Background Note on Discounting[1]

Revised December 18, 2023

Advice on Background Note on Returns: These background notes are for informational purposes for modelers. They are not intended for publication and are not publication quality. Some of the details are sketched and not derived in detail in this document. They may be cited with the warning, "Background notes are for informational purposes and are not published." This note supersedes the previously distributed Note of July 2023.

[1] This background document replaces the earlier one dated October 30, 2023. The approach has been revised to reflect suggestions from several readers, especially William Hogan and Brian Prest. The latter provided valuable advice on the background calculations of the precautionary effect especially in NPP (2022); the underlying simulations and results from NPP have been useful in developing the estimates. We are grateful for suggestions for improvement from Christian Gollier and William Pizer. File is BackgroundNoteOnDiscounting-121823a.docx.

Part A. Summary of approach to discounting

Previous versions of DICE used different approaches to discounting. Upon the advice of modelers and readers, we have revised the treatment to employ an approach known as the "certainty equivalent" rate of return. This approach has been developed using suggestions of William Hogan, theoretical approaches developed by Gollier (particularly 2016), recommendations of the National Academy committee (2017), and empirical implementation of the correction for growth uncertainty by Newell, Pizer, and Prest (2022), hereafter NPP. This summary provides a full discussion of the approach, with other aspects in further Parts of this *Background Note.*

The variables in the analysis are the following. All rates are average annual returns. All time variables are per year.

$R_T =$ discount rate from 0 to T

$r_t =$ discount rate from t-1 to t

$R_T^f, R_T^K, R_T^{CLIM} =$ risk-free, capital, and climate discount rates

$\rho =$ pure rate of time preference

$\varphi =$ elasticity of utility with respect to consumption

ρ_T^* = risk-adjusted rate of time preference

$g_T =$ average growth of per capita consumption from 0 to T

$P_T =$ precautionary effect rate from 0 to T

σ_C^2= variance of trend growth rate of per capita consumption

$\pi =$ capital premium

$\beta^{CLIM} =$ climate beta

$\tilde{x}_T =$ deterministic version of variable x_T

As in earlier versions of the DICE model, discounting continues to follow the approach of the Ramsey-Cass-Koopmans growth model in determining real rates of return. In this approach, the continuous-time equilibrium deterministic long-run rate of return from 0 to T $\left(\tilde{R}_T\right)$ is given by the pure rate of time preference *(ρ)* plus the product of the deterministic growth rate of per capita consumption from t = 0 to T $\left(\tilde{g}_T\right)$ times the elasticity of the marginal utility of consumption *(φ)*. Note that the "~" over a variable indicates a deterministic concept.

$$(A.1)\quad \tilde{R}_T = \rho + \varphi \tilde{g}_T$$

In many applications, the consumption elasticity *(φ)* is also assumed to equal the relative rate of risk aversion. This is *not* assumed in the DICE treatment of discounting. That assumption would lead to a capital risk premium that is far below the observed rate, as is discussed in the literature on the equity-premium puzzle. Instead, we rely on the CAPM estimates of the capital premium.

Our implementation of the modeling continues to rely on the Ramsey model. However, we interpret the elasticity of consumption *(φ)* as applying to relative valuations of consumption over time or the rate of inequality aversion (RIA) but not, as is often commonly assumed, to the relative rate of risk aversion (RRRA). For clarity, we label this the "Ramsey growth model."

Often, IAMs employ the "Ramsey/C-CAPM" approach in which the elasticity of consumption *(φ)* represents both the RIA and the RRRA. This is the approach of NPP and Rennert et al. (2022), for example. The Ramsey/C-CAPM approach is theoretically appealing because it unifies choice over time and over uncertain states of the world. However, that approach fails to generate a realistic risk premium (hence, the equity premium puzzle), and for this reason is not used in DICE-2023. Other extensions, such as the Epstein-Zin specification, introduce different RIA and RRRA, but these do not solve the equity premium puzzle without raising new complications. (This topic is discussed in detail below in this section and in Part E.)

As we use the term, the Ramsey growth model does not include any risk aversion in deriving what is called "the precautionary effect." Rather, alternative growth paths generate alternative discount rates (as per Weitzman, 1998). While we use the terminology of the precautionary effect, the interpretation is that it is an adjustment for uncertain growth and for inequality aversion. While we could add a separate approach to account for risk aversion, we have found no satisfactory unified model and choose to take the simpler CAPM approach, which is not rigorously connected to the Ramsey growth model but has a firm empirical foundation.

The modeling relies upon discount rates and their associated discount factors to calculate present values, optimal policies, and variables such as the social cost of carbon. The "discount factor," D_T, is the factor applied to future values to obtain the present value of a value in time T discounted back to time

0. In a deterministic framework, the discount factor is the product of the one-period discount factors. In this discussion, r_t are period-to-period rates of return from period *(t-1)* to *t*, while R_T are long rates of return from period *0* to period *T* (all in compound annual rates).

$$\text{(A.2)} \quad D_T = \left[\frac{1}{\left(1+R_T\right)^T} \right] = \left[\frac{1}{\left(1+r_1\right)} \right] \left[\frac{1}{\left(1+r_2\right)} \right] \cdots \left[\frac{1}{\left(1+r_T\right)} \right]$$

Because of uncertainty about future growth, the *expected* discount factor will differ from the deterministic discount factor by a term called the "precautionary effect." For example, with two interest rate paths which differ by a constant 4% per year for 100 years, the precautionary effect is to lower the average 100-year long rate by 1.3% points. The effect on the near term is small, with a precautionary effect of only 0.04%/year in the second period (2025). However, with long horizons, the impact of uncertain growth can be substantial, and for that reason the precautionary effect has a major impact on climate policy.

In the approach taken here, we assume that the major uncertainty is about the long-run *trend* rate of growth of per capita consumption. More precisely, we assume that the trend rate of growth of per capita consumption is normally distributed with a constant variance of $\sigma_C{}^2$.. Note that the variance of log consumption will grow as $\sigma_C{}^2 T^2$, which is different from the usual model of the equity premium where the variance of log consumption is constant over time. The next section shows a numerical example to show the impact of random trend growth.

The precautionary component for this distribution of trend growth rates is given by

$$\text{(A.3)} \quad P_T = -½\, \sigma_C{}^2 \varphi^2 T$$

where P_T = is the precautionary effect from time *0* to *T* and $\sigma_C{}^2$ is the variance of the trend growth rate of consumption. In a process where there is uncertainty about the trend rate of growth, the precautionary effect will be larger as the length of period increases because the variance of log consumption increases. For a deterministic model like DICE, we therefore correct the deterministic Ramsey equation to reflect growth uncertainty through adding the precautionary effect.

This procedure generates a sequence of "certainty-equivalent discount rates." This term is used to designate the single discount rate delivering the same discount factor as the expected value from the distribution of uncertain future discount rates (NPP, p. 1019). From (A.1) and (A.3), the certainty-equivalent risk-free discount rates $\left(R_T^f\right)$ are given by (A.4):[2]

$$\text{(A.4)} \quad R_T^f = \rho + \varphi\, \tilde{g}_T - ½\, \sigma_C{}^2 \varphi^2 T$$

To calculate the precautionary effect, we examine two procedures. The first is based on the calculations of NPP. These take estimated future growth rates from their Monte Carlo draws and the implied future interest-rate structure to estimate numerically the precautionary component. A second approach takes the standard formula in (A.3) for the precautionary effect from a model with a normal distribution of trend growth rates. The two approaches give reasonably similar estimates of the precautionary effect, and we therefore take equation (A.3) as computationally simpler and easier to implement and test. For a comparison of the two approaches, see the derivation of the precautionary effect in Part D.

The key parameters of the precautionary effect are the variance of the consumption growth rate and the consumption elasticity. The variance is estimated in several studies (e.g., Christensen. Gillingham, and Nordhaus 2018 and Müller, Stock, and Watson 2022). The studies have estimates of the standard deviation of trend per capita consumption growth to 2100 in the range of 1.0% to 1.2% per year. For our modeling, we assume that trend growth of consumption per capita follows a normal distribution with a mean of 2% per year and standard deviation of 1 percentage point per year. Part D describes the calculation of the precautionary effect in detail. Part F provides updated estimates of future economic growth.

In making calculations for DICE-2023, we rely on two components of the discount rate: a risk-free rate and an adjustment for investment risk. A broad consensus exists that the risk-free real return on investment is in the range of

[2] See, e.g., Prest (2023) equation (1) or Gollier (2016) equation (37), where we note in reference to the latter that our approach to discounting uses the CAPM rather than CCAPM approach for adjustments to the risk profile of climate investments, as discussed below.

0 to 2% per year over the last century. We take 2% per year to be the rate for long-term risk-free investments, which is the rate that has prevailed over the last century or so except for the most recent period.

Empirical evidence indicates that the return to risky assets (such as corporate capital or an unleveraged portfolio of corporate equities) is substantially higher than the risk-free rate. For example, the post-tax average rate of return on US corporate capital has averaged around 7% per year over the period from 1948 to 2022. The real return on a deleveraged portfolio of large US public corporations was 6% per year for the same period. The underlying data are presented in Part E.

At this point, we confront the "equity premium puzzle." This puzzle is that the volatility of consumption cannot rationalize the high risk premium (of 5% per year in our estimates) within the standard model (the C-CAPM model). Most studies examine the equity premium, but the puzzle remains for capital as well as equity. Given the failure of the C-CAPM model, we adopt the estimates from the CAPM model, which examines the correlation of investment risks with the market risk rather than the consumption risk. This leads to the assumption of a risk premium of 5% per year in the DICE-2023 model. Note that our approach differs from those that take the C-CAPM approach (such as Gollier 2014 and NPP).

In the new DICE-2023 specification, the discount rate includes an adjustment for the non-diversifiable risk of climate investments. Risky climate investments, primarily those to reduce emissions and reduce future damages, are introduced through the concept of the climate beta. The climate beta measures the extent to which climate investments (such as renewable power) share the non-diversifiable risk characteristics of the economy's aggregate investments. A climate beta of zero indicates that the risks on climate investments are uncorrelated with market returns; a climate beta of one indicates that climate investments have risk properties similar to those of the aggregate economy. We take our estimate of the climate beta from Dietz et al. (2020), which estimates a long-run climate beta of 0.5, so one with an intermediate correlation with market risks. A more extensive discussion of the climate beta and the reason for our estimate is given in Part C.

For our purposes, we assume that the near-term risk-free long-term rate is 2% per year and the capital risk premium is π = 5% per year. With a climate beta of 0.5, this implies a near-term risk-adjusted certainty-equivalent discount rate on climate investments of 2% + 0.5 x 5% = 4.5% per year. We note that the precautionary adjustment is taken to be zero in this illustrative calculation as near-term consumption growth trend uncertainty is minimal.

To calibrate the model requires estimates of φ and ρ. These are estimated by first calculating the deterministic risk-free rate of return from (A-1) above (where we note that using the certainty-equivalent rate equation (A-4) would yield equivalent results again due to the small level of near-term growth uncertainty). We constrain $\rho \geq 0.1\%$ per year to ensure long-run convergence and for consistency with the estimates in NPP; we then incorporate the DICE estimates of near-term growth in per capita consumption of 2% per year. These parameters lead to the bound that ρ = 0.1% per year. Solving for φ gives the following:

$$(A.5) \quad \varphi = \frac{\left(R^f_{2020} - \rho\right)}{g_{2020}} = \frac{(0.02 - 0.001)}{0.02} = 0.95$$

This then implies that the average annual discount rate on climate investments (R_T^{CLIM}) from *0* to *T* is:

$$(\mathrm{A.6}) \quad R_T^{CLIM} = \rho + \varphi \tilde{g}_T - ½\varphi^2 {\sigma_C}^2 T + \beta^{CLIM}\pi$$

With an estimated climate beta of 0.5 as well as other estimates, this leads to a near-term *(T = 0)* rate of return on climate investments of 4.5% per year.

$$\begin{aligned}(A.7) \quad R_0^{CLIM} &= 0.001 + (0.95)(0.02) - ½(0.95)^2(.01)^2(0) + (0.5)(0.05) \\ &= 0.045\end{aligned}$$

This calculation gives an estimate of 4.5% per year for the near term.

In order to implement (A.6) in DICE, we replace the pure rate of social time preference in equation (A.1) with a “risk-adjusted rate of time preference” designated by $\rho_T^* = \rho - ½\varphi^2 {\sigma_C}^2 T + \beta^{CLIM}\pi$.

Growth v level precautionary effect: An example

For those used to the standard equity-premium model, the precautionary calculation used here may be unfamiliar. In the standard model (such as Mehra and Prescott 1985), the growth of log consumption is an i.i.d. random variable. We can illustrate the difference between the level effect and the growth effect on the precautionary term with a simple example. For this example, we assume that the consumption elasticity is $\varphi = 1$. In the *level effect,* we assume that growth of log consumption has a zero trend with i.i.d. random normal disturbances over time with a standard deviation of $\sigma_{C,L} = 0.02$. The precautionary effect in this case is $P_T = -½\, \sigma_{C,L}{}^2\varphi^2 = -½ * 0.0004 * 1 = -0.0002$, which is constant over time.

With the *growth effect*, we assume that the trend rate of growth has a normal distribution with a zero mean and a standard deviation of $\sigma_{C,g} = 0.01$ per year. In the growth case, the precautionary effect is

$$P_T = -½\, \sigma_{C,g}{}^2\varphi^2 T = -½ * (0.0001) * 1 * T = -0.00005 * T.$$

Thus, while the level precautionary effect is constant at -0.02%/year, the growth precautionary effect starts out at -0.005%/year in year T = 1 and increases to -1.5%/year after 300 years. The reason is that the variance of log consumption across paths is constant with the level effect and increases with time with the growth effect.

Figure A-1 shows six paths, which are displayed on a logarithmic vertical scale. The three paths that are closely packed in the middle are the three randomly chosen paths with only the level effect. The three dispersed paths are three paths with randomly selected constant growth rates. As is clear from this example, the standard deviation of the growth paths is linear in time, while the standard deviation of the level paths is roughly constant over time.

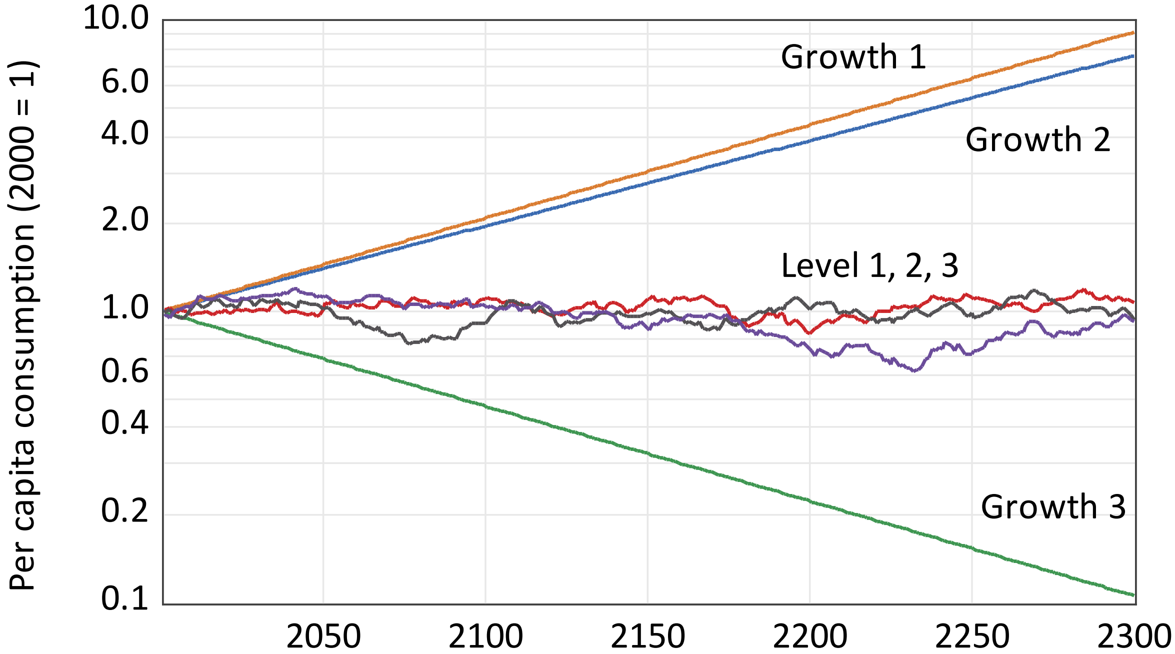

Figure A-1. Figure shows per capita consumption with randomly selected paths. These show three paths with random level effects and three paths with random growth effects. Note that the three growth paths diverge over time and have a standard deviation that is approximately linear in time. This indicates why the precautionary effect is linear in time in DICE under the assumption of random trend growth rates.

Part B. **GAMS programming for discounting**

The following is the GAMS programming code in DICE-2023 that determines discounting (file is “DICE2023-b-4-3-10.gms”).

```
PARAMETERS
[other]
** Preferences, growth uncertainty, and timing
        betaclim  Climate beta                                       / 0.5  /
        elasmu    Elasticity of marginal utility of consumption      / 0.95 /
        prstp     Pure rate of social time preference                /.001/
        pi        Capital risk premium                               / .05  /
        rartp       Risk-adjusted rate of time preference
        k0        Initial capital stock calibrated (1012 2019 USD)   / 295  /
        siggc1    Annual standard deviation of consumption growth    / .01 /
** Scaling so that MU(C(1)) = 1 and objective function = PV consumption
        tstep       Years per Period                                / 5  /
        SRF         Scaling factor discounting                      /1000000/
[other]
PARAMETERS
[other]
** Precautionary dynamic parameters
        varpcc(t)         Variance of per capita consumption
        rprecaut(t)       Precautionary rate of return
        RR(t)             STP with precautionary factor
        RR1(t)            STP factor without precautionary factor;
** Time preference for climate investments and precautionary effect
        rartp            = exp( prstp + betaclim*pi)-1;
        varpcc(t)        =  min(Siggc1**2*5*(t.val-1),Siggc1**2*5*47);
        rprecaut(t)      = -0.5*varpcc(t)*elasmu**2;
        RR1(t)           = 1/((1+rartp)**(tstep*(t.val-1)));
        RR(t)            = RR1(t)*(1+rprecaut(t))**(-tstep*(t.val-1));
[other]
VARIABLES
[other]
        TOTPERIODU(t)    Period utility
        UTILITY          Welfare function
        RFACTLONG(t)
        RSHORT(t)        Real interest rate with precautionary(per annum year on year)
        RLONG(t)         Real interest rate from year 0 to T
;
[other]
*Economic variables
        RSHORTEQ(t)       Short-run interest rate equation
        RLONGeq(t)        Long-run interest rate equation
        RFACTLONGeq(t)    Long interest factor
* Utility
        TOTPERIODUEQ(t)   Period utility
        PERIODUEQ(t)      Instantaneous utility function equation
        UTILEQ            Objective function      ;

[other]
**** Equations of the model
[other]
**Economic variables
 RFACTLONGeq(t+1)..    RFACTLONG(t+1) =E= SRF*(cpc(t+1)/cpc('1'))**(-elasmu)*rr(t+1);
 RLONGeq(t+1)..        RLONG(t+1)     =E= -log(RFACTLONG(t+1)/SRF)/(5*t.val);
 RSHORTeq(t+1)..       RSHORT(t+1)    =E= -log(RFACTLONG(t+1)/Rfactlong(t))/5;
** Welfare functions
 periodueq(t)..        PERIODU(t)     =E= ((C(T)*1000/L(T))**(1-elasmu)-1)/(1-elasmu)-1;
 totperiodueq(t)..     TOTPERIODU(t)  =E= PERIODU(t) * L(t) * RR(t);
 utileq..              UTILITY        =E= tstep * scale1 * sum(t,  TOTPERIODU(t)) + scale2;
```

Part C. **Calculation of the climate beta**

Estimates of the climate beta are scarce. The most comprehensive estimate is from Dietz et al. (2018), which uses a combination of theory and integrated assessment modelling (IAM) to estimate the climate beta. While their IAM empirical estimates rely on an earlier version of DICE, the study uses parametric uncertainties for the DICE model and adds further potential parametric uncertainties, such as those for catastrophic damages. Their long-run estimate (to 2215) for all uncertainties is beta = 0.49 (see their Table 3). However, depending upon the combination of uncertainties, the estimates range as high as 1.10. The surprisingly high beta is driven largely by the dominance of the uncertainty about TFP growth, which they assume to be normal with a standard deviation of 0.9% per year. This estimate is close to the estimate we use (1% per year) for determining the precautionary effect in DICE-2023.

Note that from an analytical point of view, as they show, the climate beta is likely to be at least 1 if the uncertainties are driven largely by uncertainty of the growth of productivity (and therefore per capita consumption). The surprising result is, as Dietz et al. clearly explain, "the positive effect on the climate beta of uncertainty about exogenous, emissions-neutral technological progress overwhelms the negative effect on the climate beta of uncertainty about the carbon-climate-response, particularly the climate sensitivity, and the damage intensity of warming." (p. 258)

A different approach to estimating the climate beta comes from estimates of industry betas for sectors where the major mitigation investments are likely to be made. For example, replacing electricity generated by fossil fuel will require generation by renewables. If we take power, utilities, and air transport as three highly carbon-intensive industries, we can examine estimates of CAPM betas from Professor Aswath Damodaran (https://pages.stern.nyu.edu/~adamodar/). For these three industries, the average unleveraged betas are calculated to be 0.49, which is reasonably close to the estimates we use from Dietz et al.

We conclude that we assume a climate beta of 0.5 for the present version of DICE-2023. However, we emphasize that this estimate contains considerable uncertainty both in terms of its empirical basis (since it is model-based rather than historically-based) and also that the estimate is strictly speaking appropriate for the C-CAPM framework where it was derived.

Part D. **Estimates of the precautionary effect**

For long-term projections, it is important to include the impacts of the uncertainty about future economic growth on discounting, a factor that has been ignored in earlier vintages of the DICE model. This note uses the approach of Newell, Pizer, and Prest, "NPP" (2022) to estimate the impact of growth uncertainty. We call this higher-order impact "the precautionary term." Note, however, as discussed in the introduction to this Note, that this term is widely used but in the present context refers to the impact of growth uncertainty on discounting that operates through the intertemporal elasticity, not risk aversion.

The basic analysis is well known but is usefully described in Gollier (2016). In the standard Ramsey model where P_T is the precautionary effect from t = 0 to T, φ is the intertemporal consumption elasticity, and the trend consumption growth rate follows a normal distribution with a variance of σ_c^2, the precautionary effect is given by:

$$(D.1) \quad P_T = -½\varphi^2\sigma_c^2 T$$

The precautionary effect is introduced as an exogenous variable since there is no uncertainty in the DICE model. This term is calculated using estimates of the uncertainty of trend growth of per capita consumption along with assumptions about the key parameter of φ. There are several estimates of the long-run path of growth uncertainty, but they are all reasonably consistent.

Monte Carlo with lognormal output

The simplest approach uses a Monte Carlo simulation. For this simulation, we assume that trend growth in consumption is distributed as $\mathcal{N}$(mean growth, standard deviation of trend growth) = $\mathcal{N}$(0.02, 0.01), ρ = .001, φ = .95, and N = 10,000 replications. To be clear, draw one might have constant consumption growth rates of 1.6% per year, draw 2 perhaps 2.5% per year, and so forth. Table D-1 shows the results for long-run real returns. The parametric assumptions are in the legend of the table. Here are the key points:

- The deterministic risk-free discount rate is a constant 2% per year. This reflects the constant expected rate of growth of output with the assumed values of ρ and φ.

- The certainty-equivalent discount rate is lower than the deterministic rate by the precautionary rate. At 80 years (2100 in the model), with the empirical assumptions above, the certainty-equivalent discount rate would be 36 basis points lower than the deterministic rate as defined here and more generally in the literature. By 280 years (2300), the precautionary effect is 121 basis points. Because of the assumptions, the precautionary rate is linear in time.
- The numerical calculation using the Monte Carlo is virtually identical to the formulaic calculation of the precautionary effect.

Period from present	R^{CE}	R^{DETER}	$R^{PRECAUT}$	$R^{PRECAUT}$calc	$R^{PRECAUT}$ minus RP^{RECAUT}calc	Sample	sigma(g)	ρ	φ
1	1.99%	2.00%	0.004%	0.005%	0.000%	10,000	0.01	0.001	0.95
30	1.86%	2.00%	0.134%	0.135%	-0.001%	10,000	0.01	0.001	0.95
80	1.64%	2.00%	0.358%	0.361%	-0.003%	10,000	0.01	0.001	0.95
180	1.20%	2.00%	0.798%	0.812%	-0.014%	10,000	0.01	0.001	0.95
280	0.78%	2.00%	1.215%	1.264%	-0.048%	10,000	0.01	0.001	0.95

ρ =	0.1%
φ =	0.95
N =	10,000
g (p.c. cons)=	N(.02, .01)
R^{CE} =	Certainty equivalent discount rate
R^{DETER} =	Deterministic rate (rate implied by average growth rate)
$R^{PRECAUT}$ =	Precautionary component determined by concavity ($-\frac{1}{2}\varphi^2\sigma_T^2$)
$R^{PRECAUT}$_CALC =	Precautionary component determined by lognormal formula ($-\frac{1}{2}\varphi^2(\sigma_{C,T})^2$)

Table D-1. Estimates of the precautionary effect using a normal distribution for trend growth of consumption[i].

Note that φ in the Table refers to the consumption elasticity.

NPP Estimates

A second approach uses the growth estimates from NPP (2022). For these, we use the results of the NPP Monte Carlo of consumption growth based on Muller et al. (2022) provided by the authors to calculate the precautionary effect.[ii] Note that these estimates differ in the levels from Christensen et al. (2018) or Rennert et al. (2022) but the variance is reasonably close. These use the same parameters as the first approach, but in NPP there is a random element to the level of consumption along with a slightly changing variance in the trend growth rate.

Figure D-1 shows the variance over time of the analytical approach above ("DICE variance") along with the variance determined by the NPP draws. The NPP has a higher short run variance because of the random element in the level of consumption, while the trend is slightly lower in NPP, with a cross-over at about 120 years. Note as well that the DICE variance is linear by assumption.[iii] Since the difference between the two precautionary effect is close to half of the difference between the two variance estimates, the precautionary effect with be close to 10 basis points higher for the NPP estimates in the first 50 years, then approximately the same for the next century or so, then lower after about 200 years.

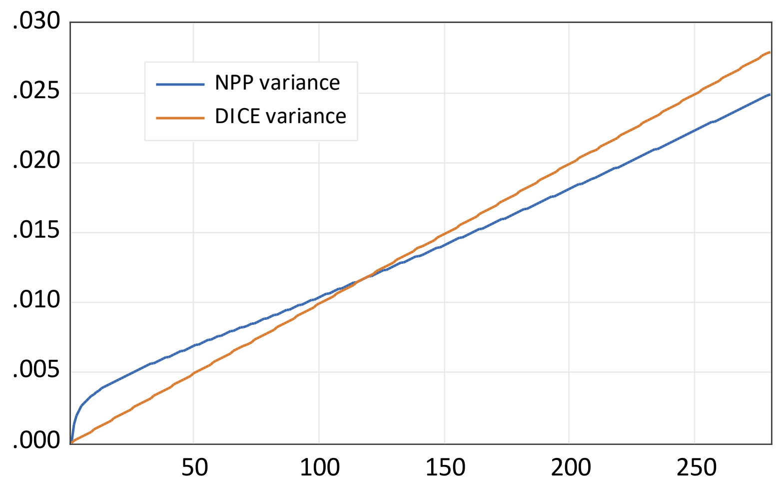

Figure D-1. Comparison of the variance of the growth rate of consumption under the two approaches.

Part E. **Historical rates of return**

We have gathered in Table E-1 data on total returns for major assets classes in the US for the 1927- 2022 period with thanks to the Stern School. Note that these are total returns including dividends, interest, and capital gains or losses.

Additionally, Table E-2 provides estimates of the rate of return (total earnings on capital, i.e., profits plus interest) as a percentage of the current replacement cost of the net stock of fixed assets of the US corporate sector based on data from the US BEA.

Year	*S&P 500*	*3-month T.Bill*	*US T. Bond*	*Baa Bond*	*Real Estate*
Arithmetic Average Historical Return					
1928-2022	11.5%	3.3%	4.9%	7.0%	4.4%
1973-2022	11.7%	4.4%	6.6%	8.8%	5.5%
2013-2022	13.6%	0.8%	0.5%	3.8%	7.7%
Arithmetic Average Real Return					
1928-2022	8.3%	0.3%	1.9%	3.9%	1.3%
1973-2022	7.6%	0.4%	2.6%	4.7%	1.5%
2013-2022	10.8%	-1.8%	-1.9%	1.3%	4.9%

Table E- 1. Total returns on US financial assets

Source: Data from NYU, Stern School, https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html in file "stern-returns-2023.xlsx"

[Real returns are not available as geometric averages from the source.]

	Return before taxes (all corps)	Return after taxes (all corps)	Return before taxes (non-fin corps)	Return after taxes (non-fin corps)
1948 - 2022	11.0%	7.6%		
1992 - 2022	10.6%	8.6%		
1998 - 2017	10.2%	7.9%	8.8%	7.2%
2012 - 2022	11.1%	9.3%		

Table E- 2. Rates of returns on capital of US corporations
Rates of return are total capital income divided by replacement cost of capital, both in current prices. They are conceptually real returns. Data from the BEA. These include S corporations. A correction for the share of S corporations reduces the return over the last two decades by about 50 basis points.[iv]

Based on these findings, we conclude the following:

1. The *short-run* risk-free return (measured as the real return on short Treasury securities) has averaged close to zero per year for most of the last century, although it has been lower in recent years.
2. The *long-run* risk-free rate of return (measured on 10-year Treasury bonds) has averaged around 2% per year over the total period, although it has been sharply lower in the last decade. The real interest rate on 10-year TIPS has a shorter period and has recently risen back to the earlier pre-financial crisis level of about 2% per year real.
3. The best estimate of the after-tax real return on capital (measured in the US corporate sector) has been 7 – 9% per year over the last half-century. Unlike financial returns there has been no major change in these returns in the last two decades.
4. Corporate equities are currently unleveraged with respect to total bond-type assets. The aggregate US corporate non-financial balance sheet has a long-debt/equity ratio of approximately 30% with a roughly equal short-debt/equity ratio. We therefore treat corporate equities as unleveraged.

The capital premium

We define the "capital premium" as the difference between the expected return on aggregate economy-wide assets and the risk-free rate of return. Based on the current balance sheet of the corporate sector, we assume that the capital premium is the same as the well-studied equity premium. [3] We observe these data only for the US non-financial corporate sector and assume that these values apply to the entire global financial structure. Based on the estimates above, we assume that the rate of return on risky assets in the US is 7% per year and has been relatively stable. That rate applies not only to financial returns but also to corporate capital. Based on the estimate of a risk-free long-run rate of return of 2% per year, we calculate the capital premium to be 5% per year.

We note at this point the difficulty of estimating various rates of return given the volatility of the series and the limited sample size. As an example, we can calculate the capital premium as the difference between the risky rate of return on corporate equities and the risk-free return. For 1928 – 2022, the geometric average capital premium is 5.7% per year, with a standard deviation of 2.0% per year. If we assume that the capital premium is i. i. d. normal, then the (5, 95) %ile for the mean return is (3.7%, 7.7%) per year. While this range is well above zero and above the estimates of the equity or capital premium from C-CAPM models, it is clear that there is considerable uncertainty about the estimate of the capital premium.

[3] The literature generally deals with the "equity premium puzzle," dating back to 1985 with Rajnish Mehra, and Edward C. Prescott (1985) and Rajnish Mehra (2008). We use the term capital rather than equity to emphasize that it applies to the return on capital more generally as is appropriate for Ramsey-type models.

Part F. **Revised estimates of economic growth**

Projections of future growth of per capita output and consumption have been revised in October 2023 in light of recent data and research. The results relative to earlier models and versions yield a slightly lower growth rate in the early years (to 2050), but more rapid growth in per capita global output over the model horizon. Note that the growth rates after 2150 make little difference in scenarios with strong policies, but they can affect the base (current policy). The reason is that most strong policies have virtually 100% emissions control after a century, so growth projections after that will have little to no effect on emissions, concentrations, and temperature.

Historical data and current projections are shown in Table F-1. The DICE-2023 projections in the last column are intermediate between the CGN/MSW results and the Rennert et al. (2022) blended statistical and expert elicitations. Figure F-1 visualizes the comparison between DICE-2023, Rennert et al. ("RFF-SPs"), and the NPP projections along with their respective 5-95th percentile ranges.

	Historical data (geometric mean, percent per year)						
	MSW	IMF	CGN	MSW	Renn-med	Renn-avg	DICE-2023
1901-1931	1.3						
1931 - 1960	1.9						
1961 - 1990	2.1	2.3					
1991 - 2022		2.0					
	Projections (geometric mean, percent per year)						
2020 - 2050			2.6	1.9	1.5	1.5	1.9
2020 - 2100			2.0	1.9	1.5	1.5	1.8
2020 - 2200				1.9	0.9	1.1	1.7
2020 - 2300				1.9	0.9	1.1	1.6

MSW = Muller, Stock, and Watson
IMF = International Monetary Fund from historical data base
CGN = Christensen, Gillingham, and Nordhaus
MSW = Muller, Stock, and Watson
Renn-med= Rennert et al., Figure 6 (from background data)
Renn-avg = Rennert et al., Figure 6 (from our calculations)
DICE-2023 = from base run of version b-4-3-6

Table F-1. Estimates of the growth in per capita output from different studies.[v]

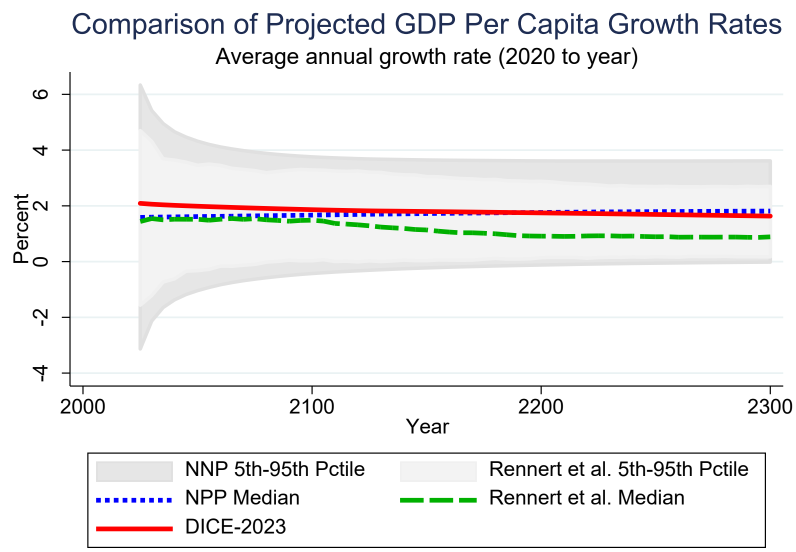

Figure F-1: Comparison of DICE-2023 GDP per capita growth projections with NPP and Rennert et al. medians and 5-95th percentile ranges. Growth is measured as annual average percent growth from 2020 through each depicted year.

A key parameter for calculating the precautionary effect on discounting is the uncertainty about future growth. For this parameter, we examined the dispersion of forecasts. Standard deviations or quantiles of the distributions of growth rates were tabulated in different studies, and these yielded the estimated standard deviations of trend growth in per capita global output shown in Table F-2.

	Estimated standard deviation of trend growth (% points/year)				
	CGN	MSW	NPP	Renn	DICE-2023
2020 - 2050	1.1	1.0	1.0	0.9	1.0
2020 - 2100	1.1	1.0	0.8	0.8	1.0
2020 - 2200			0.8	0.7	1.0
2020 - 2300			0.9	0.6	1.0

CGN = Christensen, Gillingham, and Nordhaus
MSW = Muller, Stock, and Watson
NPP = Newell, Pizer, Prest
Renn = Rennert et al., Figure 6, calculated from (5,95) percentiles.
DICE-2023 = from base run of version b-4-3-6

Table F-2. Estimates of the uncertainty of the growth in per capita output from different studies. [vi]

The estimated standard deviation of the growth rate from 2020 to 2100 ranges from 0.8% to 1.1% points per year. The standard deviation for longer periods ranges between 0.6% to 0.9% points per year depending on period and study. These are significantly larger than the historical variability in growth rates, such as the difference in long-period growth rates in MSW of around 0.4% point per year or estimates of the 1950 – 2022 standard deviation of the growth rate of 0.3% - 0.5% point per year.

For our estimates, we have chosen the estimates from GGN, NPP, and MSW because of our preference for statistical techniques in deriving variability estimates. The main effect of the higher estimate of the uncertainty in the trend growth rate will be to increase the precautionary effect and thereby to lower long-run discount rates, primarily after 2100. For modeling purposes, we choose a constant uncertainty of 1.0% point per year because of the simplicity of modeling and transparency of interpretation. This rate is close to the near-term uncertainty for most estimates but lower than estimates after 2100. Since the precautionary impact is proportional to the variance times the squared time-from-present, this assumption will tend to overestimate the precautionary impact after 2100.

Background

Table F-3 shows the estimates from Christensen et al. (2018). The estimates that are used are the expert results for the world.

Table 1. Expert and low-frequency estimates by region and time horizon

		2010–2050							2010–2100						
Region	Statistic	10th	25th	50th	75th	90th	μ	σ	10th	25th	50th	75th	90th	μ	σ
World	Expert TM	1.17	1.80	2.59	3.23	3.92	2.54	1.07	0.60	1.36	2.03	2.85	3.47	2.06	1.12
	Expert (SD)	1.37	0.97	0.75	0.85	0.92	—	—	2.14	1.14	0.84	0.94	1.06	—	—
	Low freq	1.2	1.7	2.2	2.7	3.3	2.23	0.99	1.2	1.7	2.2	2.7	3.3	2.23	0.98
High	Expert TM	0.56	1.23	1.76	2.30	2.75	1.72	0.84	0.27	0.95	1.46	2.08	2.57	1.47	0.88
	Expert (SD)	1.38	0.82	0.68	0.69	0.77	—	—	1.55	0.92	0.62	0.73	0.84	—	—
	Low freq	0.7	1.4	2.0	2.5	3.0	1.90	0.99	1.0	1.5	2.0	2.4	2.8	1.92	0.89
Middle	Expert TM	0.93	1.76	2.67	3.36	4.11	2.57	1.23	0.34	1.30	1.98	2.72	3.45	1.96	1.18
	Expert (SD)	1.47	0.91	0.77	0.68	0.77	—	—	2.15	0.83	0.81	0.64	0.97	—	—
	Low freq	0.5	1.2	1.9	2.6	3.4	1.92	1.27	0.5	1.3	1.9	2.6	3.4	1.94	1.41
Low	Expert TM	1.05	2.23	3.41	4.25	5.12	3.21	1.57	0.62	1.72	2.53	3.45	4.57	2.58	1.49
	Expert (SD)	1.70	1.25	0.78	0.95	1.26	—	—	2.10	1.23	1.10	1.05	1.55	—	—
	Low freq	2.8	4.3	6.1	8.1	10.2	6.34	3.00	1.8	3.5	5.5	7.9	10.7	5.95	3.74
United States	Expert TM	0.60	1.14	1.75	2.18	2.63	1.66	0.79	0.49	0.91	1.53	2.04	2.64	1.52	0.84
	Expert (SD)	1.18	0.76	0.75	0.69	0.68	—	—	1.28	0.76	0.76	0.65	0.78	—	—
	Low freq	0.9	1.5	2.2	2.8	3.4	2.14	1.09	1.2	1.7	2.1	2.5	2.9	2.03	0.84
China	Expert TM	1.51	2.81	4.23	5.19	6.31	4.01	1.85	0.89	2.02	2.93	3.87	4.87	2.92	1.51
	Expert (SD)	1.83	1.57	1.11	1.18	1.35	—	—	2.29	1.59	1.25	1.15	1.38	—	—
	Low freq	1.6	3.9	6.6	9.5	12.7	6.93	4.61	0.7	3.1	5.7	8.9	12.7	6.33	5.36

Note: Expert and low-frequency estimates by region and time horizon. Expert TM and SD are the trimmed mean and SD of expert forecasts at each quantile. Low-frequency forecasts are Bayes estimates at each quantile. Notation is that μ and σ are the means and SDs of the respective forecast distributions. Expert μ and σ are estimated using a fitted normal distribution (see *SI Appendix* for details).

Table F-3. Projections from Christensen et al. (2018).

Simulations by NPP using Muller et al. (2022) produce the results in Figure F-1 for the distribution of per capita growth, slightly below 2% per year.:

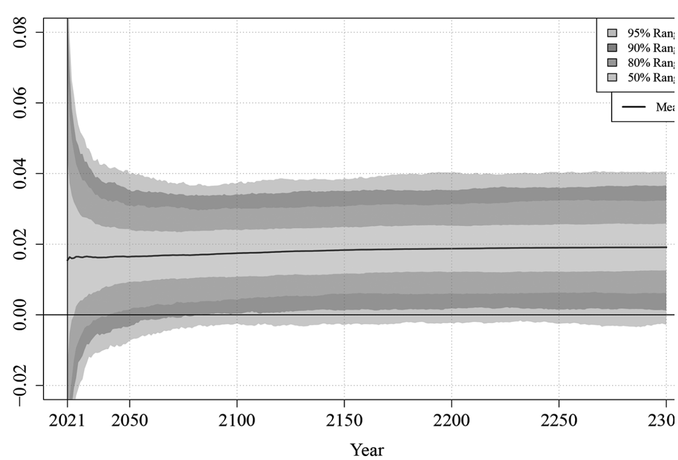

Figure F-1. Distribution of projections of per capita output from NPP

We can compare this to Rennert et al. (2022). The estimates use the "RFF-SPs" to calculate both the median and the uncertainty of future growth rates. These are shown in Figure F-2. Table F-4 shows estimates of the growth rate from Muller et al.

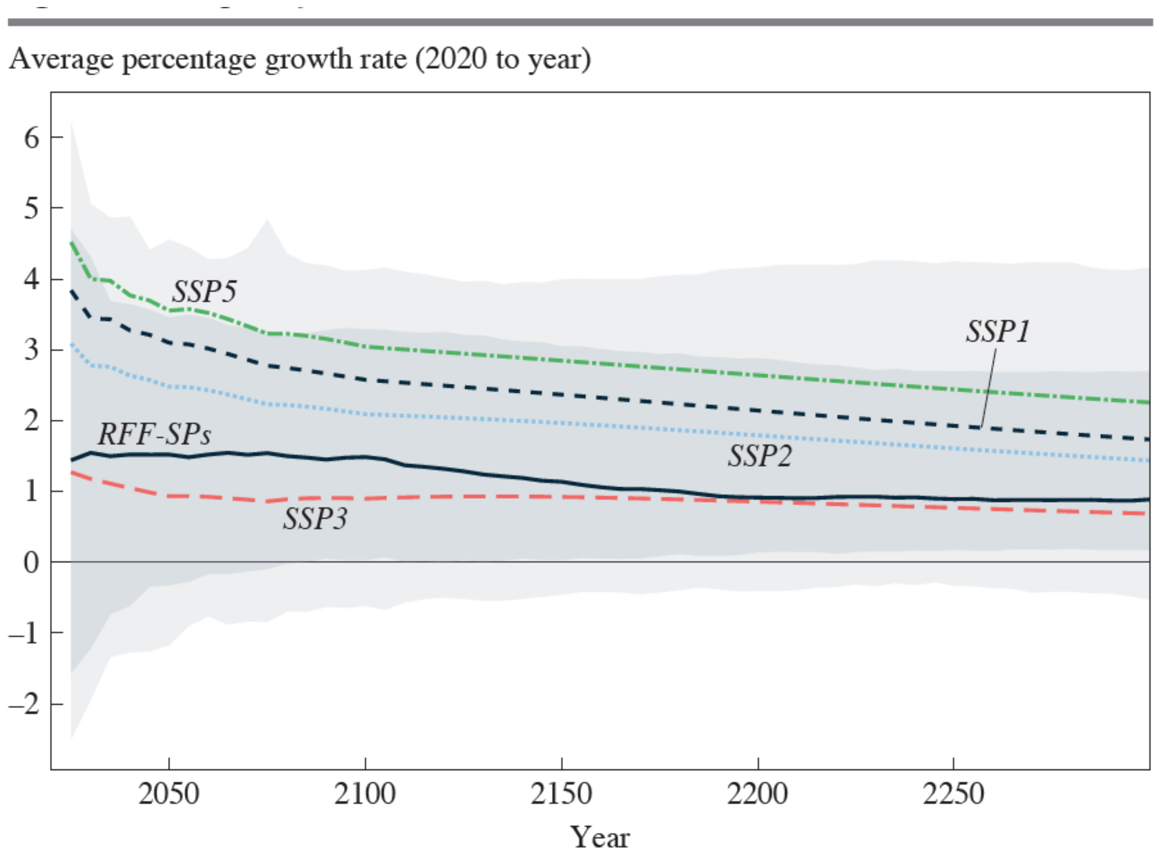

Figure F-2. Distribution of projections of per capita output from Rennert et al.

TABLE 3.—PERCENTILES OF PREDICTIVE DISTRIBUTIONS FOR AVERAGE GROWTH OVER NEXT 50 AND 100 YEARS: POPULATION WEIGHTED AVERAGE OF COUNTRY GROWTH RATES, 2017

	Percentiles: 50-year horizon			Percentiles: 100-year horizon		
	0.17	0.50	0.84	0.17	0.50	0.84
Global factor (f_t)	0.92	1.86	2.70	0.92	1.87	2.72
			Global aggregates			
All countries	1.03	2.05	3.00	1.06	2.04	2.96
OECD	0.74	1.69	2.62	0.79	1.73	2.62
Non-OECD	1.05	2.13	3.11	1.11	2.10	3.04

Table F-4. Estimate of the distribution of trend growth (σ_m) from MSW.

Note that the estimates of the uncertainty of the growth rate are reasonably consistent across the different approaches and studies.

Additionally, we look at a simple econometric forecast of future growth rates using the combined IMF and Maddison estimates for 1950 – 2022. We use four different specifications for projecting the global growth rate, g_t. For each, we fit over the 1950 – 2022 period and then forecast using dynamic forecasts through 2200. For example, in equation (*Spec* 1), the estimate of the coefficient is $\alpha = 2.15 \pm 0.179$. If we calculate the forecast errors to 2100, they produce an error in the log level of output of 1.62. Converting this to the standard deviation of the growth rate, this yields 0.019% points per year in Table F-5.

$$(Spec\ 1) \quad g_t = \alpha + \varepsilon_t$$

$$(Spec\ 2) \quad g_t - g_{t-1} = \alpha + \varepsilon_t$$

$$(Spec\ 3) \quad g_t = \alpha + \beta g_{t-1} + \varepsilon_t$$

$$(Spec\ 4) \quad TSLS: g_t = \alpha + \beta g_{t-1} + \varepsilon_t$$

Table F-5 shows the forecast standard errors of the growth rates from the estimates. They are substantially lower than the techniques shown in Table F-2, but these are overly simple specifications compared to the techniques in Muller et al. Also, perhaps the last seven decades were a tranquil period.

From 2020 to	Spec 1	Spec 2	Spec 3	Spec 4
2050	0.051	0.454	0.297	0.297
2100	0.019	0.332	0.216	0.216
2150	0.012	0.299	0.194	0.194
2200	0.008	0.283	0.184	0.184

Table F-5. Standard errors of forecasts using global growth rates, 1950 – 2022, in percentage points per year[vii]

Part G. **Dual Discounting**

The macroeconomic structure of the DICE model poses difficulties because it has two alternative discount rates, the first on standard capital and the second on climate investments. Because the discount rate on climate investments is the same as that on standard investments in current DICE model (which we call a one-stage approach), we describe in this section a two-stage investment strategy to determine the difference from the one-stage investment strategy. The concern is that the approach used in the model implicitly adopts the same beta for general and climate investments, despite their empirically different risk properties.

In order to gauge the quantitative importance of this simplification, we run a “dual discounting” version of the model which first optimizes only savings rates with the β parameter in equation (A-6) in Part A set to 1 as is most appropriate for general capital investments. In this first stage, emissions control rates are set at their baseline values. In a second stage run, we then fix savings rates at the optimized level from the first stage and optimize over mitigation rates with $\beta^{clim} = 0.5$ as is appropriate for climate investments. This can be done either with a base or an optimal scenario for calculating the optimal savings rate with $\beta = 1$, but the resulting optimal savings rates ratios differ only by a miniscule amount.

Figures G-1 through G-3 show the key results obtained from dual discounting. The optimized control rate, temperature trajectory, and social cost of carbon are slightly different between the one-stage (“benchmark”) and the two-stage (“dual discounting”) estimates. With dual discounting, the investment rates, emissions-control rates, and output levels are all lower than with the one-stage approach. Because of lower output and emissions, the year 2100 atmospheric temperature is lower with dual discounting: 2.58 °C in the one-stage discounting compared to 2.55 °C in the dual discounting. While the differences between the two approaches are small, the sign of the impact on warming may be surprising.

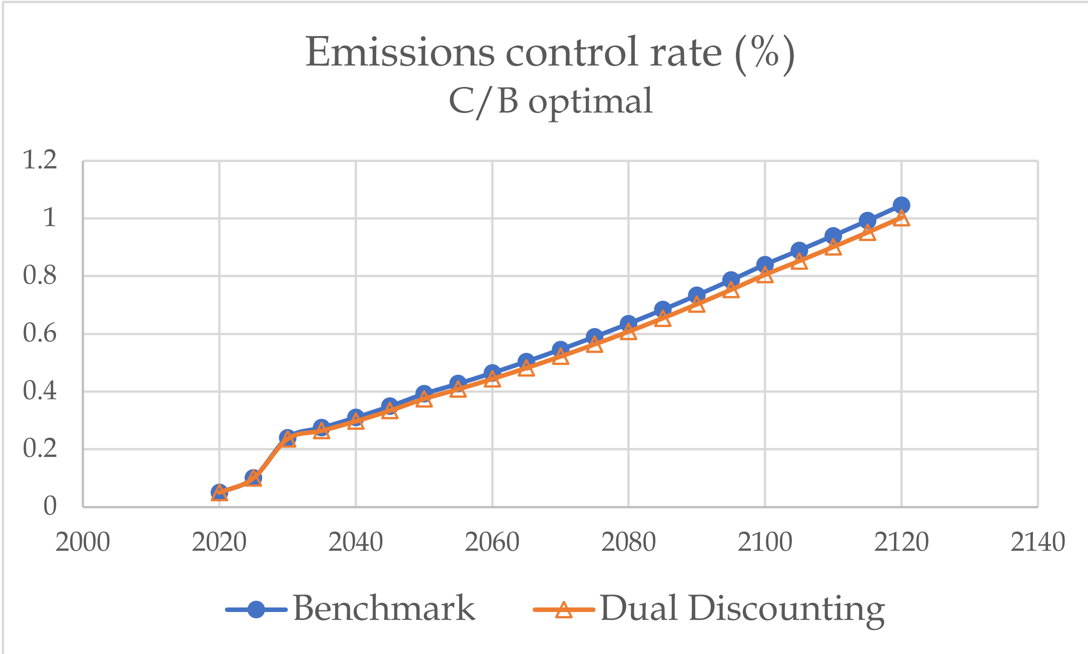

Figure G-1: Emissions control rates in the C/B optimal scenario with the benchmark and dual discounting approaches

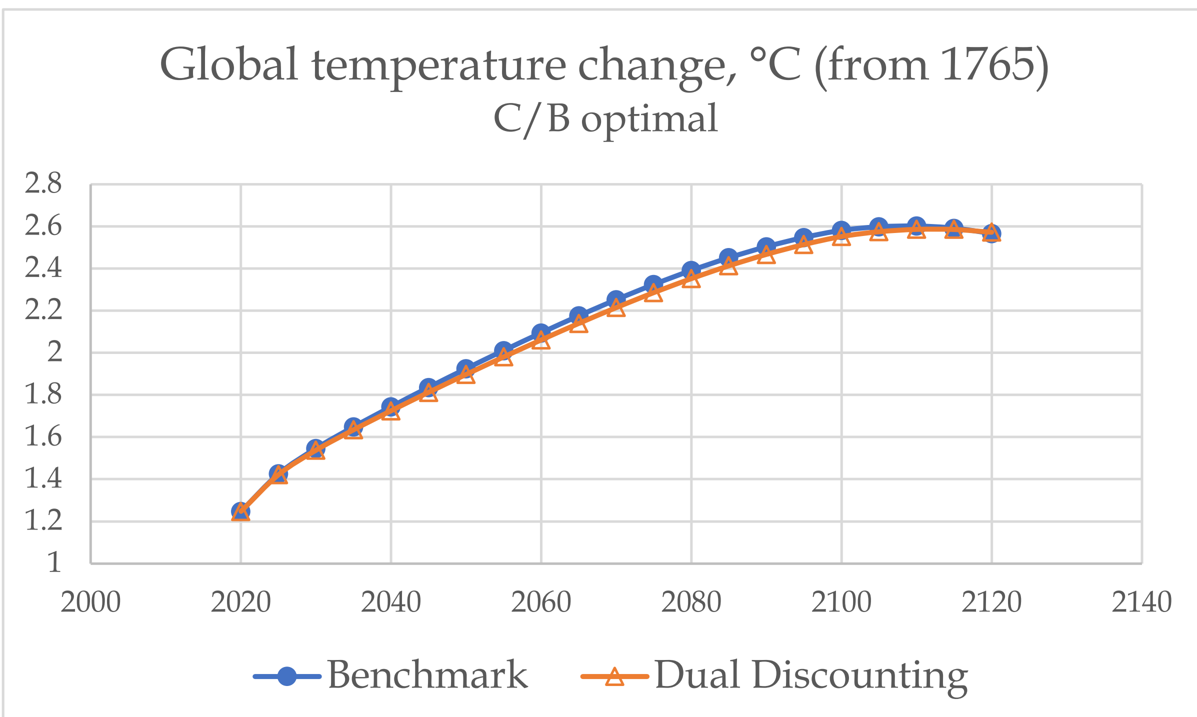

Figure G-2: Global temperature change in the C/B optimal scenario with the benchmark and dual discounting approaches

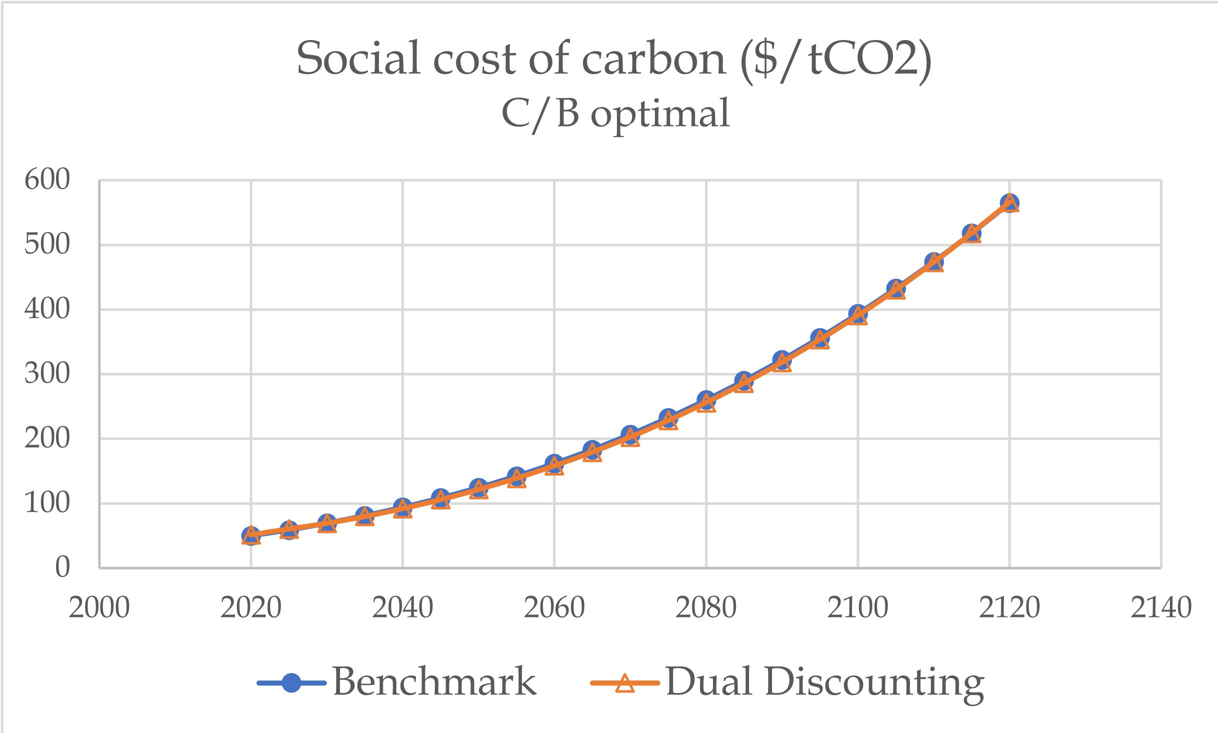

Figure G-3: Social cost of carbon in the C/B optimal scenario with the benchmark and dual discounting approaches

Part H. References

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These endnotes provide the sources for figures and tables.

[i] Source: test-dist-092923.wf1; sdf-exp-v10-npp-02.prg. Tabulated in Results-MOnte-Carlo-090523-u092923a.xlsx.

[ii] Source: figure1_draws-avggrowth.xls.

[iii] Also see Copy of GIVE_discount_rate_decomposition; Results-MOnte-Carlo-090523;

[iv] Source: Capital rates of returns 070623, page "wn-ror-nipa"

[v] Source: gdp-compar-100423.xlsx

[vi] Source: gdp-compar-100423.xlsx

[vii] Source: figure is "fig_forecast_growth" in from program long-term- g-reg-092923.prg and data ghghistory-081223.wf1.