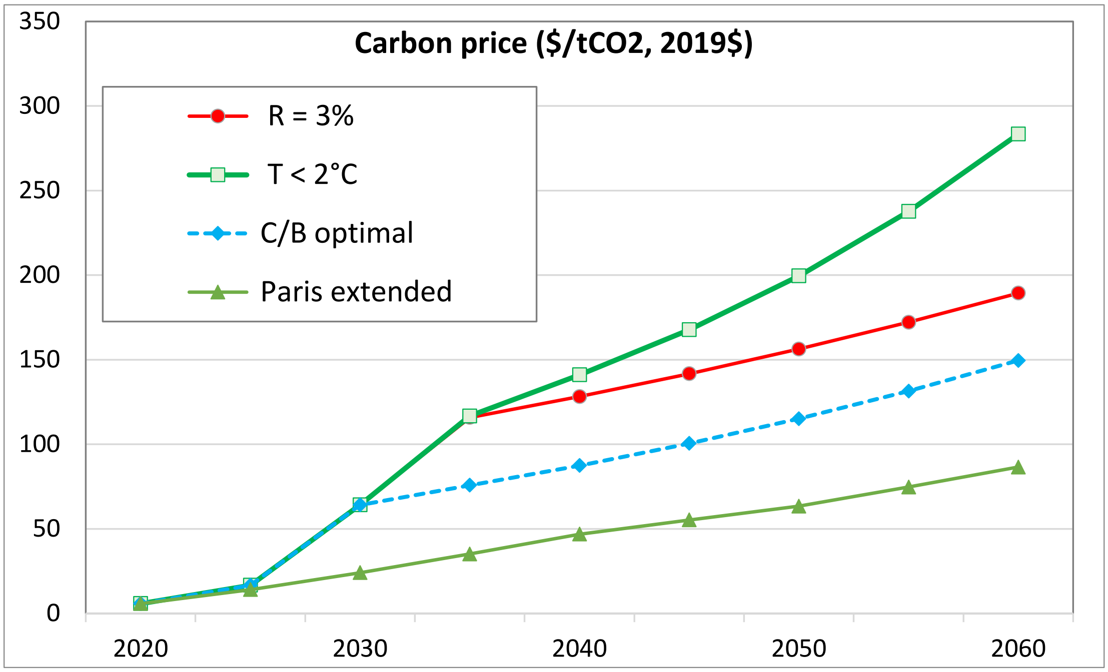

Figure 6. Price of CO_2 emissions (2019 $/tCO₂)

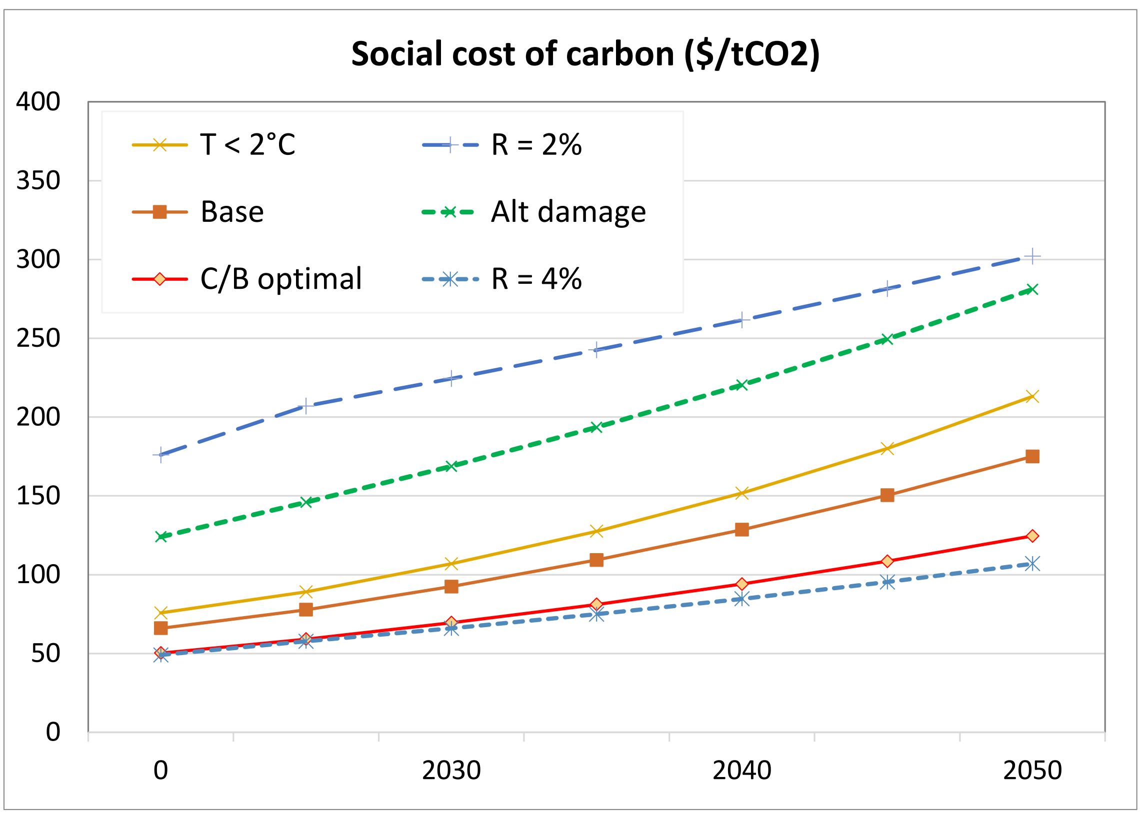

Figure 7. Social cost of carbon, alternative scenarios (2019$/$tCO_2$)

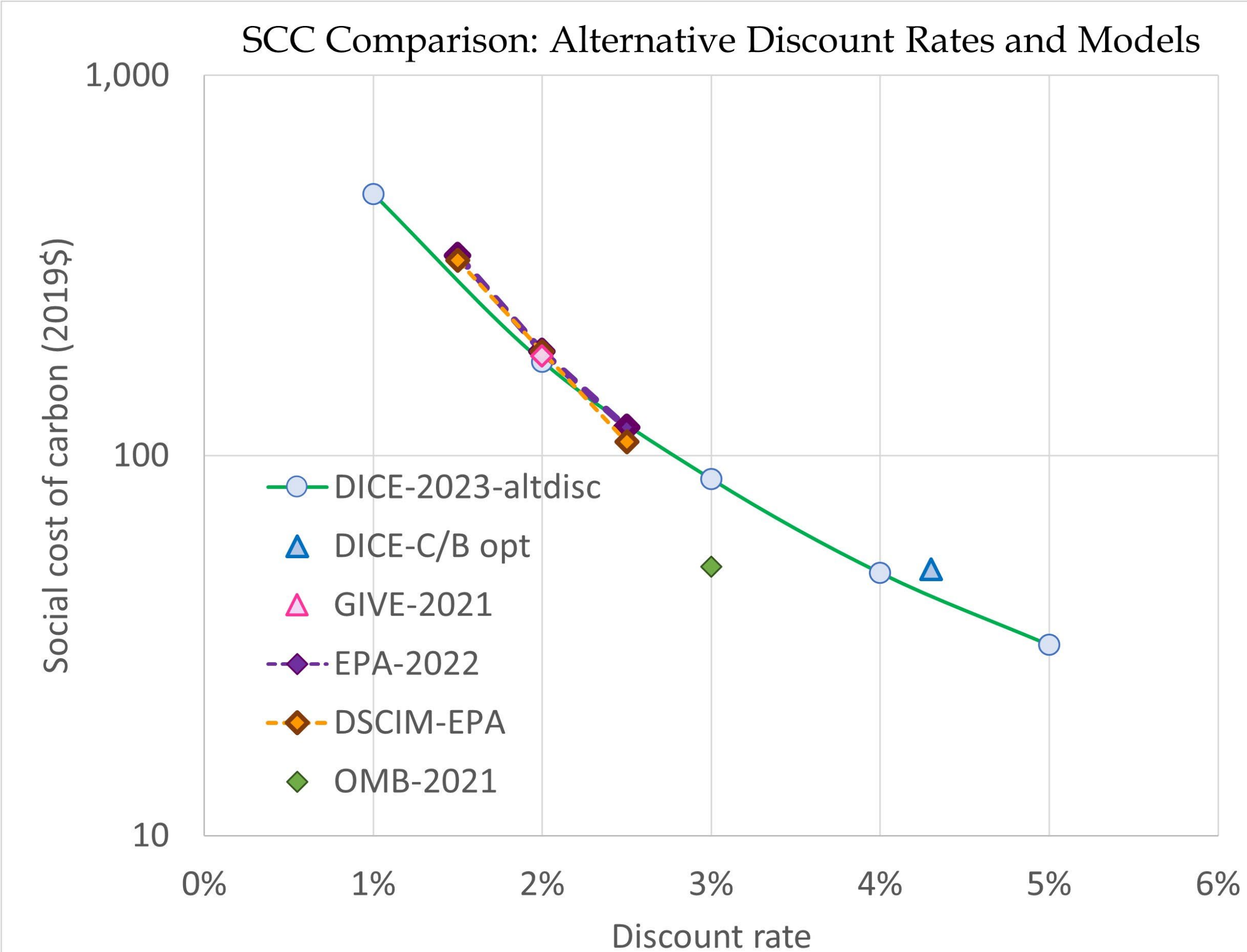

Figure 8. Social cost of carbon, 2020, alternative discount rates and models (2019$/t$CO_2$)

The figure shows the relationship between the discount rate on goods and the SCC in different scenarios of the DICE-2023 model and several other models. Results in order of the list are:

- DICE 2023-altdisc is the solid green line connecting the runs for constant discount rates in DICE-2023
- DICE- C/B-opt is the DICE-2023 estimate for the C/B optimal scenario along with the average discount rate for the period 2020 – 2050
- GIVE-2021 is the estimate from the GIVE model (K. Rennert, et al. 2022)
- EPA-2022 are the draft EPA social costs of greenhouse gas estimates based on an overall assessment (EPA 2022)
- DSCIM-EPA are the estimates specific to a damage module based on the DSCIM framework (Climate Impact Lab, 2022)
- OMB-2021(OMB 2021) is a preliminary OMB estimate based on earlier methods which did not reflect the changes introduced in 2022. Discount rates for EPA values correspond to near-term rates in their assessment.

	CO2 emissions, GtCO2/year			
Scenario	2020	2025	2050	2100
C/B optimal	42.9	42.9	37.1	15.9
T < 2°C	42.9	42.9	27.2	1.2
T < 1.5 °C	42.9	13.1	5.7	0.0
Alt damage	42.9	42.7	20.9	0.0
Paris extended	42.9	43.3	44.4	42.3
Base	42.9	44.9	54.6	75.7
R = 5%	42.8	42.5	42.2	37.6
R = 4%	44.1	43.9	39.3	28.9
R = 3%	45.6	45.3	33.5	15.4
R = 2%	46.8	46.7	22.5	0.0
R = 1%	46.8	46.9	19.2	0.0

Table 1. Results for CO_2 emissions in different scenarios.

Note that the 1.5 °C scenario has a catastrophic reduction in output to reduce emissions.

	CO2 concentrations, ppm				
Scenario	2020	2025	2050	2100	2150
C/B optimal	416.2	429.9	487.8	569.2	497.9
T < 2°C	416.2	429.9	474.7	474.7	437.9
Alt damage	416.2	429.8	466.7	458.5	401.0
Paris extended	416.2	430.1	501.3	652.5	763.5
Base	416.2	430.9	517.7	774.9	1,144.0
R = 5%	416.2	429.7	495.7	635.5	671.6
R = 4%	416.2	430.4	491.8	605.3	592.0
R = 3%	416.2	431.2	484.0	555.1	494.2
R = 2%	416.2	431.9	473.8	484.9	419.3
R = 1%	416.2	432.0	473.3	449.5	389.3

Table 2. CO_2 concentrations, parts per million (ppm) by scenario

Scenario	Global temperature, °C relative to 1765				
	2020	2025	2050	2100	2150
C/B optimal	1.25	1.42	1.92	2.58	2.29
T < 2°C	1.25	1.42	1.85	2.00	1.86
Alt damage	1.25	1.42	1.81	1.89	1.58
Paris extended	1.25	1.43	2.01	3.00	3.61
Base	1.25	1.43	2.10	3.55	4.91
R = 5%	1.25	1.42	1.97	2.93	3.24
R = 4%	1.25	1.43	1.95	2.77	2.84
R = 3%	1.25	1.43	1.90	2.49	2.26
R = 2%	1.25	1.43	1.84	2.07	1.73
R = 1%	1.25	1.43	1.84	1.81	1.49

Table 3. Global temperature increases under different scenarios

	Emissions control rate (%)					
	2020	2030	2040	2050	2060	2100
C/B optimal	5%	24%	31%	39%	46%	84%
T < 2°C	5%	24%	42%	55%	69%	99%
Alt damage	5%	24%	48%	65%	76%	100%
Paris extended	5%	13%	21%	27%	33%	57%
Base	5%	6%	8%	10%	12%	22%
R = 5%	5%	19%	23%	29%	34%	60%
R = 4%	5%	24%	29%	36%	42%	70%
R = 3%	5%	24%	39%	47%	54%	85%
R = 2%	5%	24%	48%	66%	73%	100%
R = 1%	5%	24%	48%	72%	90%	100%

Table 4. Emissions control rate for CO_2 and abatable GHGs (percent of no control)

Scenario	Emissions control rate			Present value of consumption	Difference from base
	2020	2050	2100	(Trillions of 2019 US international $)	
Base	5%	10%	22%	6,540	0.0
C/B optimal	5%	39%	84%	6,659	119.7
T < 2 °C	5%	55%	99%	6,647	107.3
Paris, updated 2022	5%	27%	57%	6,625	85.4

"Wealth" is the present value of global consumption of goods and services. They are benchmarked so that the present value of consumption in that scenario is the value of the objective function in the baseline scenario.

Table 5. Emissions control rates (percent of both CO2 and abatable non-CO2 emissions avoided) and total global wealth (present value of consumption, 2019 US$) across policy scenarios.

Note: "Wealth" is the present value of global consumption of goods and services. Values are benchmarked so that the present value of consumption in that scenario is the value of the objective function in the baseline scenario.

	Social cost of carbon ($/tCO$_2$, 2019$)		
Scenario	2020	2025	2050
C/B optimal	50	59	125
T < 2 °C	75	89	213
T < 1.5 °C	3,557	4,185	16,552
Alt damage	124	146	281
Paris extended	61	72	159
Base	66	78	175
R = 5%	32	37	74
R = 4%	49	58	107
R = 3%	87	102	172
R = 2%	176	207	302
R = 1%	485	571	695

This table shows the importance of discounting and alternative damage estimates on the SCC. It includes the SCC for the 1.5 °C scenario to indicate the cost induced by the catastrophic loss of output to reach the target. The label "R = X%" is scenario with a constant discount rate of X% per year.

Table 6. Social cost of carbon, alternative scenarios (2019$/tCO$_2$)

This table shows the importance of discounting and alternative damage estimates on the SCC. It includes the SCC for the 1.5 °C scenario to indicate the cost induced by the catastrophic loss of output to reach the target. The label "R = X%" is scenario with a constant discount rate of X% per year.

VII. The Recommendation for a Cumulative Emissions Limit

Scientists have suggested a new approach to climate change targets of cumulative emissions limits. In the Sixth Assessment Report on Mitigation, the estimate was that a cumulative limit of 3550 GtCO2 would be the 67% probability of limiting warming to 2 °C. Since cumulative emissions to 2010 are, by our estimates, 2323 GtCO2, this would allow 1227 GtCO2 of additional emissions in the future. This contains a background estimate for mitigation of non-CO2 GHGs, although that is not usually included. We take this to apply to CO2 only. (See IPCC Sixth Assessment, Mitigation. 2022, "Summary for Policymakers," p. 11, and Chapter 2.)

This proposal is easily implemented in the DICE-2023 model. It involves putting constrains on future emissions by the recommended amount. The result is a temperature path that is slightly lower than the 2 °C limit path. The reason is that the path allows for negative emissions, which keeps the cumulative emissions on the target level. This is partially because the DICE model is conceptually a 50%ile rather than a 67%ile. [6]

We can move up the cumulative emissions to 1070 GtCO2 to attain the 2 °C target. While the trajectory of emissions reductions is slightly more front-ended than the efficient 2 °C limit path, the difference from the efficient 2 °C trajectory is small, about $2 trillion in present value.

This suggests that the cumulative emissions approach – which is much simpler than a full carbon-climate model – is a reasonable approximation to a full model. One thorny issue is how to deal with non-CO2 GHGs, which are not easily measured and modeled. There are also serious issues involved in negotiating cumulative emissions limits, similar to those of annual emissions limits, which doomed the Kyoto Protocol. However, this is another useful idea to consider among alternative architectures. This comparison deserves more scrutiny in different IAMs to validate the findings.

[6] The plan was modeled in a file named "DICE2023-b-4-3-10-cumlimit.gms."

VIII. Revisions in DICE-2023

The current version of the model is DICE2023-b-4-3-10.gms, October 16, 2023. This is implemented in the GAMS modeling system. This note provides an overview of the revisions between DICE-2016R3 and DICE2023-b-4-3-10.gms. More details are provided in Appendix A's Background Notes.

A. Major changes

There are four major revisions in DICE-2023. The first and most significant change in DICE-2023 are new carbon and climate cycles. We adopted the FAIR model (Millar et al. 2017) for both the carbon and climate modules. The carbon cycle is methodologically different from earlier approaches in including saturation of the non-atmospheric reservoirs. The climate module is a two-box model, similar to D2016 but with different parameters. The TSC and ESC are largely unchanged from D2016 but brought in line with the latest IPCC Sixth Assessment Report (2021). The parameters of the model are shown in Appendix A. Note that the major adjustment from the standard FAIR model is to determine the initial conditions for the four reservoirs of the carbon cycle for 2020 and to calibrate the climate parameters.

A second methodological change is the treatment of discounting. We review data on real interest rates and the return to capital. Moreover, we explicitly account for investment risk in climate abatement. Data on real returns are drawn from market interest rates and equity yields as well as data on the rates of return to non-financial capital from the US Bureau of Economic Analysis. We incorporate an explicit risk premium on risky capital and a climate-investment beta of 0.5 from Dietz et al. (2018). Additionally, we incorporate an estimate of the precautionary effect due to uncertainty about future growth. On the basis of these revisions, we estimate that the real return on climate investments is slightly lower than in D2016 in the near term (4.5%/year in DICE-2023 v 5.0%/year in DICE-2016 for 2020), and significantly lower in the longer run.

A third major change is the damage function. We update the damage function in three ways. First, we extend the analysis of Nordhaus and Moffat (2017) by adding studies of global aggregate climate change impacts that have been published in recent years, based on a review by Piontek et al. (2021) Second, we add a new component to account for tipping points based on estimates in Dietz et al. (2021). Third, we add a judgmental adjustment factor to reflect non-monetarized impacts as explained in Appendix A. The updated damage estimates take the form of a quadratic damage function with a 3.1% loss in global output from 3 °C warming over preindustrial temperatures, up from 1.6% in DICE-2016. We emphasize that this estimate is subject to ongoing further refinements of the different components.

A fourth change is to add abatement for forcings other than industrial CO2. The procedure is to (i) estimate current and future GHG emissions and forcings; (ii) convert them to CO2-equivalent forcings; (iii) determine the fractions that are abatable; and (iv) use the estimates of cost of abatement for those from the existing literature. The estimates for (iii) and (iv) are from Harmsen et al. (2019). Total abatable emissions are equal to all CO2 emissions plus abatable non-CO2 GHG (in CO2 equivalent emissions). The abatement cost and controls are then applied uniformly to total abatable emissions. This version implies that 95% of total forcings are abatable compared to 80% of forcings in the D2016 version. See the discussion in Appendix A.

B. Other changes

Data

This section provides a summary of data changes and revisions. Each period is five years, with further details in the following appendices. A period is calculated as the average of the five years centered on the given year. Thus, "2020" is the average for 2018-2022. Note that the stocks are at the end of period, so "2020" is conceptually end-2022 for stocks.

All important input parameters are updated on the basis of the most recent information, usually through 2022. The historical and contemporaneous economic data include updates for world output, population, CO2 emissions, non-CO2 greenhouse gas forcings, and the emissions-output ratio (σ, sigma). Data are all stated in 2019 US international dollars measured at PPP exchange rates. Note that the initial period (2020) has a major anomaly because it contains the highly depressed pandemic year of 2020. We have included actual data for 2020, but exclusion would make little difference for the projections.

Projections of these data are as follows: Population projections are from the UN (https://population.un.org/wpp/). Output is measured in 2019 PPP US$, with the historical and current estimates from the IMF and the World Bank being virtually identical. Projections of output per capita are based on several studies including Christensen et al. (2018) , Müller et al. (2022), Rennert et al. (2022), and Newell et al. (2022). The no-controls emissions-output ratio for industrial CO2 is assumed to continue to decline at its historical rate of 1.5% per year. The CO2 emissions from land use are from IPCC AR6. 2021 (Physical Science), and projections of non-CO2 GHGs are on the basis of projections from MAGICC6.

The backstop technology is derived from the simulations of the ENGAGE project (Riahi et al. 2021). It has a backstop cost that is similar to estimates of other model comparisons and has an estimated decline in the zero-emissions carbon price of 1% per year until 2050. This is further discussed in Appendix A.

C. Concepts and functions

The abatement cost function is derived from the backstop cost for industrial CO2 (see the last paragraph and Appendix A). The functional form for abatement cost is the same as in DICE-2016.

The “baseline” policy is changed from earlier versions to include a low level of carbon prices and regulation, and emissions are therefore lower than the no-control level. In current policy, it is assumed that emissions are initially about 5% lower than the no-control level, which is equivalent to a carbon price of $6 per ton, and that the carbon price will grow by 2.5% per year. This is half due to carbon pricing and half due to regulations.

Major data and information about the climate and carbon cycle are from the IPCC Sixth Assessment Report for science IPCC AR6. 2021 (Physical Science). Additionally, data on scenarios are drawn from the IPCC socio-economic storylines (SSPs). We particularly referred to SSP2, which is characterized as a “middle of the road” scenario. For calibration purposes, we also used IPCC Representative Concentration Pathways (RCPs), particularly RCP45 and RCP85, IPCC AR6. 2021 (Physical Science).

The model is scaled so that utility in the baseline run has marginal value of the objective function of 1 for a change of 1 unit of consumption in 2019$. The additive scaling was set so that the present value of consumption equals the objective function for a base run of 400 years.

IX. Computational and algorithmic aspects

A. Analytical background

As we discuss in the next section, IAMs are generally computationally complex compared to physical science models, such as climate models, that use recursive time-stepped algorithms. Among IAMs, the DICE model is relatively simple because it is a straightforward non-linear optimization problem. The DICE model has generally been solved using the CONOPT or NLP solver in the GAMS modeling system (see Brooke et al. 2005). This is based on the generalized reduced gradient

(GRG) algorithm. The details of the algorithm are available in the user manual for the CONOPT solver.

CONOPT is generally a local solver and cannot ensure that the solution is a global optimum. However, the DICE-2023 version has also been solved using the BARON solver, which determines whether the solution is a global optimum. The BARON solver is much slower than the CONOPT or other local solvers, but it can solve most examples within an hour or less. A number of runs of different versions (Base, Optimal, as well as a version with a highly concave-convex damage function) indicates that the solutions with CONOPT are in all examined cases also the global optima.

Over the last decade, we have also used the EXCEL Solver. Using EXCEL Solver is also much easier to understand and to detect programming errors. It is also easier to use Excel when introducing new variables and models as the graphics can be employed to find problems. Recent versions of GAMS using auxiliary software such as "R" make graphics easier but still cumbersome. EXCEL has the shortcoming of having much longer and more complex coding. Excel also occasionally gives incorrect answers, particularly when hard constraints (such as those for temperature) are included.

By contrast with the DICE model, the RICE model (with multiple optimizing agents in equilibrium) is conceptually a fixed point problem. Many integrated assessment models today use a Negishi algorithm to solve this, and this is the approach followed in the RICE solutions. The origins of the Negishi approach date from work of Takashi Negishi, Alan Manne, Peter Dixon, Victor Ginsberg, Jean Waelbroeck, and Thomas Rutherford. The Negishi theorem is essentially an application of the second theorem of welfare economics. Several authors implemented this in the mid-1990s, particularly Nordhaus and Yang (1996) in the first version of the RICE model, although the actual implementations were and continue to differ among IAMs.

B. Solution concepts

We can summarize the points in this section in one paragraph: The DICE model is relatively simple compared to many integrated assessment models. Nonetheless, solving the model – particularly when optimizing emissions reductions – requires modern algorithms for solving non-linear optimization problems. The current DICE model is available in two different platforms. The simplest one is Excel (free with Excel) or Excel Risk Solver Platform (available for $640 to academic users). The second platform is the GAMS software system (General Algebraic Modeling System). This can be accessed only with proprietary software (available to academics for around $1000). For those who have limited research budgets, the Excel version is the most convenient platform.

Here is a more complete discussion: Optimization IAMs are generally computationally complex compared to physical science models, such as climate models, that use recursive time-stepped algorithms. Optimization problems are computationally complex because (from a mathematical point of view) they require solving a set of equilibrium conditions, such as first-order conditions. While some optimization problems can be solved quickly and efficiently, in general the computational costs rise as a polynomial or exponential function of the number of variables. By contrast, recursive problems (such as climate models) are linear in the number of variables.

The DICE model traditionally was solved using the GAMS modeling system (see Brooke et al. 2005). GAMS is a high-level modeling system for mathematical programming and optimization. It contains a high-level language and several high-performance solvers. We usually employ NLP solver in solving the DICE-RICE models. This is based on the generalized reduced gradient (GRG) algorithm. This is an algorithm that in practice has proven very efficient at solving large non-linear optimization problems where the constraints are smooth. If the model is "almost linear," it can use inner linear-programming-like iterations to achieve a rapid solution.

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A compendium of studies in several areas with many illuminating articles of CGE modeling is contained in Dixon and Jorgenson (2012)

C. Software architecture

A major issue in the design of integrated assessment models is the proper design of software. This issue has been largely ignored in the IAM community.

The GAMS code is small, although it depends upon a largely invisible translation into actual computational steps. The GAMS code, particularly for the DICE model, is easy to read over and check for mistakes. However, it requires great care in examining the equations and results to make sure they perform correctly. By contrast, all the Excel programs are huge, although many of the cells are duplicates. A major difficulty in all versions is to assure that there are no mistakes arising from interactions across the equations.

Table 7 shows the size of the source code for different versions of the DICE/RICE models.

Model	Cells or line of code
DICE-2023: GAMS without Put statements	
Total lines of code:	431
Total removing comments	327
DICE-2016R: GAMS	
Total lines of code:	262
Total removing comments	222
DICE-2023: Excel	
Total cells of code:	60,000
RICE-2005: GAMS	
Total lines of code (approx.)	2000
RICE-2010: EXCEL	
Total cells of code:	104,795

Table 7. Size of code for different DICE/RICE models

Note the huge size of code for Excel versions. These are largely copied cells because each time period is a cell. GAMS is much smaller but uses a high-level idiosyncratic language.

One of the thorny issues in developing IAMs is their computational complexity. This concern arises because of the increasing size and complexity of computerized modeling in environmental sciences and economics. Specialists in software architecture have studied the issues involved in developing large programs and emphasize the difficulties of ensuring that software is reliable and well-tested. A rule of thumb is that well-developed software contains in the order of 1 error per source line of code (SLOC). Since many computerized climate and integrated assessment models contain between 10,000 and 1 million SLOC, there is the prospect of many bugs contained in our code. [7]

This proposition is not just theoretical. There are many examples of catastrophically bad software, such as the errors that led to the crashing of a spacecraft because of insertion of a period instead of a comma in a FORTRAN statement; or inappropriate shutdown of five nuclear power reactors because of an incorrect formula programmed. Current luxury automobiles have millions of lines of code and probably contain untold thousands of bugs.

I take it as a given that large IAMs have a variety of errors, some consequential, some not. I did a routine check of one of my large models (the RICE 2011 in development at the time). I found a high error rate in terms of stranded code, poor definitions, and mistaken references. For the lines I examined, there were no substantive mistakes, but I suspect that had I gone further some would have turned up. In the RICE-2010 version, there was a mistaken reference in the sea-level rise module that led to small errors in the numerical projections. This was discovered by an interested user and corrected. Another example of coding issues was from the OECD Green model (discussed in Nordhaus 2012).

I will explore one error in depth because it is so subtle that it was found only after an intensive examination. The FUND model is one of the leading models used by researchers and governments to understand the economics of global warming. It has been used to calculate the social cost of carbon for the U.S., which calculation affects tens of billions of dollars of regulations.

The problem with the FUND model arose because of a formula for one of the components of the damage function in an early version (since corrected). The specification had agricultural damages, which were calculated with a formula having a normal variable in both the numerator and the denominator. This looks unnecessarily complex but innocuous. In fact, it turned out to be a serious error. This was pointed out in an article by Ackerman and Munitz (2012), which made the following statement: "The manner in which the optimum temperature effect is modeled in FUND 3.5 could cause division by zero for a plausible value of a Monte Carlo parameter."

It will be useful to examine the issue from a statistical point of view. The details are the following: In FUND 3.5, according to the model description, the damages for

[7] This section draws on a lecture presented at the Prague meetings of the EAREA in June 2012 and the debate about software design that ensued after that. References are contained in that discussion.

the level of temperature on agriculture have two terms. The first term of the damage component can be written as $y = az/(b\text{-}z)T$, where y = damages, T = temperature (an endogenous variable); a and b are parameters ; and z is a random variable, which in the FUND model is $(T\text{-}T^{opt,r})$. The variable $T^{opt,r}$ represents the optimal temperature in region r and is a normally distributed random variable, so y is the ratio of two normal variables.

The ratio of two normal distributions with non-zero means is a non-central Cauchy distribution. A non-central Cauchy distribution has a standard Cauchy term and another complicated term, but we can focus on the Cauchy term. This distribution is "fat tailed" and has both infinite mean and infinite variance. So the damages from agriculture in FUND 3.5 (from a statistical point of view) will dominate both the mean and dispersion of the estimated damages. Taken literally, the expected value of damages to agriculture are infinite at every temperature increase. This is subject to sampling error in finite samples of any size, but the sampling error is infinite since the moments do not exist, so any numerical calculations with finite samples are (infinitely) inaccurate. There is also a coding issue because it is not possible to get an accurate estimate of the distribution of a variable with infinite mean and variance in finite samples. The most troubling impact of this specification is the estimate of the distribution of outcomes (such as the social cost of carbon or SCC). If the damages are a fat tailed distribution, then the SCC is also fat- tailed. In finite samples, of course, all the moments are finite, but the estimates are unreliable or fragile and depend upon the sample.

I assume that this strange distribution was not intended, and in any case is easily corrected. My point was not to dwell on the shortcomings of our models. Rather, we need to recognize that most economists and environmental scientists are amateurs at software design and architecture. As computers get faster, as software packages get more capable, as our theories get more elaborate – there is a tendency to develop models that increase in parallel with the rapidly expanding frontier of computational abilities. This leads to increasingly large and complex models. We need also to ask, do we fully understand the implication of our assumptions? Is disaggregation really helping or hurting?

There is another lesson here about uncertainty analyses. Deterministic IAMs are already complex non-linear systems. Introducing uncertainty through a set of complicated functions of random variables adds yet another layer of complexity. Modelers need to be especially careful that they have not changed the properties and outcomes of the models because of strange behavior or interactions of the added random variables. The properties of linear stochastic systems are moderately well-understood, but that is not the case for all non-linear stochastic systems.

The conclusions here are four and apply to the DICE/RICE and other large IAMs. First, we modelers need to recognize the importance of good software architecture. Second, we should restrain the urge to develop ever larger and more complex computational models unless there is a clear and convincing case that they will improve our understanding or are necessary to understand the phenomena at hand. Third, we need to undertake special scrutiny when we add random elements to non-linear dynamic models. Finally, we need to take the extra time and effort to examine, re-examine, and test our software.

X. *Revisions over past vintages*

Earlier studies examined the sources of changes in model inputs and outputs since the first DICE model in 1992 (see Nordhaus 2017a). This section updates those results with an overview of the changes over three vintages, 1992, 2016, and 2023.

There have been multiple changes in the DICE model over its lifetime from 1992 to 2023. The following Table 8 shows the levels and changes for the most important variables for the model year 2015 (that is, projections for that year). This is mainly interesting because 2015 was the distant future for the first DICE model (which relied on data from the 1970s), and so this allows an analysis of the errors for a period where we have actual data. Some interesting results are these:

- CO2 emissions and total forcings were significantly overestimated in the 1992 model. The counterpart of this is that CO2/output ratio declined by 40%.
- Atmospheric concentrations and temperature were very close to the mark in the 1992 model.
- The major changes were in the economic variables. Output was revised up by 25%, while population was underestimated by 7%. The key economic variable of per capita consumption was underestimated by about 10%.
- The major revision came in the social cost of carbon, which was underestimated by a factor of more than 10. This was due to a number of different factors including discounting, the damage function, and the level of output.

We also show in Table 9 the changes from DICE-2016 to DICE-2023 for the projection for 2100 for the base run (current policy). The last column shows the change in the projection. Here are some key findings:

- Projections of 2100 temperature and atmospheric concentrations are significantly lower in the 2023 model. This is largely because of the revision in the carbon cycle.
- The social cost of carbon is way up in the 2023 model, as noted above.
- The 2100 baseline emissions/output ratio has increased, but with a higher assumed control rate, industrial CO2 emissions are only slightly up.
- Most economics variables are little changed.
- The control rate and carbon price are higher because baseline policies have become more restrictive.

Vintage	D1992	D2016R	D2023	Percentage change D1992 to D2016	D2016 to D2023	D2016 to D2023
Year	***2015***	***2015***	***2015***	***2015***	***2015***	***2015***
Emissions Control Rate (%)	10.4%	3.0%	4.6%	-71%	54%	-56%
CO2/output ratio (tCO2/000 2010$)	0.55	0.34	0.35	-38%	3%	-36%
Industrial Emissions (GTCO2 per year)	42.29	35.74	34.86	-15%	-2%	-18%
Total forcings (W/m2)	3.04	2.46	2.58	-19%	5%	-15%
Atmospheric Temperature (°C)	1.16	0.85	1.09	-27%	28%	-7%
Carbon Price (per t CO2)	4.55	2.00	4.57	-56%	128%	0%
Atmospheric concentration C (ppm)	398.81	399.53	402.01	0%	1%	1%
Short real interest rate (% per year)	4.2%	5.1%	4.5%	20%	-11%	7%
Population (billions)	6,868	7,403	7,387	8%	0%	8%
Consumption per capita (000, 2010$)	9.20	10.50	10.05	14%	-4%	9%
Output (trillions 2010$)	77.56	105.00	99.82	35%	-5%	29%
Capital stock (trillions, 2010$)	162.9	223.0	217.8	37%	-2%	34%
Savings rate (%)	18.6%	26.0%	25.7%	40%	-1%	38%
Investment (trillions, 2010$)	14.41	27.26	25.62	89%	-6%	78%
Climate Damages (% output)	0.2%	0.2%	0.4%	-15%	141%	105%
Social cost of carbon ($/tCO2 2010$)	4.54	30.98	48.36	582%	56%	964%

Table 8. Changes in variable for the year 2015 for different vintages
[Source: dicecompinitial-v14-u051424.xls; page tab2023]

Vintage	DICE-2016	DICE-2023		Change from 2016 to 2023
Period of model	18	17		
Year	2100	2100		
Atmospheric temperaturer (deg c above preind)	4.10	3.55		-13%
Atmospheric concentration C (ppm)	826.5	774.9		-6%
Population	11,069	10,534		-5%
Short real Interest rate, %/yr	3.6%	3.5%		-3%
Savings rate, fraction gross output	23.9%	23.6%		-1%
Capital stock, 2019$	1,832.39	1,814.67		-1%
Gross investment, 2019$	181.1	182.1		1%
Total forcings w/m2	6.82	6.88		1%
Output, net net trill 2019$	757.2	772.9		2%
Y gross-net, 2019$	757.3	773.3		2%
Consumption	576.2	590.8		3%
Output, gross-gross, 2019$	788.6	808.8		3%
Industrial CO2 GtCO2/yr	70.9	74.8		6%
TFP	15.4	16.4		6%
Per capita GDP	68,410	73,369		7%
Consumption per capita, 2019$	52.1	56.1		8%
Climate damages, fraction of output	4.0%	4.4%		10%
Damages, 2019$	31.4	35.4		13%
Sigmabase (CO2/output, no controls, industrial CO2)	0.10	0.12		17%
Change TFP, %/year	0.05	0.06		18%
Emissions control rate	11.2%	21.9%		96%
Social cost of carbon $/tCO2	245.5	644.8		163%
Land emissions, GtCO2/year	0.33	1.09		236%
Carbon price (2019 $ per t CO2)	10.77	43.26		302%
Abatement, 2019$	0.04	0.42		1044%

Table 9. Changes in variable for the year 2100 for DICE-2016 and 2023
[Source: dicecompinitial-v14-u051424.xls; page d16base]

XI. Conclusion

The present manual is intended to provide users as well as those interested in integrated assessment modeling a self-contained document for understanding and using the DICE/RICE family of models.

As with all large-scale models of this kind, they must be continuously updated. Additionally, they are prone to errors in the software and structure. Data and scientific views evolve, and users must be attentive to the potential for large and small changes in the economics and natural sciences. Problems arise particularly when modifications are made to models (such as alternative parameterizations) and the model is not carefully tested to make sure that the changes do not alter the behavior or introduce instabilities.

The DICE model has evolved significantly over the years since its development. The vast changes in the projections of different variables might lead some to conclude that these undermine the credibility of the modeling approach.

My response would be different, however, and can best be summarized by a remark made in another context. The economist John Maynard Keynes was criticized for changing his views on monetary policy during the Great Depression. His response is reported to be, "When the facts change, I change my mind. Pray, sir, what do you do?" This is a reminder of the need to be constantly attentive to changing economic and scientific findings. Even more important is to resist getting dug into an intellectual Maginot Line of particular views or projections.

Error is human, but humility is divine.

References to User's Manual 2024

Note that this list contains several historically useful background studies beyond those cited in the current text.

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Appendix A. Four Background Notes on DICE-2023
Lint Barrage and William Nordhaus

The following contain four parts: Background Note on Discounting, Background Note on Damages, Background Note on DFAIR, and Background Note on Non-CO2 Forcings.

These background notes are for informational purposes for modelers. They are not intended for publication and are not publication quality. Some of the details are sketched and not derived in detail in this document. They may be cited with the warning, “Background notes are for informational purposes and are not published.”

Note that references are at the end of the main document (before Appendix A).

<u>Table of contents for Background Notes:</u>

Background Note on Discounting (December 18, 2023)

Part A. Summary of approach to discounting

Previous versions of DICE used different approaches to discounting. Upon the advice of modelers and readers, we have revised the treatment to employ an approach known as the "certainty equivalent" rate of return. This approach has been developed using suggestions of William Hogan, theoretical approaches developed by Gollier (particularly 2016), recommendations of the National Academy committee (2017), and empirical implementation of the correction for growth uncertainty by Newell, Pizer, and Prest (2022), hereafter NPP. This summary provides a full discussion of the approach, with other aspects in further Parts of this *Background Note.*

The variables in the analysis are the following. All rates are average annual returns. All time variables are per year.

$R_T =$ discount rate from *0* to T

$r_t =$ discount rate from *t-1* to t

$R_T^f, R_T^K, R_T^{CLIM} =$ risk-free, capital, and climate discount rates

ρ = pure rate of time preference

φ = elasticity of utility with respect to consumption

ρ_T^* = risk-adjusted rate of time preference

g_T = average growth of per capita consumption from *0* to T

$P_T =$ precautionary effect rate from *0* to T

σ_C^2= variance of trend growth rate of per capita consumption

π = capital premium

β^{CLIM} = climate beta

$\tilde{x}_T$ = deterministic version of variable x_T

As in earlier versions of the DICE model, discounting continues to follow the approach of the Ramsey-Cass-Koopmans growth model in determining real rates of return. In this approach, the continuous-time equilibrium deterministic long-run rate of return from 0 to $T\left(\tilde{R}_T\right)$ is given by the pure rate of time preference *(ρ)* plus the product of the deterministic growth rate of per capita consumption from t = 0 to T $\left(\tilde{g}_T\right)$ times the elasticity of the marginal utility of consumption *(φ)*. Note that the "~" over a variable indicates a deterministic concept.

$$(A.1) \quad \tilde{R}_T = \rho + \varphi \tilde{g}_T$$

In many applications, the consumption elasticity *(φ)* is also assumed to equal the relative rate of risk aversion. This is *not* assumed in the DICE treatment of discounting. That assumption would lead to a capital risk premium that is far below the observed rate, as is discussed in the literature on the equity-premium puzzle. Instead, we rely on the CAPM estimates of the capital premium.

Our implementation of the modeling continues to rely on the Ramsey model. However, we interpret the elasticity of consumption *(φ)* as applying to relative valuations of consumption over time or the rate of inequality aversion (RIA) but not, as is often commonly assumed, to the relative rate of risk aversion (RRRA). For clarity, we label this the "Ramsey growth model."

Often, IAMs employ the "Ramsey/C-CAPM" approach in which the elasticity of consumption *(φ)* represents both the RIA and the RRRA. This is the approach of NPP and Rennert et al. (2022), for example. The Ramsey/C-CAPM approach is theoretically appealing because it unifies choice over time and over uncertain states of the world. However, that approach fails to generate a realistic risk premium (hence, the equity premium puzzle), and for this reason is not used in DICE-2023. Other extensions, such as the Epstein-Zin specification, introduce different RIA and RRRA, but these do not solve the equity premium puzzle without raising new complications. (This topic is discussed in detail below in this section and in Part E.)

As we use the term, the Ramsey growth model does not include any risk aversion in deriving what is called "the precautionary effect." Rather, alternative growth paths generate alternative discount rates (as per Weitzman, 1998). While we use the terminology of the precautionary effect, the interpretation is that it is an adjustment for uncertain growth and for inequality aversion. While we could add a separate approach to account for risk aversion, we have found no satisfactory unified model and choose to take the simpler CAPM approach, which is not rigorously connected to the Ramsey growth model but has a firm empirical foundation.

The modeling relies upon discount rates and their associated discount factors to calculate present values, optimal policies, and variables such as the social cost of carbon. The "discount factor," D_T, is the factor applied to future values to obtain the present value of a value in time T discounted back to time 0. In a deterministic framework, the discount factor is the product of the one-period discount factors. In this discussion, r_t are period-to-period rates of return from period $(t-1)$ to t, while R_T are long rates of return from period 0 to period T (all in compound annual rates).

$$\text{(A.2)}\quad D_T = \left[\frac{1}{\left(1+R_T\right)^T}\right] = \left[\frac{1}{(1+r_1)}\right]\left[\frac{1}{(1+r_2)}\right]\cdots\left[\frac{1}{(1+r_T)}\right]$$

Because of uncertainty about future growth, the *expected* discount factor will differ from the deterministic discount factor by a term called the "precautionary effect." For example, with two interest rate paths which differ by a constant 4% per year for 100 years, the precautionary effect is to lower the average 100-year long rate by 1.3% points. The effect on the near term is small, with a precautionary effect of only 0.04%/year in the second period (2025). However, with long horizons, the impact of uncertain growth can be substantial, and for that reason the precautionary effect has a major impact on climate policy.

In the approach taken here, we assume that the major uncertainty is about the long-run *trend* rate of growth of per capita consumption. More precisely, we assume that the trend rate of growth of per capita consumption is normally distributed with a constant variance of $\sigma_C{}^2$.. Note that the variance of log consumption will grow as $\sigma_C{}^2T^2$, which is different from the usual model of the equity premium where the variance of log consumption is constant over time. The next section shows a numerical example to show the impact of random trend growth.

The precautionary component for this distribution of trend growth rates is given by

$$\text{(A.3)}\quad P_T = -½\,\sigma_C{}^2\varphi^2 T$$

where P_T = is the precautionary effect from time 0 to T and $\sigma_C{}^2$ is the variance of the trend growth rate of consumption. In a process where there is uncertainty about the trend rate of growth, the precautionary effect will be larger as the length of period increases because the variance of log consumption increases. For a deterministic model like DICE, we therefore correct the deterministic Ramsey equation to reflect growth uncertainty through adding the precautionary effect.

This procedure generates a sequence of "certainty-equivalent discount rates." This term is used to designate the single discount rate delivering the same discount factor as the expected value from the distribution of uncertain future discount rates (NPP, p. 1019). From (A.1) and (A.3), the certainty-equivalent risk-free discount rates $\left(R_T^f\right)$ are given by (A.4):[1]

$$(\mathrm{A.4}) \quad R_T^f = \rho + \varphi \, \tilde{g}_T - ½ \, \sigma_C{}^2 \varphi^2 T$$

To calculate the precautionary effect, we examine two procedures. The first is based on the calculations of NPP. These take estimated future growth rates from their Monte Carlo draws and the implied future interest-rate structure to estimate numerically the precautionary component. A second approach takes the standard formula in (A.3) for the precautionary effect from a model with a normal distribution of trend growth rates. The two approaches give reasonably similar estimates of the precautionary effect, and we therefore take equation (A.3) as computationally simpler and easier to implement and test. For a comparison of the two approaches, see the derivation of the precautionary effect in Part D.

The key parameters of the precautionary effect are the variance of the consumption growth rate and the consumption elasticity. The variance is estimated in several studies (e.g., Christensen. Gillingham, and Nordhaus 2018 and Müller, Stock, and Watson 2022). The studies have estimates of the standard deviation of trend per capita consumption growth to 2100 in the range of 1.0% to 1.2% per year. For our modeling, we assume that trend growth of consumption per capita follows a normal distribution with a mean of 2% per year and standard deviation of 1 percentage point per year. Part D describes the calculation of the precautionary effect in detail. Part F provides updated estimates of future economic growth.

[1] See, e.g., Prest (2023) equation (1) or Gollier (2016) equation (37), where we note in reference to the latter that our approach to discounting uses the CAPM rather than CCAPM approach for adjustments to the risk profile of climate investments, as discussed below.

In making calculations for DICE-2023, we rely on two components of the discount rate: a risk-free rate and an adjustment for investment risk. A broad consensus exists that the risk-free real return on investment is in the range of 0 to 2% per year over the last century. We take 2% per year to be the rate for long-term risk-free investments, which is the rate that has prevailed over the last century or so except for the most recent period.

Empirical evidence indicates that the return to risky assets (such as corporate capital or an unleveraged portfolio of corporate equities) is substantially higher than the risk-free rate. For example, the post-tax average rate of return on US corporate capital has averaged around 7% per year over the period from 1948 to 2022. The real return on a deleveraged portfolio of large US public corporations was 6% per year for the same period. The underlying data are presented in Part E.

At this point, we confront the "equity premium puzzle." This puzzle is that the volatility of consumption cannot rationalize the high risk premium (of 5% per year in our estimates) within the standard model (the C-CAPM model). Most studies examine the equity premium, but the puzzle remains for capital as well as equity. Given the failure of the C-CAPM model, we adopt the estimates from the CAPM model, which examines the correlation of investment risks with the market risk rather than the consumption risk. This leads to the assumption of a risk premium of 5% per year in the DICE-2023 model. Note that our approach differs from those that take the C-CAPM approach (such as Gollier 2014 and NPP).

In the new DICE-2023 specification, the discount rate includes an adjustment for the non-diversifiable risk of climate investments. Risky climate investments, primarily those to reduce emissions and reduce future damages, are introduced through the concept of the climate beta. The climate beta measures the extent to which climate investments (such as renewable power) share the non-diversifiable risk characteristics of the economy's aggregate investments. A climate beta of zero indicates that the risks on climate investments are uncorrelated with market returns; a climate beta of one indicates that climate investments have risk properties similar to those of the aggregate economy. We take our estimate of the climate beta from Dietz et al. (2020), which estimates a long-run climate beta of 0.5, so one with an intermediate correlation with market risks. A more extensive discussion of the climate beta and the reason for our estimate is given in Part C.

For our purposes, we assume that the near-term risk-free long-term rate is 2% per year and the capital risk premium is π = 5% per year. With a climate beta of 0.5, this implies a near-term risk-adjusted certainty-equivalent discount rate on climate investments of 2% + 0.5 x 5% = 4.5% per year. We note that the precautionary adjustment is taken to be zero in this illustrative calculation as near-term consumption growth trend uncertainty is minimal.

To calibrate the model requires estimates of φ and ρ. These are estimated by first calculating the deterministic risk-free rate of return from (A-1) above (where we note that using the certainty-equivalent rate equation (A-4) would yield equivalent results again due to the small level of near-term growth uncertainty). We constrain ρ ≥ 0.1% per year to ensure long-run convergence and for consistency with the estimates in NPP; we then incorporate the DICE estimates of near-term growth in per capita consumption of 2% per year. These parameters lead to the bound that ρ = 0.1% per year. Solving for φ gives the following:

$$(A.5) \quad \varphi = \frac{\left(R^f_{2020} - \rho\right)}{g_{2020}} = \frac{(0.02 - 0.001)}{0.02} = 0.95$$

This then implies that the average annual discount rate on climate investments (R_T^{CLIM}) from *0* to *T* is:

$$(\mathrm{A}.6) \quad R_T^{CLIM} = \rho + \varphi \tilde{g}_T - ½\varphi^2 {\sigma_C}^2 T + \beta^{CLIM}\pi$$

With an estimated climate beta of 0.5 as well as other estimates, this leads to a near-term *(T = 0)* rate of return on climate investments of 4.5% per year.

$$\begin{aligned}(A.7) \quad R_0^{CLIM} &= 0.001 + (0.95)(0.02) - ½(0.95)^2(.01)^2(0) + (0.5)(0.05)\\ &= 0.045\end{aligned}$$

This calculation gives an estimate of 4.5% per year for the near term.

In order to implement (A.6) in DICE, we replace the pure rate of social time preference in equation (A.1) with a "risk-adjusted rate of time preference" designated by $\rho_T^* = \rho - ½\varphi^2 {\sigma_C}^2 T + \beta^{CLIM}\pi$.

Growth v level precautionary effect: An example

For those used to the standard equity-premium model, the precautionary calculation used here may be unfamiliar. In the standard model (such as Mehra and Prescott 1985), the growth of log consumption is an i.i.d. random variable. We can illustrate the difference between the level effect and the growth effect on the precautionary term with a simple example. For this example, we assume that the consumption elasticity is φ = 1. In the *level effect,* we assume that growth of log consumption has a zero trend with i.i.d. random normal disturbances over time with a standard deviation of $\sigma_{C,L} = 0.02$. The precautionary effect in this case is $P_T = -½\, \sigma_{C,L}{}^2 \varphi^2 = -½ * 0.0004 * 1 = -0.0002$, which is constant over time.

With the *growth effect*, we assume that the trend rate of growth has a normal distribution with a zero mean and a standard deviation of $\sigma_{C,g} = 0.01$ per year. In the growth case, the precautionary effect is

$P_T = -½\, \sigma_{C,g}{}^2 \varphi^2 T = -½ * (0.0001) * 1 * T = -0.00005 * T.$

Thus, while the level precautionary effect is constant at -0.02%/year, the growth precautionary effect starts out at -0.005%/year in year T = 1 and increases to -1.5%/year after 300 years. The reason is that the variance of log consumption across paths is constant with the level effect and increases with time with the growth effect.

Figure A-1 shows six paths, which are displayed on a logarithmic vertical scale. The three paths that are closely packed in the middle are the three randomly chosen paths with only the level effect. The three dispersed paths are three paths with randomly selected constant growth rates. As is clear from this example, the standard deviation of the growth paths is linear in time, while the standard deviation of the level paths is roughly constant over time.

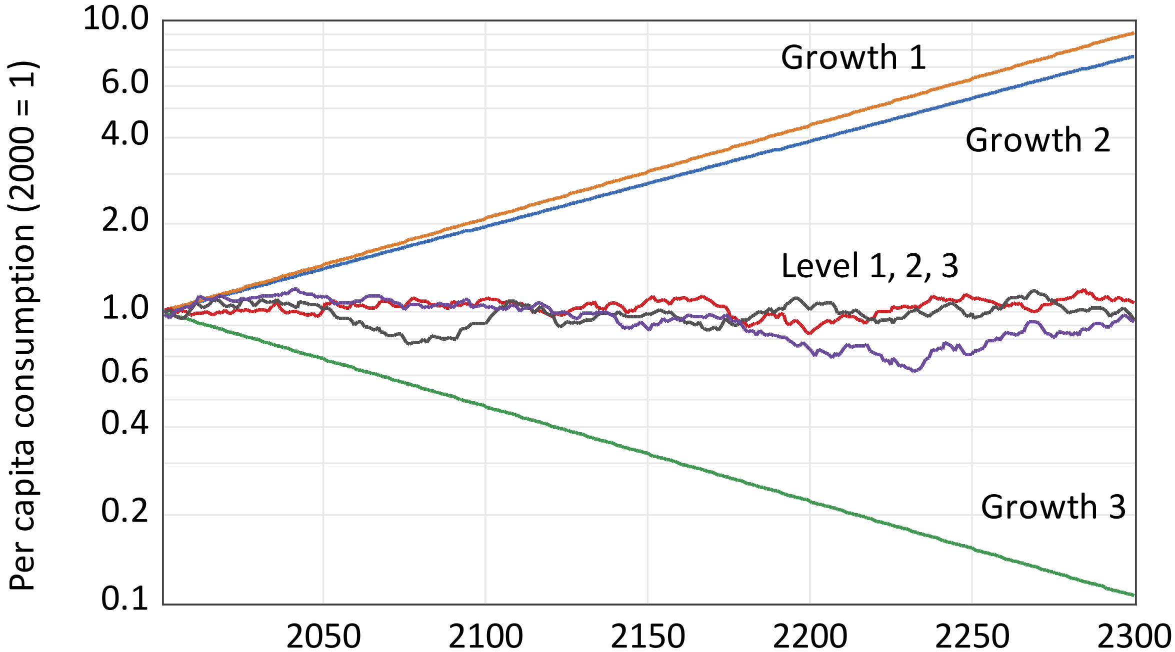


Figure A-1. Figure shows per capita consumption with randomly selected paths. These show three paths with random level effects and three paths with random growth effects. Note that the three growth paths diverge over time and have a standard deviation that is approximately linear in time. This indicates why the precautionary effect is linear in time in DICE under the assumption of random trend growth rates.

Part B. **GAMS programming for discounting**

The following is the GAMS programming code in DICE-2023 that determines discounting (file is "DICE2023-b-4-3-10.gms").

```
PARAMETERS
[other]
** Preferences, growth uncertainty, and timing
        betaclim  Climate beta                                      / 0.5  /
        elasmu    Elasticity of marginal utility of consumption     / 0.95 /
        prstp     Pure rate of social time preference               /.001/
        pi        Capital risk premium                              / .05  /
        rartp       Risk-adjusted rate of time preference
        k0        Initial capital stock calibrated (1012 2019 USD)  / 295  /
        siggc1    Annual standard deviation of consumption growth   / .01 /
** Scaling so that MU(C(1)) = 1 and objective function = PV consumption
        tstep       Years per Period                               / 5  /
        SRF         Scaling factor discounting                     /1000000/
[other]
PARAMETERS
[other]
** Precautionary dynamic parameters
        varpcc(t)         Variance of per capita consumption
        rprecaut(t)       Precautionary rate of return
        RR(t)             STP with precautionary factor
        RR1(t)            STP factor without precautionary factor;
** Time preference for climate investments and precautionary effect
        rartp           = exp( prstp + betaclim*pi)-1;
        varpcc(t)       =  min(Siggc1**2*5*(t.val-1),Siggc1**2*5*47);
        rprecaut(t)     = -0.5*varpcc(t)*elasmu**2;
        RR1(t)          = 1/((1+rartp)**(tstep*(t.val-1)));
        RR(t)           = RR1(t)*(1+rprecaut(t))**(-tstep*(t.val-1));
[other]
VARIABLES
[other]
        TOTPERIODU(t)   Period utility
        UTILITY         Welfare function
        RFACTLONG(t)
        RSHORT(t)       Real interest rate with precautionary(per annum year on year)
        RLONG(t)        Real interest rate from year 0 to T
;
[other]
*Economic variables
        RSHORTEQ(t)      Short-run interest rate equation
        RLONGeq(t)       Long-run interest rate equation
        RFACTLONGeq(t)   Long interest factor
* Utility
        TOTPERIODUEQ(t)  Period utility
        PERIODUEQ(t)     Instantaneous utility function equation
        UTILEQ           Objective function      ;

[other]
**** Equations of the model
[other]
**Economic variables
 RFACTLONGeq(t+1)..   RFACTLONG(t+1) =E= SRF*(cpc(t+1)/cpc('1'))**(-elasmu)*rr(t+1);
 RLONGeq(t+1)..       RLONG(t+1)     =E= -log(RFACTLONG(t+1)/SRF)/(5*t.val);
 RSHORTeq(t+1)..      RSHORT(t+1)    =E= -log(RFACTLONG(t+1)/Rfactlong(t))/5;
** Welfare functions
 periodueq(t)..       PERIODU(t)     =E= ((C(T)*1000/L(T))**(1-elasmu)-1)/(1-elasmu)-1;
 totperiodueq(t)..    TOTPERIODU(t)  =E= PERIODU(t) * L(t) * RR(t);
 utileq..             UTILITY        =E= tstep * scale1 * sum(t,  TOTPERIODU(t)) + scale2;
```

Part C. **Calculation of the climate beta**

Estimates of the climate beta are scarce. The most comprehensive estimate is from Dietz et al. (2018), which uses a combination of theory and integrated assessment modelling (IAM) to estimate the climate beta. While their IAM empirical estimates rely on an earlier version of DICE, the study uses parametric uncertainties for the DICE model and adds further potential parametric uncertainties, such as those for catastrophic damages. Their long-run estimate (to 2215) for all uncertainties is beta = 0.49 (see their Table 3). However, depending upon the combination of uncertainties, the estimates range as high as 1.10. The surprisingly high beta is driven largely by the dominance of the uncertainty about TFP growth, which they assume to be normal with a standard deviation of 0.9% per year. This estimate is close to the estimate we use (1% per year) for determining the precautionary effect in DICE-2023.

Note that from an analytical point of view, as they show, the climate beta is likely to be at least 1 if the uncertainties are driven largely by uncertainty of the growth of productivity (and therefore per capita consumption). The surprising result is, as Dietz et al. clearly explain, "the positive effect on the climate beta of uncertainty about exogenous, emissions-neutral technological progress overwhelms the negative effect on the climate beta of uncertainty about the carbon-climate-response, particularly the climate sensitivity, and the damage intensity of warming." (p. 258)

A different approach to estimating the climate beta comes from estimates of industry betas for sectors where the major mitigation investments are likely to be made. For example, replacing electricity generated by fossil fuel will require generation by renewables. If we take power, utilities, and air transport as three highly carbon-intensive industries, we can examine estimates of CAPM betas from Professor Aswath Damodaran (https://pages.stern.nyu.edu/~adamodar/). For these three industries, the average unleveraged betas are calculated to be 0.49, which is reasonably close to the estimates we use from Dietz et al.

We conclude that we assume a climate beta of 0.5 for the present version of DICE-2023. However, we emphasize that this estimate contains considerable uncertainty both in terms of its empirical basis (since it is model-based rather than historically-based) and also that the estimate is strictly speaking appropriate for the C-CAPM framework where it was derived.

Part D. **Estimates of the precautionary effect**

For long-term projections, it is important to include the impacts of the uncertainty about future economic growth on discounting, a factor that has been ignored in earlier vintages of the DICE model. This note uses the approach of Newell, Pizer, and Prest, "NPP" (2022) to estimate the impact of growth uncertainty. We call this higher-order impact "the precautionary term." Note, however, as discussed in the introduction to this Note, that this term is widely used but in the present context refers to the impact of growth uncertainty on discounting that operates through the intertemporal elasticity, not risk aversion.

The basic analysis is well known but is usefully described in Gollier (2016). In the standard Ramsey model where P_T is the precautionary effect from t = 0 to T, φ is the intertemporal consumption elasticity, and the trend consumption growth rate follows a normal distribution with a variance of σ_c^2, the precautionary effect is given by:

$$(D.1) \quad P_T = -½\varphi^2\sigma_c^2 T$$

The precautionary effect is introduced as an exogenous variable since there is no uncertainty in the DICE model. This term is calculated using estimates of the uncertainty of trend growth of per capita consumption along with assumptions about the key parameter of φ. There are several estimates of the long-run path of growth uncertainty, but they are all reasonably consistent.

Monte Carlo with lognormal output

The simplest approach uses a Monte Carlo simulation. For this simulation, we assume that trend growth in consumption is distributed as N(mean growth, standard deviation of trend growth) = N(0.02, 0.01), ρ = .001, φ = .95, and N = 10,000 replications. To be clear, draw one might have constant consumption growth rates of 1.6% per year, draw 2 perhaps 2.5% per year, and so forth. Table D-1 shows the results for long-run real returns. The parametric assumptions are in the legend of the table. Here are the key points:

- The deterministic risk-free discount rate is a constant 2% per year. This reflects the constant expected rate of growth of output with the assumed values of ρ and φ.

- The certainty-equivalent discount rate is lower than the deterministic rate by the precautionary rate. At 80 years (2100 in the model), with the empirical assumptions above, the certainty-equivalent discount rate would be 36 basis points lower than the deterministic rate as defined here and more generally in the literature. By 280 years (2300), the precautionary effect is 121 basis points. Because of the assumptions, the precautionary rate is linear in time.
- The numerical calculation using the Monte Carlo is virtually identical to the formulaic calculation of the precautionary effect.

Period from present	R^{CE}	R^{DETER}	$R^{PRECAUT}$	$R^{PRECAUT}$calc	$R^{PRECAUT}$ minus RP^{RECAUT}calc	Sample	sigma(g)	ρ	φ
1	1.99%	2.00%	0.004%	0.005%	0.000%	10,000	0.01	0.001	0.95
30	1.86%	2.00%	0.134%	0.135%	-0.001%	10,000	0.01	0.001	0.95
80	1.64%	2.00%	0.358%	0.361%	-0.003%	10,000	0.01	0.001	0.95
180	1.20%	2.00%	0.798%	0.812%	-0.014%	10,000	0.01	0.001	0.95
280	0.78%	2.00%	1.215%	1.264%	-0.048%	10,000	0.01	0.001	0.95

ρ =	0.1%
φ =	0.95
N =	10,000
g (p.c. cons)=	N(.02, .01)
R^{CE} =	Certainty equivalent discount rate
R^{DETER} =	Deterministic rate (rate implied by average growth rate)
$R^{PRECAUT}$ =	Precautionary component determined by concavity ($-\frac{1}{2}\varphi^2\sigma_T^2$)
$R^{PRECAUT}$_CALC =	Precautionary component determined by lognormal formula ($-\frac{1}{2}\varphi^2(\sigma_{C,T})^2$)

Table D-1. Estimates of the precautionary effect using a normal distribution for trend growth of consumption.

Note that φ in the Table refers to the consumption elasticity.

NPP Estimates

A second approach uses the growth estimates from NPP (2022). For these, we use the results of the NPP Monte Carlo of consumption growth based on Muller et al. (2022) provided by the authors to calculate the precautionary effect.[2] Note that these estimates differ in the levels from Christensen et al. (2018) or Rennert et al. (2022) but the variance is reasonably close. These use the same parameters as the first approach, but in NPP there is a random element to the level of consumption along with a slightly changing variance in the trend growth rate.

Figure D-1 shows the variance over time of the analytical approach above ("DICE variance") along with the variance determined by the NPP draws. The NPP has a higher short run variance because of the random element in the level of consumption, while the trend is slightly lower in NPP, with a cross-over at about 120 years. Note as well that the DICE variance is linear by assumption.[3] Since the difference between the two precautionary effect is close to half of the difference between the two variance estimates, the precautionary effect with be close to 10 basis points higher for the NPP estimates in the first 50 years, then approximately the same for the next century or so, then lower after about 200 years.

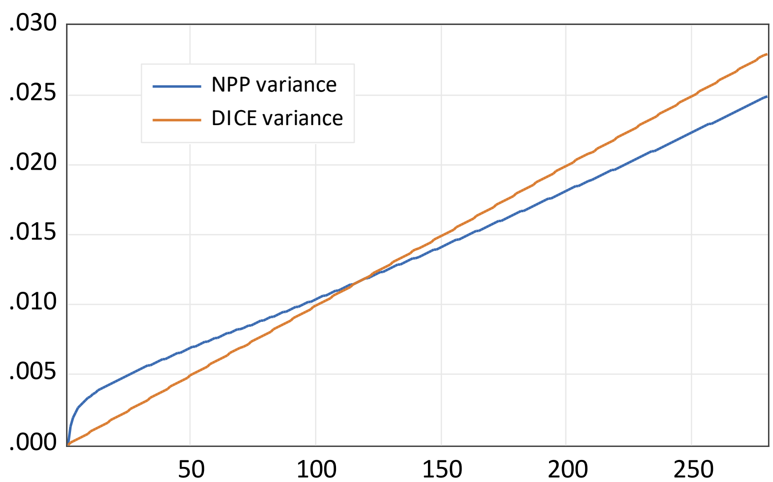


Figure D-1. Comparison of the variance of the growth rate of consumption under the two approaches.

[2] Source: figure1_draws-avggrowth.xls.

[3] Also see Copy of GIVE_discount_rate_decomposition; Results-MOnte-Carlo-090523;

Part E. **Historical rates of return**

We have gathered in Table E-1 data on total returns for major assets classes in the US for the 1927- 2022 period with thanks to the Stern School. Note that these are total returns including dividends, interest, and capital gains or losses.

Additionally, Table E-2 provides estimates of the rate of return (total earnings on capital, i.e., profits plus interest) as a percentage of the current replacement cost of the net stock of fixed assets of the US corporate sector based on data from the US BEA.

Year	*S&P 500*	*3-month T.Bill*	*US T. Bond*	*Baa Bond*	*Real Estate*
Arithmetic Average Historical Return					
1928-2022	11.5%	3.3%	4.9%	7.0%	4.4%
1973-2022	11.7%	4.4%	6.6%	8.8%	5.5%
2013-2022	13.6%	0.8%	0.5%	3.8%	7.7%
Arithmetic Average Real Return					
1928-2022	8.3%	0.3%	1.9%	3.9%	1.3%
1973-2022	7.6%	0.4%	2.6%	4.7%	1.5%
2013-2022	10.8%	-1.8%	-1.9%	1.3%	4.9%

Table E- 1. Total returns on US financial assets

Source: Data from NYU, Stern School, https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html in file "stern-returns-2023.xlsx"

[Real returns are not available as geometric averages from the source.]

	Return before taxes (all corps)	Return after taxes (all corps)	Return before taxes (non-fin corps)	Return after taxes (non-fin corps)
1948 - 2022	11.0%	7.6%		
1992 - 2022	10.6%	8.6%		
1998 - 2017	10.2%	7.9%	8.8%	7.2%
2012 - 2022	11.1%	9.3%		

Table E- 2. Rates of returns on capital of US corporations
Rates of return are total capital income divided by replacement cost of capital, both in current prices. They are conceptually real returns. Data from the BEA. These include S corporations. A correction for the share of S corporations reduces the return over the last two decades by about 50 basis points.[4]

Based on these findings, we conclude the following:

1. The *short-run* risk-free return (measured as the real return on short Treasury securities) has averaged close to zero per year for most of the last century, although it has been lower in recent years.
2. The *long-run* risk-free rate of return (measured on 10-year Treasury bonds) has averaged around 2% per year over the total period, although it has been sharply lower in the last decade. The real interest rate on 10-year TIPS has a shorter period and has recently risen back to the earlier pre-financial crisis level of about 2% per year real.
3. The best estimate of the after-tax real return on capital (measured in the US corporate sector) has been 7 – 9% per year over the last half-century. Unlike financial returns there has been no major change in these returns in the last two decades.
4. Corporate equities are currently unleveraged with respect to total bond-type assets. The aggregate US corporate non-financial balance sheet has a

[4] Source: Capital rates of returns 070623, page “wn-ror-nipa”

long-debt/equity ratio of approximately 30% with a roughly equal short-debt/equity ratio. We therefore treat corporate equities as unleveraged.

The capital premium

We define the "capital premium" as the difference between the expected return on aggregate economy-wide assets and the risk-free rate of return. Based on the current balance sheet of the corporate sector, we assume that the capital premium is the same as the well-studied equity premium. [5] We observe these data only for the US non-financial corporate sector and assume that these values apply to the entire global financial structure. Based on the estimates above, we assume that the rate of return on risky assets in the US is 7% per year and has been relatively stable. That rate applies not only to financial returns but also to corporate capital. Based on the estimate of a risk-free long-run rate of return of 2% per year, we calculate the capital premium to be 5% per year.

We note at this point the difficulty of estimating various rates of return given the volatility of the series and the limited sample size. As an example, we can calculate the capital premium as the difference between the risky rate of return on corporate equities and the risk-free return. For 1928 – 2022, the geometric average capital premium is 5.7% per year, with a standard deviation of 2.0% per year. If we assume that the capital premium is i. i. d. normal, then the (5, 95) %ile for the mean return is (3.7%, 7.7%) per year. While this range is well above zero and above the estimates of the equity or capital premium from C-CAPM models, it is clear that there is considerable uncertainty about the estimate of the capital premium.

[5] The literature generally deals with the "equity premium puzzle," dating back to 1985 with Rajnish Mehra, and Edward C. Prescott (1985) and Rajnish Mehra (2008). We use the term capital rather than equity to emphasize that it applies to the return on capital more generally as is appropriate for Ramsey-type models.

Part F. **Revised estimates of economic growth**

Projections of future growth of per capita output and consumption have been revised in October 2023 in light of recent data and research. The results relative to earlier models and versions yield a slightly lower growth rate in the early years (to 2050), but more rapid growth in per capita global output over the model horizon. Note that the growth rates after 2150 make little difference in scenarios with strong policies, but they can affect the base (current policy). The reason is that most strong policies have virtually 100% emissions control after a century, so growth projections after that will have little to no effect on emissions, concentrations, and temperature.

Historical data and current projections are shown in Table F-1. The DICE-2023 projections in the last column are intermediate between the CGN/MSW results and the Rennert et al. (2022) blended statistical and expert elicitations. Figure F-1 visualizes the comparison between DICE-2023, Rennert et al. ("RFF-SPs"), and the NPP projections along with their respective 5-95th percentile ranges.

	Historical data (geometric mean, percent per year)						
	MSW	IMF	CGN	MSW	Renn-med	Renn-avg	DICE-2023
1901-1931	1.3						
1931 - 1960	1.9						
1961 - 1990	2.1	2.3					
1991 - 2022		2.0					
	Projections (geometric mean, percent per year)						
2020 - 2050			2.6	1.9	1.5	1.5	1.9
2020 - 2100			2.0	1.9	1.5	1.5	1.8
2020 - 2200				1.9	0.9	1.1	1.7
2020 - 2300				1.9	0.9	1.1	1.6

MSW = Muller, Stock, and Watson
IMF = International Monetary Fund from historical data base
CGN = Christensen, Gillingham, and Nordhaus
MSW = Muller, Stock, and Watson
Renn-med= Rennert et al., Figure 6 (from background data)
Renn-avg = Rennert et al., Figure 6 (from our calculations)
DICE-2023 = from base run of version b-4-3-6

Table F-1. Estimates of the growth in per capita output from different studies.[6]

[6] Source: gdp-compar-100423.xlsx

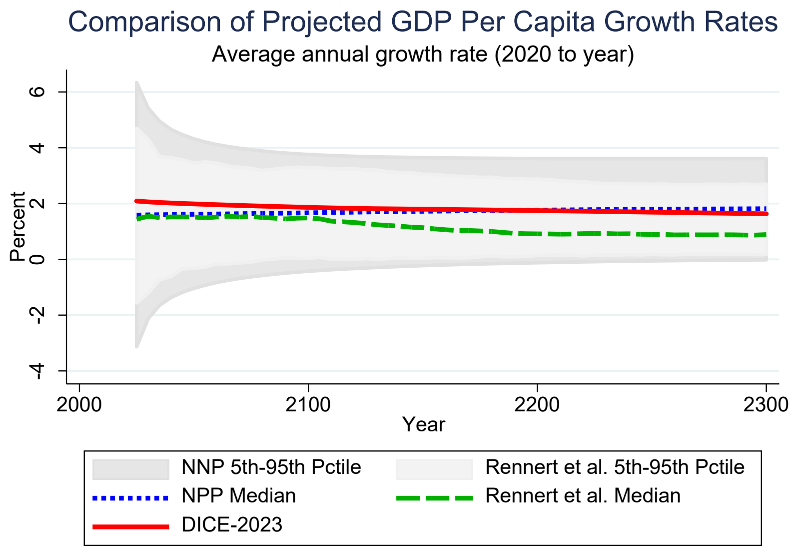


Figure F-1: Comparison of DICE-2023 GDP per capita growth projections with NPP and Rennert et al. medians and 5-95th percentile ranges. Growth is measured as annual average percent growth from 2020 through each depicted year.

A key parameter for calculating the precautionary effect on discounting is the uncertainty about future growth. For this parameter, we examined the dispersion of forecasts. Standard deviations or quantiles of the distributions of growth rates were tabulated in different studies, and these yielded the estimated standard deviations of trend growth in per capita global output shown in Table F-2.

	Estimated standard deviation of trend growth (% points/year)				
	CGN	MSW	NPP	Renn	DICE-2023
2020 - 2050	1.1	1.0	1.0	0.9	1.0
2020 - 2100	1.1	1.0	0.8	0.8	1.0
2020 - 2200			0.8	0.7	1.0
2020 - 2300			0.9	0.6	1.0

CGN = Christensen, Gillingham, and Nordhaus
MSW = Muller, Stock, and Watson
NPP = Newell, Pizer, Prest
Renn = Rennert et al., Figure 6, calculated from (5,95) percentiles.
DICE-2023 = from base run of version b-4-3-6

Table F-2. Estimates of the uncertainty of the growth in per capita output from different studies. [7]

The estimated standard deviation of the growth rate from 2020 to 2100 ranges from 0.8% to 1.1% points per year. The standard deviation for longer periods ranges between 0.6% to 0.9% points per year depending on period and study. These are significantly larger than the historical variability in growth rates, such as the difference in long-period growth rates in MSW of around 0.4% point per year or estimates of the 1950 – 2022 standard deviation of the growth rate of 0.3% - 0.5% point per year.

For our estimates, we have chosen the estimates from GGN, NPP, and MSW because of our preference for statistical techniques in deriving variability estimates. The main effect of the higher estimate of the uncertainty in the trend growth rate will be to increase the precautionary effect and thereby to lower long-run discount rates, primarily after 2100. For modeling purposes, we choose a constant uncertainty of 1.0% point per year because of the simplicity of modeling and transparency of interpretation. This rate is close to the near-term uncertainty for most estimates but lower than estimates after 2100. Since the precautionary impact is proportional to the variance times the squared

[7] Source: gdp-compar-100423.xlsx

time-from-present, this assumption will tend to overestimate the precautionary impact after 2100.

Background

Table F-3 shows the estimates from Christensen et al. (2018). The estimates that are used are the expert results for the world.

Table 1. Expert and low-frequency estimates by region and time horizon

		2010–2050							2010–2100						
Region	Statistic	10th	25th	50th	75th	90th	μ	σ	10th	25th	50th	75th	90th	μ	σ
World	Expert TM	1.17	1.80	2.59	3.23	3.92	2.54	1.07	0.60	1.36	2.03	2.85	3.47	2.06	1.12
	Expert (SD)	1.37	0.97	0.75	0.85	0.92	—	—	2.14	1.14	0.84	0.94	1.06	—	—
	Low freq	1.2	1.7	2.2	2.7	3.3	2.23	0.99	1.2	1.7	2.2	2.7	3.3	2.23	0.98
High	Expert TM	0.56	1.23	1.76	2.30	2.75	1.72	0.84	0.27	0.95	1.46	2.08	2.57	1.47	0.88
	Expert (SD)	1.38	0.82	0.68	0.69	0.77	—	—	1.55	0.92	0.62	0.73	0.84	—	—
	Low freq	0.7	1.4	2.0	2.5	3.0	1.90	0.99	1.0	1.5	2.0	2.4	2.8	1.92	0.89
Middle	Expert TM	0.93	1.76	2.67	3.36	4.11	2.57	1.23	0.34	1.30	1.98	2.72	3.45	1.96	1.18
	Expert (SD)	1.47	0.91	0.77	0.68	0.77	—	—	2.15	0.83	0.81	0.64	0.97	—	—
	Low freq	0.5	1.2	1.9	2.6	3.4	1.92	1.27	0.5	1.3	1.9	2.6	3.4	1.94	1.41
Low	Expert TM	1.05	2.23	3.41	4.25	5.12	3.21	1.57	0.62	1.72	2.53	3.45	4.57	2.58	1.49
	Expert (SD)	1.70	1.25	0.78	0.95	1.26	—	—	2.10	1.23	1.10	1.05	1.55	—	—
	Low freq	2.8	4.3	6.1	8.1	10.2	6.34	3.00	1.8	3.5	5.5	7.9	10.7	5.95	3.74
United States	Expert TM	0.60	1.14	1.75	2.18	2.63	1.66	0.79	0.49	0.91	1.53	2.04	2.64	1.52	0.84
	Expert (SD)	1.18	0.76	0.75	0.69	0.68	—	—	1.28	0.76	0.76	0.65	0.78	—	—
	Low freq	0.9	1.5	2.2	2.8	3.4	2.14	1.09	1.2	1.7	2.1	2.5	2.9	2.03	0.84
China	Expert TM	1.51	2.81	4.23	5.19	6.31	4.01	1.85	0.89	2.02	2.93	3.87	4.87	2.92	1.51
	Expert (SD)	1.83	1.57	1.11	1.18	1.35	—	—	2.29	1.59	1.25	1.15	1.38	—	—
	Low freq	1.6	3.9	6.6	9.5	12.7	6.93	4.61	0.7	3.1	5.7	8.9	12.7	6.33	5.36

Note: Expert and low-frequency estimates by region and time horizon. Expert TM and SD are the trimmed mean and SD of expert forecasts at each quantile. Low-frequency forecasts are Bayes estimates at each quantile. Notation is that μ and σ are the means and SDs of the respective forecast distributions. Expert μ and σ are estimated using a fitted normal distribution (see *SI Appendix* for details).

Table F-3. Projections from Christensen et al. (2018).

Simulations by NPP using Muller et al. (2022) produce the results in Figure F-1 for the distribution of per capita growth, slightly below 2% per year.:

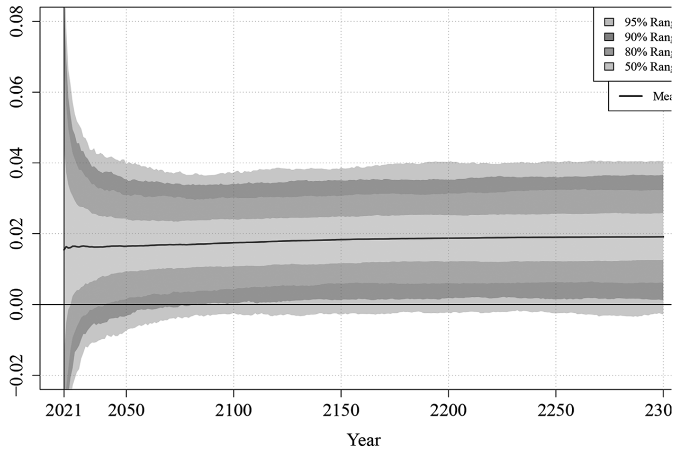


Figure F-1. Distribution of projections of per capita output from NPP

We can compare this to Rennert et al. (2022). The estimates use the "RFF-SPs" to calculate both the median and the uncertainty of future growth rates. These are shown in Figure F-2. Table F-4 shows estimates of the growth rate from Muller et al.

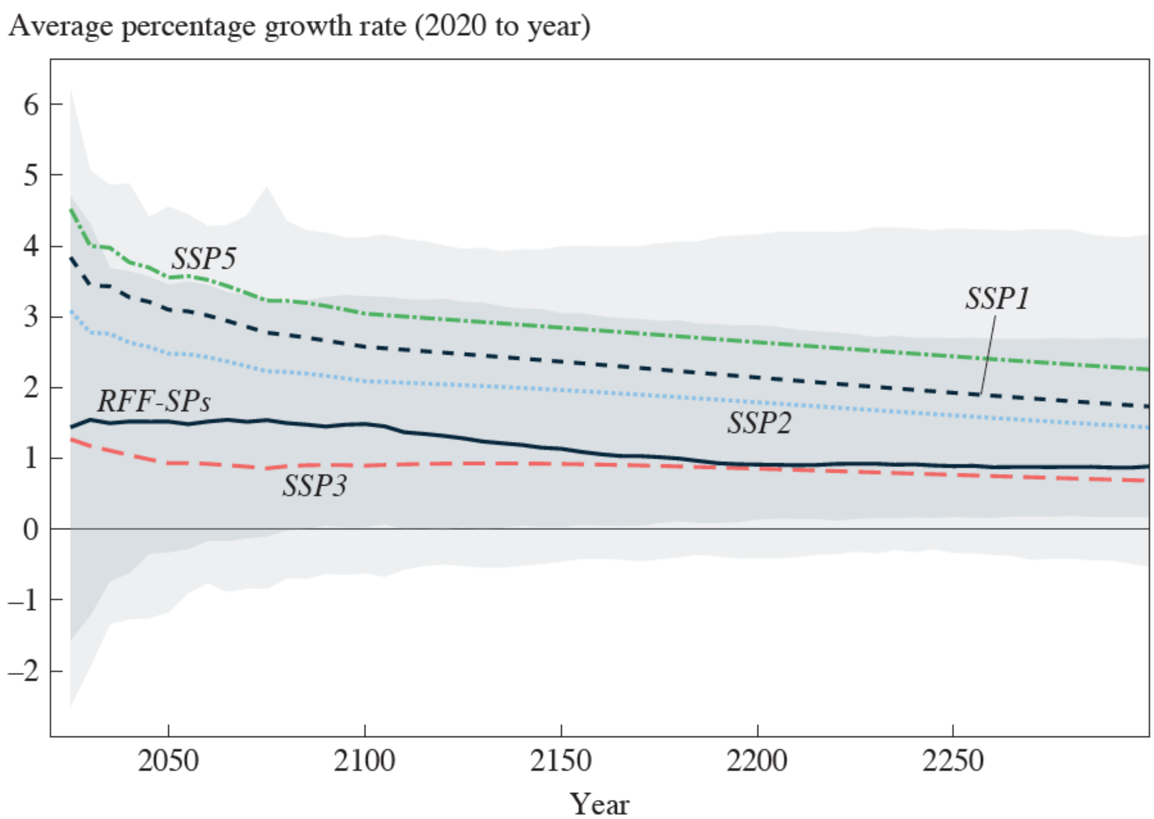


Figure F-2. Distribution of projections of per capita output from Rennert et al.

TABLE 3.—PERCENTILES OF PREDICTIVE DISTRIBUTIONS FOR AVERAGE GROWTH OVER NEXT 50 AND 100 YEARS: POPULATION WEIGHTED AVERAGE OF COUNTRY GROWTH RATES, 2017

	Percentiles: 50-year horizon			Percentiles: 100-year horizon		
	0.17	0.50	0.84	0.17	0.50	0.84
Global factor (f_t)	0.92	1.86	2.70	0.92	1.87	2.72
Global aggregates						
All countries	1.03	2.05	3.00	1.06	2.04	2.96
OECD	0.74	1.69	2.62	0.79	1.73	2.62
Non-OECD	1.05	2.13	3.11	1.11	2.10	3.04

Table F-4. Estimate of the distribution of trend growth (σ_m) from MSW.

Note that the estimates of the uncertainty of the growth rate are reasonably consistent across the different approaches and studies.

Additionally, we look at a simple econometric forecast of future growth rates using the combined IMF and Maddison estimates for 1950 – 2022. We use four different specifications for projecting the global growth rate, g_t. For each, we fit over the 1950 – 2022 period and then forecast using dynamic forecasts through 2200. For example, in equation (*Spec* 1), the estimate of the coefficient is α = 2.15 ± 0.179. If we calculate the forecast errors to 2100, they produce an error in the log level of output of 1.62. Converting this to the standard deviation of the growth rate, this yields 0.019% points per year in Table F-5.

$$(Spec\ 1)\quad g_t = \alpha + \varepsilon_t$$

$$(Spec\ 2)\quad g_t - g_{t-1} = \alpha + \varepsilon_t$$

$$(Spec\ 3)\quad g_t = \alpha + \beta g_{t-1} + \varepsilon_t$$

$$(Spec\ 4)\quad TSLS : g_t = \alpha + \beta g_{t-1} + \varepsilon_t$$

Table F-5 shows the forecast standard errors of the growth rates from the estimates. They are substantially lower than the techniques shown in Table F-2, but these are overly simple specifications compared to the techniques in Muller et al. Also, perhaps the last seven decades were a tranquil period.

From 2020 to	Spec 1	Spec 2	Spec 3	Spec 4
2050	0.051	0.454	0.297	0.297
2100	0.019	0.332	0.216	0.216
2150	0.012	0.299	0.194	0.194
2200	0.008	0.283	0.184	0.184

Table F-5. Standard errors of forecasts using global growth rates, 1950 – 2022, in percentage points per year

Part G. **Dual Discounting**

The macroeconomic structure of the DICE model poses difficulties because it has two alternative discount rates, the first on standard capital and the second on climate investments. Because the discount rate on climate investments is the same as that on standard investments in current DICE model (which we call a one-stage approach), we describe in this section a two-stage investment strategy to determine the difference from the one-stage investment strategy. The concern is that the approach used in the model implicitly adopts the same beta for general and climate investments, despite their empirically different risk properties.

In order to gauge the quantitative importance of this simplification, we run a "dual discounting" version of the model which first optimizes only savings rates with the β parameter in equation (A-6) in Part A set to 1 as is most appropriate for general capital investments. In this first stage, emissions control rates are set at their baseline values. In a second stage run, we then fix savings rates at the optimized level from the first stage and optimize over mitigation rates with $\beta^{clim} = 0.5$ as is appropriate for climate investments. This can be done either with a base or an optimal scenario for calculating the optimal savings rate with β = 1, but the resulting optimal savings rates ratios differ only by a miniscule amount.

Figures G-1 through G-3 show the key results obtained from dual discounting. The optimized control rate, temperature trajectory, and social cost of carbon are slightly different between the one-stage ("benchmark") and the two-stage ("dual discounting") estimates. With dual discounting, the investment rates, emissions-control rates, and output levels are all lower than with the one-stage approach. Because of lower output and emissions, the year 2100 atmospheric temperature is lower with dual discounting: 2.58 °C in the one-stage discounting compared to 2.55 °C in the dual discounting. While the differences between the two approaches are small, the sign of the impact on warming may be surprising.

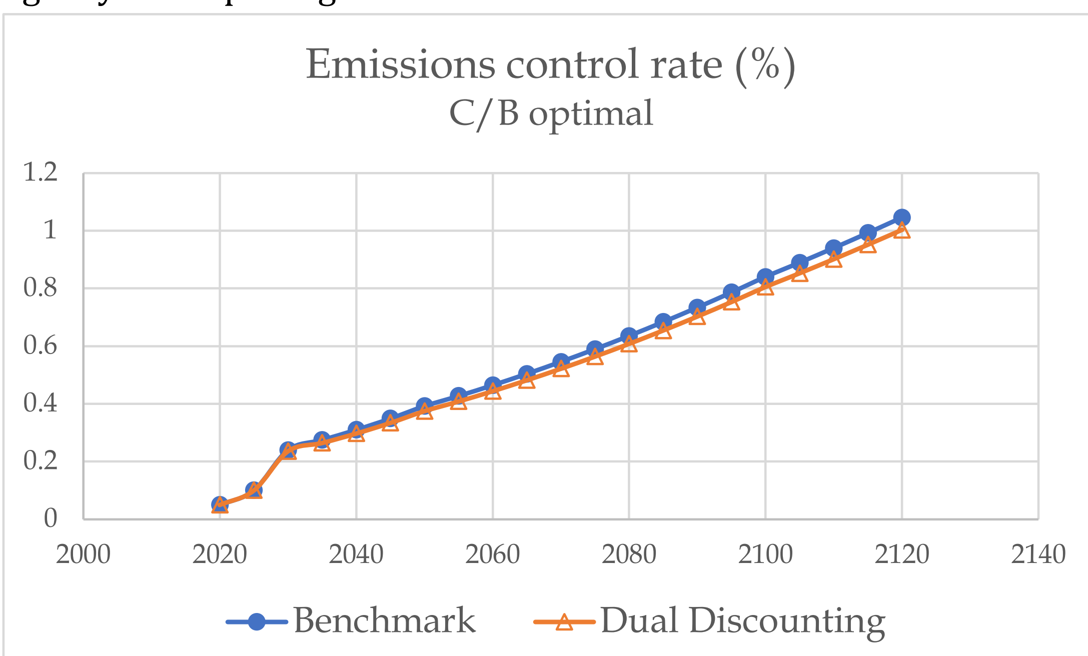


Figure G-1: Emissions control rates in the C/B optimal scenario with the benchmark and dual discounting approaches

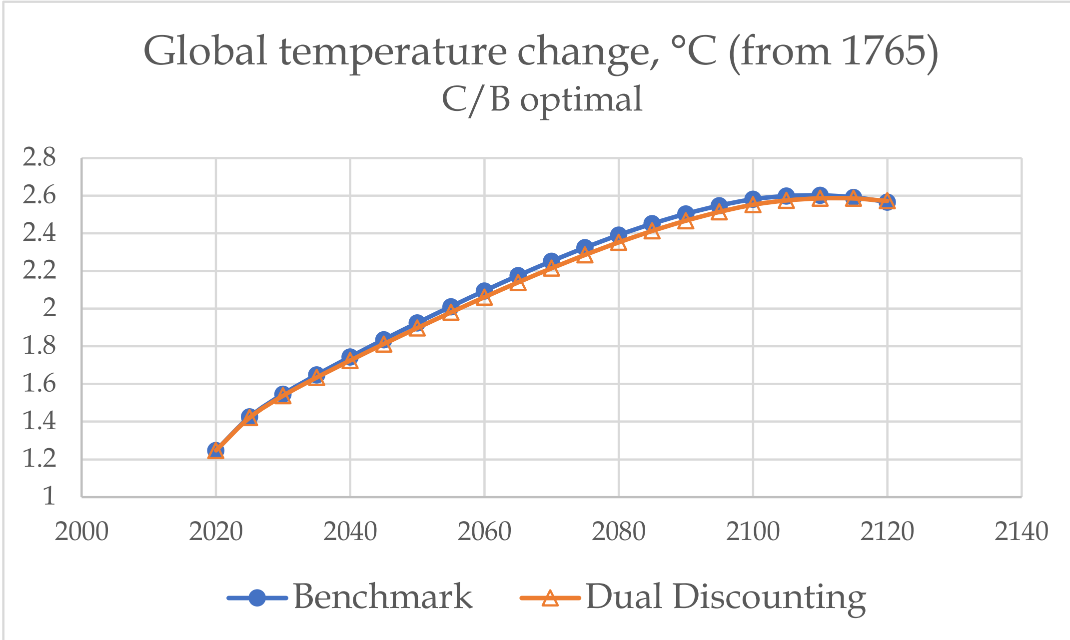


Figure G-2: Global temperature change in the C/B optimal scenario with the benchmark and dual discounting approaches

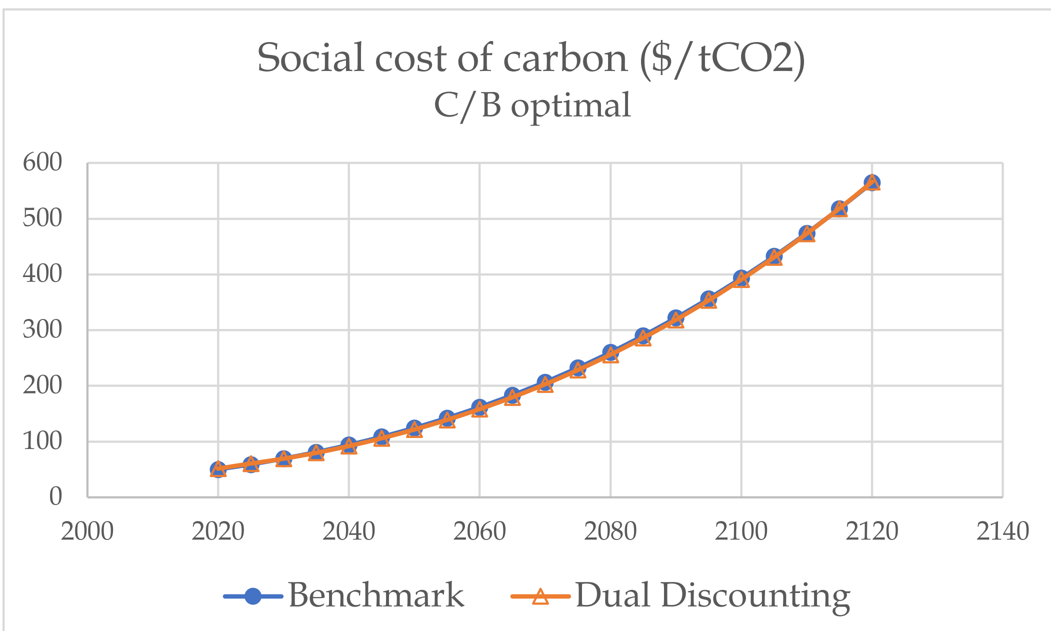


Figure G-3: Social cost of carbon in the C/B optimal scenario with the benchmark and dual discounting approaches

Background Note on Damages (March 12, 2023)

SUMMARY

This note describes the calibration of the damage function for DICE-2023. The DICE-2023 damage function is constructed from three elements: An updated literature-synthesis estimate of global aggregate warming impacts, suggesting a loss of 1.62% GDP-equivalent impact at 3°C warming over pre-industrial temperatures. An adjustment for tipping points based on estimates from Dietz et al. (2021), adding +1% output loss due to 3°C warming. A judgmental adjustment for further omitted impacts and uncertainty of +0.5% loss at 3°C warming. Item #1 updates the synthesis of literature damage estimates from Nordhaus and Moffat (2017). The remainder of this note describes this update.

Adjustments to Nordhaus and Moffat (2017) Data

We use the review of global aggregate climate change impact estimates from Nordhaus and Moffatt (2017, "NM") as a starting point. They estimate a base damage function utilizing 38 impact estimates from 25 studies (see Table 1). NM's preferred specification is a weighted quantile regression with weights based on considerations such as study recency and quality. We first make three adjustments to NM's original data:

We reduce the weight given to Cline's (1992) damage estimate for 10°C warming from 0.1 in NM to 0.0. This change is motivated by the speculative nature of the 10°C estimate especially relative to growing literature and understanding of impacts for lower levels of warming.

We reduce the weight given to Dellink (2012) from 1.0 to 0.25 to reflect the fact that our updated review includes a newer version of the ENV-Linkages model in Dellink et al. (2019).

We correct the temperature level corresponding to the Bosello et al. (2012) estimate from a pre-industrial to a 1920-40 temperature baseline to align with the other estimates (from 1.92°C to 1.52°C).

New Estimates added to Nordhaus and Moffat (2017) Data

We next update the NM data by adding estimates of global aggregate climate change impacts that have been published since the NM review. Piontek et al. (2021) presents a review of competing methodologies and recent studies quantifying aggregate impacts of climate change. We utilize their review as a basis for this update. This approach was chosen because Piontek et al. (2021) exhibits good coverage and alignment with a similar recent review by the IPCC.[8] We note that the inclusion criteria for new estimates in our analysis are thus not formally the same as for the original NM review.

We specifically consider the studies listed in Piontek et al.'s (2021) Supplementary Materials Table (Section 3). Among these, we focus our attention on new bottom-up and top-down aggregate damage assessments. That is, we exclude studies which use prior impact estimates to propose new damage functions but do not themselves produce new climate change impact estimates. We further restrict our attention to studies that are published and provide estimates of aggregate climate change impacts at the global level. We also exclude one study for which we were unable to replicate temperature projections and received no reply to an inquiry with the corresponding author. These restrictions leave us with 11 new studies from which we extract 18 new estimates as shown in Table 1 below.

We note as well that these estimates differ from those in Howard and Sterner (2017, 2022) used as our alternative damage estimates. Our reservations on the estimates in the 2017 study are that it did not weight studies and additionally it included studies that, in our view, were not empirically based. The new study has not published when the current estimates were prepared, and we will review those when completed.

A. Temperature Baseline Harmonization Notes

[8] The IPCC's 6th Assessment Report presents a synthesis of "Global aggregate economic impact estimates by global warming level" in Figure Cross-Working Group Box ECONOMIC.1 (O'Neill et al., 2022). Our updated damage function includes all studies listed in this Figure except for two review-based estimates (Tol 2018 and Howard and Sterner 2017, where the latter is still considered in our analysis in the "alt. damages" scenario) and two studies that are arguably not independent from other prior literature that is included in the damage function (specifically Burke et al. (2018) which builds on Burke et al. (2015) and Rose et al. (2017) which builds on DICE, PAGE, and FUND).

The following are general procedures for our estimates:

We harmonize all impact estimates' corresponding temperatures to a 1920-40 baseline so as to align with the original NM data.

We generally use warming estimates from within papers when available, including for adjustments to alternate baselines. Otherwise, we assume 0.4 °C warming between 1920-40 and pre-industrial, and use an average of GISS, Hadley, and NCDC temperature anomaly series to infer warming between additional periods as needed (0.5 °C warming between 1980-2004 and 1920-40, 0.55 °C warming between 1985-2005 and 1920-40, 0.56 °C warming between 1980-2010 and 1920-40, and 1.05 °C warming from 1920-40 and 2015-19).

We note that the data sources used for temperature adjustments matter: RCP 8.5 warming in 2100 may be considered as 3.91 °C or 4.25 °Cover 1920-40 levels depending on whether one uses IPCC or the three-averaged data series, for example. These differences will affect the estimated damage function as well.

B. Weights

In line with the NM approach, we also consider weights for each of the new estimates based on our evaluation of each estimate's quality, novelty, and independence (see study notes below). We note that there is not one unique way to construct such weights and that one could create reasonable alternative weighting schemes (including more formal adjustments for considerations such as sectoral coverage of each underlying study). While we prefer the weighted regressions, we also show results for unweighted regressions (Table 2). Both weighted and unweighted regressions yield results of the same order of magnitude but estimates for weighted impacts are generally larger.

C. Updated Data

Study	Year	Temp (°C)	Impact (%)	New	Weight
Cline	1992	2.5	-1.1	0	0.9
Cline	1992	10	-6	0	0
Nordhaus a	1994	3	-1.33	0	0
Nordhaus b	1994	3	-3.6	0	0.5
Nordhaus b	1994	6	-10.4	0	0.5
Frankhauser	1995	2.5	-1.4	0	1
Tol	1994	2.5	-1.9	0	0.1
Nordhaus and Yang	1996	2.5	-1.7	0	0.1
Mendelsohn et al.	2000	2.2	0.03	0	0.1
Mendelsohn et al.	2000	2.2	0.07	0	0.1
Mendelsohn et al.	2000	2	0.08	0	0.1
Mendelsohn et al.	2000	3.5	0.01	0	0.1
Nordhaus and Boyer	2000	2.5	-1.5	0	1
Tol	2002	1	2.3	0	0.1
Maddison	2003	3.1	-2.2	0	0.1
Rehdanz and Maddison	2005	1.24	-0.32	0	0.1
Rehdanz and Maddison	2005	0.84	-0.32	0	0.1
Hope	2006	4.085	-3.04	0	0.25
Nordhaus	2006	3	-1.05	0	1
Nordhaus	2008	3	-2.49	0	0.25
Nordhaus	2010	3.4	-2.8	0	0.25
Maddison and Rehdanz	2011	4	-17.8	0	0.1
Bosello et al.	2012	1.52	0.5	0	1
Ronson and Mensbrugghe	2012	3.1	-2.14	0	0.1

Ronson and Mensbrugghe	2012	5.5	-6.05	0	0.1
Dellink	2012	2.5	-1.1	0	0.25
Kemfert	2012	0.25	-0.17	0	0.1
Hambel	2012	1	-0.3	0	0.1
Nordhaus	2013	3	-2.25	0	0
FUND	2015	2	0.2	0	0.3
FUND	2015	3	-0.17	0	0.4
FUND	2015	4	-0.85	0	0.3
WITCH	2015	2	-1.84	0	0.3
WITCH	2015	3	-3.72	0	0.4
WITCH	2015	4	-6.25	0	0.3
PAGE09	2017	2	-0.72	0	0.3
PAGE09	2017	4	-2.9	0	0.4
PAGE09	2017	6	-6.51	0	0.3
Dellink et al.	2019	2.1	-2	1	0.75
Dellink et al.	2019	3.6	-6	1	0.25
Roson and Sartori	2016	3.55	-1.72	1	0.1
Takakura et al.	2019	2.6	-2.24	1	0.33
Takakura et al.	2019	3.6	-4.69	1	0.33
Takakura et al.	2019	1.6	-1.02	1	0.33
Kompas et al.	2018	3.55	-3	1	0.2
Kompas et al.	2018	4.55	-7.24	1	0.2
Kompas et al.	2018	2.55	-1.77	1	0.2
Zhao et al.	2019	3.1	-2.42	1	0
Letta and Tol	2018	4.412	-1.88	1	0.5
Burke et al.	2015	4.86	-23	1	0.5
Newell et al.	2021	3.91	-2.53	1	1
Pretis et al.	2018	1.6	-13	1	0.35
Pretis et al.	2018	1.1	-8	1	0.15
Kahn et al.	2021	3.91	-7.64	1	1
Kalkuhl and Wenz	2020	4.25	-7.4	1	0.05
Kalkuhl and Wenz	2020	4.25	-13.4	1	0.05

Color Key:

	Adjusted NM data point
	New bottom-up studies
	New top-down studies

Table 1: Updated data of global climate change impact estimates. “Temp” refers to °C warming over 1920-40 levels. “Impact” refers to estimated global GDP impact in percent. “New” is an indicator equal to 0 for studies already featured in NM and equal to 1 for new studies.

D. Estimation Results

Table 1 summarizes predicted impacts at 3°C and 6°C warming over 1920-40 levels for different data samples and regression specifications (OLS or median, denoted "Qtile"). All estimates are based on a simple quadratic function in temperature (without a constant). We present results for different data samples that exclude damage estimates above certain temperature levels (e.g., “T limit T<10°C” implies that only estimates for less than 10 °C warming are included). The updated preferred estimate – using weighted quantile regression - is highlighted in bold. It suggests around 30% larger impacts (-2.16%) than the corresponding estimate in NM (-1.63%).

			Predicted Impacts at:	
Method	Weights?	T limit	3°C over 1920-40	6°C over 1920-40
OLS	-	-	-1.71%	-6.84%
OLS	-	T<10°C	-2.94%	-11.74%
OLS	-	T<5°C	-3.46%	-13.84%
OLS	Yes	-	-2.94%	-11.74%
OLS	Yes	T<5°C	-3.35%	-13.38%
Qtile	-	-	-1.80%	-7.20%
Qtile	Yes	-	-2.16%	-8.63%

Table 2: Predicted impacts based on updated regression results.

E. Detailed Study Notes

This section describes how impact estimates were extracted from each of the studies added to the NM data, as well as brief notes on weights.

Dellink et al. (2019)

Climate impact projections:

Headline: 2.5 °C warming by 2060 induces average global GDP loss of 2% (by 2060). We infer that the temperature baseline must be preindustrial based on the paper's use of the MAGICC Model and the fact that, in Figure 3, warming levels appear to be 1 °C at 2015.

The paper also makes a damage prediction post 2060. We infer a 6% loss at 4 °C based on the "central projection - full damages" line in Figure 12. Global GDP losses of 2% for 2.1 °C and 6% for 3.6 °C increases over 1920-40 temperature levels.

Weights: 1 total (0.75 on 2.1 °C estimate and 0.25 on 3.6 °C estimate)

Notes: Solid methodology. Put higher weight on the 2.1 °C estimate since the ENV-Linkages model is more detailed and there is more uncertainty about the extrapolation to the AD-DICE model used for the 3.6 °C projection.

Takakura et al. (2019)

Climate impact projections:

Study considers impacts across 4 RCP scenarios x 5 climate models x 5 SSP scenarios.

Temperature baseline is pre-industrial as per SI Figure 2.

We use the paper's Supplementary Information to calculate averages of global impact estimates across years and scenarios for three temperatures of interest: 2 °C warming over preindustrial (by averaging across years and scenarios with warming between 1.98 °C <T<2.02 °C), 3 °C warming over preindustrial (averaging across observations with 2.98 °C <T<3.02 °C), and 4 °C warming over preindustrial (averaging across observations 3.98 °C <T<4.02 °C). We note that we compute aggregate impacts by aggregating the sectoral impacts at the "World" level.

Global GDP losses of 1.02% for 1.6 °C , 2.24% for 2.6 °C , and 4.69% for 3.6 °C over 1920-40 levels.

Weights: 1 total (0.33 on each temperature estimate)

Notes: Independent estimate.

Roson and Sartori (2016)

Climate impact projections:

Paper presents country-level GDP impact estimates for a +3.0 °C increase in average temperature.

The temperature baseline is 1985-2005 for sea level rise, 1980-2004 for agriculture method 2, and not explicitly stated for some of the other impacts, although several infer warming damages relative to current average temperatures. We thus assume a 1985-2005 baseline overall.

No global impact estimate is provided. We constructed one using the country level GDP impact estimates for 3 °C warming presented in Appendix Table A1.1 using 2019 GDP weights (World Bank, PPP, constant 2017 dollars). Note: For countries with missing estimates, we adopt the relevant regional "Rest of Region" estimate (e.g., "Rest of Oceania" for Vanuatu, etc.)

Global GDP loss of 1.72% for 3.55 °C over 1920-40.

Weight: 0.1

Notes: Questions over methodology especially in going from damage functions to GDP impacts.

Zhao et al. (2020)

Climate impact projections:

Damages are reported as the percentage of cumulative discounted climate damages relative to cumulative discounted Gross World Product from 2011-2100. For their "Business as usual" scenario, this percentage is 2.42%. We are inferring associated temperature increase based on Figure 5 as 3.5 °C .

The temperature baseline seems to be preindustrial based on the definition of T variable in equation (3).

Global GDP loss of 2.42% for 3.1 °C warming over 1920-40.

Weight: 0

Notes: Very close to FUND & methodological concerns such as over addition of earthquake and volcano damages in climate impacts.

Kompas et al. (2018)

Climate impact projections:

Project global GDP losses of 3% for 3 °C warming (stated in text).

Temperature baseline is 1985-2005 (confirmed by author in correspondence).

Table A1 gives projected losses in dollars for 4 °C and 2 °C . We calculate the corresponding GDP percentages by backing out the assumed global GDP in 2100 (319.79 trillion). This yields predicted losses of 1.77% for 2 °C and 7.24% for 4 °C over the presumed 1985-2005 baseline.

Global GDP loss of 1.77% for 2.55 °C , 3% for 3.55 °C , and 7.24% for 4.55 °C over 1920-40.

Weight: 0.6 (divided evenly over each estimate)

Notes: Improved methods over but lack of independence from Roson and Sartori (2016).

Kalkuhl and Wenz (2018)

Climate impact projections:

Focus on GRP-weighted aggregate results (Table 7).

Climate impacts are calculated relative to no additional warming beyond 2015-19 base period.

Assume RCP 8.5. The implied average temperature increase from 2015-2019 and 2095-99 is 3.2 °C (population-weighted).

- Cross-sectional estimates imply 7.4% global GRP loss in 2099.
- Panel estimates imply 13.4% global GRP loss in 2099.

Global GDP losses of 7.4% or 13.4% from 3.2 °C +1.05 °C = 4.25 °C over 1920-40 levels.

Weight: 0.1 (divided evenly over each estimate)

We were unable to replicate the data on real output using data for several countries. We are concerned about the study because the damage estimates appear to be based on nominal rather than real output. They will include differences in price levels in the estimates, which is non-standard in damage estimates. This approach was confirmed with the authors.

Kahn et al. (2021)

Climate impact projections:

Their benchmark estimates measure damages relative to a baseline scenario under which temperature in each country continues to increase according to its historical trend from 1960-2014. This is not ideal for our purposes.

For one scenario, the authors also estimate damages relative to a "no warming" beyond 2015 baseline. We use this figure as it is closest to what we want to measure.

The results suggest a global GDP-weighted average income loss of 7.64% in RCP 8.5 by 2100.

As a corresponding temperature increase, we utilize the mean global surface temperature change from RCP 8.5 across models as reported in IPCC (2014), which is 3.7 °C +0.61 °C = 4.31 °C over 1850-1900 levels.

Global GDP losses of 7.64% from 4.31 °C -0.4 °C =3.91 °C over 1920-40.

Weight: 1

Notes: This study has high weight because it uses a different methodology from most other top-down estimates.

<u>Burke et al. (2015)</u>

Climate impact projections:

Headline result is 23% global output loss in 2100 in RCP 8.5 scenario resulting in (population-weighted) average global average temperature change in 2100 of 4.3 °C .

Base period is 1980-2010.

Global GDP losses of 23% for 4.3 °C +0.56 °C = 4.86 °C warming over 1920-40 levels.

Weight: 0.5

Notes: Partial superseded by Newell et al. (2021) and methodological similarities with other top-down estimates.

<u>Pretis et al. (2018)</u>

Climate impact projections:

Compare, relative to a baseline of "no additional warming" over 2006-15 levels, 0.6 °C additional warming or 1.5 over preindustrial, and 1.1 °C additional warming or 2 °C over preindustrial.

Headline numbers:

Median projected global GDP per capita is 8% lower in 2100 for 1.5 °C (imprecisely estimated).

Median projected global GDP per capita is 13% lower in 2100 for 2 °C .

Global GDP loss of 8% for 1.5 °C -0.4 °C = 1.1 °C and 13% for 2 °C -0.4C = 1.6 °C over 1920-40 levels.

Weight: 0.5 (0.35 on 1.6 °C estimate and 0.15 on 1.1 °C estimate)

Notes: Methodological similarities to other top-down estimates. Higher weight on 1.6 °C estimate due to higher statistical precision.

<u>Letta and Tol (2019)</u>

Climate impact projections:

Consider impacts for RCP 8.5 resulting in sample average warming of 3.912 °C over reference period of 1980-2004.

We infer aggregate impacts based on the regression results for the full sample (Table 2 Column 1) and calculate average cumulative impacts, which would suggest 1.88% lower TFP levels by 2095.

Global GDP loss of 1.88% for 3.912 °C +0.5 °C = 4.412 °C over 1920-40.

Weight: 0.5

Notes: Methodological similarities to other top-down estimates.

<u>Newell et al. (2021)</u>

Climate impact projections:

We calculate the mean of the distribution of predicted climate impacts across the bootstrapped "levels effects" models in the MCSs (Figure 6 bottom panel). The authors kindly shared their data for us to be able to perform this calculation.

We focus on the "levels effects" since they are precisely estimated.

The results imply a GDP loss of 2.53% in RCP 8.5 year 2100, which they note to have an associated CMIP5 average warming level of 4.31 °C over pre-industrial.

Global GDP loss of 2.53% from 4.31 °C -0.4C = 3.9 °C over 1920-40 levels.

Weight: 1

Notes: Comprehensive analysis nesting hundreds of potential top-down specifications including those of other studies.

Background Note on DFAIR (October 26, 2023)

I. Summary

This note describes the calibration of the Finite Amplitude Impulse-Response (FAIR) model for DICE-2022, labeled “DFAIR.” Part I is the calibration to climate-carbon models. Part II is harmonization with historical data. The appendix shows details from the calculations.

The DFAIR carbon-cycle model differs from the earth-science literature in using a five year step. Otherwise, it is essentially equivalent to the Millar et al. (2017) implementation and tracks closely the adaptation of Dietz et al. (2021). For the climate module, it uses the calibration of Millar et al. (2017) with a simple two-box climate model. We have tested alternative parameterizations. They perform differently for different tests, and we have found the Millar et al. (2017) specification sufficiently close to adopt that for the DFAIR model.

The key findings are the following:

First, the Millar et al. (2017) specification of FAIR tracks closely both carbon cycle models and the historical data on emissions and concentrations. The DFAIR model tends to underestimate concentrations relative to the Joos et al. (2013) multi-model simulations. Additionally, it tends to underpredict concentrations slightly in both the carbon cycle models and for the historical data.

Second, for large pulses (the 5000 GtC pulse estimate by Joos), all versions of the FAIR model behave poorly in the short run (up to 100 years), but they are reasonably close for the longer run.

Third, looking at the two major simulations (100GtC and 5000 GtC), the FAIR model is successful in capturing the saturation in carbon absorption that results in higher atmospheric retention in the large compared to the small pulse. This is a notable advance over the linear carbon models used in earlier vintages of the DICE and most other IAMs.

The calibration of initial conditions for FAIR is difficult because it is path-dependent. We have created a “1765 model” of the DFAIR GAMS version that runs from 1765 to 2020. Using this, we made a successful splice with the 2020 initial conditions based on the 1765 model.

Fourth, the climate model does a reasonable job of capturing the dynamics and does not need structural revisions from earlier versions of DICE. However, the DFAIR model uses the equation structure of the Millar version, which makes interpretation easier.

Finally, it should be emphasized that the carbon cycle models underlying the multi-model calibrations in Joos et al. (2003) are quite divergent. For example, the 5% - 95% confidence interval for the 100-year time integrated impulse response function for concentrations is 30 – 75 years.

The major open issue is whether the FAIR model's parameters should be adjusted to reflect its shortcomings. This is a larger project and awaits further research.

II. Calibration to climate/carbon cycle models.

The DFAIR model uses the structure derived by Millar et al. (2017), with five year steps. It takes all parameters from the original Millar estimates. The only difference is that the DFAIR model has an equilibrium temperature sensitivity (ETS) of 3.0 °C per CO2 doubling.

We then examined three estimates for the carbon cycle and one test for the climate model. Two of the calibrations were to Joos et al. (2013) multimodel study. Note that according to AR6: "Although there has been greater understanding since AR5 of the carbon cycle responses to CO2 emissions 27 (Chapter 5, Sections 5.4 and 5.5), there has been no new quantification of the response of the carbon-cycle to an instantaneous pulse of CO2 emission since Joos et al. (2013)." Note as well that the models underlying Joos appear to have a neutral biosphere, so that feature misses any interaction with increased carbon uptake in that sector. However, Millar appears to capture land uptake, but the impact of this uptake on parameters is unclear.

A. <u>Carbon cycle</u>

The testing of alternative approaches to the carbon cycle used three pulse tests (100 GtC from pre-industrial conditions, 5000 GtC from pre-industrial conditions, and 100 GtC from 2010 levels) and one run that tested the accuracy of matching historical data on CO2 emissions and concentrations.

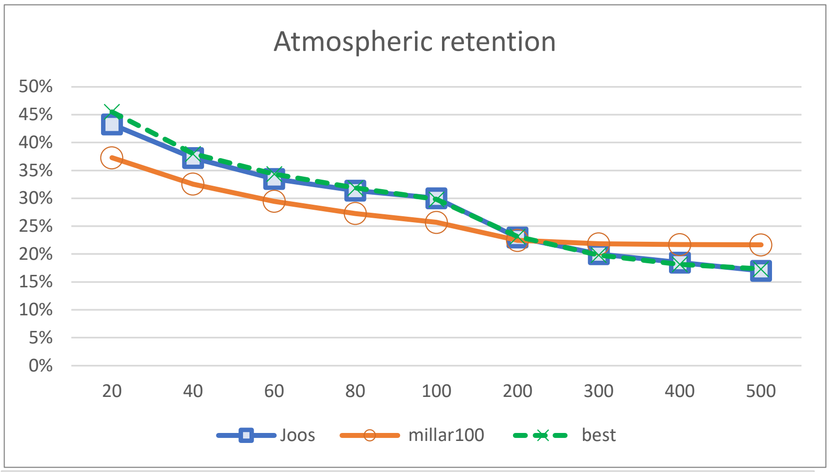
i. Pulse tests for 100 GtC from pre-industrial conditions

We began with two pulses from pre-industrial concentrations and compared with Joos et al. (2013). We used the Millar specification of the FAIR model, DFAIR, with five year steps for the comparisons. These tests were not precise for Joos because of the divergence among models. One pulse was 100GtC from pre-industrial conditions (PIC) and the second was 5000GtC from PIC. We also tested alternative parameters to see which would be a better fit (labeled "Best").

Figure 1 shows the key results for the atmospheric retention. The Millar et al. (2017) parameters have lower atmospheric retention than Joos et al. (2013) or the other specifications, particularly at short horizons. The "best" parameters match very closely.

Figure 2 shows the temperature response to the same pulse. The temperature results are reversed, with Millar et al. (2017) having lower temperature than Joos et al. (2013) even though the atmospheric concentrations are lower. This result comes from the difference in the climate models in Joos et al. (2013) and the FAIR 2-equation climate model. There are two versions of Millar. One uses an ETS of 3.0 °C, while the other uses a lower value of 2.75 °C. Both are low relative to the Joos models.

While the temperature models relative to Joos are divergent, the DFAIR version of the climate model matches the IPCC AR6 climate estimates exactly, so this suggests that the problem is the Joos climate models rather than the DFAIR climate model.



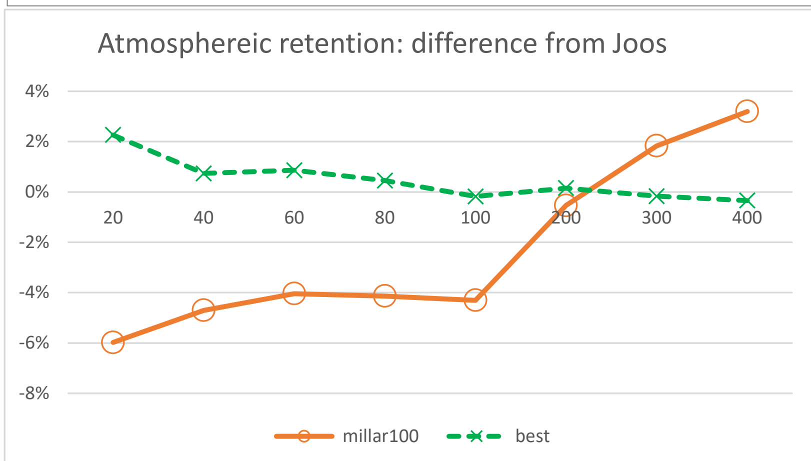


Figure **DFAIR-**1. Atmospheric retention, alternative specifications, pulse of 100 GtC from PIC (years).

(Source: Tests=FAIR-sept0122.xls)

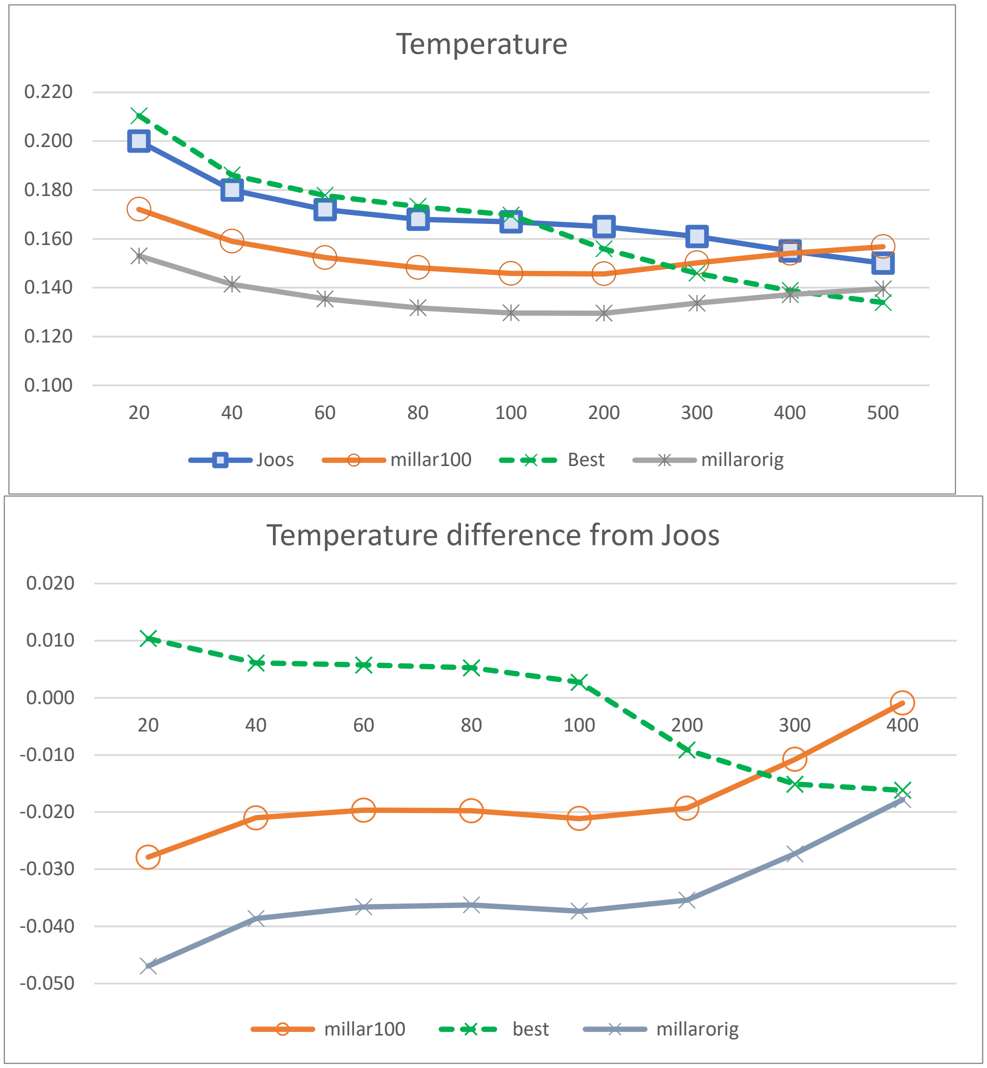


Figure **DFAIR-**2. Results of 100GtC pulse on temperature at different time horizons (years).

(Source: Tests=FAIR-sept0122.xls)

ii. Pulse test for 5000 GtC from pre-industrial conditions

We performed identical runs for a very large pulse, 5000 GtC. For reference, cumulative emissions to date are approximately 2000 GtC by 2100 in the base run and 1400 GtC in the optimal run. The point of this test is to see how well FAIR reproduces the saturation of larger models.

Figure 3 shows the results for the atmospheric retention. FAIR does reasonably well from about 60 years forward, although it tends to have lower convexity than the full earth-system models in Joos et al. (2013). This non-convexity seems to hold for all model parameters, and FAIR therefore tends to understate concentrations for the first half-century (as was the case for the 100 GtC pulse as well). Figure 4 shows the results for temperature (using the higher ETS). The simple climate model fails to capture the shape of the full models after 100 years. The reason for the difference is unclear and is probably not due to the error in the atmospheric retention.

However, the most important result is that the model definitely captures the saturation with high pulses. Note, comparing figures 1 and 3, that the atmospheric retention rises from 30% with the small shock to 70% with the large shock at 100 years. Linear carbon cycle models, such as those used in earlier DICE models and other studies, would have identical percentages.

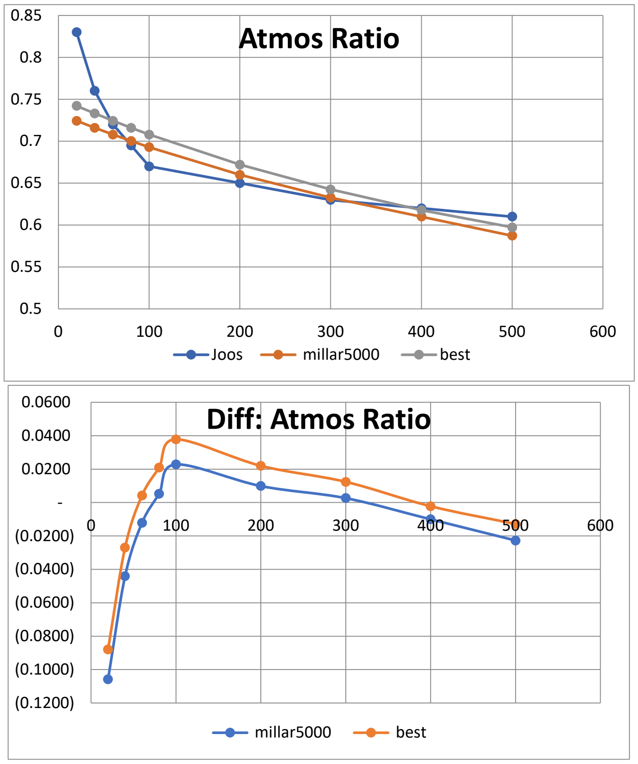


Figure **DFAIR-**3. Atmospheric retention of CO2 for 5000 GtC pulse

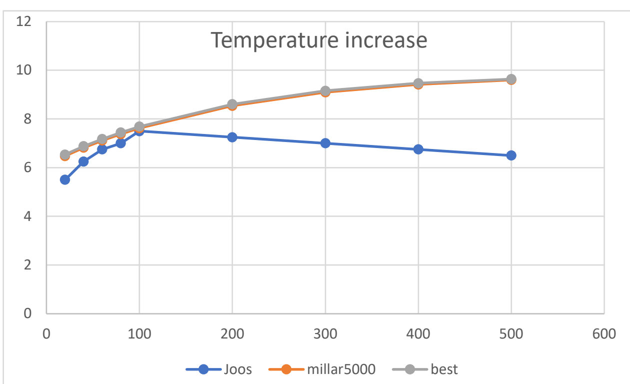


Figure **DFAIR-**4. Temperature increase (°C) for 5000 GtC pulse

iii. *<u>100 GtC from 2010 levels (389 ppm)</u>*

The third pulse test considered 100 GtC again but against background concentrations which are otherwise held constant at 2010 levels assumed to be 389 ppm (829.42 GtC), in line with both Joos et al. (2013) and the key results reported also in Dietz et al. (2021). For this test, we modify the finel

DFAIR model from DICE-2023 based on the experimental conditions of Joos et al. and Dietz et al. In particular, we (i) follow Dietz et al. in distributing the corresponding initial excess atmospheric CO2 concentrations over pre-industrial levels (829.42 – 588 = 241.4 GtC) into the four carbon reservoirs according to 52.9% in box 1; 34.3% in box 2; 11.1% in box 3; 1.6% in box 4; (ii) assuming 0.85 ∘ C as initial atmospheric warming relative to pre-industrial and lower ocean warming of 0.22°C. (by setting initial temperature in Box 1 to 0.22°C and in Box 2 to 0.63°C so that TATM0 = 0.85°C); (iii) assuming 531 GtC as initial cumulative emissions as in Dietz et al.; (iv) adopting the baseline emissions scenario from Dietz et al. to keep concentrations absent the pulse approximately constant (specifically by adding up annual emissions underlying Dietz et al. into 5-year totals and taking the average per period as value for ECO2); (v) assuming zero non-CO2 forcings as in Dietz et al., noting that experimentation with assuming initial year non-CO2 forcings to be constant at initial year levels had only a minimal impact (+0.001-0.002°C) on the estimated temperature response. One notable difference from Dietz et al. is that we do not assume an equilibrium climate sensitivity of 3.1°C (which they impose across models) but retain the DFAIR benchmark value of 3.0°C in line with IPCC AR6.

The results of the 100 GtC pulse test in this environment are shown in Appendix Figure D-1. The results indicate that the DFAIR model addresses the critique of Dietz et al. (2021) and others that prior vintages of DICE exhibited excessive warming inertia in response to emissions impulses compared to recent climate model estimates as shown in Joos et al. (2013). It does, however, again also indicate that DFAIR initially underpredicts warming levels slightlyl in the near term, and overpredicts slightly in the longer run.

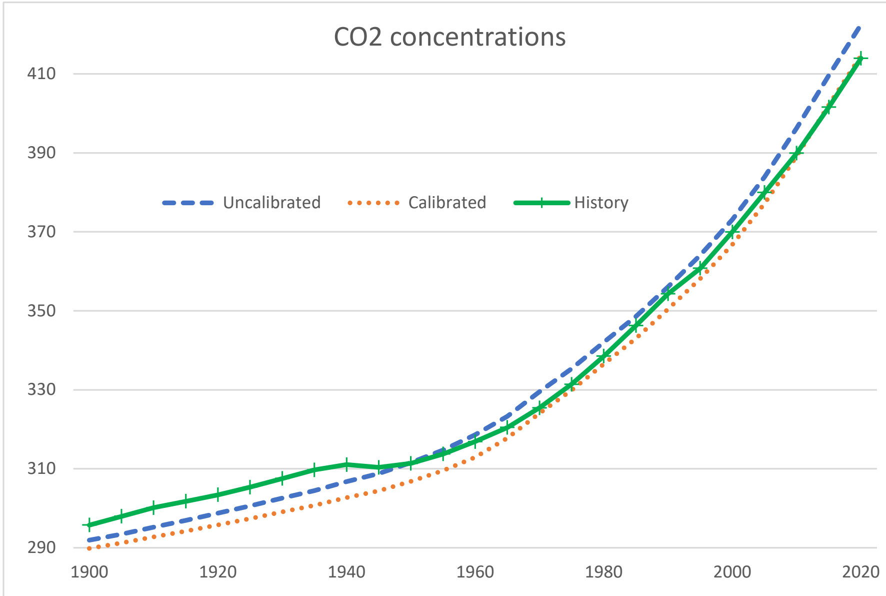
iv. *Comparison with history*

The final comparison is to use FAIR to project concentrations from 1765 to 2020. For this purpose, we used actual CO2 emissions as best coult be reconstructed from IPCC, CDIAC, and EDGAR. We then constrained both CO2 emissions and non-CO2 forcings using the data from AR6. The emissions were from Table 5.1, while the non-CO2 forcings were from Table AIII.3, and were interpolated. We then ran the model from pre-industrial concentrations starting in 1765.

This specification overpredicted temperature signficantly in 2020 (1.515 °C v. 1.25 °C from IPCC). Additionally, it overpredicted ppm but only slightly (422.5 v 417.1 ppm).

We then created a "1765 run" to match CO2 concentrations. The 1765 run adjusted both emissions and non-CO2 forcings to better track history. This run slightly underpredicted 2020 ppm (414.7 v 417.1 ppm actual) but matched tempeature (1.248). We then used the 2020 values here for calibration of the DICE-2020 model. The following shows the assumptions and results for the different scenarios and history.

		1900	1950	2000	2020
CO2 emissions (GtCO2)					
	Uncalibrated	6.2	11.2	30.0	40.5
	Calibrated	5.4	9.9	30.0	40.5
	IPCC total	3.5	7.6	27.4	36.6
	DICE	2.0	6.2	25.7	37.7
Non-CO2 forcings (W/m2)					
	Uncalibrated	0.00	0.06	0.46	0.68
	Calibrated	-0.10	-0.04	0.06	0.28
	History	-0.01	-0.06	0.46	0.68
Concentrations (ppm)					
	Uncalibrated	291.9	311.6	373.1	422.5
	Calibrated	289.8	306.8	366.7	414.7
	History	295.7	311.3	370.0	414.0
Temperature (from 1765)					
	Uncalibrated	0.19	0.39	1.05	1.52
	Calibrated	0.11	0.29	0.80	1.25
	History	0.13	0.30	0.87	1.25



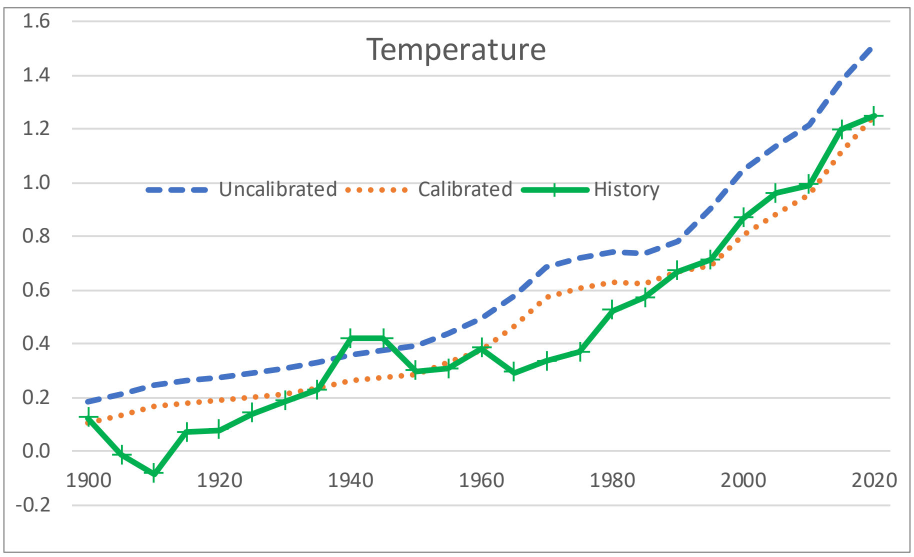


Table DFAIR-1 and Figure DFAIR-5. Comparison of FAIR model variants with history and two scenarios for emissions and forcings

Source: DICE2022A-base-3-11-1765-Runs.xlsx.

B. Climate Module

The FAIR climate module is analytically similar to that in DICE-2016. It is a two-equation model with five parameters (see appendix for the equations). We have adopted four of the five, which represent the dynamics, but have changed the temperature sensitivity from 2.75 °C to 3.0 °C.

We then ran a standard test for the dynamics of the model using stylized CO2 concentrations. This involved 1% concentration growth from pre-industrial concentrations to doubling and then constant concentrations after that. Table 2 shows the transient and equilibrium temperature sensitivity (transient is average around 70 after the start of the growth). The DFAIR does fine here.

	DFAIR	IPCC-AR6
TSC	1.80	1.80
ESC	3.00	3.00

TSC = transitory sensitivity coefficient = temperature increase at 60-80 years
ESC = equilibrium sensitivity coefficient
Units are degrees C from pre-industrial conditions

Table DFAIR-2. Behavior of DFAIR climate module compared to Sixth IPCC Assessment report

[Source: ESC-table-092522u122822.xls]

C. Calibration of history with projections.

The final step is to calculate the initial conditions. (These are listed at the end of this section.) These are the initial stocks in the four carbon reservoirs, cumulative carbon emissions, and the initial temperatures in the two boxes.

To estimate these, we used the “1765 model.” This starts in equilibrium in 1765, with all initial stocks in equilibrium. It then puts historical CO2 emissions into the DFAIR model starting in 1765. The simulation uses the calibrated version discussed above.

We then calibrated the DICE model (starting in 2020) with the 1765 model and compared the outputs over the splice period from 2020 to 2035. [update using Millar (adj)]. Table 3 compares the outputs of 1765 model and 2020+ model and shows the splice is essentially perfect.

Period	1	2	3	1
Year	2020	2025	2030	2035
Total CO2 Emissions, GTCO:	0.08%	0.48%	1.69%	2.42%
Atmospheric concentration	0.00%	0.02%	0.10%	0.19%
Atmospheric temperaturer (	0.00%	0.03%	0.12%	0.26%
Total forcings w/m2	7.72%	0.05%	0.17%	0.34%
Forcings, exogenous w/m2				
CO2 forcings w/m2	0.00%	0.06%	0.20%	0.38%
Total CO2 Emissions, GTCO:	0.08%	0.48%	1.69%	2.42%
Permanent C box	0.00%	0.04%	0.15%	0.29%
Slow C box	0.00%	0.06%	0.21%	0.41%
Medium C box	0.00%	0.15%	0.57%	1.06%
Fast C box	0.00%	0.42%	1.49%	2.34%
Temp Box 1	0.00%	0.01%	0.03%	0.06%
Temp Box 2	0.00%	0.03%	0.13%	0.28%
Alpha	0.00%	-0.02%	-0.03%	0.06%
IFR	0.00%	0.00%	-0.01%	0.01%
cacc	0.00%	-0.05%	-0.14%	-0.14%
ccatot	0.00%	0.01%	0.04%	0.15%

Table DFAIR-3. Quality of splice between 1765 model and DICE model.

Percentage difference between the values of 1765 model and DICE-2022 for overlap of splice.

[Source: compare-1765-dice2022-u122822.xls]

The following are the initial conditions for the calibrated version (note that the high resolution figures are from the output of the 1765 model). One important note is that the temperature change is from 1765 and therefore does not always match either the standard calculations from preindustrial or those in damage studies.

```
** INITIAL CONDITIONS TO BE CALIBRATED TO HISTORY
** CALIBRATION
     mat0   Initial concentration in atmosphere in 2020 (GtC)     /886.5128014/

     res00  Initial concentration in Reservoir 0 in 2020 (GtC)     /150.093 /
     res10  Initial concentration in Reservoir 1 in 2020 (GtC)     /102.698 /
     res20  Initial concentration in Reservoir 2 in 2020 (GtC)     /39.534  /
     res30  Initial concentration in Reservoir 3 in 2020 (GtC)     / 6.1865 /

     mateq      Equilibrium concentration atmosphere  (GtC)          /588   /
     tbox10    Initial temperature box 1 change in 2020 (C from 1765)  /0.1477
/
     tbox20    Initial temperature box 2 change in 2020 (C from 1765)
/1.099454/
     tatm0     Initial atmospheric temperature change in 2020        /1.24715 /
```

Appendix DFAIR-1. Parameters of different models.

The basic parameters from Millar et al. (2017) are reproduced in Table A-1.

Table 1. Default parameter values for the simple impulse-response climate–carbon-cycle models used in this paper.

Parameter	Value – AR5-IR	Value – PI-IR	Value – FAIR	Guiding analogues
a_0	0.2173	0.1545	0.2173	Geological re-absorption
a_1	0.2240	0.1924	0.2240	Deep ocean invasion/equilibration
a_2	0.2824	0.2424	0.2824	Biospheric uptake/ocean thermocline invasion
a_3	0.2763	0.4108	0.2763	Rapid biospheric uptake/ocean mixed-layer invasion
τ_0 (year)	1×10^6	1×10^6	1×10^6	Geological re-absorption
τ_1 (year)	394.4	276.7	394.4	Deep ocean invasion/equilibration
τ_2 (year)	36.54	30.75	36.54	Biospheric uptake/ocean thermocline invasion
τ_3 (year)	4.304	4.459	4.304	Rapid biospheric uptake/ocean mixed-layer invasion
q_1 ($\mathrm{KW^{-1}\,m^2}$)	0.33	0.33	0.33	Thermal equilibration of deep ocean
q_2 ($\mathrm{KW^{-1}\,m^2}$)	0.41	0.41	0.41	Thermal adjustment of upper ocean
d_1 (year)	239.0	239.0	239.0	Thermal equilibration of deep ocean
d_2 (year)	4.1	4.1	4.1	Thermal adjustment of upper ocean
r_0 (year)	–	–	32.40	Preindustrial $\mathrm{iIRF_{100}}$
r_C (year $\mathrm{GtC^{-1}}$)	–	–	0.019	Increase in $\mathrm{iIRF_{100}}$ with cumulative carbon uptake
r_T (year $\mathrm{K^{-1}}$)	–	–	4.165	Increase in $\mathrm{iIRF_{100}}$ with warming

Table DFAIR-A-1. Parameters in Millar version of FAIR model.

(Source: Millar et al. (2017) , Table 1)

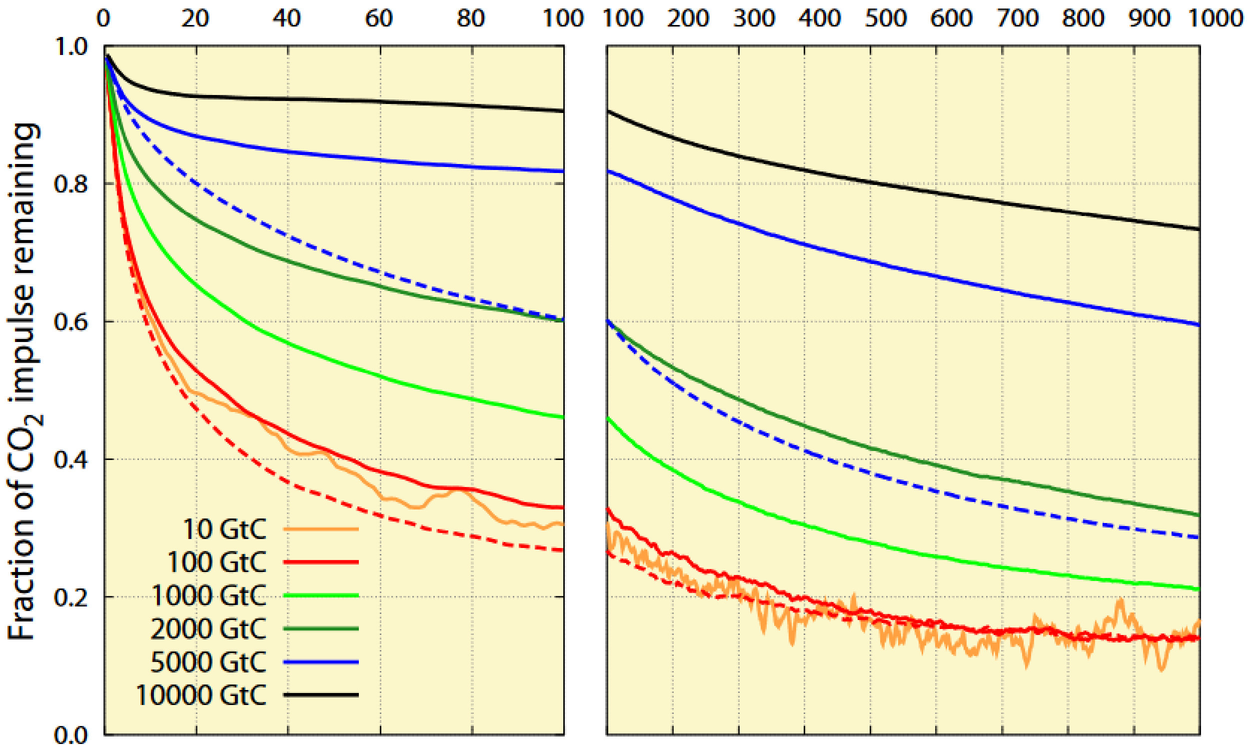
Appendix DFAIR-2. Parameters of Millar Model and Alternatives

Parameters	Millar (orig)	Dietz	DFAIR
emshare0	0.2173	0.2173	0.2173
emshare1	0.2240	0.2240	0.2240
emshare2	0.2824	0.2824	0.2824
emshare3	0.2763	0.2763	0.2763
tau0	1,000,000	1,000,000	1,000,000
tau1	394.4000	394.4000	394.4000
tau2	36.5300	36.5300	36.5300
tau3	4.3040	4.3040	4.3040
teq1	0.3300	0.3300	0.3240
teq2	0.4100	0.4100	0.4400
d1	239.0000	239.0000	236.0000
d2	4.1000	4.1000	4.0700
IRF0	32.4000	34.4000	32.4000
irC	0.0190	0.0190	0.0190
irT	4.1650	4.1650	4.1650
fco22x	3.7400	4.2000	3.9300
mat0	588.0000	588.0000	588.0000

Variable	Units	Definition
emshare0	Fraction	Geological re-absorption
emshare1	Fraction	Deep ocean invasion/equilibration
emshare2	Fraction	Biospheric uptake/ocean thermocline invasion
emshare3	Fraction	Rapid biospheric uptake/ocean mixed-layer invasion
tau0	Year	Geological re-absorption
tau1	Year	Deep ocean invasion/equilibration
tau2	Year	Biospheric uptake/ocean thermocline invasion
tau3	Year	Rapid biospheric uptake/ocean mixed-layer invasion
teq1	$KW^{-1}m2$	Thermal equilibration of deep ocean
teq2	KW-1m2	Thermal adjustment of upper ocean
d1	Year	Thermal equilibration of deep ocean
d2	Year	Thermal adjustment of upper ocean
IRF0	Year	Preindustrial iIRF100
irC	$YearGtC^{-1}$	Increase in iIRF100 with cumulative carbon uptake
irT	$YearK^{-1}$	Increase in iIRF100 with warming
fco22x	$KW^{-1}m2$	Forcings for CO2 doubling
mat0	GtC	Initial carbon stock

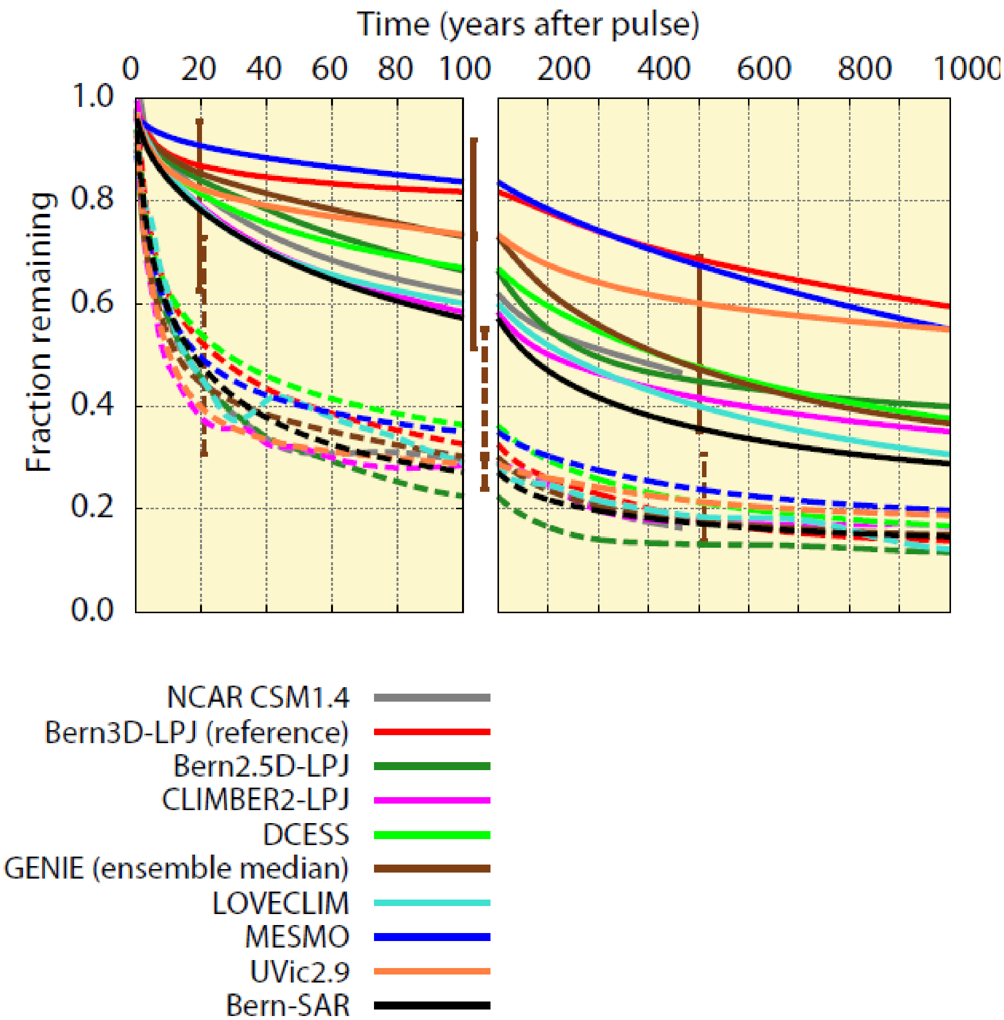
Appendix DFAIR-3.. Literature on Carbon Cycle

The following show the results from the Joos et al. (2013) study that are used in the calibrations discussed above.



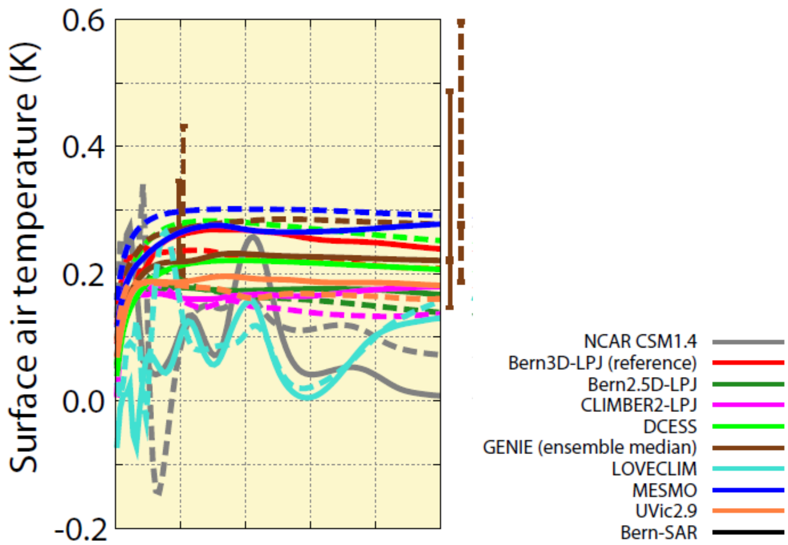
[From Joos et al. (2013): "Fig. 7. Influence of pulse size and climate-carbon cycle feedback on the response in atmospheric CO2 and the time-integrated IRFCO2 as simulated with the Bern3D-LPJ model (standard setup). Pulse emissions, ranging from 10 to 10 000 GtC in the individual simulations, are added to the atmosphere under preindustrial conditions. Dashed lines represent simulations where climate was kept constant in the model."]

A. Results for atmospheric retention of pulses in multimodel:

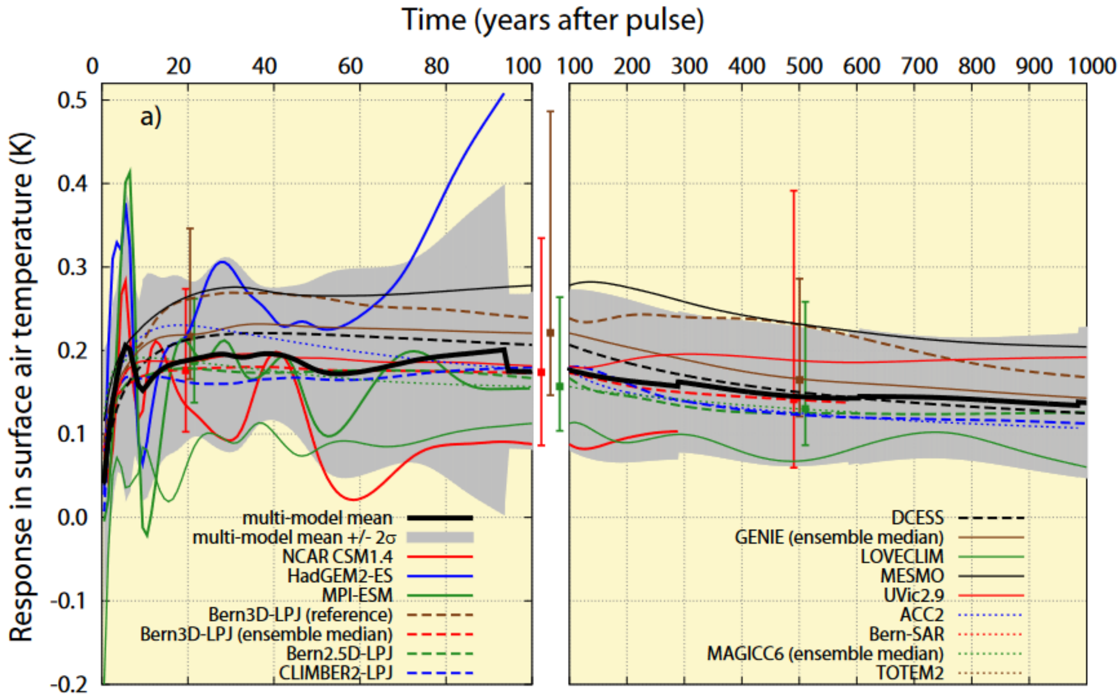


[From Joos et al. (2013): " Fig. 6. Response of the carbon cycle-climate system to a pulse emission of 5000 GtC (solid, PI5000) and 100 GtC (dashed, PI100) added to the atmosphere under preindustrial conditions. The responses in surface air temperature, ocean heat content, steric sea level rise, and in carbon fluxes for PI5000 are scaled by a factor of 50 for a better comparison with the 100 GtC pulse."]

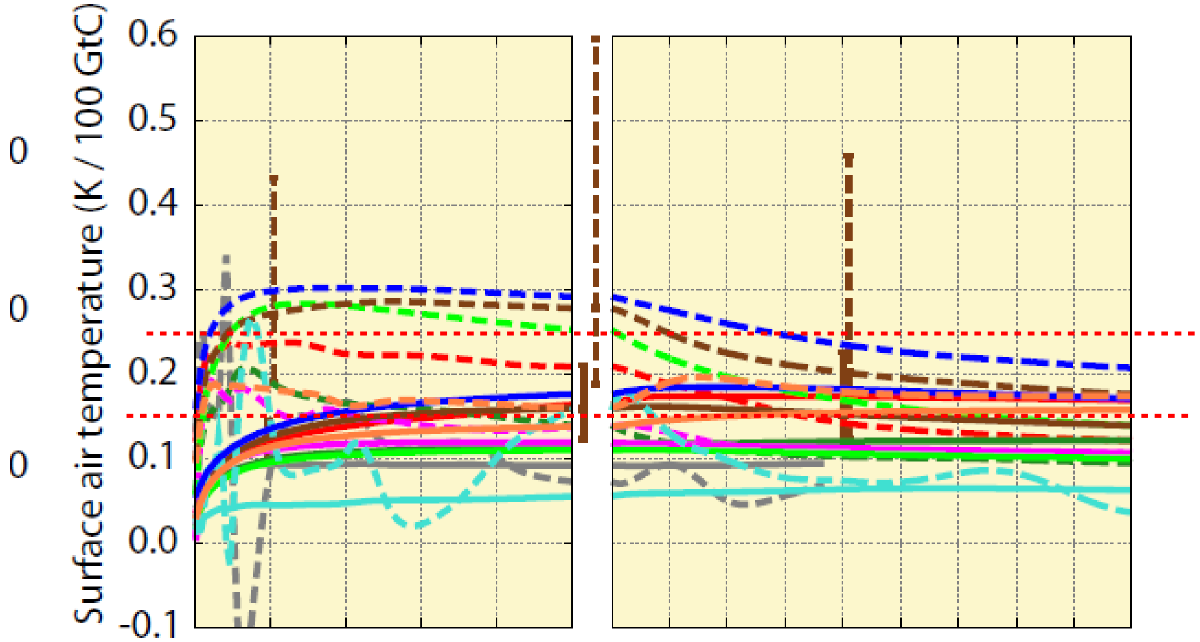
B. Results for temperature to pulses



"From Joos et al. (2013): Fig. 4. Influence of the background conditions on the climate-carbon cycle response to a pulse emission of 100 GtC into the atmosphere. Solid lines are for current conditions (CO2, ref = 389 ppm, PD100) and dashed lines for preindustrial conditions (CO2, ref = 280 ppm, PI100)."



[From Joos et al. (2013): "Fig. 2. As Fig. 1 but for the perturbation in global mean surface air temperature (a), in ocean heat content (b), and in steric sea level rise (c). Results are for a CO_2 emission pulse of 100 GtC added to a current CO_2 concentration of 389 ppm (PD100).We note that the signal-to-noise ratio is small for the models that feature a dynamic atmosphere (HadGEM2-ES, MPI-ESM, NCAR-CSM1.4, and LOVECLIM) and the plotted evolutions for these models represent both the forced response and a contribution from the models' internal (unforced) climate variability. Small abrupt changes in the multi-model mean and confidence range arise from a change in the number of model simulations; different groups run their model over different periods, pending on CPU availability."]



[From Joos et al. (2013): "Fig. 6. Response of the carbon cycle-climate system to a pulse emission of 5000 GtC (solid, PI5000) and 100 GtC (dashed, PI100) added to the atmosphere under preindustrial conditions. The responses in surface air temperature, ocean heat content, steric sea level rise, and in carbon fluxes for PI5000 are scaled by a factor of 50 for a better comparison with the 100 GtC pulse."]

NOTE that Millar et al. (2017) finds roughly the same results in his comparison.

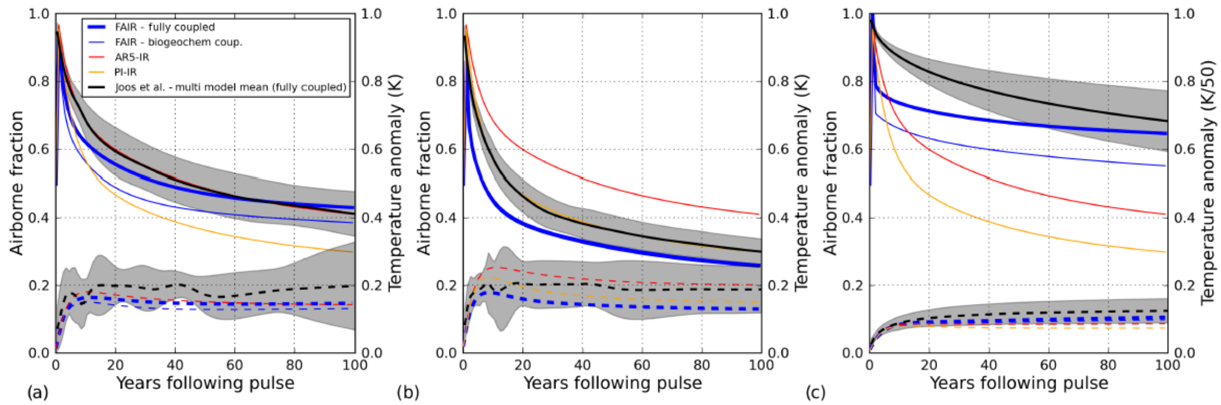
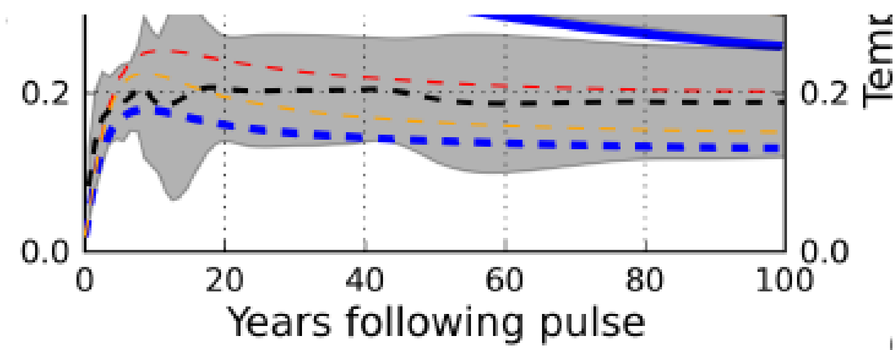
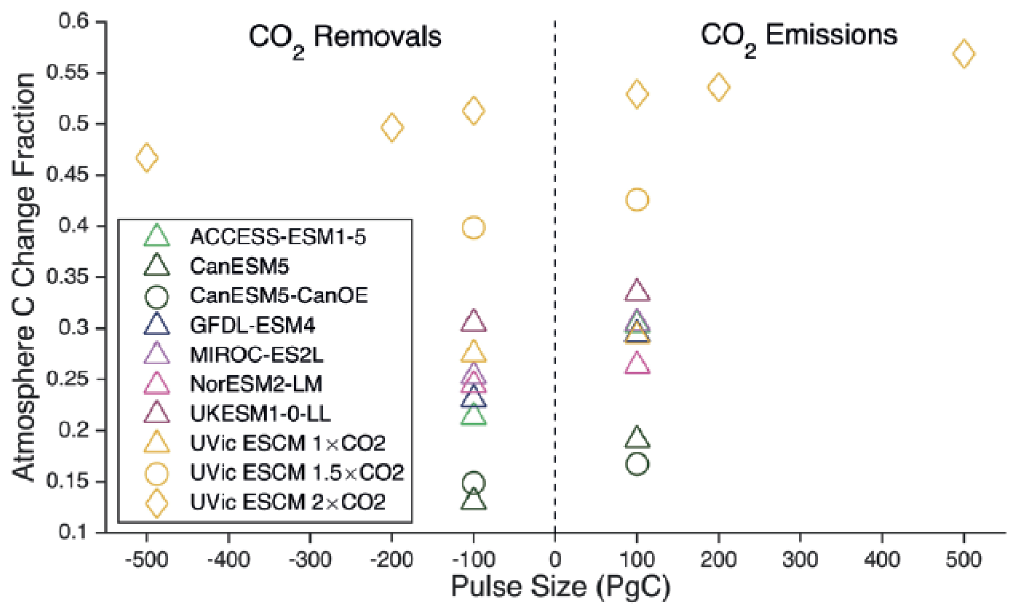


Figure 3. Response to pulse emission experiments of Joos et al. (2013). **(a)** shows the response to a 100 GtC imposed on present-day (389 ppm) background conditions (PD100 experiment), **(b)** the response to a 100 GtC pulse in preindustrial conditions (PI100 experiment) and **(c)** the response to a 5000 GtC pulse in preindustrial conditions (PI5000 experiment) with the warming normalised by the increase in pulse size between **(b)** and **(c)**. Airborne fraction (left-hand axis) is represented by solid lines in all panels and warming (right-hand axis) by dashed lines. FAIR is shown as thick blue lines, AR5-IR as red, and PI-IR as orange. The black lines in all panels shows the Joos et al. (2013) multi-model mean for airborne fraction (solid) and warming (dashed), with the grey shading indicating 1 standard deviation uncertainty across the ensemble. Thin blue lines denote the biogeochemically coupled version of FAIR.

Following is blowup of (b), 100GtC from preindustrial.

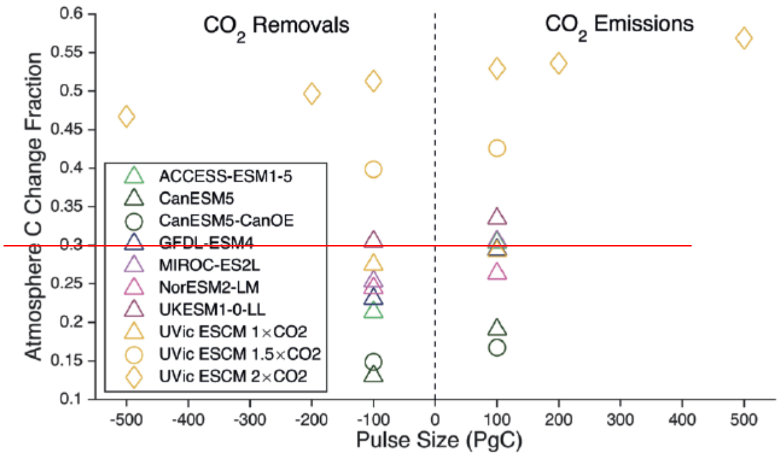


Although there is little discussion of the basic carbon cycle in IPCC AR6, there is discussion of the asymmetry. Figure 5.35 from IPCC AR6 is consistent with the Joos et al. (2013) findings. Median model is 30% retention for a 100GtC pulse from PI conditions.

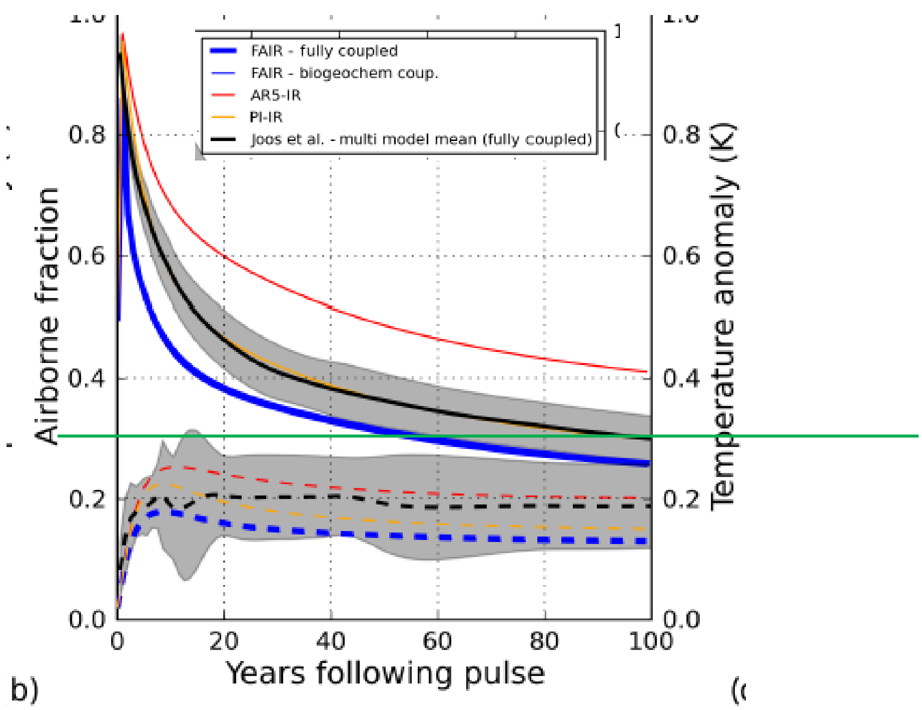


"Figure 5.35 | Asymmetry in the atmospheric carbon dioxide (CO2) response to CO2 emissions and removals. Shown are the fractions of total CO2 emissions remaining in the atmosphere (right-hand side) and CO2 removals remaining out of the atmosphere (left-hand side) 80–100 after a pulse emission/removal. Triangles and green circles denote results for seven Earth system models (ESMs) and the UVic ESCM model of intermediate complexity forced with ±100 PgC pulses applied from a pre-industrial state (1 × CO2) (Carbon Dioxide Removal Model Intercomparison Project (CDRMIP) experiment CDR-pi-pulse; Keller et al., 2018b). Yellow circles and diamonds indicate UVic ESCM results for CO2 emissions/removals applied at 1.5 times (1.5 × CO2) and 2 times (2 × CO2) the pre-industrial CO2 concentration, respectively. Pulses applied from a 2 × CO2 state span the magnitude ±100 PgC to ±500 PgC. UVic ESCM data is from Zickfeld et al. (2021). Further details on data sources and processing are available in the chapter data table (Table 5.SM.6)."

Median model is 30% retention for a 100GtC pulse from PI conditions.



Same as Joos et al. (2013) below:



NOTE from AR6: “Although there has been greater understanding since AR5 of the carbon cycle responses to CO2 emissions 27 (Chapter 5, Sections 5.4 and 5.5), there has been no new quantification of the response of the carbon-cycle 28 to an instantaneous pulse of CO2 emission since Joos et al. (2013).”

So, best is to start with calibrating to the 100 year, 100GtC pulse. Here are parameters for best (v1), Millar et al. (2017) , and others. The adjustments necessary are both (1) decrease the permanent share by about .05; and (2) increase the IFR0 by about 7 years. The latter was used by Dietz et al, and the former is necessary to get the asymptote correct.

Parameters		millar100	v1	v2	v3	v4
emshare0 =		0.2173	0.1650	0.1650	0.2173	0.1650
emshare1 =		0.2240	0.2740	0.3040	0.2240	0.2240
emshare2 =		0.2824	0.3324	0.3824	0.2824	0.3824
emshare3 =		0.2763	0.2286	0.1486	0.2763	0.2286
tau0 =		1.00E+06	1.00E+06	1.00E+06	1.00E+06	1.00E+06
tau1 =		394.4000	394.4000	394.4000	394.4000	394.4000
tau2 =		36.5300	36.5300	36.5300	36.5300	36.5300
tau3 =		4.3040	4.3040	4.3040	4.3040	4.3040
teq1 =		0.3300	0.3300	0.3300	0.3300	0.3300
teq2 =		0.4100	0.4100	0.4100	0.4100	0.4100
d1 =		239.0000	239.0000	239.0000	239.0000	239.0000
d2 =		4.1000	4.1000	4.1000	4.1000	4.1000
IRF0 =		32.4000	39.0000	39.0000	39.0000	39.0000
irC =		0.0190	0.0190	0.0190	0.0190	0.0190
irT =		4.1650	4.1650	4.1650	4.1650	4.1650
fco22x =		4.2000	4.2000	4.2000	4.2000	4.2000

Appendix DFAIR-4

The equations of the DFAIR model as of February 2023 are the following. (FAIR-beta-4-3-1.gms)

** Equals old FAIR with recalibrated parameters for revised F2xco2 and Millar model.
** Deletes nonnegative reservoirs. See explanation below

sets tfirst(t), tlast(t);

PARAMETERS
yr0 Calendar year that corresponds to model year zero /2020/
emshare0 Carbon emissions share into Reservoir 0 /0.2173/
emshare1 Carbon emissions share into Reservoir 1 /0.224/
emshare2 Carbon emissions share into Reservoir 2 /0.2824/
emshare3 Carbon emissions share into Reservoir 3 /0.2763/
tau0 Decay time constant for R0 (year) /1000000/
tau1 Decay time constant for R1 (year) /394.4/
tau2 Decay time constant for R2 (year) /36.53/
tau3 Decay time constant for R3 (year) /4.304/

teq1 Thermal equilibration parameter for box 1 (m^2 per KW) /0.324/
teq2 Thermal equilibration parameter for box 2 (m^2 per KW) /0.44/
d1 Thermal response timescale for deep ocean (year) /236/
d2 Thermal response timescale for upper ocean (year) /4.07/

irf0 Pre-industrial IRF100 (year) /32.4/
irC Increase in IRF100 with cumulative carbon uptake (years per GtC) /0.019/
irT Increase in IRF100 with warming (years per degree K) /4.165/
fco22x Forcings of equilibrium CO2 doubling (Wm-2) /3.93/

** INITIAL CONDITIONS TO BE CALIBRATED TO HISTORY

** CALIBRATION
mat0 Initial concentration in atmosphere in 2020 (GtC)
/886.5128014/
res00 Initial concentration in Reservoir 0 in 2020 (GtC) /150.093 /
res10 Initial concentration in Reservior 1 in 2020 (GtC) /102.698 /
res20 Initial concentration in Reservoir 2 in 2020 (GtC) /39.534 /
res30 Initial concentration in Reservoir 3 in 2020 (GtC) / 6.1865 /
mateq Equilibrium concentration atmosphere (GtC) /588 /
tbox10 Initial temperature box 1 change in 2020 (C from 1765) /0.1477
/
tbox20 Initial temperature box 2 change in 2020 (C from 1765)
/1.099454/
tatm0 Initial atmospheric temperature change in 2020 /1.24715 /
;
VARIABLES
*Note: Stock variables correspond to levels at the END of the period
FORC(t) Increase in radiative forcing (watts per m2 from 1765)
TATM(t) Increase temperature of atmosphere (degrees C from 1765)
TBOX1(t) Increase temperature of box 1 (degrees C from 1765)
TBOX2(t) Increase temperature of box 2 (degrees C from 1765)
RES0(t) Carbon concentration in Reservoir 0 (GtC from 1765)
RES1(t) Carbon concentration in Reservoir 1 (GtC from 1765)
RES2(t) Carbon concentration in Reservoir 2 (GtC from 1765)
RES3(t) Carbon concentration in Reservoir 3 (GtC from 1765)
MAT(t) Carbon concentration increase in atmosphere (GtC from 1765)
CACC(t) Accumulated carbon in ocean and other sinks (GtC)
IRFt(t) IRF100 at time t
alpha(t) Carbon decay time scaling factor
SumAlpha Placeholder variable for objective function;

**** IMPORTANT PROGRAMMING NOTE. Earlier implementations has reservoirs as non-negative.
**** However, these are not physical but mathematical solutions.
**** So, they need to be unconstrained so that can have negative emissions.
NONNEGATIVE VARIABLES TATM, MAT, IRFt, alpha
EQUATIONS

```
    FORCE(t)       Radiative forcing equation
    RES0LOM(t)     Reservoir 0 law of motion
    RES1LOM(t)     Reservoir 1 law of motion
    RES2LOM(t)     Reservoir 2 law of motion
    RES3LOM(t)     Reservoir 3 law of motion
    MMAT(t)        Atmospheric concentration equation
    Cacceq(t)      Accumulated carbon in sinks equation
    TATMEQ(t)      Temperature-climate equation for atmosphere
    TBOX1EQ(t)     Temperature box 1 law of motion
    TBOX2EQ(t)     Temperature box 2 law of motion
    IRFeqLHS(t)    Left-hand side of IRF100 equation
    IRFeqRHS(t)    Right-hand side of IRF100 equation
;
** Equations of the model
  res0lom(t+1)..   RES0(t+1) =E=
(emshare0*tau0*alpha(t+1)*(Eco2(t+1)/3.667))*(1-exp(-
tstep/(tau0*alpha(t+1))))+Res0(t)*exp(-tstep/(tau0*alpha(t+1)));
  res1lom(t+1)..   RES1(t+1) =E=
(emshare1*tau1*alpha(t+1)*(Eco2(t+1)/3.667))*(1-exp(-
tstep/(tau1*alpha(t+1))))+Res1(t)*exp(-tstep/(tau1*alpha(t+1)));
  res2lom(t+1)..   RES2(t+1) =E=
(emshare2*tau2*alpha(t+1)*(Eco2(t+1)/3.667))*(1-exp(-
tstep/(tau2*alpha(t+1))))+Res2(t)*exp(-tstep/(tau2*alpha(t+1)));
  res3lom(t+1)..   RES3(t+1) =E=
(emshare3*tau3*alpha(t+1)*(Eco2(t+1)/3.667))*(1-exp(-
tstep/(tau3*alpha(t+1))))+Res3(t)*exp(-tstep/(tau3*alpha(t+1)));
  mmat(t+1)..      MAT(t+1)  =E=
mateq+Res0(t+1)+Res1(t+1)+Res2(t+1)+Res3(t+1);
  cacceq(t)..      Cacc(t)   =E=  (CCATOT(t)-(MAT(t)-mateq));
  force(t)..       FORC(t)    =E=  fco22x*((log((MAT(t)/mateq))/log(2))) +
F_Misc(t)+F_GHGabate(t);

  tbox1eq(t+1)..   Tbox1(t+1) =E=  Tbox1(t)*exp(-
tstep/d1)+teq1*Forc(t+1)*(1-exp(-tstep/d1));
  tbox2eq(t+1)..   Tbox2(t+1) =E=  Tbox2(t)*exp(-
tstep/d2)+teq2*Forc(t+1)*(1-exp(-tstep/d2));
```

```
   tatmeq(t+1)..    TATM(t+1)  =E=   Tbox1(t+1)+Tbox2(t+1);
   irfeqlhs(t)..    IRFt(t)   =E=  ((alpha(t)*emshare0*tau0*(1-exp(-
100/(alpha(t)*tau0))))+(alpha(t)*emshare1*tau1*(1-exp(-
100/(alpha(t)*tau1))))+(alpha(t)*emshare2*tau2*(1-exp(-
100/(alpha(t)*tau2))))+(alpha(t)*emshare3*tau3*(1-exp(-
100/(alpha(t)*tau3)))));
   irfeqrhs(t)..    IRFt(t)   =E=  irf0+irC*Cacc(t)+irT*TATM(t);
**  Upper and lower bounds for stability
MAT.LO(t)       = 10;
TATM.UP(t)      = 20;
TATM.lo(t)      = .5;
alpha.up(t) = 100;
alpha.lo(t) = 0.1;
* Initial conditions
MAT.FX(tfirst)    = mat0;
TATM.FX(tfirst)   = tatm0;
Res0.fx(tfirst) = Res00;
Res1.fx(tfirst) = Res10;
Res2.fx(tfirst) = Res20;
Res3.fx(tfirst) = Res30;
Tbox1.fx(tfirst) = Tbox10;
Tbox2.fx(tfirst) = Tbox20;
** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
```

Background Note on Non-CO2 Forcings (October 26, 2023)

I. Overview

The treatment of land emissions and non-CO2 greenhouse gases is changed in the current DICE model. In earlier treatments, land emissions and non-CO2 GHGs were exogenous. In the current treatment, land emissions are endogenous and included with industrial CO2, while non-CO2 GHG emissions are divided into abatable and non-abatable. The abatable non CO2 greenhouse gases are included as a CO2 equivalent. Because the stock-flow-forcings relationship for non-CO2 emissions is simplified, there are small discrepancies in the results relative to complete earth-systems models.

II. Emissions data and projections

Land CO2 emissions were excluded from abatable emissions in earlier versions. Additionally, a major problem with the earlier treatment was that it assumed that land emissions decline sharply over time. If we look at IPCC estimates, we see a 0.7% per year increase in land per year over last forty years. This changes the outlook considerably for non-industrial CO2. In the new simulations, the baseline land emissions are changed to minus 2% per year.

	1750–2019 Cumulative (PgC)	1850-2019 Cumulative (PgC)	1980–1989 Mean Annual Growth Rate (PgC yr^{-1})	1990–1999 Mean Annual Growth Rate (PgC yr^{-1})	2000–2009 Mean Annual Growth Rate (PgC yr^{-1})	2010–2019 Mean Annual Growth Rate (PgC yr^{-1})
Emissions						
Fossil fuel combustion and cement production	445 ± 20	445 ± 20	5.4 ± 0.3	6.3 ± 0.3	7.7 ± 0.4	9.4 ± 0.5
Net land use change	240 ± 70	210 ± 60	1.3 ± 0.7	1.4 ± 0.7	1.4 ± 0.7	1.6 ± 0.7
Total emissions	685 ± 75	655 ± 65	6.7 ± 0.8	7.7 ± 0.8	9.1 ± 0.8	10.9 ± 0.9
Partition						
Atmospheric increase	285 ± 5	265 ± 5	3.4 ± 0.02	3.2 ± 0.02	4.1 ± 0.02	5.1 ± 0.02
Ocean sink[c]	170 ± 20	160 ± 20	1.7 ± 0.4	2.0 ± 0.5	2.1 ± 0.5	2.5 ± 0.6
Terrestrial sink	230 ± 60	210 ± 55	2.0 ± 0.7	2.6 ± 0.7	2.9 ± 0.8	3.4 ± 0.9
Budget imbalance	0	20	–0.4	–0.1	0	–0.1

Table Note-NC-1. IPCC AR6 Calculations on Emissions (Table 5.1), with label as follows:

Global anthropogenic CO_2 budget accumulated since the industrial revolution (onset in 1750) and averaged over the 1980s, 1990s, 2000s, and 2010s. By convention, a negative ocean or land to atmosphere CO_2 flux is equivalent to a gain of carbon by these reservoirs. The table does not include natural exchanges (e.g. rivers, weathering) between reservoirs. Uncertainties represent the 68% confidence interval (Friedlingstein et al., 2020).

Next, if we look at historical emissions of CO2 and CO2-2, we see that CO2-e has grown a little more slowly than fossil CO2 over period 1970 – 2018 (0.2% per year more slowly).

	Average annual growth rate		
Period	CO2-e	CO2	Difference
1970-2021	1.5%	1.7%	-0.19%
2000-2021	1.6%	1.8%	-0.20%
Source: non-c02vco2 history-010323.xls			

Table Note-NC-2. Historical growth CO2 and CO2-e emissions.

Item	2020	2025	2050	2100
Base/NC industrial emissions (GtCO2/yr)	39.55	43.38	61.12	78.54
Base/NC land emissions (GtCO2/yr)	5.90	5.31	3.14	1.09
Abateable nonCO2 emissions GHG (GtCO2e/yr)	9.24	9.56	11.14	14.30
Total, CO2-e abateable emissions (GtCO2e/yr)	**54.69**	**58.24**	**75.39**	**93.93**
Base/NC CO2 forcings (W/m2)	2.43	2.69	4.04	6.62
Abateable other forcings (w/m2)	1.16	1.10	1.04	1.28
Exogenous forcings (w/m2)	(0.20)	(0.16)	0.04	0.44
Total abateable forcings (w/m2)	**3.39**	**3.63**	**5.12**	**8.34**

Base/NC = with zero emissions control

Source: non-co2GHG-MAC-up010323.xls

Table Note-NC-3. Projections CO2 and CO2-e emissions.

Item	Growth, 2020-2100
Base/NC industrial emissions (GtCO2/yr)	0.86%
Base/NC land emissions (GtCO2/yr)	-2.09%
Abateable nonCO2 emissions GHG (GtCO2e/yr)	0.55%
Total, CO2-e abateable emissions (GtCO2e/yr)	0.68%
Base/NC CO2 forcings (W/m2)	1.26%
Abateable other forcings (w/m2)	0.12%
Exogenous forcings (w/m2)	na
Total abateable forcings (w/m2)	1.13%

Base/NC = with zero emissions control

Source: non-co2GHG-MAC-up010323.xls

Table Note-NC-4. Projections of growth of CO2 and CO2-e emissions.

III. Abatement costs

The abatement cost function is drawn from Harmsen 2019. The maximum reduction at the CO2 backstop of $480 is 57% in 2050; and about 69% in 2100. The curve is more convex that for CO2. However, this may be because it is constructed using engineering approaches. We assume that the mitigation cost function is the same and that the abatable part is 65% of non-CO2-GHG.

The following shows the marginal cost function for CO2 and non-CO2 GHGs from DICE and Harmsen. Note that the price/marginal cost curve for non-CO2 GHG's is much more convex than for CO2.

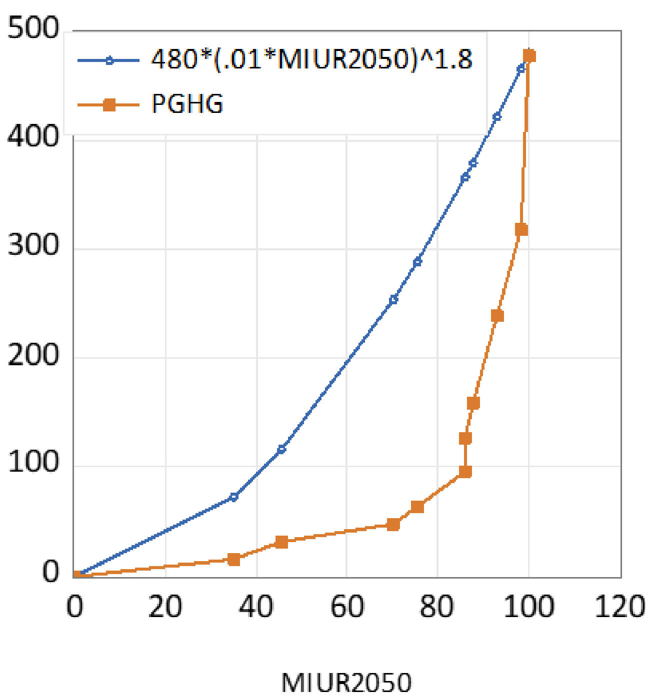


Figure Note-NC-1. Projections of growth of CO2 and CO2-e emissions.

[Source; Lab Notes for DICE.]

The following is the regression of the marginal cost on the emissions control rate for the Harmsen data, showing the higher convexity.

Dependent Variable: LOG(PGHG)
Method: Least Squares
Date: 12/20/21 Time: 14:00
Sample: 2 11
Included observations: 10

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-7.442956	1.881006	-3.956901	0.0042
LOG(MIUR2050)	2.800396	0.435560	6.429421	0.0002

R-squared	0.837851	Mean dependent var	4.615397
Adjusted R-squared	0.817583	S.D. dependent var	1.065225
S.E. of regression	0.454961	Akaike info criterion	1.439648
Sum squared resid	1.655918	Schwarz criterion	1.500165
Log likelihood	-5.198238	Hannan-Quinn criter.	1.373261
F-statistic	41.33745	Durbin-Watson stat	0.581986
Prob(F-statistic)	0.000203		

Table Note-NC-5. Projections of growth of CO2 and CO2-e emissions.

[Source; Lab Notes for DICE.]

IV. Forcings and carbon price error

The issue with the non-CO2 gases is that they cannot easily be translated from forcings into CO2e and the reverse. I had originally thought that we could multiple CO2 by a forcings ratio to get CO2e. However, this is inaccurate. The reason is that dF/dCO2 differs with the level of concentrations. A little experimentation also shows that it is not obvious how to put in a correction factor. The calculation of dF/dCO2 is not the natural = constant/M(t) (from logarithmic derivative).

Additionally, the treatment leads to error in calculation of the carbon price and SCC. Some experimentation showed that this was because of the addition of GHGs other than industrial CO2. See the following table:

Ratio SCC/cprice							
Year	2020	2025	2030	2035	2040	2045	2050
NO NON-CO2 GHG	1.19	0.89	0.91	0.92	0.93	0.94	0.95
ALL GHG	1.33	1.03	1.04	1.05	1.06	1.06	1.05
ONLY IND CO2	1.16	1.00	1.00	1.00	1.00	1.00	1.00
NO NON-CO2 GHG	19.4%	-10.9%	-9.2%	-7.8%	-6.6%	-5.6%	-4.9%
ALL GHG	33.2%	3.3%	4.5%	5.2%	5.6%	5.6%	5.4%
ONLY IND CO2	16.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Table Note-NC-6. Error in carbon price due to non-CO2 GHGs.

[Source; Lab Notes for DICE.]

Appendix B. Nuts and Bolts of Running the Models

This appendix has been prepared primarily by Paul Sztorc of the Yale Department of Economics from 2010 to 2013. It applied originally to the 2013 version but has been updated to DICE-2023. The instructions for the GAMS program structure is essentially unchanged from the earlier version. This appendix is designed to help new users get started.

Where Do I Find the Models?

The models for DICE-2023 are on the DICE web page at https://bit.ly/3TwJ5nO. If this is no longer operational, please consult William Nordhaus's webpage.

Required Software

The latest DICE-2023 comes in two formats: a GAMS software program and an EXCEL spreadsheet. Previous versions of DICE/RICE were either excel-only or GAMS-only:

DICE Version	Software Required
2023, 2016, 2013, 2008, 1999, 1994, DICE123	GAMS (General Algebraic Modeling System)[1]
2023, 2016, 2013, 2010	Microsoft Excel (in a macro-enabled workbook).

[1] The GAMS software codes for DICE versions 2023, 2016, 2008, 1999, and 1994 are available in Appendix C of this manual.

Basics for running GAMS

- A GAMS license is needed to run a model the size and complexity of DICE 2023. With an academic discount, such a license is available for around $1000 from the GAMS site.
- You can read the GAMS manual to run. It is relatively easy after a couple of hours of practice.

Basics for running the EXCEL version

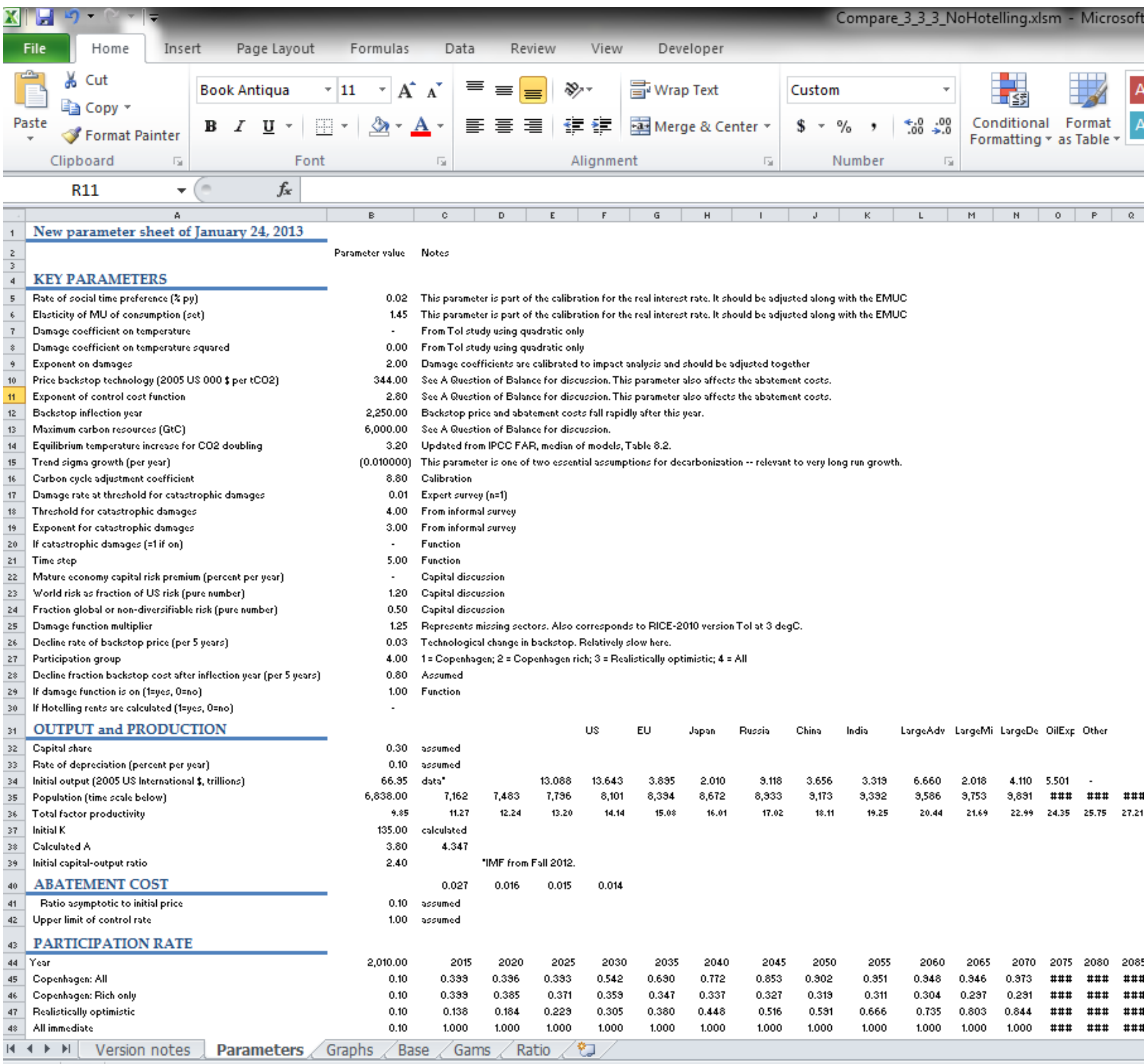
The basic EXCEL version can be used without any further software with standard Excel software. To do the optimization, we used proprietary software called "Risk Solver Platform," developed by FrontlineSolvers. With an academic discount, this costs about $1000.

Viewing the Model Inputs

An important set of variables are the input parameters (rate of growth of population, climate parameters, etc.). These can be varied, although care must be taken to make sure that they do not change the structure in an inappropriate way.

DICE 2010 [Excel]

In Excel parameters are entered in standard spreadsheet format in "Parameters" sheet.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	New parameter sheet of January 24, 2013																
2		Parameter value	Notes														
3																	
4	KEY PARAMETERS																
5	Rate of social time preference (% py)	0.02	This parameter is part of the calibration for the real interest rate. It should be adjusted along with the EMUC														
6	Elasticity of MU of consumption (set)	1.45	This parameter is part of the calibration for the real interest rate. It should be adjusted along with the EMUC														
7	Damage coefficient on temperature	-	From Tol study using quadratic only														
8	Damage coefficient on temperature squared	0.00	From Tol study using quadratic only														
9	Exponent on damages	2.00	Damage coefficients are calibrated to impact analysis and should be adjusted together														
10	Price backstop technology (2005 US 000 $ per tCO2)	344.00	See A Question of Balance for discussion. This parameter also affects the abatement costs.														
11	Exponent of control cost function	2.80	See A Question of Balance for discussion. This parameter also affects the abatement costs.														
12	Backstop inflection year	2,250.00	Backstop price and abatement costs fall rapidly after this year.														
13	Maximum carbon resources (GtC)	6,000.00	See A Question of Balance for discussion.														
14	Equilibrium temperature increase for CO2 doubling	3.20	Updated from IPCC FAR, median of models, Table 8.2.														
15	Trend sigma growth (per year)	(0.010000)	This parameter is one of two essential assumptions for decarbonization -- relevant to very long run growth.														
16	Carbon cycle adjustment coefficient	8.80	Calibration														
17	Damage rate at threshold for catastrophic damages	0.01	Expert survey (n=1)														
18	Threshold for catastrophic damages	4.00	From informal survey														
19	Exponent for catastrophic damages	3.00	From informal survey														
20	If catastrophic damages (=1 if on)	-	Function														
21	Time step	5.00	Function														
22	Mature economy capital risk premium (percent per year)	-	Capital discussion														
23	World risk as fraction of US risk (pure number)	1.20	Capital discussion														
24	Fraction global or non-diversifiable risk (pure number)	0.50	Capital discussion														
25	Damage function multiplier	1.25	Represents missing sectors. Also corresponds to RICE-2010 version Tol at 3 degC.														
26	Decline rate of backstop price (per 5 years)	0.03	Technological change in backstop. Relatively slow here.														
27	Participation group	4.00	1 = Copenhagen; 2 = Copenhagen rich; 3 = Realistically optimistic; 4 = All														
28	Decline fraction backstop cost after inflection year (per 5 years)	0.80	Assumed														
29	If damage function is on (1=yes, 0=no)	1.00	Function														
30	If Hotelling rents are calculated (1=yes, 0=no)	-															
31	OUTPUT and PRODUCTION					US	EU	Japan	Russia	China	India	LargeAdv	LargeMi	LargeDe	OilExp	Other	
32	Capital share	0.30	assumed														
33	Rate of depreciation (percent per year)	0.10	assumed														
34	Initial output (2005 US International $, trillions)	66.95	data*		13.088	13.643	3.895	2.010	9.118	3.656	3.319	6.660	2.018	4.110	5.501	-	
35	Population (time scale below)	6,838.00	7,162	7,483	7,796	8,101	8,394	8,672	8,933	9,173	9,392	9,586	9,753	9,891	###	###	###
36	Total factor productivity	9.85	11.27	12.24	13.20	14.14	15.08	16.01	17.02	18.11	19.25	20.44	21.69	22.99	24.35	25.75	27.21
37	Initial K	135.00	calculated														
38	Calculated A	3.80	4.347														
39	Initial capital-output ratio	2.40		*IMF from Fall 2012.													
40	ABATEMENT COST		0.027	0.016	0.015	0.014											
41	Ratio asymptotic to initial price	0.10	assumed														
42	Upper limit of control rate	1.00	assumed														
43	PARTICIPATION RATE																
44	Year	2,010.00	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060	2065	2070	2075	2080	2085
45	Copenhagen: All	0.10	0.399	0.396	0.393	0.542	0.690	0.772	0.853	0.902	0.951	0.948	0.946	0.973	###	###	###
46	Copenhagen: Rich only	0.10	0.399	0.385	0.371	0.359	0.347	0.337	0.327	0.319	0.311	0.304	0.297	0.291	###	###	###
47	Realistically optimistic	0.10	0.138	0.184	0.229	0.305	0.380	0.448	0.516	0.591	0.666	0.735	0.803	0.844	###	###	###
48	All immediate	0.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	###	###	###

Dice 2013 or later (GAMS)

In GAMS, parameters are entered as in-line software code.

DICE2013_050313_mup_base.gms

```
$ontext
This is the DICE-2013 model. It is matched Excel version DICE_NR_032813.xlsm.
This has been revised from SCC version.
This version is DICE2013_032813.gms
It needs the include files.
$offtext

$title        DICE-2013        April 26, 2013 TFR Version

set         t  Time periods (5 years per period) /1*60/   ;


parameters

**Time Step
        tstep Years per Period                                /5/

** Preferences
        elasmu   Elasticity of marginal utility of consumption     /  1.45    /
        prstp    Initial rate of social time preference per year   / .015    /

** Population and technology
        gama     Capital elasticity in production function        /.300 /
        pop0     Initial world population (millions)              /6838/
        popadj   Growth rate to calibrate to 2050 pop projection  /0.134490 /
        popasym  Asymptotic population                            /10500/
        dk       Depreciation rate on capital (per year)          /.100/
        q0       Initial world gross output (trill 2005 USD)      /66.950/
        k0       Initial capital value (trill 2005 USD)           /135/
        a0       Initial level of total factor productivity       /3.7976214 /
        ga0      Initial growth rate for TFP per 5 years          /0.079/
        dela     Decline rate of TFP per 5 years                  /0.006/

** Emissions parameters
        sig0     Initial Sigma (industrial MTCO2 per thous 2005 USD )    /0.488982524/
        gsigma1  Initial growth of sigma (coninuous per year )           /-0.01 /
        dsig     Decline rate of decarbonization per period              /-0.001/
        eland0   Initial Carbon emissions from land (GtCO2 per period)   / 1.53972/
        deland   Decline rate of land emissions (per period)             / .2 /

** Carbon cycle
* Initial Conditions
        mat0   Initial Concentration in atmosphere 2010 (GtC)       /818.985/
        mu0    Initial Concentration in upper strata 2010 (GtC)     /1527.000/
```

Viewing the Model Structure

DICE Excel

The model structure of DICE Excel is contained within the formulas entered on each sheet. For example, the atmospheric concentrations of CO_2 are shown as a formula below. There are about 28,000 'lines of code' (discrete cells) in the Excel version, so the model is possibly subject to errors. Users should be very careful when changing parameters to make sure that they have checked the results. The following is the screenshot of DICE Excel 2013.

	A	B	C	D	E	F
1	Global	1.000	2.000	3.000	4.000	5.000
2		2,010.000	2,015.000	2,020.000	2,025.000	2,030.000
31	CLIMATE MODULE TRANSITION PARAMETERS (per 5 years)					
32	Speed of adjustment parameter for atmospheric temperature	**0.104**				
33	Coefficient of heat loss from atmosphere to oceans	**0.088**				
34	Coefficient of heat gain by deep oceans	**0.025**				
35	**Global Environmental Variables**					
36	**CARBON CYCLE (beginning of period)**					
37	Atmospheric concentration of carbon (GTC)	818.985	849.920	=Base!$B65*C87/100+Base!$B66*C89/100+C113*5/3.666		
38	Atmospheric concentration of carbon (ppm)	384.500	399.023	414.989	432.459	451.454
39	Concentration in biosphere and upper oceans (GTC)	1,527.000	1,540.103	1,555.394	1,573.053	1,593.266
90	Concentration in deep oceans (GTC)	10,010.000	10,010.439	10,010.911	10,011.421	10,011.974
91	**CUMULATIVE EMISSIONS/HOTELLING RENTS**					
92	Cumulative Emissions to date	90.000	132.377	180.468	234.504	294.653
93	Ratio to Max	0.000	0.000	0.000	0.000	0.000

Dice 2023 [GAMS]

The GAMS code is written in a high-level language and is much more succinct. For the base and optimal runs, it consists of about 330 lines of code (of which about half are comments, blank, or otherwise trivial). The following shows the way the model structure (equation set) is defined and written in DICE-2013. There is no difference in coding in DICE-2023.

DICE2013_050313_mup_base.gms

```
EQUATIONS
        CCACCA(t)          Cumulative carbon emissions
        UTIL               Objective function
        YY(t)              Output net equation
        YNETEQ(t)          Output net of damages equation
        YGROSSEQ(t)        Output gross equation
        DAMEQ(t)           Damage equation
        ABATEEQ(t)         Cost of emissions reductions equation
        CC(t)              Consumption equation
        KK(t)              Capital balance equation
        CPCE(t)            Per capita consumption definition
        EEQ(t)             Emissions equation
        EINDEQ(t)          Industrial emissions
        SEQ(t)             Savings rate equation
        RIEQ(t)            Interest rate equation
        FORCE(t)           Radiative forcing equation
        MMAT(t)            Atmospheric concentration equation
        MMU(t)             Shallow ocean concentration
        MML(t)             Lower ocean concentration
        TATMEQ(t)          Temperature-climate equation for atmosphere
        TOCEANEQ(t)        Temperature-climate equation for lower oceans
        DAMFRACEQ(t)       Equation for damage fraction
        MCABATEEQ(t)       Equation for MC abatement
        CARBPRICEEQ(t)     Carbon price equation from abatement
        CEMUTOTPEREQ(t)    Period utility
        PERIODUEQ(t)       Instantaneous utility function equation;
*        MIUPART(t)         MIU Adjusted for participation fraction;
*        CPRICEPARTICIPANTS(t) Carbon price Adjusted for participation fraction;

** Equations of the model

ccacca(t+1)..          CCA(t+1)        =E=  CCA(t)+ EIND(t)*5/3.666;
kk(t+1)..              K(t+1)          =L=  (1-dk)**tstep * K(t) + tstep * I(t);
eindeq(t)..            EIND(t)         =E=  sigma(t) * YGROSS(t) * (1-(MIU(t)));
eeq(t)..               E(t)            =E=  EIND(t) + etree(t);
force(t)..             FORC(t)         =E=  fco22x * ((log((MAT(t)/588.000))/log(2))) + forcoth(t);
mmat(t+1)..            MAT(t+1)        =E= MAT(t)*b11 + MU(t)*b21 + (E(t)*(5/3.666));
mml(t+1)..             ML(t+1)         =E= ML(t)*b33  + MU(t)*b23;
mmu(t+1)..             MU(t+1)         =E= MAT(t)*b12 + MU(t)*b22 + ML(t)*b32;
tatmeq(t+1)..          TATM(t+1)       =E=  TATM(t) + c1 * ((FORC(t+1)-(fco22x/t2xco2)*TATM(t))-(c3*(TATM(t)-TO
toceaneq(t+1)..        TOCEAN(t+1)     =E= TOCEAN(t) + c4*(TATM(t)-TOCEAN(t));
ygrosseq(t)..          YGROSS(t)       =E= (al(t)*(L(t)/1000)**(1-GAMA))*(K(t)**GAMA);
dameq(t)               DAMAGES(t)      =E= YGROSS(t) * DAMFRAC(t);
```

Definitions of variables

This section answers the questions: What does this variable mean? Where are these terms defined?

DICE [Excel]

In Excel, the explanations are written in column A.

2		2,010.000	2,015.000	2,020
99	CLIMATE MODULE			
100	Atmospheric temperature (degrees Celsius above preindustrial)	0.830	0.934	1
101	Total increase in radiative forcing since preindustrial (Watts per square meter)	1.824	2.062	2
102	Lower ocean temperature (degrees Celsius above preindustrial)	0.007	0.027	0
103	**Economic Endogenous Variables**			
104	OUTPUT			
105	Output gross of abatement cost and climate damage ($trill)	63.542	75.806	89

DICE 2023 [GAMS]

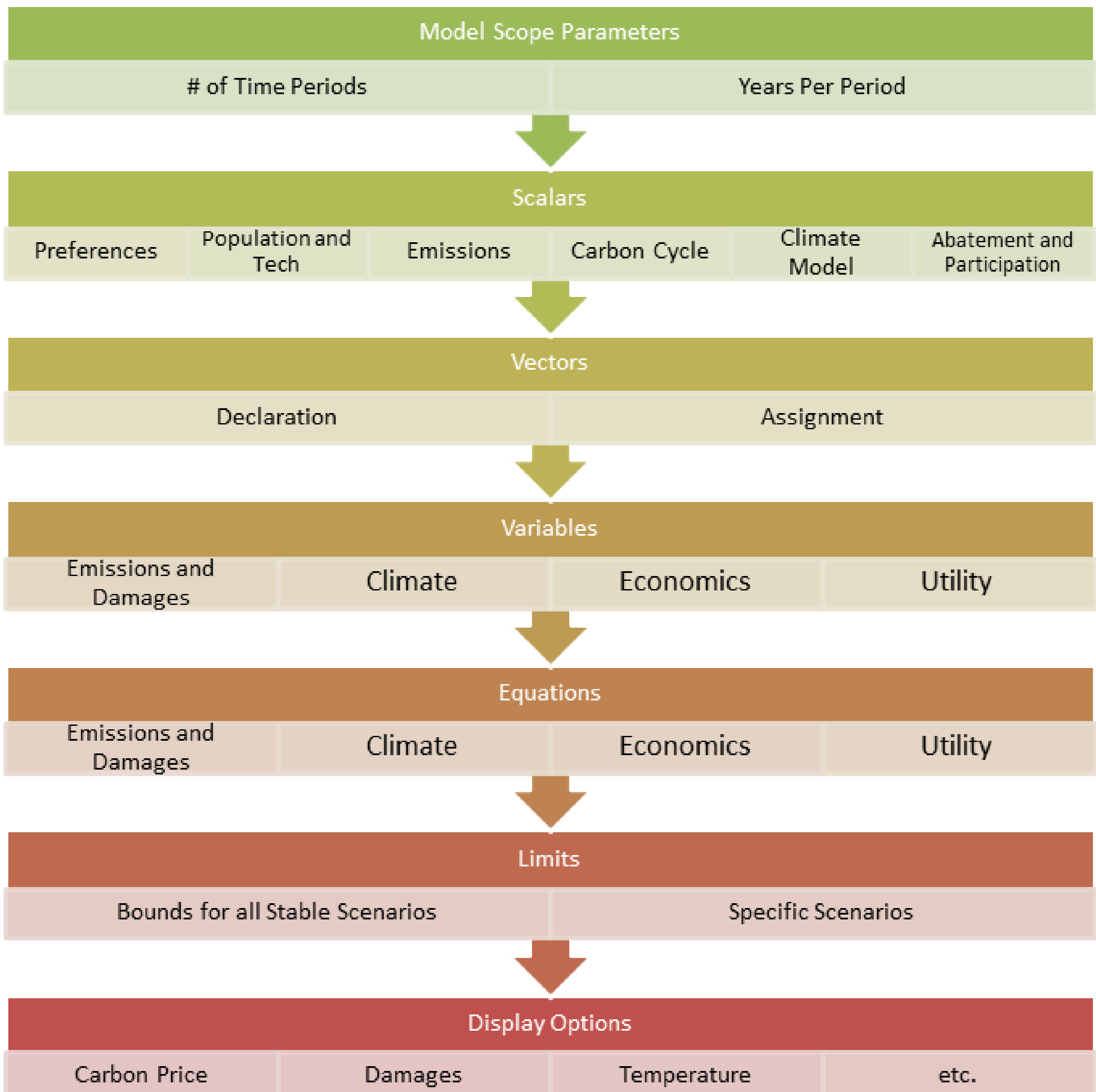
In GAMS, the definitions are given when a variable is declared:

DICE2013_050313_mup_base.gms

```

PARAMETERS
        L(t)            Level of population and labor
        al(t)           Level of total factor productivity
        sigma(t)        CO2-equivalent-emissions output ratio
        rr(t)           Average utility social discount rate
        ga(t)           Growth rate of productivity from 0 to T
        forcoth(t)      Exogenous forcing for other greenhouse gases
        gl(t)           Growth rate of labor (0 to T)
        gcost1          Growth of cost factor
        gsig(t)         Change in sigma (cumulative improvement of energy efficiency)
        etree(t)        Emissions from deforestation
        cost1(t)        Adjusted cost for backstop
        partfract(t)    Fraction of emissions in control regime
        lam             Climate model parameter
```

Structure of DICE 2023 Software Code

The flow of information through the DICE model can be visualized as follows:



Time Periods

The current version of DICE-2023 has a five-year time step. The following shows the structure in an earlier version.

```
$ontext
This is the DICE-2013R model, version DICE2013R_090713.gms, revised from April version.
This version just includes the optimal and base depending upon the "Ifopt" control.
This version does not have any output statements.
$offtext

$title        DICE-2013R September 2013

set          t   Time periods (5 years per period)                    /1*60/ ;

parameters

**Time Step
        tstep    Years per Period                                    /5/

** If optimal control
        ifopt    If optimized 1 and if base is 0                     /1/

```

In the Excel version, the years are given and obvious. In using the models, it is not obvious what the year is in GAMS. DICE-2023 starts in year 2020, and runs in 5 year periods for 81 periods in the published version, spanning the timeframe from 2020 to 2425. The notation /1*60/ is interpreted as "1, 2,..., 59, 60 ." This corresponds to the years "2020, 2015, 2020, ..., 2315."

Changing the time periods is a common and useful edit. However, the only number than can be reasonably changed is the ending time period (60 in this case), as the data inputs for DICE-2023 have been configured for initial conditions in 2020, and for time step of 5 years (for example, for population growth).

Scalars

Scalars are the easiest to change. Simply replace the number in between "/ /" with its desired value. For example, replace "/6838/" with "/7000/" to start off the world with a higher world population.

```
parameters

**Time Step
        tstep Years per Period                                  /5/

** Preferences
        elasmu   Elasticity of marginal utility of consumption     /  1.45    /
        prstp    Initial rate of social time preference per year   / .015    /

** Population and technology
        gama     Capital elasticity in production function        /.300 /
        pop0     Initial world population (millions)              /6838/
```

Vectors

Vectors are slightly more complicated to change, as they are not directly observed until the model runs. Vectors are declared as parameters which are indexed along a set, for example:

```
PARAMETERS
        L(t)            Level of population and labor
        al(t)           Level of total factor productivity
```

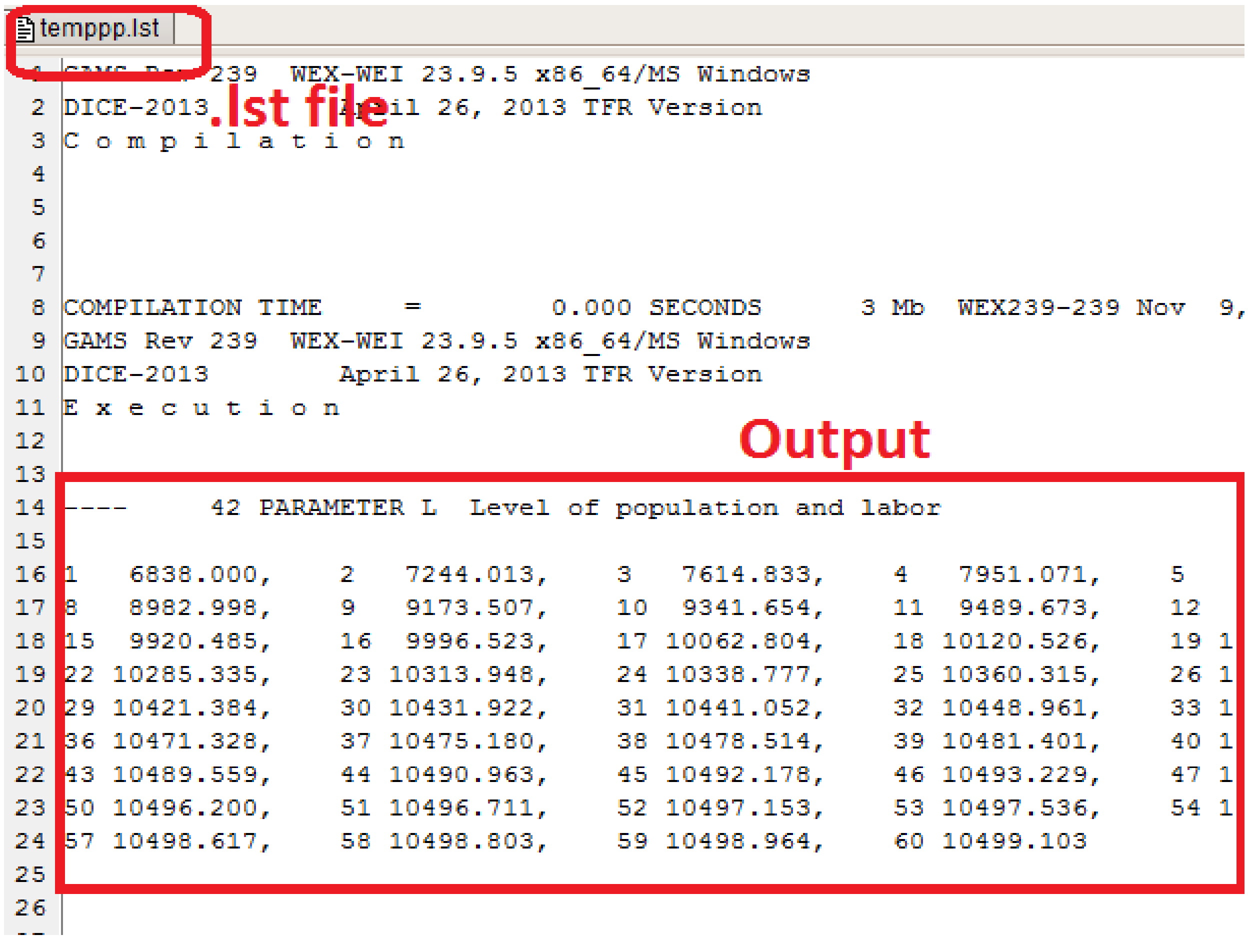
After being declared, they are assigned a value using simple equality statements, typically within a loop function over the indexed set (t 'time period' in this example).

```
L("1") = pop0;
loop(t, L(t+1)=L(t););
loop(t, L(t+1)=L(t)*(popasym/L(t))**popadj ;);
```

To change a vector, it highly advisable to test the declaration in a new GAMS window. What follows is an example of a simple program with the sole purpose of calculating and displaying the parameter L (the population level):

```
$title        DICE-2013        April 26, 2013 TFR Version

set         t  Time periods (5 years per period) /1*60/    ;
parameters

** Population and technology
        gama     Capital elasticity in production function        /.300 /
        pop0     Initial world population (millions)              /6838/
        popadj   Growth rate to calibrate to 2050 pop projection  /0.134490 /
        popasym  Asymptotic population                            /10500/


PARAMETERS
        L(t)          Level of population and labor;

L("1") = pop0;
loop(t, L(t+1)=L(t););
loop(t, L(t+1)=L(t)*(popasym/L(t))**popadj ;);

display L;
```

You might run the program and display the answer to test that the change is correctly made, as in the following.




```
GAMS Rev 239  WEX-WEI 23.9.5 x86_64/MS Windows
DICE-2013          April 26, 2013 TFR Version
C o m p i l a t i o n




COMPILATION TIME     =        0.000 SECONDS      3 Mb  WEX239-239 Nov  9,
GAMS Rev 239  WEX-WEI 23.9.5 x86_64/MS Windows
DICE-2013          April 26, 2013 TFR Version
E x e c u t i o n


----     42 PARAMETER L  Level of population and labor

1   6838.000,    2   7244.013,    3   7614.833,    4   7951.071,    5
8   8982.998,    9   9173.507,    10  9341.654,    11  9489.673,    12
15  9920.485,    16  9996.523,    17 10062.804,    18 10120.526,    19 1
22 10285.335,    23 10313.948,    24 10338.777,    25 10360.315,    26 1
29 10421.384,    30 10431.922,    31 10441.052,    32 10448.961,    33 1
36 10471.328,    37 10475.180,    38 10478.514,    39 10481.401,    40 1
43 10489.559,    44 10490.963,    45 10492.178,    46 10493.229,    47 1
50 10496.200,    51 10496.711,    52 10497.153,    53 10497.536,    54 1
57 10498.617,    58 10498.803,    59 10498.964,    60 10499.103
```

Variables and Equations

With knowledge of GAMS syntax, it is easy to edit the variables and equations. However, editing variables and equations is by far the most perilous change because one runs the risk of unintentionally producing a model with an entirely new structure, or one with unstable or unrealistic solutions, or an infeasible model with no valid solutions whatsoever. Often, we will do the edits in both GAMS and EXCEL to ensure that no major error has been made.

With that caveat, here is the setup: Variables are first declared. Secondly, they may be constrained as positive. Thirdly, the variables are used in a list of equations: Equalities are set using "=E=" (for "equal to"); or as inequalities using "=L=" for "less than" or =G= for "greater than").

Notably, this list of equations appears twice: There is literally a list of equations (and their descriptions); and then a second list in which each equation is defined mathematically.

Here is a first list of equations. The first line defines which variables are non-negative. Then, there follows a list of equation names.

```
;
NONNEGATIVE VARIABLES  MIU, TATM, MAT, MU, ML, Y, YNET, YGROSS, C, K, I, RFACTLONG;
EQUATIONS
*Emissions and Damages
        CCATOTEQ(t)       Cumulative total carbon emissions
        DAMFRACEQ(t)      Equation for damage fraction
        DAMEQ(t)          Damage equation
        ABATEEQ(t)        Cost of emissions reductions equation
        MCABATEEQ(t)      Equation for MC abatement
        CARBPRICEEQ(t)    Carbon price equation from abatement
*Economic variables
        YGROSSEQ(t)       Output gross equation
        YNETEQ(t)         Output net of damages equation
        YY(t)             Output net equation
        CC(t)             Consumption equation
        CPCE(t)           Per capita consumption definition
        SEQ(t)            Savings rate equation
        KK(t)             Capital balance equation
        RSHORTEQ(t)       Short-run interest rate equation
        RLONGeq(t)        Long-run interest rate equation
        RFACTLONGeq(t)    Long interest factor
```

After that, we show the actual equations. Notice, again, the equations are first declared under the header "EQUATIONS" and then secondarily defined in a lower list. Then the actual equation is given directly. For example, the name of the gross output equation is ygrosseq(t), and the equation is

"ygrosseq(t).. YGROSS(t) =E= (AL(t)*(L(t)/1000)**(1-gama))*(K(t)**gama); "

```
**Economic variables
 ygrosseq(t)..          YGROSS(t)       =E=  (AL(t)*(L(t)/1000)**(1-gama))*(K(t)**gama);
 yneteq(t)..            YNET(t)         =E= YGROSS(t)*(1-damfrac(t));
 yy(t)..                Y(t)            =E= YNET(t) - ABATECOST(t);
 cc(t)..                C(t)            =E= Y(t) - I(t);
 cpce(t)..              CPC(t)          =E= 1000 * C(t) / L(t);
 seq(t)..               I(t)            =E= S(t) * Y(t);
 kk(t+1)..              K(t+1)          =L=  (1-dk)**tstep * K(t) + tstep * I(t);
 RFACTLONGeq(t+1)..     RFACTLONG(t+1) =E= SRF*(cpc(t+1)/cpc('1'))**(-elasmu)*rr(t+1);
 RLONGeq(t+1)..         RLONG(t+1)      =E= -log(RFACTLONG(t+1)/SRF)/(5*t.val);
 RSHORTeq(t+1)..        RSHORT(t+1)     =E= -log(RFACTLONG(t+1)/Rfactlong(t))/5;
** Welfare functions
 periodueq(t)..         PERIODU(t)      =E=  ((C(T)*1000/L(T))**(1-elasmu)-1)/(1-elasmu)-1;
 totperiodueq(t)..      TOTPERIODU(t)   =E= PERIODU(t) * L(t) * RR(t);
 utileq..               UTILITY         =E= tstep * scale1 * sum(t,  TOTPERIODU(t)) + scale2;
```

Limits and Scenarios

Finally, the model requires certain bounds for reasons of stability, logical coherence, or to calibrate a specific scenario. For example, to limit temperature [TATM(t)] increase to 2 °C, you would add the following limit constraint:

```
TATM.up(t)   = 2 ;
```

The GAMS version of DICE 2023 has several scenarios in the "PUT" statements. These include "OPTIMAL", "LIMIT2T", "BASE", and several fixed discount rates. You can see the "scenarios" description above for the important ones.

To these meaningful constraints, we add a group of simpler constraints that work to speed up the solution. For example, the first constraint ensures that the capital stock is at least $1 trillion. Often, a value of 0 for an economic variable will lead to computational issues. The last line sets the initial value of capital at a value initialized earlier.

```
K.LO(t)             = 1;
C.LO(t)             = 2;
CPC.LO(t)           = .01;
RFACTLONG.lo(t)  =.0001;
s.fx(t)$(t.val > 37)        =.27;
ccatot.fx(tfirst)           = CumEmiss0;
K.FX(tfirst)                = k0;
```

Display Options

Model outputs can be viewed in two ways: as put files or as displayed in the *.lst file. The simplest way is to use the display command, followed by a list of relevant outputs.

```
display aa1,L,e.l,eeq.m;
```

Parameters need no suffix, but variables (e), and equations (eeq), require suffixes (of ".l" for "level", and ".m" for "marginal", in this example).

The above statement produces the following results in the .lst file:

DICE2013_041213.lst

```
E x e c u t i o n


----   2034 PARAMETER aa1                  =        0.000  Damage intercept

----   2034 PARAMETER L  Level of population and labor

1   6838.000,    2   7244.013,    3   7614.833,    4   7951.071,    5   8254.051,    6
8   8982.998,    9   9173.507,    10  9341.654,    11  9489.673,    12  9619.676,    1
15  9920.485,    16  9996.523,    17 10062.804,    18 10120.526,    19 10170.752,    2
22 10285.335,    23 10313.948,    24 10338.777,    25 10360.315,    26 10378.992,    2
29 10421.384,    30 10431.922,    31 10441.052,    32 10448.961,    33 10455.811,    3
36 10471.328,    37 10475.180,    38 10478.514,    39 10481.401,    40 10483.901,    4
43 10489.559,    44 10490.963,    45 10492.178,    46 10493.229,    47 10494.140,    4
50 10496.200,    51 10496.711,    52 10497.153,    53 10497.536,    54 10497.867,    5
57 10498.617,    58 10498.803,    59 10498.964,    60 10499.103


----   2034 VARIABLE E.L  CO2-equivalent emissions (GtC)

1        32.611,    2        29.237,    3        31.481,    4        33.569,    5       35
7        38.222,    8        39.048,    9        39.438,    10       39.355,    11      38
13       36.021,    14       33.826,    15       31.074,    16       27.765,    17      23
19       14.552,    20        9.088,    21        3.125,    22        0.014,    23       0
25        0.007,    26        0.006,    27        0.005,    28        0.004,    29       0
31        0.002,    32        0.002,    33        0.001,    34 9.759135E-4,    35 7.80730
37 4.996677E-4,    38 3.997342E-4,    39 3.197873E-4,    40 2.558299E-4,    41 2.04663
43 1.309849E-4,    44 1.047879E-4,    45 8.383033E-5,    46 6.706426E-5,    47 5.36514
49 3.433690E-5,    50 2.746952E-5,    51 2.197562E-5,    52 1.758049E-5,    53 1.40644
55 9.001213E-6,    56 7.200971E-6,    57 5.760776E-6,    58       10.875,    59      51


----   2034 EQUATION EEQ.M  Emissions equation

1        -0.081,    2        -0.076,    3        -0.071,    4        -0.066,    5       -0
7        -0.053,    8        -0.049,    9        -0.045,    10       -0.041,    11      -0
13       -0.032,    14       -0.030,    15       -0.027,    16       -0.025,    17      -0
19       -0.019,    20       -0.017,    21       -0.016,    22       -0.015,    23      -0
25       -0.011,    26       -0.010,    27       -0.009,    28       -0.008,    29      -0
31       -0.006,    32       -0.006,    33       -0.005,    34       -0.004,    35      -0
37       -0.003,    38       -0.003,    39       -0.003,    40       -0.002,    41      -0
```

"Parameter L" shows population, where "1" is 2010 is 6838 million, and so forth. "E.L" is the emissions of CO_2 in billions (G or giga) of tons of CO_2. The "CPRICE.L" is the market price of CO_2 emissions in the optimized runs, also the social cost of carbon. Note that these numbers are not necessarily correct and were created to serve as an example.

Put statements

We generally construct "PUT" statements in programs. These allow the output to be converted to a *.csv file. For example, in the main program for DICE-2023, we write:

```

*** STATEMENTS FOR DEFINITIONS AND PUT STATEMENTS FOR MAJOR SCENARIOS
$include Include\Putlong-5-2-1.gms

```

Include files

Include files are like subroutines. The "Include" calls a subroutine or file, here "Putlong-5-2-1.gms". That file constructs a number of further files. In the end, it produces a file with the following first few lines. This can then be further analyzed. These PUT statements were used to create the tables and figures in the text above.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2	Results of DICE2023-5-2-1.csv: May 3, 2024												
3	SCENARIO: OPTIMAL												
4	Results of DICE2023-opt-b-4-3-10												
5	OPTIMAL												
6	This is optimal if ifopt = 1 and baseline if ifopt = 0												
7	ifopt =	1											
8													
9	Period	1	2	3	4	5	6	7	8	9	10	11	
10	Year	2020	2025	2030	2035	2040	2045	2050	2055	2060	2065	2070	20
11	Objective function (2019$)	6648.802											
12	Industrial CO2 GtCO2/yr	37.3362	38.12701	34.50727	34.68866	35.04359	35.01005	34.53887	34.33523	33.81985	32.96677	31.74603	30.123
13	Atmospheric concentration C (ppm)	416.2032	430.0223	441.4078	452.9416	464.5833	476.1195	487.3585	498.5138	509.4199	519.9224	529.8559	539.03
14	Atmospheric concentrations GtC	886.5128	915.9476	940.1986	964.7655	989.5625	1014.134	1038.074	1061.834	1085.064	1107.435	1128.593	1148
15	Atmospheric temperaturer (deg c above preind)	1.24715	1.455172	1.588237	1.695259	1.795336	1.893025	1.98824	2.081208	2.171714	2.258857	2.341622	2.418
16	Total forcings w/m2	2.887838	3.109288	3.297353	3.470575	3.644249	3.815152	3.978893	4.136407	4.286926	4.427947	4.557267	4.672
17	Forcings, exogenous w/m2	-0.6	-0.574	-0.548	-0.522	-0.496	-0.47	-0.444	-0.418	-0.392	-0.366	-0.34	-0.3
18	CO2 forcings w/m2	2.327838	2.513034	2.661197	2.807443	2.95133	3.090398	3.222681	3.350995	3.473698	3.589401	3.696704	3.7941
19	CO2 forcings w/m2	2.327838	2.513034	2.661197	2.807443	2.95133	3.090398	3.222681	3.350995	3.473698	3.589401	3.696704	3.7941
20	Carbon price (2019 $ per t CO2)	5.760333	16.61043	64.12156	79.71019	91.60719	104.9632	119.9046	136.5417	154.9939	175.3721	197.7827	222.32
21	Emissions control rate	0.05	0.1	0.24	0.283695	0.319288	0.358672	0.402154	0.437541	0.475096	0.514834	0.556758	0.600
22	Social cost of carbon $/tCO2	52.62582	61.91273	72.95246	85.23959	98.99496	114.36	131.4568	150.3879	171.2695	194.2058	219.2954	246.63
23	Output, net net trill 2019$	134.1581	155.5383	178.5719	203.6843	230.9628	260.5068	292.4547	327.0032	364.3178	404.6008	448.0776	494.9
24	Short Interest rate, %/yr	0	0.045346	0.044344	0.04356	0.042838	0.042128	0.041436	0.04075	0.040072	0.039402	0.038741	0.038
25	Population	7752.9	8137.367	8481.171	8786.618	9056.484	9293.783	9501.598	9682.961	9840.769	9977.733	10096.35	10198
26	TFP	5.84	6.252677	6.690982	7.156263	7.649925	8.173428	8.728293	9.3161	9.938492	10.59718	11.29392	12.030
27	Output, gross-gross, 2019$	134.8914	156.7257	180.5162	206.3132	234.3693	264.8699	297.9873	333.9054	372.8573	415.0823	460.8444	510.43

Further readings

Further readings on the GAMS model can be found on the GAMS site at http://www.gams.com/docs/document.htm.

Manuals for Solver can be found at http://www.solver.com/user-guides-frontline-systems-excel-solvers.

One of the leading scholars who has developed GAMS for energy and economic modeling is Tom Rutherford. See for example his "Solution Software for Computable General Equilibrium Modeling," with Mark Horridge, Alex Meeraus and Ken Pearson in *Handbook of Computable General Equilibrium Modeling* (ISBN: 9780444595683), Peter B. Dixon and Dale W. Jorgenson (eds.), Elsevier, 1331–1381, 2013.

The GAMS code for different vintages is shown in Appendix C.

Appendix C. GAMS Code for Different Vintages of the DICE Model

This appendix contains the GAMS codes for four vintages of models: 1992-94, 1999, 2007, and 2023. There were intermediate vintages as well, but these were the most thoroughly documented and form the basis of most of the publications. The RICE model program is published in Nordhaus and Boyer (2000) as well. These can be run by simply creating a *.gms file, copying the text in, and running. There may be some formatting problems, but these should be easily corrected.

A. 1992-1994 version of DICE model

```
* DICE123a
* July 28, 1994

*    This is an optimal growth model to calculate the optimal control
* rate and timing for the abatement of CO2 and other Greenhouse Gases.
*    This is the standard model used for Science (Nov. 1992) and for
* the base model in W. Nordhaus, Managing the Global Commons,
* MIT Press, 1994, forthcoming.

SETS     T         Time periods        /1*40/
        TFIRST(T) First period
        TLAST(T)  Last period

SCALARS  BET       Elasticity of marginal utility                /0/
        R         Rate of social time pref per year             /.03/
        GL0       Growth rate of population per decade          /.223/
        DLAB      Decline rate of pop growth per dec            /.195/
        DELTAM    Removal rate carbon per decade                /.0833/
        GA0       Initial growth rate for technology per dec    /.15/
        DELA      Decline rate of technology per dec            /.11  /
        SIG0      CO2-equiv-GNP ratio                           /.519/
        GSIGMA    Growth of sigma per decade                    /-.1168/
        DK        Depreciation rate on capital per year         /.10/
        GAMA      Capital elasticity in output                  /.25/
        M0        CO2-equiv concentrations 1965 bill t C        /677/
        TL0       Lower stratum temperature (C) 1965            /.10/
        T0        Atmospheric temperature (C) 1965              /.2/
        ATRET     Marginal atmospheric retention rate           /.64/
        Q0        1965 world gross output trillions 89 US dol   /8.519/
        LL0       1965 world population millions                /3369/
        K0        1965 value capital billions 1989 US dollars   /16.03/
        C1        Coefficient for upper level                   /.226/
        LAM       Climate feedback factor                       /1.41/
        C3        Coefficient trans upper to lower stratum      /.440/
        C4        Coeff of transfer for lower level             /.02/
        A0        Initial level of total factor productivity    /.00963/
        A1        Damage coeff for co2 doubling (frac GWP)      /.0133/
        B1        Intercept control cost function               /.0686/
        B2        Exponent of control cost function             /2.887/
        PHIK      transversality coeff capital                  /140 /
        PHIM      Transversality coeff carbon ($ per ton)       /-9/
        PHITE     Transversalit coeff temper (bill $ per deg C) /-7000 /

PARAMETERS   L(T)         Level of population and labor
           AL(T)        Level of Total factor productivity
           SIGMA(T)     Emissions-output ratio
           RR(T)        Discount factor
           GA(T)        Growth rate of T. F. P. from 0 to T
```

```
            FORCOTH(T)    Exogenous forcing other greenhouse gases
            GL(T)         Growth rate of labor 0 to T
            GSIG(T)       Cumulative improvement of energy efficiency
            DUM(T)        dummy variable 0 except 1 in last period;

TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T)  = YES$(ORD(T) EQ CARD(T));
DISPLAY TFIRST, TLAST;

GL(T) = (GL0/DLAB)*(1-exp(-DLAB*(ord(t)-1)));
L(T)=LL0*exp(GL(t));
GA(T)  = (GA0/DELA)*(1-exp(-DELA*(ord(t)-1)));
AL(T) =  a0*exp(GA(t));
GSIG(T) = (GSIGMA/DELA)*(1-exp(-DELA*(ord(t)-1)));
SIGMA(T)=SIG0*exp(GSIG(t));
DUM(T)=1$(ord(T)  eq card(T));

RR(T) = (1+R)**(10*(1-ord(t)));
FORCOTH(T) = 1.42;
FORCOTH(T)$(ord(t) lt 15) = .2604+.125*ord(T)-.0034*ord(t)**2;

VARIABLES            MIU(T)        Emission control rate GHGs
                FORC(T)       Radiative forcing, W per m2
                TE(T)         Temperature, atmosphere C
                TL(T)         Temperature, lower ocean C
                M(T)          CO2-equiv concentration bill t
                E(T)          CO2-equiv emissions bill t
                C(T)          Consumption trill US dollars
                K(T)          Capital stock trill US dollars
                CPC(T)        Per capita consumption thousands US dol
                PCY(t)        Per capita income thousands US dol
                I(T)          Investment trill US dollars
                S(T)          Savings rate as fraction of GWP
                RI(T)         Interest rate per annum
                TRANS(T)      transversality variable last period
                Y(T)          OUTPUT
                UTILITY;

POSITIVE VARIABLES MIU, E, TE, M, Y, C, K, I;

EQUATIONS     UTIL      Objective function
            YY(T)     Output
            CC(T)     Cconsumption
            KK(T)     Capital balance
            KK0(T)    Initial condition of K
            KC(T)     Terminal condition of K
            CPCE(t)   Per capita consumption
            PCYE(T)   Per capita income equation
            EE(T)     Emissions process
            SEQ(T)    Savings rate equation
            RIEQ(T)   Interest rate equation
            FORCE(T)  Radiative forcing equation
            MM(T)     CO2 distribution equation
            MM0(T)    Initial condition for M
            TTE(T)    Temperature-climate equation for atmosphere
            TTE0(T)   Initial condition for atmospheric temp
            TLE(T)    Temperature-climate equation for lower oceans
            TRANSE(t) Transversality condition
            TLE0(T)   Initial condition for lower ocean;

KK(T)..       K(T+1) =L= (1-DK)**10 *K(T)+10*I(T);
KK0(TFIRST)..  K(TFIRST) =E= K0;
KC(TLAST)..    R*K(TLAST) =L= I(TLAST);
```

```
EE(T)..       E(T)=G=10*SIGMA(T)*(1-MIU(T))*AL(T)*L(T)**(1-GAMA)*K(T)**GAMA;
FORCE(T)..    FORC(T) =E=  4.1*(log(M(T)/590)/log(2))+FORCOTH(T);
MM0(TFIRST).. M(TFIRST) =E= M0;
MM(T+1)..     M(T+1) =E= 590+ATRET*E(T)+(1 - DELTAM)*(M(T)-590);

TTE0(TFIRST).. TE(TFIRST) =E= T0;
TTE(T+1)..    TE(T+1) =E= TE(t)+C1*(FORC(t)-LAM*TE(t)-C3*(TE(t)-TL(t)));
TLE0(TFIRST).. TL(TFIRST) =E= TL0;
TLE(T+1)..    TL(T+1) =E= TL(T)+C4*(TE(T)-TL(T));

YY(T)..       Y(T) =E= AL(T)*L(T)**(1-GAMA)*K(T)**GAMA*(1-B1*(MIU(T)**B2))
              /(1+(A1/9)*SQR(TE(T)));
SEQ(T)..      S(T) =e= I(T)/(.001+Y(T));
RIEQ(T)..     RI(T) =E= GAMA*Y(T)/K(T)- (1-(1-DK)**10)/10  ;

CC(T)..       C(T) =E= Y(T)-I(T);
CPCE(T)..     CPC(T) =e= C(T)*1000/L(T);
PCYE(T)..     PCY(T) =e= Y(T)*1000/L(T);

TRANSE(TLAST).. TRANS(TLAST)=E=RR(TLAST)
  *(PHIK*K(TLAST)+PHIM*M(TLAST)+PHITE*TE(TLAST));

UTIL..        UTILITY =E=
      SUM(T, 10 *RR(T)*L(T)*LOG(C(T)/L(T))/.55 +TRANS(T)*DUM(T));

*  Upper and Lower Bounds: General for stability
MIU.up(T) = 0.99;
MIU.lo(T) = 0.01;
K.lo(T) = 1;
TE.up(t) = 20;
M.lo(T) = 600;
C.LO(T) = 2;

* Upper and lower bounds for historical constraints

MIU.fx('1')=0.;
MIU.fx('2')=0.;
MIU.fx('3')=0.;

* Solution options

option iterlim = 99900;
option reslim = 99999;
option solprint = off;
option limrow = 0;
option limcol = 0;
model CO2 /all/;
solve CO2 maximizing UTILITY using nlp ;
display Y.l, C.l, S.l, K.l, MIU.l, E.l, M.l, TE.l, FORC.l, RI.l,
 CC.m, EE.m, KK.m, MM.m, TTE.m, CPC.l, TL.l, PCY.l, i.l;
display SIGMA, RR, L, AL, DUM, FORCOTH;
```

B. 1999 version of DICE model

```
** DICE 1999. Optimal carbon policy
** New optimal DICE as of 5/5/99
** Calibrated to RICE99 of 5/3/99

SETS  T              Time periods                 /1*40/
      TFIRST(T)        First period
      TLAST(T)         Last period
      tearly(T)        First 20 periods
      TLATE(T)  Second 20 periods;

SCALARS
 SRTP     Initial rate of social time preference per year /.03/
 DR       Decline rate of social time preference per year /.0025719/
 GL0      Growth rate of population per decade           /.157/
 DLAB     Decline rate of pop growth per decade          /.2220/
 A0       Initial level of total factor productivity      /.01685/
 GA0      Initial growth rate for technology per decade   /.038/
 DELA     Decline rate of technol. change per decade      /.00000001/
 SIG0     CO2-equivalent emissions-GNP ratio              /.274/
 GSIGMA   Growth of sigma per decade                     /-.158854/
 desig    Decline rate of decarbonization               /.02358711/
 desig2   Quadratic term in decarbonization             /-.00085/
 DK       Depreciation rate on capital per year          /.10/
 GAMA     Capital elasticity in production function       /.30/
 MAT1990  Concentration in atmosphere 1990 (b.t.c.)       /735/
 MU1990   Concentration in upper strata 1990 (b.t.c)      /781/
 ML1990   Concentration in lower strata 1990 (b.t.c)      /19230/
 b11      Carbon cycle transition matrix                 /0.66616/
 b12      Carbon cycle transition matrix                 /0.33384/
 b21      Carbon cycle transition matrix                 /0.27607/
 b22      Carbon cycle transition matrix                 /0.60897/
 b23      Carbon cycle transition matrix                 /0.11496/
 b32      Carbon cycle transition matrix                 /0.00422/
 b33      Carbon cycle transition matrix                 /0.99578/
 TL0      1985 lower strat. temp change (C) from 1900     /.06/
 T0       1985 atmospheric temp change (C)from 1900       /.43/
 Q0       1990 world gross output trill 90 US dollars     /21.08/
 LL0      1990 world population millions                 /5632.7/
 K0       1990 value capital trill 1990 US dollars        /47/
 C1       Climate-equation coefficient for upper level    /.226/
 LAM      Climate feedback factor                        /1.41/
 C3       Transfer coeffic. upper to lower stratum        /.440/
 C4       Transfer coeffic for lower level               /.02/
 A1       Damage coeff linear term                       /-.0045/
 A2       Damage coeff quadratic term                     /.0035/
 COST10   Intercept control cost function                 /.03/
 COST2    Exponent of control cost function               /2.15/
 ET0      C Emiss from deforest (bill tons per dec)       /11.28/
 dmiufunc Decline in cost of abatement function (per decade) /-.08/
 decmiu   Change in decline of cost function              /.005/
 coefopt1     Scaling coefficient in the objective function  /.333187/
 coefopt2     Scaling coefficient in the objective function  /5135680.6/  ;


PARAMETERS
  L(T)         Level of population and labor
  AL(T)        Level of total factor productivity
  SIGMA(T)     CO2-equivalent-emissions output ratio
  R(T)         Instantaeous rate of social time preference
  RR(T)        Average utility social discount rate
```

```
  GA(T)        Growth rate of productivity from 0 to T
  FORCOTH(T)   Exogenous forcing for other greenhouse gases
  GL(T)        Growth rate of labor 0 to T
  gcost1
  GSIG(T)      Cumulative improvement of energy efficiency
  ETREE(T)     Emissions from deforestation
  cost1(t)     cost function for abatement ;

TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T)  = YES$(ORD(T) EQ CARD(T));
TEARLY(T) = YES$(ORD(T) LE 20);
TLATE(T)  = YES$(ORD(T) GE 21);
DISPLAY TFIRST, TLAST;

GL(T) = (GL0/DLAB)*(1-exp(-DLAB*(ord(t)-1)));
L(T)=LL0*exp(GL(t));


ga(T)=ga0*EXP(-dela*10*(ORD(T)-1));
al("1") = a0;
LOOP(T,
al(T+1)=al(T)/((1-ga(T)));
);

gsig(T)=gsigma*EXP (  -desig*10*(ORD(T)-1) - desig2*10* ((ord(t)-1)**2)  )   ;
sigma("1")=sig0;
LOOP(T,
sigma(T+1)=(sigma(T)/((1-gsig(T+1))));
);
gcost1(T)=dmiufunc*EXP(-decmiu*10*(ORD(T)-1));
cost1("1")=cost10;
LOOP(T,
cost1(T+1)=cost1(T)/((1+gcost1(T+1)));
);

ETREE(T) = ET0*(1-0.1)**(ord(T)-1);

R(T)=srtp*EXP(-DR*10*(ORD(T)-1));
RR("1")=1;
LOOP(T,
RR(T+1)=RR(T)/((1+R(T))**10);
);
FORCOTH(T)=(-0.1965+.149*(ORD(T)-1)-.0019*(ORD(T)-1)**2)$
    (ORD(T) LT 12) + 1.15$(ORD(T) GE 12);

VARIABLES
 MIU(T)   Emission control rate GHGs
 FORC(T)  Radiative forcing, W per m2
 TE(T)    Temperature, atmosphere C
 TL(T)    Temperature, lower ocean C
 MAT(T)   Carbon concentration in atmosphere (b.t.c.)
 MU(T)    Carbon concentration in shallow oceans (b.t.c.)
 ML(T)    Carbon concentration in lower oceans (b.t.c.)
 E(T)     CO2-equivalent emissions bill t
 C(T)     Consumption trill US dollars
 K(T)     Capital stock trill US dollars
 CPC(T)   Per capita consumption thousands US dol
 PCY(t)   Per capita income thousands US dol
 I(T)     Investment trill US dollars
 S(T)     Savings rate as fraction of GWP
 RI(T)    Real interest rate per annum
 Y(T)     Output
```

```
 UTILITY;

POSITIVE VARIABLES MIU, TE, E, Mat, mu, ml, Y, C, K, I ;

EQUATIONS
 UTIL      Objective function
 YY(T)     Output equation
 CC(T)     Consumption equation
 KK(T)     Capital balance equation
 KK0(T)    Initial condition for K
 KC(T)     Terminal condition for K
 CPCE(t)   Per capita consumption definition
 PCYE(T)   Per capita income definition
 EE(T)     Emissions process
 SEQ(T)    Savings rate equation
 RIEQ(T)   Interest rate equation
 FORCE(T)  Radiative forcing equation
 MMAT0(T)  Starting atmospheric concentration
 MMAT(T)   Atmospheric concentration equation
 MMU0(T)   Initial shallow ocean concentration
 MMU(T) Shallow ocean concentration
 MML0(T)   Initial lower ocean concentration
 MML(T)    Lower ocean concentration
 TTE(T)    Temperature-climate equation for atmosphere
 TTE0(T)   Initial condition for atmospheric temperature
 TLE(T)    Temperature-climate equation for lower oceans
 TLE0(T)   Initial condition for lower ocean
 ;

** Equations of the model

KK(T)..        K(T+1) =L= (1-DK)**10 *K(T)+10*I(T);
KK0(TFIRST)..   K(TFIRST) =E= K0;
KC(TLAST)..     .02*K(TLAST) =L= I(TLAST);

EE(T)..        E(T)=G=10*SIGMA(T)*(1-MIU(T))*AL(T)*L(T)**(1-GAMA)*K(T)**GAMA + ETREE(T);
FORCE(T)..      FORC(T) =E=  4.1*((log(Mat(T)/596.4)/log(2)))+FORCOTH(T);

MMAT0(TFIRST).. MAT(TFIRST) =E= MAT1990;
MMU0(TFIRST)..  MU(TFIRST) =E= MU1990;
MML0(TFIRST)..  ML(TFIRST) =E= ML1990;
MMAT(T+1)..     MAT(T+1) =E= MAT(T)*b11+E(T)+MU(T)*b21;
MML(T+1)..      ML(T+1) =E= ML(T)*b33+b23*MU(T);
MMU(T+1)..      MU(T+1) =E= MAT(T)*b12+MU(T)*b22+ML(T)*b32;

TTE0(TFIRST)..  TE(TFIRST) =E= T0;
TTE(T+1)..      TE(T+1) =E= TE(t)+C1*(FORC(t)-LAM*TE(t)-C3*(TE(t)-TL(t)));
TLE0(TFIRST)..  TL(TFIRST) =E= TL0;
TLE(T+1)..      TL(T+1) =E= TL(T)+C4*(TE(T)-TL(T));

YY(T)..        Y(T) =E= AL(T)*L(T)**(1-GAMA)*K(T)**GAMA*(1-cost1(t)*(MIU(T)**cost2))
            /(1+a1*TE(T)+ a2*TE(T)**2);

SEQ(T)..        S(T) =e= I(T)/(.001+Y(T));
RIEQ(T)..       RI(T) =E= GAMA*Y(T)/K(T)- (1-(1-DK)**10)/10  ;
CC(T)..         C(T) =E= Y(T)-I(T);
CPCE(T)..       CPC(T) =e= C(T)*1000/L(T);
PCYE(T)..       PCY(T) =e= Y(T)*1000/L(T);

UTIL..         UTILITY =E= SUM(T, 10 *RR(T)*L(T)*LOG(C(T)/L(T))/coefopt1)+ coefopt2 ;

**  Upper and Lower Bounds: General conditions imposed for stability
```

```
MIU.up(T)      = 1.0;
MIU.lo(T)      = 0.000001;
K.lo(T)        = 1;
TE.up(t)       = 12;
MAT.lo(T)      = 10;
MU.lo(t)       = 100;
ML.lo(t)       = 1000;
C.lo(T)        = 2;

** Emissions control policy. Current setting is for optimal policy.
** Reinstate equation "Miu.fx(t) = .0" for no-control run.

* For base, remove the * from the following to get zero controls
*MIU.fx(t)=.0;

** Solution options

option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;

model CO2 /all/;

solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;

** Display of results

display y.l, c.l, s.l, k.l, miu.l, e.l, te.l, forc.l, ri.l;
display cc.m, ee.m, kk.m, tte.m, cpc.l, tl.l, pcy.l, i.l;
display sigma, rr, l, al, forcoth, etree;
display mat.l,mu.l,ml.l;


Parameters
Year(t)        Date
Indem(t)       Industrial emissions (b.t.c. per year)
Wem(t)         Total emissions (b.t.c. per year);
Year(t)        = 1995 +10*(ord(t)-1);
Indem(t)       = e.l(t)-etree(t);
Wem(t)         = e.l(t);Parameters
 Tax(t)        Carbon tax ($ per ton)
 damtax(t)     Concentration tax ($ per ton)
 dam(t)        Damages
 cost(t)       Abatement costs;
tax(t)  = -1*ee.m(t)*1000/(kk.m(t));
damtax(t)      = -1*mmat.m(t)*1000/kk.m(t);
dam(t)  = y.l(t)*(1-1/(1+a1*te.l(t)+ a2*te.l(t)**2));
cost(t)        = y.l(t)*(cost1(t)*(miu.l(t)**cost2));

display gsig, sigma;
display ga, al, cost1, gcost1, tax,miu.l,e.l,te.l;
```

C. 2008 version of DICE model

```
$ontext
DICE delta version 8
July 17, 2008.
This version is used for the DICE book, A Question of Balance (YUP, 2008).
We have included only the base, Hotelling, and optimal runs.
Exclude statements are removed so that it can run as a self-contained program.
$offtext

SETS  T            Time periods                 /1*60/ ;

SCALARS

** Preferences
 B_ELASMU   Elasticity of marginal utility of consumption    /  2.0    /
 B_PRSTP    Initial rate of social time preference per year   / .015    /

** Population and technology
 POP0     2005 world population millions              /6514     /
 GPOP0    Growth rate of population per decade          /.35      /
 POPASYM  Asymptotic population                       / 8600    /
 A0       Initial level of total factor productivity     /.02722   /
 GA0      Initial growth rate for technology per decade   /.092      /
 DELA     Decline rate of technol change per decade       /.001      /
 DK       Depreciation rate on capital per year          /.100     /
 GAMA     Capital elasticity in production function       /.300     /
 Q0       2005 world gross output trill 2005 US dollars   /61.1      /
 K0       2005 value capital trill 2005 US dollars        /137.     /

** Emissions
 SIG0     CO2-equivalent emissions-GNP ratio 2005        /.13418    /
 GSIGMA   Initial growth of sigma per decade            /-.0730    /
 DSIG     Decline rate of decarbonization per decade      /.003    /
 DSIG2    Quadratic term in decarbonization              / .000    /
 ELAND0   Carbon emissions from land 2005(GtC per decade) / 11.000  /

** Carbon cycle
 MAT2000  Concentration in atmosphere 2005 (GtC)         /808.9   /
 MU2000   Concentration in upper strata 2005 (GtC)       /1255     /
 ML2000   Concentration in lower strata 2005 (GtC)       /18365     /
 b11      Carbon cycle transition matrix               /0.810712 /
 b12      Carbon cycle transition matrix               /0.189288 /
 b21      Carbon cycle transition matrix               /0.097213 /
 b22      Carbon cycle transition matrix               /0.852787 /
 b23      Carbon cycle transition matrix               /0.05     /
 b32      Carbon cycle transition matrix               /0.003119 /
 b33      Carbon cycle transition matrix               /0.996881 /

** Climate model
 T2XCO2   Equilibrium temp impact of CO2 doubling oC      / 3    /
 FEX0     Estimate of 2000 forcings of non-CO2 GHG       / -.06   /
 FEX1     Estimate of 2100 forcings of non-CO2 GHG       / 0.30   /
 TOCEAN0  2000 lower strat. temp change (C) from 1900     /.0068   /
 TATM0    2000 atmospheric temp change (C)from 1900       /.7307   /
 C1       Climate-equation coefficient for upper level    /.220    /
 C3       Transfer coeffic upper to lower stratum         /.300    /
 C4       Transfer coeffic for lower level               /.050    /
 FCO22X   Estimated forcings of equilibrium co2 doubling  /3.8     /

** Climate damage parameters calibrated for quadratic at 2.5 C for 2105
 A1       Damage intercept                            / 0.00000    /
```

```
 A2      Damage quadratic term                       /  0.0028388 /
 A3      Damage exponent                           / 2.00      /

** Abatement cost
 EXPCOST2   Exponent of control cost function              /2.8   /
 PBACK      Cost of backstop 2005 000$ per tC 2005         /1.17  /
 BACKRAT    Ratio initial to final backstop cost           / 2    /
 GBACK      Initial cost decline backstop pc per decade     /.05   /
 LIMMIU     Upper limit on control rate                    / 1    /

** Participation
 PARTFRACT1  Fraction of emissions under control regime 2005 /1      /
 PARTFRACT2  Fraction of emissions under control regime 2015 /1      /
 PARTFRACT21 Fraction of emissions under control regime 2205 /1      /
 DPARTFRACT  Decline rate of participation                   /0      /

** Availability of fossil fuels
 FOSSLIM  Maximum cumulative extraction fossil fuels        / 6000  /

** Scaling and inessential parameters
  scale1 Scaling coefficient in the objective function       /194    /
  scale2 Scaling coefficient in the objective function       /381800 / ;

* Definitions for outputs of no economic interest
SETS
     TFIRST(T)
     TLAST(T)
     TEARLY(T)
     TLATE(T);

PARAMETERS
  L(T)          Level of population and labor
  AL(T)         Level of total factor productivity
  SIGMA(T)      CO2-equivalent-emissions output ratio
  R(T)          Instantaeous rate of social time preference
  RR(T)         Average utility social discount rate
  GA(T)         Growth rate of productivity from 0 to T
  FORCOTH(T)    Exogenous forcing for other greenhouse gases
  GL(T)         Growth rate of labor 0 to T
  GCOST1        Growth of cost factor
  GSIG(T)       Cumulative improvement of energy efficiency
  ETREE(T)      Emissions from deforestation
  COST1(t)      Adjusted cost for backstop
  PARTFRACT(T)  Fraction of emissions in control regime
  AA1           Variable A1
  AA2           Variable A2
  AA3           Variable A3
  ELASMU        Variable elasticity of marginal utility of consumption
  PRSTP         Variable nitial rate of social time preference per year
  LAM           Climate model parameter
  Gfacpop(T)    Growth factor population ;

PARAMETERS
  L(T)          Level of population and labor
  AL(T)         Level of total factor productivity
  SIGMA(T)      CO2-equivalent-emissions output ratio
  RR(T)         Average utility social discount factor
  GA(T)         Growth rate of productivity from 0 to T
  FORCOTH(T)    Exogenous forcing for other greenhouse gases
  GL(T)         Growth rate of labor 0 to T
  GCOST1        Growth of cost factor
  GSIG(T)       Cumulative improvement of energy efficiency
  ETREE(T)      Emissions from deforestation
```

```
 COST1(t)      Adjusted cost for backstop
 PARTFRACT(T)  Fraction of emissions in control regime
 AA1           Variable A1
 AA2           Variable A2
 AA3           Variable A3
 ELASMU        Variable elasticity of marginal utility of consumption
 PRSTP         Variable nitial rate of social time preference per year
 LAM           Climate model parameter
 Gfacpop(T)    Growth factor population ;

* Unimportant definitions to reset runs
TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T)  = YES$(ORD(T) EQ CARD(T));
TEARLY(T) = YES$(ORD(T) LE 20);
TLATE(T)  = YES$(ORD(T) GE 21);
AA1 = A1;
AA2 = A2;
AA3 = A3;
ELASMU = B_ELASMU;
PRSTP  = B_PRSTP;

b11 = 1 - b12;
b21 = 587.473*B12/1143.894;
b22 = 1 - b21 - b23;
b32 = 1143.894*b23/18340;
b33 = 1 - b32 ;


* Important parameters for the model
LAM     = FCO22X/ T2XCO2;
Gfacpop(T) =   (exp(gpop0*(ORD(T)-1))-1)/exp(gpop0*(ORD(T)-1));
L(T)=POP0* (1- Gfacpop(T))+Gfacpop(T)*popasym;
ga(T)=ga0*EXP(-dela*10*(ORD(T)-1));
al("1") = a0;
LOOP(T, al(T+1)=al(T)/((1-ga(T))););
gsig(T)=gsigma*EXP(-dsig*10*(ORD(T)-1)-dsig2*10*((ord(t)-1)**2));sigma("1")=sig0;LOOP(T,sigma(T+1)=(sigma(T)/((1-
gsig(T+1)))););
cost1(T) = (PBACK*SIGMA(T)/EXPCOST2)* ( (BACKRAT-1+ EXP (-gback* (ORD(T)-1) ) )/BACKRAT);
ETREE(T) = ELAND0*(1-0.1)**(ord(T)-1);
RR(t)=1/((1+prstp)**(10*(ord(T)-1)));
FORCOTH(T)= FEX0+ .1*(FEX1-FEX0)*(ORD(T)-1)$(ORD(T) LT 12)+ 0.36$(ORD(T) GE 12);
partfract(t) = partfract21;
PARTFRACT(T)$(ord(T)<25) = Partfract21 + (PARTFRACT2-Partfract21)*exp(-DPARTFRACT*(ORD(T)-2));
partfract("1")= PARTFRACT1;


VARIABLES
 MIU(T)          Emission control rate GHGs
 FORC(T)         Radiative forcing in watts per m2
 TATM(T)         Temperature of atmosphere in degrees C
 TOCEAN(T)       Temperatureof lower oceans degrees C
 MAT(T)          Carbon concentration in atmosphere GtC
 MATAV(T)        Average concentrations
 MU(T)           Carbon concentration in shallow oceans Gtc
 ML(T)           Carbon concentration in lower oceans GtC
 E(T)            CO2-equivalent emissions GtC
 C(T)            Consumption trillions US dollars
 K(T)            Capital stock trillions US dollars
 CPC(T)          Per capita consumption thousands US dollars
 PCY(t)          Per capita income thousands US dollars
 I(T)            Investment trillions US dollars
 S(T)            Gross savings rate as fraction of gross world product
 RI(T)           Real interest rate per annum
```

```
 Y(T)           Gross world product net of abatement and damages
 YGROSS(T)      Gross world product GROSS of abatement and damages
 YNET(T)        Output net of damages equation
 DAMAGES(T)     Damages
 ABATECOST(T)   Cost of emissions reductions
 CCA(T)         Cumulative industrial carbon emissions GTC
 PERIODU(t)     One period utility function
 UTILITY;

POSITIVE VARIABLES MIU, TATM, TOCE, E, MAT, MATAV, MU, ML, Y, YGROSS, C, K, I, CCA ;

EQUATIONS

 CCTFIRST(T)     First period cumulative carbon
 CCACCA(T)       Cumulative carbon emissions
 UTIL            Objective function
 YY(T)           Output net equation
 YNETEQ(T)       Output net of damages equation
 YGROSSEQ(T)     Output gross equation
 DAMEQ(T)        Damage equation
 ABATEEQ(T)      Cost of emissions reductions equation
 CC(T)           Consumption equation
 KK(T)           Capital balance equation
 KK0(T)          Initial condition for capital
 KC(T)           Terminal condition for capital
 CPCE(t)         Per capita consumption definition
 PCYE(T)         Per capita income definition
 EE(T)           Emissions equation
 SEQ(T)          Savings rate equation
 RIEQ(T)         Interest rate equation
 FORCE(T)        Radiative forcing equation
 MMAT0(T)        Starting atmospheric concentration
 MMAT(T)         Atmospheric concentration equation
 MMATAVEQ(t)     Average concentrations equation
 MMU0(T)         Initial shallow ocean concentration
 MMU(T)          Shallow ocean concentration
 MML0(T)         Initial lower ocean concentration
 MML(T)          Lower ocean concentration
 TATMEQ(T)       Temperature-climate equation for atmosphere
 TATM0EQ(T)      Initial condition for atmospheric temperature
 TOCEANEQ(T)     Temperature-climate equation for lower oceans
 TOCEAN0EQ(T)    Initial condition for lower ocean temperature
 PERIODUEQ(t)    Instantaneous utility function equation  ;

** Equations of the model

CCTFIRST(TFIRST).. CCA(TFIRST)=E=0;
CCACCA(T+1)..     CCA(T+1)=E=CCA(T)+ E(T);
KK(T)..           K(T+1) =L= (1-DK)**10 *K(T)+10*I(T);
KK0(TFIRST)..     K(TFIRST) =E= K0;
KC(TLAST)..       .02*K(TLAST) =L= I(TLAST);
EE(T)..           E(T)=E=10*SIGMA(T)*(1-MIU(T))*AL(T)*L(T)**(1-GAMA)*K(T)**GAMA + ETREE(T);
FORCE(T)..        FORC(T) =E=  FCO22X*((log((Matav(T)+.000001)/596.4)/log(2)))+FORCOTH(T);
MMAT0(TFIRST)..   MAT(TFIRST) =E= MAT2000;
MMU0(TFIRST)..    MU(TFIRST)  =E= MU2000;
MML0(TFIRST)..    ML(TFIRST)  =E= ML2000;
MMAT(T+1)..       MAT(T+1)    =E= MAT(T)*b11+MU(T)*b21 + E(T);
MMATAVEQ(t)..     MATAV(T)    =e= (MAT(T)+MAT(T+1))/2 ;
MML(T+1)..        ML(T+1)     =E= ML(T)*b33+b23*MU(T);
MMU(T+1)..        MU(T+1)     =E= MAT(T)*b12+MU(T)*b22+ML(T)*b32;
TATM0EQ(TFIRST).. TATM(TFIRST) =E= TATM0;
TATMEQ(T+1)..     TATM(T+1) =E= TATM(t)+C1*(FORC(t+1)-LAM*TATM(t)-C3*(TATM(t)-TOCEAN(t)));
TOCEAN0EQ(TFIRST).. TOCEAN(TFIRST) =E= TOCEAN0;
```

```
TOCEANEQ(T+1)..    TOCEAN(T+1) =E= TOCEAN(T)+C4*(TATM(T)-TOCEAN(T));
YGROSSEQ(T)..   YGROSS(T) =e= AL(T)*L(T)**(1-GAMA)*K(T)**GAMA;
DAMEQ(T)..      DAMAGES(t) =E= YGROSS(T)- YGROSS(T)/(1+aa1*TATM(T)+ aa2*TATM(T)**aa3);
YNETEQ(T)..     YNET(T) =E=  YGROSS(T)/(1+aa1*TATM(T)+ aa2*TATM(T)**aa3);
ABATEEQ(T)..    ABATECOST(T) =E= (PARTFRACT(T)**(1-expcost2))*YGROSS(T)*(cost1(t)*(MIU(T)**EXPcost2));
YY(T)..         Y(T) =E= YGROSS(T)*((1-(PARTFRACT(T)**(1-expcost2))*cost1(t)*(MIU(T)**EXPcost2)))/(1+aa1*TATM(T)+
aa2*TATM(T)**aa3);
SEQ(T)..        S(T)    =E= I(T)/(.001+Y(T));
RIEQ(T)..       RI(T)   =E= GAMA*Y(T)/K(T)- (1-(1-DK)**10)/10  ;
CC(T)..         C(T)    =E= Y(T)-I(T);
CPCE(T)..       CPC(T)  =E= C(T)*1000/L(T);
PCYE(T)..       PCY(T)  =E= Y(T)*1000/L(T);
PERIODUEQ(T)..  PERIODU(T)  =E=   ((C(T)/L(T))**(1-ELASMU)-1)/(1-ELASMU);
UTIL..          UTILITY =E= SUM(T, 10 *RR(T)*L(T)*(PERIODU(T))/scale1)+ scale2 ;

**  Upper and Lower Bounds: General conditions for stability

K.lo(T)         = 100;
MAT.lo(T)       = 10;
MU.lo(t)        = 100;
ML.lo(t)        = 1000;
C.lo(T)         = 20;
TOCEAN.up(T)    = 20;
TOCEAN.lo(T)    = -1;
TATM.up(t)      = 20;
miu.up(t)       = LIMMIU;
partfract("1")= 0.25372;

* First period predetermined by Kyoto Protocol
miu.fx("1")     = 0.005;

** Fix savings assumption for standardization if needed
*s.fx(t)=.22;

** Cumulative limits on carbon use at 6000 GtC
CCA.up(T) = FOSSLIM;

** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
model CO2 /all/;

* Optimal run
* Solution for optimal run

solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;

* Definition of opt results

Parameters
Year(t)         Date
opt_y(t)
opt_cpc(t)
opt_s(t)
opt_indem(t)
```

```
opt_sigma(t)
opt_tatm(t)
opt_mat(t)
opt_tax(t)
opt_ri(t)
opt_rr(t)
opt_al(t)
opt_forcoth(t)
opt_l(t)
opt_etree(t)
opt_yy(t)
opt_cc(t)
opt_miu(t)
opt_wem(t)
opt_ri(t)
opt_dam(t)
opt_abate(t)
opt_mcemis(t)
opt_utility
opt_scc(t) ;

Year(t)        = 2005 +10*(ord(t)-1);
opt_y(t)=y.l(t);
opt_cpc(t)=cpc.l(t);
opt_s(t)=s.l(t)     ;
opt_indem(t)= e.l(t)-etree(t);;
opt_sigma(t)=sigma(t) ;
opt_tatm(t)=tatm.l(t)  ;
opt_mat(t)=mat.l(t)     ;
opt_tax(t)=-1*ee.m(t)*1000/(kk.m(t)+.00000000001)       ;
opt_ri(t)=ri.l(t);
opt_rr(t)=rr(t)   ;
opt_al(t)=al(t)    ;
opt_forcoth(t)=forcoth(t);
opt_l(t)=l(t);
opt_etree(t)=etree(t);
opt_yy(t)=yy.m(t)     ;
opt_cc(t)=cc.m(t)      ;
opt_miu(t)=miu.l(t)     ;
opt_wem(t)= e.l(t);
opt_ri(t)=ri.l(t)         ;
opt_dam(t)= damages.l(t);
opt_abate(t) = abatecost.l(t);
opt_mcemis(t)= expcost2*cost1(t)*miu.l(t)**(expcost2-1)/sigma(t)*1000;
opt_utility=utility.l  ;
opt_scc(t)=-ee.m(t)/cc.m(t)*(1000)    ;

option decimals=6;
display opt_scc,y.l,opt_miu,ee.m, cc.m, yy.m;
```

D. 2016 version of DICE model

This is the central file for the version of October ???. The files for the different options are available online on the DICE-2016 website. There are two versions available. (1) The version below is the "Vanilla" version. It is self-contained and can be run without any further subroutines. It contains a full set of output statements. (2) The "Rockyroad" version has all scenarios, with a full set of outputs. It requires the subroutines ("include" programs). The results of this manual are from the Rockyroad version of October 4, ???.

For the models, see [dicemodel.net.](dicemodel.net)

```
$ontext
This is the DICE-2016 model, version DICE2016v2_102213_vanilla_v24b.gms, revised from April version.
The vanilla version includes only the optimal and baseline scenarios.
These are determined by setting the "ifopt" control at 1 (optimal) or 0 (baseline).
This version has write ("put") output but does not have subroutines ("include").

$offtext

$title      DICE-2016 October 2013

set     t  Time periods (5 years per period)                  /1*60/ ;

parameters

**Time Step
    tstep    Years per Period                                /5/

** If optimal control
    ifopt    If optimized 1 and if base is 0                 /1/

** Preferences
    elasmu   Elasticity of marginal utility of consumption    /  1.45 /
    prstp    Initial rate of social time preference per year   / .015  /

** Population and technology
    gama     Capital elasticity in production function        /.300    /
    pop0     Initial world population (millions)              /6838    /
    popadj   Growth rate to calibrate to 2050 pop projection  /0.134   /
    popasym  Asymptotic population (millions)                 /10500   /
    dk       Depreciation rate on capital (per year)          /.100    /
    q0       Initial world gross output (trill 2005 USD)      /63.69   /
    k0       Initial capital value (trill 2005 USD)           /135     /
    a0       Initial level of total factor productivity       /3.80    /
    ga0      Initial growth rate for TFP per 5 years          /0.079   /
    dela     Decline rate of TFP per 5 years                  /0.006   /

** Emissions parameters
    gsigma1  Initial growth of sigma (per year)               /-0.01   /
    dsig     Decline rate of decarbonization (per period)     /-0.001  /
    eland0   Carbon emissions from land 2010 (GtCO2 per year)   / 3.3    /
    deland   Decline rate of land emissions (per period)        / .2     /
    e0       Industrial emissions 2010 (GtCO2 per year)        /33.61   /
    miu0     Initial emissions control rate for base case 2010 /.039    /

** Carbon cycle
* Initial Conditions
    mat0   Initial Concentration in atmosphere 2010 (GtC)      /830.4   /
    mu0    Initial Concentration in upper strata 2010 (GtC)    /1527.   /
    ml0    Initial Concentration in lower strata 2010 (GtC)    /10010.  /
    mateq  Equilibrium concentration atmosphere  (GtC)         /588     /
    mueq   Equilibrium concentration in upper strata (GtC)     /1350    /
    mleq   Equilibrium concentration in lower strata (GtC)     /10000   /

* Flow paramaters
    b12      Carbon cycle transition matrix                    /.088   /
```

```
      b23     Carbon cycle transition matrix                    /0.00250/

* These are for declaration and are defined later
      b11     Carbon cycle transition matrix
      b21     Carbon cycle transition matrix
      b22     Carbon cycle transition matrix
      b32     Carbon cycle transition matrix
      b33     Carbon cycle transition matrix
      sig0    Carbon intensity 2010 (kgCO2 per output 2005 USD 2010)

** Climate model parameters
      t2xco2   Equilibrium temp impact (oC per doubling CO2)    / 2.9   /
      fex0     2010 forcings of non-CO2 GHG (Wm-2)             / 0.25  /
      fex1     2100 forcings of non-CO2 GHG (Wm-2)             / 0.70  /
      tocean0  Initial lower stratum temp change (C from 1900)  /.0068  /
      tatm0    Initial atmospheric temp change (C from 1900)    /0.80   /

      c10      Initial climate equation coefficient for upper level /0.098  /
      c1beta   Regression slope coefficient(SoA~Equil TSC)          /0.01243/

      c1      Climate equation coefficient for upper level     /0.098  /
      c3      Transfer coefficient upper to lower stratum      /0.088  /
      c4      Transfer coefficient for lower level            /0.025  /
      fco22x   Forcings of equilibrium CO2 doubling (Wm-2)      /3.8    /

** Climate damage parameters
      a10      Initial damage intercept                      /0      /
      a20      Initial damage quadratic term                    /0.00267 /
      a1       Damage intercept                              /0      /
      a2       Damage quadratic term                            /0.00267 /
      a3       Damage exponent                                /2.00    /

** Abatement cost
      expcost2  Exponent of control cost function              / 2.8  /
      pback     Cost of backstop 2005$ per tCO2 2010            / 344  /
      gback     Initial cost decline backstop cost per period   / .025 /
      limmiu    Upper limit on control rate after 2150         / 1.2  /
      tnopol    Period before which no emissions controls base  / 45   /
      cprice0   Initial base carbon price (2005$ per tCO2)      / 1.0  /
      gcprice   Growth rate of base carbon price per year       /.02   /

** Participation parameters
      periodfullpart Period at which have full participation           /21  /
      partfract2010  Fraction of emissions under control in 2010       / 1  /
      partfractfull  Fraction of emissions under control at full time  / 1  /

** Availability of fossil fuels
      fosslim        Maximum cumulative extraction fossil fuels (GtC) /6000/

** Scaling and inessential parameters
* Note that these are unnecessary for the calculations but are for convenience
      scale1      Multiplicative scaling coefficient            /0.016408662 /
      scale2      Additive scaling coefficient                  /-3855.106895/ ;

* Program control variables
sets    tfirst(t), tlast(t), tearly(t), tlate(t);

PARAMETERS
      L(t)          Level of population and labor
      al(t)         Level of total factor productivity
      sigma(t)      CO2-equivalent-emissions output ratio
      rr(t)         Average utility social discount rate
      ga(t)         Growth rate of productivity from
```

```
        forcoth(t)    Exogenous forcing for other greenhouse gases
        gl(t)         Growth rate of labor
        gcost1        Growth of cost factor
        gsig(t)       Change in sigma (cumulative improvement of energy efficiency)
        etree(t)      Emissions from deforestation
        cost1(t)      Adjusted cost for backstop
        partfract(t)  Fraction of emissions in control regime
        lam           Climate model parameter
        gfacpop(t)    Growth factor population
        pbacktime(t)  Backstop price
        optlrsav      Optimal long-run savings rate used for transversality
        scc(t)        Social cost of carbon
        cpricebase(t) Carbon price in base case
        photel(t)     Carbon Price under no damages (Hotelling rent condition);
* Program control definitions
        tfirst(t) = yes$(t.val eq 1);
        tlast(t)  = yes$(t.val eq card(t));
* Parameters for long-run consistency of carbon cycle
        b11 = 1 - b12;
        b21 = b12*MATEQ/MUEQ;
        b22 = 1 - b21 - b23;
        b32 = b23*mueq/mleq;
        b33 = 1 - b32 ;
* Further definitions of parameters
        sig0 = e0/(q0*(1-miu0));
        lam = fco22x/ t2xco2;
        L("1") = pop0;
        loop(t, L(t+1)=L(t););
        loop(t, L(t+1)=L(t)*(popasym/L(t))**popadj ;);
        ga(t)=ga0*exp(-dela*5*((t.val-1)));
        al("1") = a0; loop(t, al(t+1)=al(t)/((1-ga(t))););

        gsig("1")=gsigma1; loop(t,gsig(t+1)=gsig(t)*((1+dsig)**tstep) ;);
        sigma("1")=sig0;   loop(t,sigma(t+1)=(sigma(t)*exp(gsig(t)*tstep)););
        pbacktime(t)=pback*(1-gback)**(t.val-1);
        cost1(t) = pbacktime(t)*sigma(t)/expcost2/1000;
        etree(t) = eland0*(1-deland)**(t.val-1);
        rr(t) = 1/((1+prstp)**(tstep*(t.val-1)));
        forcoth(t) = fex0+ (1/18)*(fex1-fex0)*(t.val-1)$(t.val lt 19)+ (fex1-fex0)$(t.val ge 19);
        optlrsav = (dk + .004)/(dk + .004*elasmu + prstp)*gama;

        partfract(t)$(ord(T)>periodfullpart) = partfractfull;
        partfract(t)$(ord(T)<periodfullpart+1) = partfract2010+(partfractfull-partfract2010)*(ord(t)-1)/periodfullpart;
        partfract("1")= partfract2010;
*Transient TSC Correction ("Speed of Adjustment Parameter")
        c1 =  c10 + c1beta*(t2xco2-2.9);
*Base Case      Carbon Price
        cpricebase(t)= cprice0*(1+gcprice)**(5*(t.val-1));
VARIABLES
        MIU(t)          Emission control rate GHGs
        FORC(t)         Increase in radiative forcing (watts per m2 from 1900)
        TATM(t)         Increase temperature of atmosphere (degrees C from 1900)
        TOCEAN(t)       Increase temperatureof lower oceans (degrees C from 1900)
        MAT(t)          Carbon concentration increase in atmosphere (GtC from 1750)
        MU(t)           Carbon concentration increase in shallow oceans (GtC from 1750)
        ML(t)           Carbon concentration increase in lower oceans (GtC from 1750)
        E(t)            Total CO2 emissions (GtCO2 per year)
        EIND(t)         Industrial emissions (GtCO2 per year)
        C(t)            Consumption (trillions 2005 US dollars per year)
        K(t)            Capital stock (trillions 2005 US dollars)
        CPC(t)          Per capita consumption (thousands 2005 USD per year)
        I(t)            Investment (trillions 2005 USD per year)
        S(t)            Gross savings rate as fraction of gross world product
```

```
    RI(t)          Real interest rate (per annum)
    Y(t)           Gross world product net of abatement and damages (trillions 2005 USD per year)
    YGROSS(t)      Gross world product GROSS of abatement and damages (trillions 2005 USD per year)
    YNET(t)        Output net of damages equation (trillions 2005 USD per year)
    DAMAGES(t)     Damages (trillions 2005 USD per year)
    DAMFRAC(t)     Damages as fraction of gross output
    ABATECOST(t)   Cost of emissions reductions  (trillions 2005 USD per year)
    MCABATE(t)     Marginal cost of abatement (2005$ per ton CO2)
    CCA(t)         Cumulative industrial carbon emissions (GTC)
    PERIODU(t)     One period utility function
    CPRICE(t)      Carbon price (2005$ per ton of CO2)
    CEMUTOTPER(t)  Period utility
    UTILITY        Welfare function
;
NONNEGATIVE VARIABLES  MIU, TATM, MAT, MU, ML, Y, YGROSS, C, K, I;
EQUATIONS
*Emissions and Damages
    EEQ(t)          Emissions equation
    EINDEQ(t)       Industrial emissions
    CCACCA(t)       Cumulative carbon emissions
    FORCE(t)        Radiative forcing equation
    DAMFRACEQ(t)    Equation for damage fraction
    DAMEQ(t)        Damage equation
    ABATEEQ(t)      Cost of emissions reductions equation
    MCABATEEQ(t)    Equation for MC abatement
    CARBPRICEEQ(t)  Carbon price equation from abatement
*Climate and carbon cycle
    MMAT(t)         Atmospheric concentration equation
    MMU(t)          Shallow ocean concentration
    MML(t)          Lower ocean concentration
    TATMEQ(t)       Temperature-climate equation for atmosphere
    TOCEANEQ(t)     Temperature-climate equation for lower oceans
*Economic variables
    YGROSSEQ(t)     Output gross equation
    YNETEQ(t)       Output net of damages equation
    YY(t)           Output net equation
    CC(t)           Consumption equation
    CPCE(t)         Per capita consumption definition
    SEQ(t)          Savings rate equation
    KK(t)           Capital balance equation
    RIEQ(t)         Interest rate equation
* Utility
    CEMUTOTPEREQ(t) Period utility
    PERIODUEQ(t)    Instantaneous utility function equation
    UTIL            Objective function      ;
** Equations of the model
*Emissions and Damages
 eeq(t)..           E(t)          =E= EIND(t) + etree(t);
 eindeq(t)..        EIND(t)       =E= sigma(t) * YGROSS(t) * (1-(MIU(t)));
 ccacca(t+1)..      CCA(t+1)      =E= CCA(t)+ EIND(t)*5/3.666;
 force(t)..         FORC(t)       =E= fco22x * ((log((MAT(t)/588.000))/log(2))) + forcoth(t);
 damfraceq(t) ..    DAMFRAC(t)    =E= (a1*TATM(t))+(a2*TATM(t)**a3) ;
 dameq(t)..         DAMAGES(t)    =E= YGROSS(t) * DAMFRAC(t);
 abateeq(t)..       ABATECOST(t)  =E= YGROSS(t) * cost1(t) * (MIU(t)**expcost2) * (partfract(t)**(1-expcost2));
 mcabateeq(t)..     MCABATE(t)    =E= pbacktime(t) * MIU(t)**(expcost2-1);
 carbpriceeq(t)..   CPRICE(t)     =E= pbacktime(t) * (MIU(t)/partfract(t))**(expcost2-1);

*Climate and carbon cycle
 mmat(t+1)..        MAT(t+1)      =E= MAT(t)*b11 + MU(t)*b21 + (E(t)*(5/3.666));
 mml(t+1)..         ML(t+1)       =E= ML(t)*b33  + MU(t)*b23;
 mmu(t+1)..         MU(t+1)       =E= MAT(t)*b12 + MU(t)*b22 + ML(t)*b32;
 tatmeq(t+1)..      TATM(t+1)     =E= TATM(t) + c1 * ((FORC(t+1)-(fco22x/t2xco2)*TATM(t))-(c3*(TATM(t)-TOCEAN(t))));
 toceaneq(t+1)..    TOCEAN(t+1)   =E= TOCEAN(t) + c4*(TATM(t)-TOCEAN(t));
```

```
*Economic variables
 ygrosseq(t)..       YGROSS(t)      =E= (al(t)*(L(t)/1000)**(1-GAMA))*(K(t)**GAMA);
 yneteq(t)..         YNET(t)        =E= YGROSS(t)*(1-damfrac(t));
 yy(t)..             Y(t)           =E= YNET(t) - ABATECOST(t);
 cc(t)..             C(t)           =E= Y(t) - I(t);
 cpce(t)..           CPC(t)         =E= 1000 * C(t) / L(t);
 seq(t)..            I(t)           =E= S(t) * Y(t);
 kk(t+1)..           K(t+1)         =L= (1-dk)**tstep * K(t) + tstep * I(t);
 rieq(t+1)..         RI(t)          =E= (1+prstp) * (CPC(t+1)/CPC(t))**(elasmu/tstep) - 1;
*Utility
 cemutotpereq(t)..   CEMUTOTPER(t)  =E= PERIODU(t) * L(t) * rr(t);
 periodueq(t)..      PERIODU(t)     =E= ((C(T)*1000/L(T))**(1-elasmu)-1)/(1-elasmu)-1;
 util..              UTILITY        =E= tstep * scale1 * sum(t,  CEMUTOTPER(t)) + scale2 ;
*Resource limit
CCA.up(t)       = fosslim;
* Control rate limits
MIU.up(t)           = limmiu*partfract(t);
MIU.up(t)$(t.val<30) = 1;
**  Upper and lower bounds for stability
K.LO(t)         = 1;
MAT.LO(t)       = 10;
MU.LO(t)        = 100;
ML.LO(t)        = 1000;
C.LO(t)         = 2;
TOCEAN.UP(t)    = 20;
TOCEAN.LO(t)    = -1;
TATM.UP(t)      = 40;
CPC.LO(t)       = .01;
* Control variables
* Set savings rate for steady state for last 10 periods
set lag10(t) ;
lag10(t) =  yes$(t.val gt card(t)-10);
S.FX(lag10(t)) = optlrsav;
* Initial conditions
CCA.FX(tfirst)    = 90;
K.FX(tfirst)      = k0;
MAT.FX(tfirst)    = mat0;
MU.FX(tfirst)     = mu0;
ML.FX(tfirst)     = ml0;
TATM.FX(tfirst)   = tatm0;
TOCEAN.FX(tfirst) = tocean0;
** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
model  CO2 /all/;

* For base run, this subroutine calculates Hotelling rents
* Carbon price is maximum of Hotelling rent or baseline price
If (ifopt eq 0,
      a2 = 0;
      solve CO2 maximizing UTILITY using nlp;
      photel(t)=cprice.l(t);
      a2 = a20;
      cprice.fx(t)$(t.val<tnopol+1) = max(photel(t),cpricebase(t));
);
miu.fx('1')$(ifopt=1) = miu0;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;
```

```
** POST-SOLVE
* Calculate social cost of carbon
scc(t) = -1000*eeq.m(t)/cc.m(t);
** Display at bottom of output for visual inspection
option decimals=4;
display tatm.l,cpc.l,scc,y.l,s.l;
display ri.l,miu.l,cca.l,photel,cpricebase,cprice.l,utility.l,t2xco2 ,a2,partfract;
* Some sample results.
* Produces a file "DiceResults.csv" in the base directory
* For aLL relevant information, see 'PutOutputAllT.gms' in the Include folder.
* The statement at the end of the *.lst file "Output..." will tell you where to find the file.

file results /DiceResults.csv/;     results.nd = 10 ; results.nw = 0 ; results.pw=1200; results.pc=5;
put results;
put /"Results of DICE model run using model DICE2016_103113_vanilla_v2b4.gms";
put /"This is optimal if ifopt = 1 and baseline if ifopt = 0";
put /"ifopt =" ifopt;
put // "Period";
Loop (T, put T.val);
put / "Year" ;
Loop (T, put (2005+(TSTEP*T.val) ));
put / "Industrial Emissions (GTCO2 per year)" ;
Loop (T, put EIND.l(T));
put / "Atmospheric concentration of carbon (ppm)" ;
Loop (T, put (MAT.l(T)/2.13));
put / "Atmospheric Temperature (deg C above preindustrial)" ;
Loop (T, put TATM.l(T));
put / "Output (Net of Damages and Abatement, trillion USD pa) " ;
Loop (T, put Y.l(T));
put / "Climate Damages (fraction of gross output)" ;
Loop (T, put DAMFRAC.l(T));
put / "Consumption Per Capita (thousand USD per year)" ;
Loop (T, put CPC.l(T));
put / "Carbon Price (per t CO2)" ;
Loop (T, put cprice.l(T));
put / "Emissions Control Rate (total)" ;
Loop (T, put MIU.l(T));
put / "Social cost of carbon" ;
Loop (T, put scc(T));
put / "Interest Rate (Real Rate of Return)" ;
Loop (T, put RI.l(T));
putclose;
```

GAMS code for DICE-2023

This following shows the GAMS code for the October 2023 version of DICE-2023. Note that the first part is the main GAMS program with includes the different include files. The second part lists the code as a “*.lst” file which puts in the “Include” or subroutine files. The code is available at https://bit.ly/3TwJ5nO.