Gaussian Network 高斯网络或高斯图模型

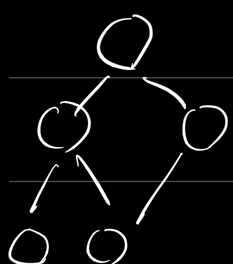
BN: bayesian network. MN: Markov Network. GN: Gaussian Network

PGM: 概率图模型 possibility graphical model

PGM: BN: (有向图. 贝叶斯网络) $\xrightarrow{\text{continuous}}$ GN.
MN: (马尔可夫随机场).

随机变量都是离散的. (BN. MN) discrete

如果随机变量是连续的 → GN. GBN. / GMN.



$x_i \sim N(\mu_i, \Sigma_i)$.

$X = (x_1, x_2 \cdots x_p)^T$ 有p个节点.

$$p(x) = \frac{1}{(2\pi)^{\frac{p}{2}} \cdot |\Sigma|^{\frac{1}{2}}} \exp\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\}$$

高斯网络是一个无向图 ⟷ 对应的 网络来高维高斯分布

每个节点都是一维的随机变量.

↓分析 μ, Σ.

$$\Sigma = (\sigma_{ij}) = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & & \sigma_{2p} \\ \vdots & \vdots & & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_{pp} \end{pmatrix}_{p \times p}$$

$x_i \perp x_j \Leftrightarrow \sigma_{ij} = 0$.

在高斯分布下成立.

边缘独立性是绝对独立.
但是条件独立直接从协方差
矩阵看不出独立性存在.

↓引出概念

$$\Lambda = \Sigma^{-1} = \begin{pmatrix} \lambda_{11} & \cdots & \lambda_{1p} \\ \lambda_{p1} & \cdots & \lambda_{pp} \end{pmatrix}_{p \times p}$$

Λ 叫做 precision Matrix ⇔ information matrix.

$x_i \perp x_j \mid -\{x_i, x_j\} \Leftrightarrow \lambda_{ij} = 0$.

变成条件独立

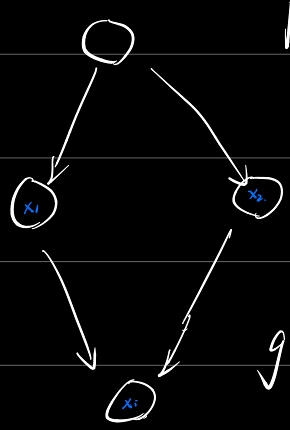
条件独立性：$x_A \perp x_B \mid x_C$. 如果没有条件独立性假设 ⇒高斯图⇔高维高斯分布

则图中所有点之间都有关系. ⇒ 完全图 没有意义

第二讲

高斯贝叶斯网络 → 连续型的 PGM.

↳ 有向图：GBN.



$P(X) = \prod_{i=1}^{p} p(x_i \mid \underbrace{x_{pa(i)}}_{\text{集合：父亲集合}})$ BN的因子分解.

GBN: is based on linear Gaussian model.

↓ global Model. ↓ local Model.

$x_{pa(x_i)} = \text{set}(x_1, x_2)$

$x = (x_1, \cdots x_p)^T \quad X \sim N(\mu, \Sigma)$.

$p(x_i \mid x_{pa(i)}) = N(x_i \mid \mu_i + w_i^T x_{pa(i)}, \sigma_i^2)$.

x_i是一维的.

$x_i = \mu_i + \sum_{j \in x_{pa(i)}} w_{ij} \cdot (x_j - \mu_j) + \sigma_i \cdot \underline{\underline{\varepsilon_i}}$.

ε_i is r.v.

$\varepsilon_i \sim N(0, 1)$.

$\Rightarrow x_i - \mu_i = \sum_{j \in x_{pa(i)}} w_{ij} \cdot (x_j - \mu_j) + \sigma_i \cdot \varepsilon_i$.

$\mu = (\mu_1, \mu_2, \cdots \mu_p)^T$

$\boxed{x_i \sim N(\mu_i, \sigma_i^2)}$

$W = [w_{ij}] \quad \varepsilon = (\varepsilon_1, \varepsilon_2 \cdots \varepsilon_p)^T$

↑平移一次以0为中心的高斯分布 线性高斯模型.

$S = \text{diag}(\sigma_i)$.

上面的可以写成：

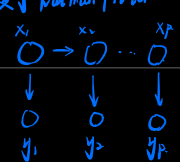
$\Rightarrow \boxed{x - \mu = W \cdot (x - \mu) + S \cdot \varepsilon}$

局部：$p(x) = N(\mu_x \mid \Sigma_x)$ ⓧ → ⓨ

$p(y \mid x) = N(y \mid Ax + b, \Sigma_y)$.

数学基础中有 卡曼 filter. → 典型的高斯线性模型

例：kalman Filter（连）(HMM)（不连）$\lambda = (\pi, A, B)$. → $p(x_t \mid x_{t-1})$ $p(y_t \mid x_t)$.



$$\begin{cases} p(x_t \mid x_{t-1}) = p(x_t \mid Ax_{t-1} + B, Q). \\ p(y_t \mid x_t) = p(y_t \mid Cx_t + D, R). \end{cases}$$

↓ 协方差矩阵.

$x_t = Ax_{t-1} + B + \varepsilon$ ↑满足随机变量 $\varepsilon \sim N(0, Q)$.

$y_t = Cx_t + D + \sigma$. $\sigma \sim N(0, R)$.

→ 特殊的GBN. 只有一个父亲.

$(I-W)\cdot(X-\mu)=S\cdot\varepsilon$

根据定义

$\Sigma = cov(X) = cov(X-\mu) = cov[(I-W)\cdot S\cdot\varepsilon]$

$X-\mu = (I-W)^{-1}\cdot S\cdot\varepsilon$

断掉了

机器学习·高斯马尔科夫随机场

Gaussian Markov Network

$p(x) = \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{\frac{1}{2}}}\cdot \exp\{-\frac{1}{2}(X-\mu)^T\cdot\Sigma^{-1}\cdot(X-\mu)\} \rightarrow pdf$