Corrigenda to: Growing Like India.

The Unequal Effects of Service-Led Growth

Tianyu Fan, Michael Peters, and Fabrizio Zilibotti

Yale University

November 2024

Abstract

We correct an error in an expression of Fan *et al.* (2023). The error has no effect on the analysis and results in the paper.

In this note, we provide a correction to Equation (5) in Proposition 1 of Fan *et al.* (2023)—henceforth, FPZ. We thank Youdan Zhang for bringing this issue to our attention. The correction does not affect any of the theoretical results in the paper (aside from the expression itself), nor does it impact the empirical or quantitative analysis.

Proposition 1 in FPZ characterizes the value-added indirect utility function and the expenditure system. The correct version of Proposition 1 is as follows:

Proposition 1. *The value-added indirect utility function of consumers in region r is given by*

$$\mathcal{V}\left(e, \mathbf{P_{rt}}, \tilde{\mathbf{p}}_{\mathbf{rt}}\right) = \frac{1}{\varepsilon}\left(\frac{e}{P_{rFt}^{\omega_F} P_{rGt}^{\omega_G} P_{rCSt}^{\omega_{CS}}}\right)^{\varepsilon} - \sum_{s\in\{F,G,CS\}} \nu_s \ln P_{rst} - \Omega\left(\tilde{\mathbf{p}}_{\mathbf{rt}}\right), \tag{5C}$$

where the price indices $\mathbf{P_{rt}} = (P_{rFt}, P_{rGt}, P_{rCSt})$ and the parameters ω_s and ν_s are defined as in Proposition 1 of FPZ, $\tilde{\mathbf{p}}_{\mathbf{rt}} = \{\tilde{p}_{nrt}\}_{n\in[0,1]}$, is a set of prices for the consumer services value-added for good n in the region r, $\tilde{p}_{nrt} = \mathcal{A}_{rnt}^{-1} w_{rt}$, and

$$\Omega\left(\tilde{\mathbf{p}}_{\mathbf{rt}}\right) = \nu_{CS} \int_n \left(\frac{\kappa_n \lambda_{nCS}}{\int_n \kappa_n \lambda_{nCS}} - \frac{\beta_n \lambda_{nCS}}{\int_n \beta_n \lambda_{nCS}}\right) \ln \tilde{p}_{rnt}\ dn.$$

The associated value-added expenditure shares are given by

$$\vartheta_{rst}\left(e, \mathbf{P_{rt}}\right) = \omega_s + \nu_s \left(\frac{e}{P_{rFt}^{\omega_F} P_{rGt}^{\omega_G} P_{rCSt}^{\omega_{CS}}}\right)^{-\varepsilon}. \tag{6C}$$

The value-added indirect utility function in Proposition 1 of FPZ omits the term $\Omega(\tilde{\mathbf{p}}_{\mathbf{rt}})$. Equation (5C) corrects this by highlighting that the price index for consumer services (CS), P_{rCSt}, is not a sufficient statistic for consumers' indirect utility. Consumers' welfare also depends on the *dispersion* of prices $\{\tilde{p}_{rnt}\}$, and hence labor productivity $\{\mathcal{A}_{rnt}\}$, across final goods n.

FPZ utilize Proposition 1 in two ways. First, we use the demand system in equation (6C) to estimate the model. Since equation (6C) remains identical to Equation (6) in FPZ, the omission of the term $\Omega(\tilde{\mathbf{p}}_{\mathbf{rt}})$ does not affect any of the estimation nor of the empirical results. Second, we use the indirect utility function to compute the welfare consequences of productivity growth. We now demonstrate that the omission of $\Omega(\tilde{\mathbf{p}}_{\mathbf{rt}})$ also leaves this analysis unchanged. The following corollaries elucidate two key properties of the Ω function that explain why.

Corollary 1 (**No-Heterogeneity Case**). *If $\mathcal{A}_{rnt} = \mathcal{A}_{rt}$, for all $n \in [0,1]$, then, $\Omega(\tilde{\mathbf{p}}_{\mathbf{rt}}) = 0$ and Proposition 1 is identical to Proposition 1 in Fan et al. (2023).*

Corollary 2 (**Homogeneity of Degree Zero**). *Ω is homogeneous of degree zero: for any scalar $z > 0$, $\Omega(\tilde{\mathbf{p}}_{\mathbf{rt}}) = \Omega(z \times \tilde{\mathbf{p}}_{\mathbf{rt}})$.*

Corollary 1 establishes that $\Omega(\tilde{\mathbf{p}}_{\mathbf{rt}})$ vanishes if there is no dispersion in CS labor productivity across final goods n *within* a region. In this case, Proposition 1 of FPZ, including Equation (5), holds fully. Our analysis does not rely on this restriction, except in one of the robustness analyses in Section 7.3, where we assume that $D(\mathbf{p_r})$ follows a CES form. For the results of this extension to hold, heterogeneity in CS labor productivity across final goods within the same region must be ruled out.

Corollary 2 establishes that Ω is invariant to proportional *changes* in CS productivity across goods within a location (recall that $\tilde{p}_{rnt} = w_{rt}/\mathcal{A}_{rnt}$). Thus, the welfare analysis in Section 6 of FPZ exactly applies as long as the counterfactual productivity changes in CS within region r arise from *proportional changes* in $\mathcal{A}_{rnt}$ across the n goods in that region. In this case, Corollary 2 guarantees that the equivalent variation as defined on page 1481 in FPZ, remains unchanged, as we prove formally in Appendix A-3 to this corrigenda.[1] There we also show that all counterfactuals involving productivity changes outside the CS sector are entirely unaffected by the omission of Ω in Proposition 1 in FPZ.

[1] In plain terms, the equivalent variation measures the share of 2011 income a household in region r would be willing to forego to avoid changes in prices and wages associated with a counterfactual return of CS productivity to 1987 levels *while holding constant the relative productivity distribution of CS across final goods within each district*. Note that, if local relative productivity of final goods was allowed to change, the welfare effect of these changes would be quantitatively small. This is because $\Omega(\tilde{\mathbf{p}}_{\mathbf{rt}})$, being defined as the difference between two weighted averages of local productivities across goods, is inherently small, as is its change over time.

References

Fan, Tianyu, Michael Peters, and Fabrizio Zilibotti (2023). "Growing Like India: The Unequal Effects of Service-Led Growth." *Econometrica*, 91(4), 1457-1494.

APPENDIX

A-1 Proof of Proposition 1

Recall that the indirect utility function over final goods is defined as $\mathcal{V}^{FE}(e, \mathbf{p}_{rt}) = \frac{1}{\varepsilon}\left(\frac{e}{B(\mathbf{p}_{rt})}\right)^{\varepsilon} - D(\mathbf{p}_{rt})$, where $B(\mathbf{p}_{rt}) = \exp\left(\int_0^1 \beta_n \ln p_{rnt} dn\right)$ and $D(\mathbf{p}_{rt}) = \left(\int_0^1 \kappa_n \ln p_{rnt} dn\right)$. Substituting the expression for $p_{rnt} = P_{rFt}^{\lambda_n F} P_{rGt}^{\lambda_n G} \tilde{p}_{rnt}^{\lambda_n CS}$ given in FPZ yields

$$\exp\left(\int_0^1 \beta_n \ln p_{rnt} dn\right) = P_{rFt}^{\omega_F} P_{rGt}^{\omega_G} P_{rCSt}^{\omega_{CS}},$$

where ω_s and P_{rCSt} are defined in Proposition 1 in FPZ. Similarly, given the definition of ν_s in Proposition 1 in FPZ, it follows that

$$\begin{aligned}
\int_n \kappa_n \ln p_{rnt} dn &= \int_n \kappa_n \lambda_{nF} \ln P_{rFt} dn + \int_n \kappa_n \lambda_{nG} \ln P_{rGt} dn + \int_n \kappa_n \lambda_{nCS} \ln \tilde{p}_{rnt} dn \\
&= \nu_F \ln P_{rFt} + \nu_G \ln P_{rGt} + \int_n \kappa_n \lambda_{nCS} \ln \tilde{p}_{rnt} dn \\
&= \sum_{s \in \{F,G,CS\}} \nu_s \ln P_{rst} + \nu_{CS} \int_n \left(\frac{\kappa_n \lambda_{nCS}}{\nu_{CS}} - \frac{\beta_n \lambda_{nCS}}{\omega_{CS}}\right) \ln \tilde{p}_{rnt} dn \\
&= \sum_{s \in \{F,G,CS\}} \nu_s \ln P_{rst} + \Omega(\tilde{\mathbf{p}}_{\mathbf{rt}}).
\end{aligned}$$

Hence,

$$\frac{1}{\varepsilon}\left(\frac{e}{B(\mathbf{p}_{rt})}\right)^{\varepsilon} - D(\mathbf{p}_{rt}) = \frac{1}{\varepsilon}\left(\frac{e}{P_{rFt}^{\omega_F} P_{rGt}^{\omega_G} P_{rCSt}^{\omega_{CS}}}\right)^{\varepsilon} - \sum_{s \in \{F,G,CS\}} \nu_s \ln P_{rst} - \Omega(\tilde{\mathbf{p}}_{\mathbf{rt}}),$$

as claimed in Proposition 1.

To derive equation (6C), we apply Roy's identity to the indirect utility function in (5C). Applying Roy's identity to this expression yields the same results as in Fan *et al.* (2023) for ϑ_{rFt} and ϑ_{rGt}. Consider, next, the expenditure share on CS , ϑ_{rCSt}. The expenditure share on consumer service value-added embedded in final good n is given by:

$$\vartheta_{rnt}^{CS}\left(e,\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)=-\frac{\frac{\partial\mathcal{V}\left(e,\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)}{\partial\tilde{p}_{rnt}}\tilde{p}_{rnt}}{\frac{\partial\mathcal{V}\left(e,\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)}{\partial e}e}=\frac{\partial\ln B\left(\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)}{\partial\ln\tilde{p}_{rnt}}+\frac{\partial D\left(\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)}{\partial\ln\tilde{p}_{rnt}}\left(\frac{e}{B\left(\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)}\right)^{-\varepsilon}$$
$$=\beta_n\lambda_{nCS}+\lambda_{nCS}\kappa_n\left(\frac{e}{P_{rFt}^{\omega_F}P_{rGt}^{\omega_G}P_{rCSt}^{\omega_{CS}}}\right)^{-\varepsilon}.$$

Integrating over the final goods $n\in[0,1]$ and using the definitions of ω_{CS} and ν_{CS} yields:

$$\vartheta_{rCSt}\left(e,\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)=\int_n\vartheta_{rnt}^{CS}\left(e,\mathbf{P_{rt}},\tilde{\mathbf{p}}_{\mathbf{rt}}\right)dn=\int_n\beta_n\lambda_{nCS}dn+\left(\frac{e}{P_{rFt}^{\omega_F}P_{rGt}^{\omega_G}P_{rCSt}^{\omega_{CS}}}\right)^{-\varepsilon}\int_n\kappa_n\lambda_{nCS}dn$$
$$=\omega_{CS}+\nu_{CS}\left(\frac{e}{P_{rFt}^{\omega_F}P_{rGt}^{\omega_G}P_{rCSt}^{\omega_{CS}}}\right)^{-\varepsilon},$$

which is Equation (6C) in Proposition 1. This is identical to Equation (6) in Fan *et al.* (2023).

A-2 Proofs of Corollaries 1 and 2

To prove Corollary 1, note that if $\mathcal{A}_{rnt}=\mathcal{A}_{rt}$, then, $\tilde{p}_{rnt}=w_{rt}/\mathcal{A}_{rt}$, for all $n\in[0,1]$. Hence,

$$\Omega\left(\tilde{\mathbf{p}}_{\mathbf{rt}}\right)=\nu_{CS}\ln\left(\frac{w_{rt}}{\mathcal{A}_{rt}}\right)\left(\frac{\int_n\beta_n\lambda_{nCS}}{\int_n\beta_n\lambda_{nCS}}-\frac{\int_n\kappa_n\lambda_{nCS}}{\int_n\kappa_n\lambda_{nCS}}\right)=0$$

To prove Corollary 2, note that

$$\begin{aligned}\Omega\left(z\tilde{\mathbf{p}}_{\mathbf{rt}}\right) &= \nu_{CS}\int_n\left(\frac{\beta_n\lambda_{nCS}}{\int_n\beta_n\lambda_{nCS}}-\frac{\kappa_n\lambda_{nCS}}{\int_n\kappa_n\lambda_{nCS}}\right)\ln\left(z\times\tilde{p}_{rnt}\right)\,dn\\ &= \nu_{CS}\ln\left(z\right)\left(\frac{\int_n\beta_n\lambda_{nCS}}{\int_n\beta_n\lambda_{nCS}}-\frac{\int_n\kappa_n\lambda_{nCS}}{\int_n\kappa_n\lambda_{nCS}}\right)\,dn+\Omega\left(\tilde{\mathbf{p}}_{\mathbf{rt}}\right)=\Omega\left(\tilde{\mathbf{p}}_{\mathbf{rt}}\right)\end{aligned}$$

A-3 Derivation of Equivalent Variation

Let $\mathscr{A}=\{A_{rF},A_{rG},\{\mathcal{A}_{rn}\}_n\}_r$ denote a vector of productivities across sectors, regions, and final goods. Indirect utility using equation (5C) in Proposition 1 is given by

$$\mathcal{V}\left(e,\mathbf{P_r}\left(\mathscr{A}\right),\tilde{\mathbf{p}}_{\mathbf{r}}\left(\mathscr{A}\right)\right)=\frac{1}{\varepsilon}\left(\frac{e}{\prod_s P_{rs}^{\omega_s}\left(\mathscr{A}\right)}\right)^{\varepsilon}-\sum_{s\in\{F,G,CS\}}\nu_s\ln P_{rs}\left(\mathscr{A}\right)-\Omega\left(\tilde{\mathbf{p}}_{\mathbf{r}}\left(\mathscr{A}\right)\right)\tag{A-1}$$

where we have highlighted the dependence of the equilibrium prices on $\mathscr{A}$. Denote by $\mathscr{A}_{2011}$ the productivity vector in 2011 and let $\mathscr{A}^{CF}$ denote any counterfactual we want to evaluate. The compensating variation of changing $\mathscr{A}_{2011}$ to $\mathscr{A}^{CF}$, $\varpi^q_{r,2011\to CF}$, is implicitly defined by

$$\mathcal{V}\left(qw_r\left(\mathscr{A}_{2011}\right)\left(1+\varpi^q_{r,2011\to CF}\right),\mathbf{P}_\mathbf{r}\left(\mathscr{A}_{2011}\right),\tilde{\mathbf{p}}_\mathbf{r}\left(\mathscr{A}_{2011}\right)\right)\equiv\mathcal{V}\left(qw_r\left(\mathscr{A}^{CF}\right),\mathbf{P}_\mathbf{r}\left(\mathscr{A}^{CF}\right),\tilde{\mathbf{p}}_\mathbf{r}\left(\mathscr{A}^{CF}\right)\right) \tag{A-2}$$

Combining (A-1) and (A-2) yields

$$1+\varpi^q_{r,2011\to CF}=\Pi_s\left(\frac{\frac{w_r\left(\mathscr{A}^{CF}\right)}{P_{rs}\left(\mathscr{A}^{CF}\right)}}{\frac{w_r\left(\mathscr{A}_{2011}\right)}{P_{rs}\left(\mathscr{A}_{2011}\right)}}\right)^{\omega_s}\times\left(1-\left(\frac{qw_r\left(\mathscr{A}^{CF}\right)}{\sum_s P_{rs}\left(\mathscr{A}^{CF}\right)^{\omega_s}}\right)^{-\varepsilon}\varepsilon\left(\sum_s\nu_s\ln\frac{P_{rs}\left(\mathscr{A}^{CF}\right)}{P_{rs}\left(\mathscr{A}_{2011}\right)}-\Lambda\right)\right)^{1/\varepsilon}$$

where

$$\Lambda\equiv\Omega\left(\tilde{\mathbf{p}}_\mathbf{r}\left(\mathscr{A}_{2011}\right)\right)-\Omega\left(\tilde{\mathbf{p}}_\mathbf{r}\left(\mathscr{A}^{CF}\right)\right). \tag{A-3}$$

The expression for $\varpi^q_{r,2011\to CF}$ is identical to equation A-5 in the appendix of Fan *et al.* (2023), except for the term Λ given in (A-3).

Define

$$\varrho_n=\frac{\beta_n\lambda_{nCS}}{\omega_{CS}}-\frac{\kappa n\lambda_{nCS}}{\nu_{CS}}.$$

Note that $\int_n\varrho_n dn=0$. Given the definition of Ω in Proposition 1, equation (A-3) then implies that

$$\Lambda=\nu_{cs}\left(\int_n\varrho_n\ln\frac{w_r(\mathscr{A}_{2011})}{\mathcal{A}_{rn2011}}dn-\int_n\varrho_n\ln\frac{w_r\left(\mathscr{A}^{CF}\right)}{\mathcal{A}^{CF}_{rn}}dn\right)=\nu_{cs}\int_n\varrho_n\ln\frac{\mathcal{A}^{CF}_{rn}}{\mathcal{A}_{rn2011}}dn. \tag{A-4}$$

Equation (A-4) proves the two properties stated in the main text:

1. For any counterfactual that involves the agricultural or the industrial sector, $\mathcal{A}^{CF}_{rn}=\mathcal{A}_{rn2011}$. In that case, $\Lambda=0$, so that the results reported in FPZ directly apply.

2. For any counterfactual that involves changes in consumer service productivity, $\Lambda=0$ as long as the change in $\mathcal{A}_{rn}$ is common across final goods n. In that case, $\mathcal{A}^{CF}_{rn}=\kappa\times\mathcal{A}_{rn2011}$ and $\int_n\varrho_n\ln\kappa dn=0$. If there is no heterogeneity in CS productivity across goods, this restriction is automatically satisfied. Hence, Corollary 1 is sufficient but by no means necessary for the results in FPZ to apply.

Corrigendum to “Growing Like India: The Unequal Effects of Service-Led Growth”

Tianyu Fan, Michael Peters, and Fabrizio Zilibotti
Yale University

June 2025

Abstract

This note corrects a typographical error in an expression in Fan et al. (2023) that Sam Kortum and Tanay Kondiparthy kindly brought to our attention. The error does not affect the analysis or results of the original paper.

This corrigendum addresses a typographical error in the expression for the function $D(\mathbf{P_r})$ on the top of page 1489 of Fan et al. (2023). The original expression was stated as:

$$D(\mathbf{P_r}) = \frac{1}{\gamma}\left(\left(\sum_{s\in\{F,G,\mathrm{CS}\}} P_{rs}^{\nu_s}\right)^{\gamma} - 1\right). \tag{1}$$

The summation should be a product. The corrected expression is:

$$D(\mathbf{P_r}) = \frac{1}{\gamma}\left(\left(\prod_{s\in\{F,G,\mathrm{CS}\}} P_{rs}^{\nu_s}\right)^{\gamma} - 1\right). \tag{2}$$

Consequently, the value-added indirect utility function is:

$$\mathcal{V}(e,\mathbf{P}_{\mathrm{r}}) = \frac{1}{\varepsilon}\left(\frac{e}{P_{rF}^{\omega_F}P_{rG}^{\omega_G}P_{r\mathrm{CS}}^{\omega_{\mathrm{CS}}}}\right)^{\varepsilon} - \frac{1}{\gamma}\left(\left(\prod_{s\in\{F,G,\mathrm{CS}\}} P_{rs}^{\nu_s}\right)^{\gamma} - 1\right). \tag{3}$$

Applying Roy’s identity, the subsequent expenditure function is correct, as derived from the generalized PIGL preferences in final expenditure in Online Appendix 2.1 of Fan et al. (2023). The welfare metric in Online Appendix 2.1 remains unaffected by this correction. Thus, the typographical error has no impact on any of the the analysis or results of the original paper.

1 Appendix

To illustrate the derivation of the expenditure function, consider the indirect utility function:

$$\mathcal{V}\left(e,\mathbf{P}\right)=\frac{1}{\varepsilon}\left(\frac{e}{P_F^{\omega_F}P_G^{\omega_G}P_{\mathrm{CS}}^{\omega_{\mathrm{CS}}}}\right)^{\varepsilon}-\frac{1}{\gamma}\left(\left(\prod_{s\in\{F,G,\mathrm{CS}\}}P_s^{\nu_s}\right)^{\gamma}-1\right). \tag{4}$$

Applying Roy's identity, we compute the partial derivatives:

$$\frac{\partial\mathcal{V}\left(e,\mathbf{P}\right)}{\partial P_s}=-\frac{\omega_s}{P_s}\left(\frac{e}{P_F^{\omega_F}P_G^{\omega_G}P_{\mathrm{CS}}^{\omega_{\mathrm{CS}}}}\right)^{\varepsilon}-\frac{\nu_s}{P_s}\left(\prod_{s\in\{F,G,\mathrm{CS}\}}P_s^{\nu_s}\right)^{\gamma}, \tag{5}$$

$$\frac{\partial\mathcal{V}\left(e,\mathbf{P}\right)}{\partial e}=\frac{1}{e}\left(\frac{e}{P_F^{\omega_F}P_G^{\omega_G}P_{\mathrm{CS}}^{\omega_{\mathrm{CS}}}}\right)^{\varepsilon}. \tag{6}$$

The demand for good s, x_s, is given by:

$$x_s=-\frac{\frac{\partial\mathcal{V}(e,\mathbf{P})}{\partial P_s}}{\frac{\partial\mathcal{V}(e,\mathbf{P})}{\partial e}}=\frac{\omega_s e}{P_s}+\frac{\nu_s e}{P_s}\left(\frac{e}{P_F^{\omega_F+\gamma\nu_F/\varepsilon}P_G^{\omega_G+\gamma\nu_G/\varepsilon}P_{\mathrm{CS}}^{\omega_{\mathrm{CS}}+\gamma\nu_{\mathrm{CS}}/\varepsilon}}\right)^{-\varepsilon}. \tag{7}$$

Thus, the expenditure share for sector s, ϑ_s, is:

$$\vartheta_s=\omega_s+\nu_s\left(\frac{e}{P_F^{\omega_F+\gamma\nu_F/\varepsilon}P_G^{\omega_G+\gamma\nu_G/\varepsilon}P_{\mathrm{CS}}^{\omega_{\mathrm{CS}}+\gamma\nu_{\mathrm{CS}}/\varepsilon}}\right)^{-\varepsilon}. \tag{8}$$

This derivation confirms that the expenditure function and expenditure shares are consistent with the corrected form of $D\left(\mathbf{P_r}\right)$.

References

Fan, T., Peters, M., and Zilibotti, F. (2023). "Growing Like India: The Unequal Effects of Service-Led Growth." *Econometrica*, 91(4):1457–1494.