The Labor Market Incidence of New Technologies

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This paper develops a general framework for evaluating the incidence of labor market shocks, focusing particularly on automation and artificial intelligence. Unequal labor market shocks are shared among workers across occupations depending on their substitutability. Central to our theory is the concept of distance-dependent elasticity of substitution (DIDES), where substitutability between occupations declines with their distance in skill space. Our analysis reveals that automation and AI cluster in skill-adjacent occupations, generating limited employment shifts but significant wage disparities. Furthermore, the dynamic model demonstrates that limited mobility persists both during the transition and in the long run, limiting the labor market's capacity to absorb rapid technological progress.

Keywords. Automation, Artificial Intelligence, Labor Market Incidence, Distance Dependent Substitutability, Skill Space.

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1. Introduction

Recent decades have witnessed transformative technological advancements, particularly in automation technologies[1] and artificial intelligence (AI). These innovations have spurred productivity gains and economic growth but have also deepened labor market inequality. While these innovations enhance efficiency and create new opportunities, they disproportionately affect specific jobs — displacing routine, manual jobs with automation and threatening cognitive, non-routine tasks with AI (Autor 2015; Restrepo 2024). The uneven labor market outcomes stem from two interrelated forces: labor-substituting technologies induce asymmetric shifts in labor demand across occupations, and worker mobility constraints limit the ability to adjust to these shifts. In this paper, we focus on the latter, presenting a general framework to evaluate the labor market incidence of new technologies and other labor demand or supply shocks.

A defining feature of automation and AI is their tendency to cluster within specific skill domains. Unlike dispersed labor shocks, these innovations target occupations with similar skill requirements. Automation — driven by industrial robots and specialized software — predominantly impacts routine, manual tasks, clustering in jobs such as manufacturing and assembly occupations. Conversely, AI is concentrated in cognitively demanding roles, including data analysis and research, where it can replicate or augment complex decision-making. This concentrated impact restricts workers' ability to transition to unaffected jobs, as skill mismatches hinder mobility. Our analysis reveals that this clustering constrains employment adjustments and drives significant wage disparities, playing a crucial role in the distributional consequences of technological change.

We develop a theoretical model in which distance-dependent elasticity of substitution (DIDES), where substitutability between occupations declines with their distance in skill space, governs labor reallocation, capturing how workers respond to technological shocks. Building on a task-based production framework and a Roy model of occupational choice, our approach integrates a correlated productivity distribution to encapsulate skill-based substitutability across occupations. Workers possess a set of skills that they can apply in various jobs; however, occupations differ in their reliance on these skills. Thus, substitutability between two occupations depends on their skill-space proximity — reflecting differences in required skills — and on within-skill as well as cross-skill substitutability, which together determine the ease of transitioning between jobs.

A key advantage of this approach is that it provides a dimension reduction mechanism, transforming the complex multi-dimensional substitution structure between occupations into a parsimonious framework. By embedding DIDES in the Roy model, we derive an elasticity of substitution matrix that naturally reflects proximity in skill space as a de-

[1] We follow Acemoglu and Restrepo (2022) for the definition of automation technologies: industrial robots, machinery, and software (no AI capability).

terminant of occupational mobility. When automation or AI disproportionately impacts skill-adjacent occupations, the constrained mobility amplifies wage dispersion, as displaced workers have fewer high-productivity alternatives. In contrast, when technological shocks are more dispersed, reallocation costs are less pronounced, allowing for greater worker mobility. This framework enables us to formally link the distribution of technological shocks to the incidence of wage and employment adjustments, highlighting the role of labor market structure in shaping inequality.

Empirically, we leverage data from the Occupational Information Network (O*NET) as the primary source for measuring occupational skill requirements and technological exposures. Approximately 200 descriptors covering skills, abilities, knowledge, work activities, and work context are extracted from O*NET, and we then apply principal component analysis (PCA) to reduce this high-dimensional information into three interpretable indices — cognitive, manual, and interpersonal skill requirements — which serve as empirical proxies for an occupation's location in the skill space. In parallel, we construct technological exposure measures by querying GPT-4o separately on the feasibility of automating each task using traditional automation methods and on the feasibility of executing tasks with AI models. For traditional automation exposure, we assess whether each task can be performed without human intervention by industrial robots and machines, whereas for AI exposure, we evaluate whether tasks can be executed by general AI models like GPT-4o. Covering 19,200 tasks across 862 occupations, GPT-4o assessments reveal that roughly one-third (about 6,000 tasks) are potentially executed by AI, similar to the scale of automation. This approach allows us to empirically capture the distinct distribution of technological shocks, highlighting that automation clusters in occupations with high manual demands and that AI predominantly targets roles with significant cognitive components.

We estimate labor supply elasticities by exploiting long-run occupational employment changes in response to automation-induced wage shifts. Using wage effects of automation estimated from the Panel Study of Income Dynamics (PSID) alongside changes in cross-sectional employment from the Census and ACS, our estimation employs a pseudo-Poisson maximum likelihood (PPML) approach—commonly used in gravity models—to jointly estimate the average elasticity (θ) and the skill-specific correlation parameters (ρ_s), which govern within-skill substitutability, under a cross-nested CES framework that parameterizes DIDES.

Under a standard CES specification (with ρ_s constrained to zero), the estimated average labor supply elasticity is around 3.12. When we relax this restriction (CNCES specification), allowing for nonzero ρ_s, the cross-skill elasticity θ falls to approximately 1.10, implying that within-skill substitutability accounts for about two-thirds of the variation in labor supply responses. Moreover, the estimated transferability parameters reveal that cognitive skills are more transferable than manual skills, highlighting significant heterogeneity in labor mobility across occupations.

By integrating our estimated labor supply elasticities with measures of occupational exposure, we assess the static incidence of automation and AI on the labor market. Our analysis reveals that, due to the clustering of technological shocks within skill-adjacent occupations, about 40% of relative labor demand changes are transmitted into wage changes — indicating that the distributional effects are only partially absorbed. Moreover, workers recover merely around 20% of their wage losses through occupational transitions, underscoring that limited mobility driven by skill dissimilarity and the proximity of automation and AI in the skill space amplifies wage inequality. In fact, the clustering nature of automation and AI exposure reduces the equilibrium wage labor mobility by one-third. Furthermore, relying on average elasticity estimates significantly overstates worker mobility and understates the unequal wage incidence.

After characterizing the static incidence of automation and AI, we extend our static Roy-based framework into a dynamic discrete choice model that incorporates DIDES to capture gradual labor market transitions. In our dynamic extension, workers are modeled as forward-looking agents who face idiosyncratic productivity shocks and incur job transition costs when switching occupations. By incorporating a correlation-adjusted transition probability and deriving a dynamic hat-algebra with correlation, our framework captures how changes in underlying fundamentals — such as technology adoption — affect occupational wages, allocations, and overall welfare over time within a rich substitution structure.

Using CPS data aggregated into 15 occupation clusters, we employ an adjusted Euler equation approach to estimate a short-run elasticity of 0.07. Our findings indicate that gradual automation adoption has led to a persistent relative wage decline—up to a 50% difference between occupations with high versus low exposure—with employment adjustments absorbing roughly two-thirds of labor demand shifts and mobility gains offsetting about half of wage losses. In contrast, under a counterfactual scenario where AI adoption rapidly reaches automation's scale by 2030, the labor market exhibits slow absorption: less than one-third of demand shifts are absorbed initially, leading to a sharper wage decline and mobility gains that recover only around one-third of wage losses. These results underscore that the clustering of technological shocks limits worker mobility, thereby exacerbating wage inequality both in transition and the long run.

This paper offers a comprehensive analysis of how automation and AI reshape the labor market, emphasizing the interplay between skill-based substitutability and the pace of technological change. By introducing DIDES and integrating it into both static and dynamic frameworks, we provide new insights into the mechanisms driving labor market inequality and the challenges posed by rapid technological transitions.

Related Literature. Our paper contributes to the extensive literature on the labor market impacts of technological changes. Early research by Katz and Murphy (1992) and Autor, Katz, and Krueger (1998) introduced the concept of skill-biased technical change (SBTC), which was later expanded by Acemoglu (2002) and Autor and Dorn (2013), showing how technological advancements disproportionately increase demand for skilled workers, thereby widening wage inequality. Building on this framework, we argue that skill-biased technologies not only reshape relative labor demand but also constrain worker mobility, limiting the labor market's ability to adjust to shocks. More recent work by Acemoglu and Restrepo (2018, 2020, 2022) has focused on the distributional consequences of automation, demonstrating how labor-replacing technologies exacerbate wage polarization by disproportionately displacing middle-skill jobs. In parallel, research on AI has explored its potential to disrupt a broader range of tasks. Webb (2019) and Acemoglu et al. (2022) highlight how AI affects both routine and non-routine cognitive tasks, while Noy and Zhang (2023) and Brynjolfsson, Li, and Raymond (2025) examine the labor market implications of generative AI, emphasizing its ability to reshape knowledge-based and creative work.

A growing literature examines the role of labor reallocation in mitigating unequal labor demand shifts, emphasizing how mobility constraints slow adjustments and amplify wage inequality. Matsuyama (1992) and Heckman, Lochner, and Taber (1998) highlight how sectoral shifts interact with skill acquisition, while Lee and Wolpin (2006) and Dvorkin and Monge-Naranjo (2019) emphasize occupational mobility frictions in wage dynamics. Traiberman (2019) further shows that limited occupational mobility worsens sector-specific shocks, particularly under globalization. In the context of transitions, Caliendo, Dvorkin, and Parro (2019) and Lehr and Restrepo (2022) demonstrate that slower reallocation prolongs wage disparities, while Adão, Beraja, and Pandalai-Nayar (2024) argues that skill specialization constrains labor market adjustments. Similarly, Feigenbaum and Gross (2024) documents how historical technological shifts reveal persistent labor mobility constraints. Our study builds on this literature by showing that the clustering nature of automation and AI further restricts worker mobility, exacerbating labor market frictions both in transition and in the long run.

The concept of distance-dependent elasticity of substitution (DIDES) is a fundamental result in assignment models, governing how substitutability between workers or tasks declines with their distance in a given skill space. This property is inherent to all assignment frameworks, including both single- and multidimensional settings, as established in the theoretical assignment literature (Sattinger 1993; Kremer 1993; Teulings 1995, 2005; Lindenlaub 2017). These models consistently show that proximity in skill space increases substitutability, shaping equilibrium wage distributions and labor market adjustments. Building on this foundation, our study enriches the empirical content of DIDES by providing an empirical formulation, measurement, and estimation, allowing us to quantify its role in labor market responses to automation and AI.

Lastly, our framework follows the Roy tradition, incorporating skill heterogeneity to analyze labor market adjustments. A well-established literature highlights the role of skill transferability in job mobility (Gathmann and Schönberg 2010) and the importance of skill heterogeneity for wage distribution (Heckman and Sedlacek 1985; Böhm, von Gaudecker, and Schran 2024). These studies show that skill selection effects obscure wage growth patterns and that changing skill prices have driven rising wage inequality. Recently, Dvorkin and Monge-Naranjo (2019) incorporate skill accumulation and transition costs into the standard Roy model to study the welfare effects of task-biased labor demand shocks. We extend the Roy framework by allowing for correlation in productivity distributions (Lind and Ramondo 2023) to formulate DIDES and examine its interaction with clustering shocks from automation and AI to study the economic incidence.

Road Map. The structure of the paper is as follows. Section 2 presents a static model featuring DIDES and examines how its interaction with the shock distribution shapes labor market incidence. Section 3 describes the data sources and empirical methodology used to estimate labor supply elasticities and analyzes the static incidence of automation and AI. Section 4 extends the static model into a dynamic discrete choice framework to capture gradual labor market transitions and assess the dynamic incidence of technological changes. Section 5 concludes by summarizing the findings and discussing policy implications.

2. Theoretical Framework

We examine how the distribution of shocks interacts with distance-dependent elasticity of substitution (DIDES) to shape the incidence of labor market shocks. Our analysis begins with a long-run perspective using a static model, which we later extend to incorporate short-term mobility frictions in the quantitative analysis. We first integrate a task-based production model with a Roy framework of occupational choice to study labor reallocation and wage responses to technological shocks. Next, we characterize the distributional effects of new technologies and apply spectral analysis to assess their incidence. Finally, we formalize DIDES through the correlation structure of the labor productivity distribution.

2.1. Technology, Occupations, and Workers

Technology and Occupations. Building on the framework introduced by Acemoglu and Restrepo (2022), we formulate a task-based production structure to examine the impact of new technologies on labor demand. This production specification is designed to capture how automation and AI affect labor demand across occupations. Output in this economy is produced by combining a set of tasks, $\mathcal{T}$, using a CES aggregator:

$$y = \left(\int_{\mathcal{T}} (y(x))^{\frac{\sigma-1}{\sigma}} \cdot dx \right)^{\frac{\sigma}{\sigma-1}}$$

where σ denotes the elasticity of substitution between task inputs, directly related to labor demand elasticity, as discussed in the following sections.

The production process is structured around a set of occupations, $\mathcal{O} = \mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_O$, where tasks are exogenously assigned to distinct occupations.[2] Formally,

$$\mathcal{T} = \bigcup_{i=1}^{O} \mathcal{T}_i \quad \text{with} \quad \mathcal{T}_i \cap \mathcal{T}_j = \varnothing \quad \text{for} \quad i \neq j$$

such that each task uniquely belongs to one occupation, with no overlap. The representative firm can produce task inputs by using capital or by hiring workers for each occupation under a linear production technology:

$$y(x) = a(x)\, k(x) + \ell_o(x), \forall x \in \mathcal{T}_o$$

where $a(x)$ represents the capital productivity at task x, and $k(x)$ and $\ell_o(x)$ denote the capital and labor input, respectively. We assume perfect substitutability between workers and capital at the task level, allowing us to isolate the role of substitutability among workers.[3]

Capital used to perform task x, denoted as $k(x)$, is produced using the final good at a constant marginal cost of one.[4]

The representative firm takes occupational wages, $\boldsymbol{w} = w_{o\,o=1}^{\;O}$, as given and optimally allocates labor and capital across tasks to minimize costs. In particular, task allocation between factors is cost-minimizing. [5] The task assignments are characterized by:

$$\mathcal{T}_o^{\ell} = \left\{ x : \frac{1}{a(x)} \geq w_o, x \in \mathcal{T}_0 \right\} \text{ and } \mathcal{T}_o^{k} = \left\{ x : \frac{1}{a(x)} < w_o, x \in \mathcal{T}_0 \right\} \tag{1}$$

where $\mathcal{T}_o^{\ell}$ denotes the set of tasks allocated to labor, and $\mathcal{T}_o^{k}$ denotes those allocated to

[2]Our study focuses on how the labor market absorbs labor demand shocks rather than how automation and AI shift labor demand. Therefore, we take the occupational structure as given.

[3]Similarly, since our focus is on labor market absorption, we take labor shares as exogenous and do not analyze the competition between technology and workers.

[4]The assumption of constant marginal cost implies that the supply of capital is perfectly elastic in the long run. This could be derived from an underlying model of consumer saving decisions, where, in a steady state, the interest rate would equal the rate of time preference.

[5]Throughout this paper, we analyze the labor market impact of a given technological adoption rather than the endogenous decision to adopt new technologies.

capital in occupation o. Consequently, the occupational labor demand is given by:

$$L_o^d = \int_{x \in \mathcal{T}_o^\ell} \ell_o(x)\, dx.$$

Workers. The economy consists of a continuum of workers, each indexed by i, who possess heterogeneous productivity across occupations, denoted by $\boldsymbol{\epsilon}(i) = \{\epsilon_o(i)\}_{o=1}^{O}$ across occupations. We assume that the joint distribution of productivity across occupations follows:

$$\Pr\left[\epsilon_1(i) \leq \epsilon_1, \ldots, \epsilon_O(i) \leq \epsilon_O\right] = \exp\left[-F\left(A_1\epsilon_1^{-\theta}, \ldots, A_O\epsilon_O^{-\theta}\right)\right] \tag{2}$$

where $A_o > 0$ denotes the occupational scale parameter with the shape $\theta > 0$ parameter determining the marginal Fréchet distribution: $\Pr[\epsilon_o(i) \leq \epsilon_o] = e^{-A_o\epsilon_o^{-\theta}}$. The occupational shifter A_o captures average occupational labor productivity, while the shape parameter θ determines productivity heterogeneity across workers, as in standard Roy models with extreme value distributions.

The function F, also known as the tail dependence function, captures the correlation structure of productivity across occupations, governing occupational substitutability. It is homogeneous of degree one and satisfies the sign-switching property [6]. The sign-switching property implies that occupations are substitutes from the workers' perspective, ensuring the gross substitutes property. Without loss of generality, we impose the normalization condition $F(1, 0, \ldots, 0) = 1$, which separates the scales from the correlation function. We initially leave the correlation function unspecified and later parameterize it to reflect that occupational substitutability declines with skill distance (DIDES).

In this static framework, workers choose the occupation that maximizes their static utility:

$$u(i) = \max_o u\left(c_o(i), \ell_o(i)\right) = \max_o \ln\left(c_o/\ell_o(i)\right)$$
$$\text{where } c_o = w_o \text{ and } \ell_o(i) = 1/\epsilon_\circ(i)$$

where $\boldsymbol{w} = \{w_o\}_{o=1}^{O}$ represents the equilibrium occupational wages. Workers supply one unit of labor and incur disutility from time usage, determined by their occupational productivity by $\ell_0(i) = 1/\epsilon_\circ(i)$. Let I_o denote the set of workers who choose occupation o, formally defined as: $I_o = \left\{i \mid w_o\epsilon_o(i) = \max_{o'} w_o'\epsilon_o'(i)\right\}$. The total occupational labor supply is then:

$$L_o^s = \int_{i \in I_o} 1 di.$$

[6]For a detailed discussion of correlation functions and max-stability, see appendix A.1.1 and Lind and Ramondo (2023).

Finally, we define $\mathcal{L}(\boldsymbol{w})$ as the implied labor supply function, mapping wages to the total labor supply across all occupations. For example, if the correlation function is additive, i.e., $F(x_1, \ldots, x_O) = \sum_o x_o$, then the resulting labor supply elasticity follows a constant elasticity of substitution (CES) structure.[7]

2.2. Market Equilibrium and Comparative Statics

Static Equilibrium. We define market equilibrium as an allocation of tasks to factors and a production plan for capital goods that maximizes utility. Given capital productivity $\{a(x)\}_{x\in\mathcal{T}}$ and occupational labor productivity $\{A_o\}_{o=1}^{O}$, the market equilibrium is characterized by the wage vector $\boldsymbol{w} = \{w_o\}_{o=1}^{O}$, capital production decisions $\{k(x)\}_{x\in\mathcal{T}}$, and a labor allocation to tasks $\{\ell_o(x)\}_{x\in\mathcal{T}}$ such that

a. The firm makes capital production decisions and hires labor to maximize profit.
b. Workers choose occupations to maximize their utility.
c. The labor market clears for each occupation, $L_o^d = L_o^s$.

PROOF. The existence and uniqueness of the equilibrium follow from the strict gross substitutability of the excess labor supply (see Appendix A.1.2). □

Comparative Statics. With the equilibrium defined, we proceed to analyze the comparative statics of new technologies. We model automation and AI as technologies that enhance capital productivity in tasks that were previously performed by labor but are now carried out by capital across occupations. The set of newly automated tasks in occupation o is denoted by $\mathcal{D}_o^j$, where $j \in \{\text{Automation, AI}\}$. The share of tasks automated is defined as

$$d\ln\Gamma_o^j = \frac{M_{\mathcal{D}_o^j}}{M_{\mathcal{T}_o^\ell}}, j \in \{\text{Automation, AI}\}$$

where $M_{\mathcal{D}_o^j}$ is the measure of tasks newly automated in occupation o by technology j, and $M_{\mathcal{T}_o^\ell}$ is the total measure of tasks in the same occupation.

In this paper, we focus on automation and AI, whose impact varies across occupations. As shown in Section 3, automation shocks are concentrated in occupations that require high manual skills and low cognitive skills, whereas AI shocks are clustered in occupations with high cognitive demands and low manual requirements. Within the task assignment framework, these technologies generate both productivity effects and replacement effects. Under the assumption of a fixed occupational structure, the productivity effect benefits all occupations, whereas displacement effects are unevenly distributed. The central question we study is how these displacement effects are shared among workers.

[7] When the productivity distribution is independent, the labor supply elasticity equals the shape parameter: $\frac{\partial \ln L_o/L_{o'}}{\partial \ln w_o/w_{o'}} = \theta$.

PROPOSITION 1. ***Automation/AI Comparative Statics****: Consider automation or AI characterized by $d\ln\Gamma_o^j$. The first-order impacts on employment, $d\ln\boldsymbol{h}$, and wages, $d\ln\boldsymbol{w}$, are determined by the following system of equations:*

- ***Labor demand equation****:*

$$d\ln\boldsymbol{w} = \frac{1}{\sigma}d\ln y\cdot\mathbf{1} - \frac{1}{\sigma}d\ln\Gamma^j - \frac{1}{\sigma}d\ln\boldsymbol{L}, \tag{3}$$

- ***Labor supply equation****:*

$$d\ln\boldsymbol{L} = \Theta d\ln\boldsymbol{w}, \quad \textit{where} \quad \Theta = \frac{\partial\ln\mathcal{L}}{\partial\ln\boldsymbol{w}^T}. \tag{4}$$

Here Θ is the matrix of partial elasticities of labor supply

$$\Theta_{oo'} \equiv \frac{\partial\ln L_o}{\partial\ln w_{o'}} = \begin{cases}\theta\frac{x_oF_{oo}}{F_o}\left|_{\{x_o=A_oW_o^\theta\}}\right. + \theta\left(1-\pi_o\right) & \textit{if } o=o' \\ \theta\frac{x_{o'}F_{oo'}}{F_o}\left|_{\{x_o=A_oW_o^\theta\}}\right. - \theta\pi_{o'} & \textit{if } o\neq o'\end{cases} \tag{5}$$

where $\pi_o = \frac{x_oF_o}{F}\left|_{\{x_o=A_oW_o^\theta\}}\right.$ is the occupational employment share, with $F_o = \frac{\partial F(x_1,\ldots,x_o)}{\partial x_o}$ and $F_{oo'} = \frac{\partial^2F(x_1,\ldots,x_o)}{\partial x_o\partial x_o'}$.

PROOF. See Appendix A.1.2. □

Proposition 1 establishes that the effects of automation and AI technologies on wages and employment are jointly determined by the labor demand equation (3) and the labor supply equation (4). The labor demand equation reflects two opposing forces: *productivity effect* $d\ln y$, which is uniform across all occupations, and *displacement effect* $d\ln\Gamma^j$, which varies between occupations. The elasticity of substitution between tasks, σ, also governs labor demand elasticity. A higher σ dampens employment responses to shocks, amplifying wage disparities.

The labor supply equation (4) captures how employment responds to unequal wage changes.[8] The elasticities of the labor supply comprise two components: *correlated substitutability*, resulting from the correlated productivity distribution $\theta\frac{x_{o'}F_{oo'}}{F_o}$, and *independent substitutability*, only determined by the shares of occupation employment. Notably, when the productivity distribution between occupations is independent, represented as $F(x_1,\ldots,x_O) = \sum_o x_o$, the correlated substitutability term becomes zero, as $F_{oo'} = 0$. Within this framework, distance-dependent substitutability will be embedded in the correlation structure between occupations and represents a specific realization of the broader concept of correlated substitutability. At this stage, we preserve the generality of the framework before parameterizing a particular correlation structure.

[8]Note that $\sum_{o'}\Theta_{oo'} = 0, \forall o$, a result of maximum stability as shown in the proof. A uniform proportional wage change, such as an increase in total output, does not induce employment shifts

The Incidence of New Technologies. We solve for the wage changes and gains from reallocation in response to automation and AI using the first-order approximation.[9]

LEMMA 1. *To the first order, the wage changes for the automation shocks are characterized by*

$$d \ln \boldsymbol{w} = \frac{1}{\sigma} d \ln y \cdot \mathbf{1} - \Delta \cdot \frac{d \ln \Gamma^j}{\sigma}, \ \textit{where}\ \Delta = \left(\mathbf{I} + \frac{1}{\sigma}\Theta\right)^{-1}. \tag{6}$$

and the average mobility gains (defined using equivalent variation) for workers in occupation o before the new technologies are given by :

$$\textit{Mobility Gains}_o = -\sum_{o'} \Theta_{oo'} \left(d \ln w_{o'} - d \ln w_o\right)^2 \mathbf{1}_{d \ln w_{o'} - d \ln w_o > 0}. \tag{7}$$

PROOF. See Appendix A.1.3. □

Equation (6) illustrates how distributional automation shocks are transmitted to wages. First, the uniform productivity effect passes fully to the wage without inducing any employment effect. In contrast, the distributional displacement effects are mediated by a dampening matrix Δ, which is positively related to demand elasticity σ and inversely related to the labor supply elasticities Θ. This relationship reflects the fundamental trade-off between wage effects and employment effects: a shock that induces greater employment adjustments results in smaller wage changes. In the extreme case where $\sigma \to \infty$ or $\Theta \to 0$, no employment shifts occur, leading to full wage pass-through and the largest unequal impacts across workers.

Importantly, as we will detail in Section 2.3, the pass-through of automation and AI shocks depends on their distribution across occupations. If automation and AI technologies are highly concentrated in occupations that are close substitutes, the resulting employment effects are limited, leading to greater wage disparities. In contrast, if the shocks are more dispersed, the substitutability between differentially affected occupations tends to be higher, altering the incidence of wage and employment adjustments. Thus, the distribution of shocks plays a crucial role in determining average substitutability and its overall impact on wages and employment.

In the economy with worker mobility, wage changes primarily reflect workers who remain in their current occupations. However, workers are heterogeneous in their outside options, and some will transition to better-aligned occupations in response to technological shocks. For instance, consider a marginal mechanical engineer who is also highly productive as an electrical engineer. When automation reduces labor demand for mechanical engineers, such a worker switches to electrical engineering and is better off than those who remain in their initial occupation.

[9] we write automation in the form of $\frac{d \ln \Gamma^j}{\sigma}$ as it represents the labor demand changes stem from the new technologies.

We can decompose these gains into two terms, as shown in Equation (8). The first term captures the effect of average substitutability across all occupations, while the second term measures the correlation between substitutability and relative wage changes. When new technologies cluster in occupations that are close substitutes, the substitutability between occupations is negatively correlated with relative wage changes. In other words, the covariance between bilateral elasticities and relative wage changes is positive, which reduces the magnitude of reallocation gains.

This decomposition highlights an important insight: relying solely on estimated average elasticity tends to overstate the gains from reallocation while underestimating the unequal effects of new technologies. By explicitly accounting for the interaction between substitutability and wage adjustments, we can better assess the true mobiliti gains of automation and AI.

$$\begin{aligned} \textit{Mobility Gains}_o = & -\bar{\Theta}_o \sum_{o'} \left(d\ln w_o - d\ln w_{o'} \right)^2 \mathbf{1}_{d\ln w_{o'} - d\ln w_o > 0} \\ & - O \cdot \operatorname{Cov}\left(\Theta_{oo'}, \left(d\ln w_o - d\ln w_{o'} \right)^2 \mathbf{1}_{d\ln w_{o'} - d\ln w_o > 0} \right) \end{aligned} \tag{8}$$

2.3. Spectral Analysis

Before parameterizing the correlation structure in the workers' productivity distribution (2) and focusing on distance-dependent substitutability, this section employs spectral analysis to better understand how shock distribution influences employment effects and its incidence.

We begin by presenting an eigendecomposition of the elasticity of substitution matrix Θ and the dampening matrix Δ. While Kleinman, Liu, and Redding (2023) employ spectral analysis to examine convergence speeds, we extend this approach by relating empirical automation shocks to eigenshocks, demonstrating how these correspond to varying degrees of employment shifts and wage pass-through. Finally, we analyze a special 2×2 example to provide further insight into these dynamics.

Eigen-decomposition and Eigen-shocks. The wage incidence of automation and AI (6) can be expressed entirely in terms of the eigenvalues and eigenvectors of the elasticity of substitution matrix Θ. Using eigen-decomposition, we write $\Theta = U\Lambda V$, where Λ is a diagonal matrix of eigenvalues arranged in increasing order, and $V = U^{-1}$. [10] For each eigenvalue λ_n, the n-th column of U, $\boldsymbol{u}_n$, and the nth row of V, $\boldsymbol{v}'_n$, are the associated right

[10] The matrix Θ is not necessarily symmetric. For an asymmetric Θ, distinct eigenvalues are sufficient to ensure the existence of an eigen-decomposition. Additionally, we normalize the 2-norm of the right eigenvectors to 1.

and left eigenvectors of Θ:

$$\lambda_n \boldsymbol{u}_n = \Theta \boldsymbol{u}_n \text{ and, } \lambda_n \boldsymbol{v}_n' = \boldsymbol{v}_n' \Theta$$

Furthermore, both the sets $\{\boldsymbol{u}_o\}$ and $\{\boldsymbol{v}_o'\}$ form bases that span the O-dimensional space. The regularity conditions of the correlation function ensure that Θ is a dominant matrix with positive diagonal elements and negative off-diagonal elements.[11] This property leads to the following characterization of eigenvalues:

LEMMA 2. *All eigenvalues are nonnegative, with at least one eigenvalue being 0 (e.g., $\lambda_0 = 0$, associated with the uniform right eigenvector $\boldsymbol{u}_0 = \frac{1}{\sqrt{O}} \cdot [1, \ldots, 1]'$).*

PROOF. See Appendix A.1.5. □

The uniform eigenvector corresponds to common shocks, such as productivity effects, that do not induce labor reallocation $\lambda_1 = 0$. On the other hand, the positivity of the eigenvalues stems from the gross substitutes property, where labor mobility counteracts relative wage changes and dampens the direct replacement effect, as detailed in the next proposition.

PROPOSITION 2. ***Spectral Analysis:*** *Consider the given empirical automation shock $\frac{d \ln \boldsymbol{\Gamma}}{\sigma}$. Its impact on wages can be expressed using the eigenvalues (λ_n) and eigenvectors ($\boldsymbol{u}_n$) of the elasticity of substitution matrix:*

$$d \ln \boldsymbol{w} = \frac{1}{\sigma} d \ln y \cdot \mathbf{1} + \sum_{n=1}^{O} \frac{b_n}{1 + \lambda_n/\sigma} \boldsymbol{u}_n, \text{ with } \frac{d \ln \boldsymbol{\Gamma}}{\sigma} = \sum_{n=1}^{O} \boldsymbol{u}_n \cdot b_n$$

where the weights b_n can be recovered as the coefficients in a linear projection of the automation shocks $\frac{d \ln \Gamma}{\sigma}$ onto the basis $\{\boldsymbol{u}_o\}$, $\boldsymbol{b} = \left(U'U\right)^{-1} U' \cdot \frac{d \ln \Gamma^j}{\sigma}$.

PROOF. See Appendix A.1.4. □

Proposition 2 highlights that we can define a set of base shocks, referred to as *eigenshocks*, which align with the right eigenvectors.[12] These eigenshocks pass through to the wage distribution, scaled by the dampening factor $\frac{1}{1+\lambda_n/\sigma} \leq 1$. The first eigenshock ($\lambda_1 = 0$) corresponds to uniform productivity shocks, which do not induce labor reallocation. The remaining eigenshocks are subject to dampening effects driven by worker mobility. When Θ is symmetric, the following lemma characterizes the variance of wage changes based on how automation shocks load onto these eigenshocks.

[11] The sign-switching property of F guarantees that occupations are gross substitutes, implying that cross-labor supply elasticities are negative.

[12] Θ depends on the scale parameters of the productivity distribution and is generally asymmetric. As a result, the eigenshocks are not orthogonal unless Θ is symmetric.

LEMMA 3. *The variance of wage changes caused by the automation shocks is characterized by*

$$\mathrm{Var}\left(d\ln\boldsymbol{w}\right)=\boldsymbol{c}^{T}\,\mathrm{Var}\left(U\right)\boldsymbol{c}\ \textit{with}\ c_n=\frac{b_n}{1+\lambda_n/\sigma}.$$

When Θ *is symmetric, it can be simplified as*

$$\mathrm{Var}\left(d\ln\boldsymbol{w}\right)=\sum_{n=2}^{O}\frac{b_n^2}{\left(1+\lambda_n/\sigma\right)^2}\leq\mathrm{Var}\left(\frac{d\ln\Gamma^j}{\sigma}\right).$$

The lemma 3 reveals that labor mobility mitigates the unequal effects of labor demand shocks caused by automation and AI. The strength of this dampening effect depends on the exact distribution of the shocks. For shocks with the same variance, if the shock loads heavily onto eigenshocks associated with larger eigenvalues, it dissipates across workers by inducing larger employment shifts. Conversely, shocks concentrated in eigenshocks with small eigenvalues lead to large wage disparities with minimal labor reallocation.

A 2x2 Example. Having discussed spectral analysis in general, we now illustrate the role of shock distribution and distance-dependent substitutability using a simple example of four occupations. These four occupations have equal employment shares and belong to two clusters: cognitive (c_1 and c_2) and manual (m_1 and m_2). Within each cluster, c_1 and c_2 (m_1 and m_2) are close in the skill space, while the distance between c_1 and m_1 is larger. We formulate DIDES using the following productivity distribution: [13]:

$$\Pr\left[\epsilon_{c_1}(i)\leq\epsilon_{c_1},\cdots,\epsilon_{m_2}(i)\leq\epsilon_{m_2}\right]=\exp\left[-\left(\epsilon_{c_1}^{\frac{-\theta}{1-\rho}}+\epsilon_{c_2}^{\frac{-\theta}{1-\rho}}\right)^{1-\rho}-\left(\epsilon_{m_1}^{\frac{-\theta}{1-\rho}}+\epsilon_{m_2}^{\frac{-\theta}{1-\rho}}\right)^{1-\rho}\right]$$

With this specification, workers' productivity in c_1 and c_2 (m_1 and m_2) are correlated with a correlation coefficient ρ, while the productivity among occupations in different clusters is independent. Consequently, c_1 and c_2 (m_1 and m_2) are close substitutes. We can then eigen-decompose the elasticity of substitution matrix[14]:

$$\boldsymbol{\lambda}=\begin{bmatrix}0\\ \theta\\ \frac{\theta}{1-\rho}\\ \frac{\theta}{1-\rho}\end{bmatrix},\quad 2U=\begin{bmatrix}1\\1\\1\\1\end{bmatrix},\begin{bmatrix}1\\1\\-1\\-1\end{bmatrix},\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix},\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}.$$

[13]the correlation function is $F=\sum_{s\in\{c,m\}}\left(\sum_{o\in\{1,2\}}x_{s_o}^{\frac{1}{1-\rho}}\right)^{1-\rho}$.

[14]We assume starting with equal employment shares for these occupations

The matrix Θ has one zero eigenvalue corresponding to the uniform eigenshock, which does not induce employment changes. The second eigenshock represents the clustering shock, where occupations within the same cluster experience identical shocks. This clustering shock is associated with the eigenvalue θ, implying that employment adjustments occur at the cluster level, while within-cluster correlations do not influence the response. Moreover, the clustering shock generates substantial unequal wage incidence. Intuitively, when closely located occupations are similarly affected, displaced workers must traverse longer distances in the skill space to transition to relatively better-off jobs, resulting in limited labor reallocation. In contrast, eigenshocks 3 and 4, associated with larger eigenvalues that increase with ρ, correspond to more dispersed shocks in the skill space. With such dispersed shocks, workers have better outside options, as closely substitutable occupations are relatively better insulated from negative impacts.

As we demonstrate in the empirical section 3, automation and AI are strongly correlated with occupational skill requirements, largely explained by clustering eigenshocks. These technologies tend to induce substantial wage disparities while causing limited worker reallocation. Intuitively, they are concentrated in occupations that are closely related within the skill space but distant from those that benefit from these technological advancements. Consequently, workers displaced by automation often struggle to transition into better off occupations, as they are less productive in those roles, leading to disproportionate adverse effects. Similarly, Adão, Beraja, and Pandalai-Nayar (2024) finds that the adoption of information technology was slower due to distinct skill requirements. This example illustrates how DIDES interacts with the distribution of shocks to shape their incidence.

2.4. Cross-Nested CES and DIDES

Ideally, we would prefer to estimate the elasticity of substitution matrix Θ nonparametrically. However, since our primary focus is on distributional consequences and understanding how shock distribution shapes incidence, a finer level of aggregation is necessary.[15] Given the high dimensionality of Θ and the limited availability of exogenous variation, we adopt a parametric approach. Our parameterization of the correlation function is based on the idea that occupational skill requirements determine workers' mobility between occupations, as emphasized in the empirical literature (Traiberman 2019; Lise and Postel-Vinay 2020).

The bilateral substitutability between two occupations depends on their relative locations in a multi-dimensional skill space, with substitutability decreasing as distance increases. In this sense, our approach provides an empirical formulation of the DIDES

[15] In the empirical section, we use a 306-occupation classification.

framework commonly featured in assignment models (Teulings 2005; Lindenlaub 2017).[16]

Cross-Nested CES. We adopt the cross-nested constant elasticity of substitution (CNCES) structure proposed by Lind and Ramondo (2023). To formulate the correlation function, we begin by describing the structure of the labor supply. Each worker i is endowed with a set of skills, $\mathcal{S}$. For each skill $s \in \mathcal{S}$, workers draw skill-specific productivity across occupations from a correlated Fréchet distribution:

$$\Pr\left[\epsilon_1^s(i) \leq \epsilon_1^s, \ldots, \epsilon_O^s(i) \leq \epsilon_O^s\right] = \exp\left[-\left(\sum_{o=1}^{O}\left(\left(\epsilon_o^s\right)^{-\theta}\right)^{\frac{1}{1-\rho_s}}\right)^{1-\rho_s}\right].$$

where $\rho_s \in [0, 1)$ is the skill-specific correlation coefficient that measures the portability of skill s. A higher ρ_s indicates that workers tend to have similar productivity draws for that skill between occupations, meaning that the substitutability between occupations is greater for skill s. For example, cognitive skills are often highly portable between occupations (a higher ρ), allowing workers to use them elsewhere without significant productivity loss.

On the other hand, occupations rely on different technologies in the production process and utilize skills differently. Let A_o^s denote the efficiency of using skill s in the production technology of the occupation o. For a given worker i, the most productive skill powered by occupational technologies will be utilized in each occupation. Consequently, the worker's occupational productivity is the maximum of her individual skill productivities, scaled by the corresponding occupational technologies:

$$\epsilon_o(i) = \max_s A_o^s \cdot \epsilon_o^s(i).$$

PROPOSITION 3. ***Cross-Nested CES:*** *The joint distribution of productivity across occupations follows*

$$\Pr\left[\epsilon_1(i) \leq \epsilon_1, \ldots, \epsilon_O(i) \leq \epsilon_O\right] = \exp\left[-F\left(A_1\epsilon_1^{-\theta}, \ldots, A_O\epsilon_O^{-\theta}\right)\right]$$
$$\textit{with } F(x_1, \ldots, x_O) = \sum_{s\in\mathcal{S}}\left[\sum_{o=1}^{O}\left(\omega_o^s x_o\right)^{\frac{1}{1-\rho_s}}\right]^{1-\rho_s}$$

where $A_o \equiv \sum_{s\in\mathcal{S}}\left(A_o^s\right)^\theta$ *is the aggregate occupational labor productivity and* $\omega_o^s \equiv \frac{\left(A_o^s\right)^\theta}{A_o}$ *denotes the share of labor productivity derived from the skill s, which corresponds to the requirement of occupational skill.*

[16] Since we focus on wage changes in response to shocks, we use a Roy model to extend DIDES in assignment models to a multi-dimensional setting. However, this approach remains agnostic about the cross-sectional income distribution.

PROOF. See Appendix A.1.6 □

The correlation function comprises S skill-specific nests, each taking a CES form. We define ω_o^s in the correlation function F as the occupational skill requirement, indicating the relative importance of each skill s for the occupation o. A high ω_o^s implies that skill s is particularly critical in o' production process. Within each nest s, the correlation in productivity across occupations is captured by the correlation coefficient ρ_s, as discussed earlier. When $\rho_k = 0$, productivity is independent and the nest k is additive. If all $\rho_k = 0$, the distribution is simplified to an independent Fréchet distribution. In contrast, as $\rho_k \to 1$, productivity becomes perfectly correlated within the nest s.

Although we use a similar functional form to Lind and Ramondo (2023), their approach adopts a latent-factor interpretation and estimates the structure based on bilateral trade flows. Their identification strategy depends on a low-dimensional structure and requires a large dataset of bilateral trade flows for factorization, which is not feasible for estimating the labor supply structure. In contrast, our approach builds from a microfoundation and incorporates a structural interpretation, allowing the weights in the correlation function $\{\omega_o^s\}$, to have a directly measurable empirical counterpart: occupational skill requirements. We also refer to $\{\omega_o^s\}$ as the occupational location in the skill space. With the measured weights, the correlation parameters are identified using labor supply and wage responses to automation shocks.

Distance-Dependent Elasticity of Substitution. As discussed above, the key to our analysis is the elasticity of substitution matrix Θ implied by the CNCES structure. The next proposition characterizes the employment shares $\boldsymbol{\pi}$ and the matrix Θ.

PROPOSITION 4. ***Employment Shares and Elasticity of Substitution.*** *With CNCES, the occupational employment shares are given by:*

$$\pi_o = \sum_{s \in \mathcal{S}} \pi_o^s, \ \ with\ \pi_o^s = \pi_o^{s,W} \cdot \pi^{s,B} \tag{9}$$

$$where\ \pi_o^{s,W} = \frac{\left(\omega_o^s A_o w_o^\theta\right)^{\frac{1}{1-\rho_s}}}{\sum_{o'=1}^{O}\left(\omega_{o'}^s A_{o'} w_{o'}^\theta\right)^{\frac{1}{1-\rho_s}}}\ and\ \pi^{s,B} = \frac{\left[\sum_{o'=1}^{O}\left(\omega_{o'}^s A_{o'} w_{o'}^\theta\right)^{\frac{1}{1-\rho_s}}\right]^{1-\rho_s}}{\sum_{s' \in \mathcal{S}}\left[\sum_{o'=1}^{O}\left(\omega_{o'}^{s'} A_{o'} w_{o'}^\theta\right)^{\frac{1}{1-\rho_{s'}}}\right]^{1-\rho_{s'}}}$$

and the partial elasticity of substitution matrix can be written as (5)*, with the correlated elasticities expressed as:*

$$\theta \frac{x_{o'} \cdot F_{oo'}}{F_o}\Big|_{\{x_o = A_o W_o^\theta\}} = -\theta \sum_{s \in \mathcal{S}} \frac{\rho_s}{1-\rho_s} \cdot \pi_o^{s,W} \pi_{o'}^{s,W} \cdot \frac{\pi^{s,B}}{\pi_o} \tag{10}$$

PROOF. See Appendix A.1.7. □

In Proposition 4, π_o^s represents the share of workers who use skill s in the occupation o. Additionally, $\pi_o^{s,W}$ denotes the within-skill share, while $\pi^{s,B}$ refers to the between-skill share. Equation (9) has a straightforward interpretation: occupational employment is the sum of workers in the occupation who use different skills. Furthermore, the employment of workers for each skill is the product of the within-skill employment share and the share of workers using that skill. Occupations with similar skill requirements are located close to each other in the skill space and tend to exhibit similar within-skill employment shares, conditional on total occupational employment.

Equation (10) characterizes how the concept of DIDES is embedded in the correlated substitutability. Occupations with similar skill requirements exhibit higher substitutability due to their skill requirement overlap, reflected in similar within-skill employment shares. Additionally, when two occupations primarily employ workers using skills that have a high correlation across occupations (ρ_s), they are more substitutable. Conversely, if two occupations have dissimilar within-skill employment shares (e.g., when they are located far apart in the skill space) or employ skills with low correlation, the elasticity is low. In the special case where $\rho_s = 0, \forall s$, the model is reduced to the CES framework with no correlated substitutability, and the partial elasticity of substitution depends solely on the size of the occupation.

These insights are particularly relevant to automation and AI, which disproportionately affect skill-adjacent occupations. This clustering leads to an unequal wage incidence of shocks among workers. Since Θ depends on realized employment shares, analytically characterizing the elasticity matrix based solely on skill requirements is infeasible. To address this, the next section applies spectral analysis and uses Equation (8) to study the incidence of these new technologies.

Generality. We develop this economic model to better capture the impact of automation technologies and AI while abstracting from labor supply shocks. The labor demand equation (3) and the labor supply equation (4) in Proposition 1 provide a general framework for analyzing equilibrium wage and employment responses in static models. The trade-off between wage and employment effects lies at the core of labor market studies, and our empirical DIDES framework goes beyond conventional assumptions of homogeneous workers and unit elasticity, allowing for a more nuanced analysis of labor market shocks, particularly their unequal effects.

3. Empirical Results

This section presents the empirical analysis in three parts. First, we discuss the measurement of occupational skill requirements (ω_o^s) and occupational exposure to automation and AI, emphasizing the clustering nature of these technologies. Second, we estimate the elasticities of labor supply $\left\{\theta, \{\rho_s\}_{s\in\mathcal{S}}\right\}$ by leveraging long-run employment changes in response to automation-induced wage shifts. Finally, we evaluate the labor market incidence of automation and AI shocks, combining their distribution with the estimated labor supply structure.

3.1. Data and Measurement

The primary data source for measuring both skill requirements and occupational exposure is O*NET (the Occupational Information Network).[17] O*NET provides two key elements: (i) skill requirements, which define an occupation's location in the skill space of labor supply , and (ii) task descriptions, which allow us to measure exposure to automation and AI.

Occupational Skill Requirements. The primary empirical objective of introducing a Roy model with skill heterogeneity is to formulate DIDES while reducing the dimensionality of the problem. Consequently, we measure occupational skill requirements directly rather than estimating them. Specifically, we use approximately 200 descriptors from O*NET covering skills, abilities, knowledge, work activities, and work context, which serve as empirical counterparts to skill requirements.

Following Lise and Postel-Vinay (2020), we apply Principal Component Analysis (PCA) to condense this large set of descriptors into three interpretable skill dimensions: cognitive, manual, and interpersonal skill requirements. We first retain the top three principal components, which collectively explain 58% of the variation. We then construct cognitive, manual, and interpersonal skill indices by recombining these components and scaling them to $r_s \in [0,1]$ [18]. Finally, we measure occupational skill requirements (ω_o^s) as the relative importance of each skill index, weighted by its contribution to overall variance: $\omega_o^s = \frac{r_o^s \times \mathrm{Var}_s}{\sum_{s\in\mathcal{S}} r_o^s \times \mathrm{Var}_s}$. Table 1 provides examples of occupations and their corresponding skill requirements. A detailed description of this procedure is provided in Appendix B.2.

[17] The O*NET database, maintained by the U.S. Department of Labor, provides comprehensive data on occupational characteristics, worker skills, and job requirements across a wide range of professions. (Link: `https://www.onetonline.org/`)

[18] We use linear transformations instead of ranking, as they preserve the relative distances between occupations in cognitive, manual, and interpersonal skill requirements.

TABLE 1. Occupations with Skill Requirements and Technological Exposures

Occupation	Skill Requirements			Technological Exposures	
	Cognitive	Manual	Interpersonal	AI	Automation
Chief executives and administrators	0.71	0.11	0.18	0.28	0.03
Electrical engineers	0.73	0.19	0.08	0.71	0.19
Economists, market and survey researchers	0.79	0.07	0.14	0.86	0.31
Licensed practical nurses	0.52	0.26	0.22	0.08	0.47
Textile sewing machine operators	0.52	0.47	0.01	0.02	0.51

This table presents examples of occupations along with their skill requirements and technological exposures. Skill requirements are categorized into *cognitive*, *manual*, and *interpersonal* dimensions, while technological exposures indicate the likelihood of tasks being performed by *AI* or *automation*. Occupations with higher cognitive requirements, such as economists and engineers, tend to be more exposed to AI, whereas those with greater manual demands, such as textile machine operators, are more susceptible to automation.

Occupational Exposure to Technologies. Several measures exist for occupational exposure to automation (Acemoglu and Restrepo 2022; Autor et al. 2024). In contrast, estimating occupational exposure to AI-driven technologies presents a challenge, as their full impact has yet to materialize. To construct a forward-looking measure, we follow Eloundou et al. (2023) and Dell (2025) to leverage GPT-4o to evaluate task-level automation and AI feasibility. Specifically, we query GPT-4o on whether each task in O*NET's database (covering 19,200 tasks across 862 occupations) can be performed without human intervention by: (i) general AI models like GPT-4o (representing AI exposure) or (ii) industrial robots, machines, and computers without AI (representing traditional automation exposure). GPT-4o estimates that approximately 6,000 tasks, or one-third of the total, can potentially be performed by AI, a similar magnitude to automation technologies.

Table 2 provides examples of task evaluations for two occupations: economists and sewing machine operators. This classification distinguishes automation-exposed tasks, which involve well-defined, rule-based processes susceptible to mechanization, from AI-exposed tasks, which primarily involve inductive reasoning, complex decision-making, and non-physical cognitive tasks. The latter aligns with Polanyi's Paradox, which states that many cognitive tasks are difficult to codify into explicit rules, making them more suitable for AI than traditional automation (Autor 2015).

Using these task-level evaluations, we compute the share of tasks within each occupation that are either automatable or AI-exposed, forming our occupational exposure measures for automation and AI. Table 1 also reports automation and AI exposure levels for selected example occupations. Additional methodological details and comparisons with existing measures are provided in Appendix B.3.

Technological Exposures in Skill-Space. We now examine how technological exposure correlates with occupational skill requirements. Existing research shows that occupations

TABLE 2. Example of Task Evaluation by GPT-4o

Occupations and Tasks	**AI**	**Automation**
Economists, Market and Survey Researchers		
Explain economic impact of policies to the public.	YES	NO
Supervise research projects and students' study projects.	NO	NO
Teach theories, principles, and methods of economics.	YES	NO
Textile Sewing Machine Operators		
Remove holding devices and finished items from machines.	NO	YES
Cut materials according to specifications, using tools.	NO	YES
Record quantities of materials processed.	YES	YES

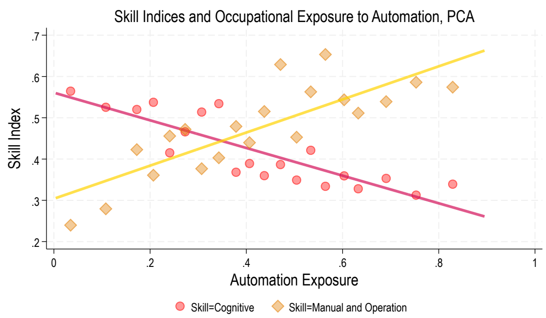
AI exposure is determined by querying GPT-4o: "Can AGI (e.g., large language models like GPT-4) potentially perform the task without human intervention?" Automation exposure is assessed by asking: "Can industrial robots, machines, and computers (no AI capability) perform the task without human intervention?" GPT-4o evaluates each task with a binary "Yes" or "No" response. The table presents examples from two occupations, each with three tasks.

requiring higher manual skills and fewer cognitive skills are more susceptible to automation (Autor, Levy, and Murnane 2003). The GPT-4o evaluation confirms this pattern, as illustrated in Panel (a) of Figure 1. Conversely, AI exposure exhibits the opposite trend: occupations requiring higher cognitive skills and fewer manual skills are more vulnerable to AI-driven displacement, as AI can increasingly perform these tasks at lower costs.

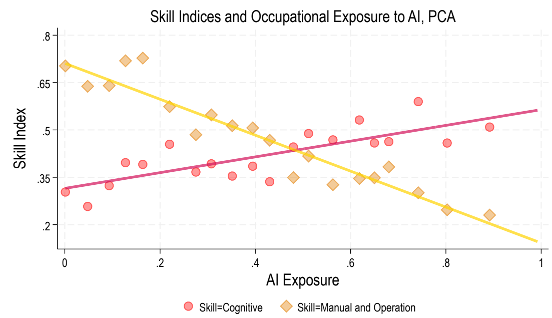
Although automation and AI affect largely distinct sets of jobs, they share a common feature: both technologies cluster within skill-adjacent occupations. Panel (c) of Figure 1 illustrates automation exposure in cognitive-manual skill space, where darker colors indicate higher exposure levels. This clustering suggests that automation is highly concentrated in manual-intensive, cognitively simpler jobs. Panel (d) shows the opposite trend for AI, with exposure clustering in cognitively demanding occupations. As highlighted in Section 2.3, this clustering significantly restricts worker mobility, leading to an unequal incidence on the wage distribution, as displaced workers also face deteriorated next-best alternative employment opportunities.

Notably, cognitive and manual skills together account for 88% of total skill requirements across occupations,[19] reinforcing their central role in shaping the labor market impact of automation and AI. Given their dominance in occupational specialization, our main analysis focuses on these two skill dimensions, while Appendix B.4 examines technological exposure along the interpersonal skill dimension.

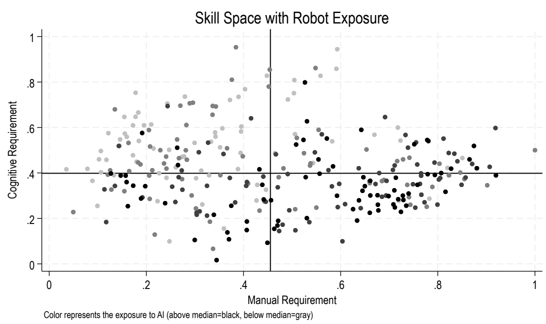
[19]Since cognitive and manual skills dominate occupational differentiation, they also largely determine substitution patterns.



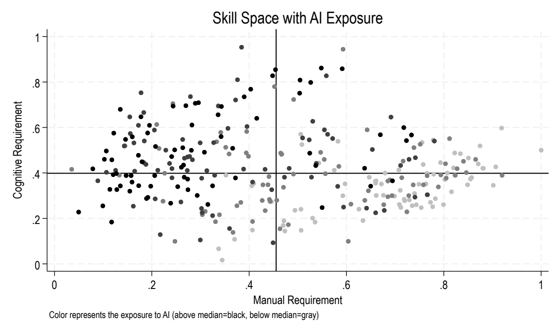
A. Skill Requirement vs. Automation Exposure



B. Skill Requirement vs. AI Exposure



C. Automation Exposure in Skill Space



D. AI Exposure in Skill Space

FIGURE 1. Technological Exposures in Skill Space

This figure illustrates the distribution of automation and AI exposures within skill space. *Panel (a)* presents a binscatter of *occupational skill indices* against *automation exposure*, while *Panel (b)* does the same for *AI exposure*. *Panels (c) and (d)* visualize the distribution of *automation and AI exposure in cognitive-manual space*, where darker dots indicate occupations with higher exposure levels.

3.2. Estimation of Labor Supply Elasticities

Having established technological exposure patterns across skill space, we now turn to estimating labor supply elasticities and formally evaluating the labor market incidence of automation and AI. In this section, we leverage long-run occupational employment changes in response to automation shocks to estimate the elasticities $\{\theta, \{\rho_s\}_{s \in \mathcal{S}}\}$.

Data and Automation Shocks. To estimate labor supply elasticities, we require data on wage and employment shifts induced by labor demand shocks. Following Cortes (2016), we use the Panel Study of Income Dynamics (PSID) to estimate relative wage trends for occupations with varying levels of automation exposure between 1985 and 2019.[20]

To account for selection effects, we exploit within-individual job spell variation using

[20] Cortes (2016) classifies occupations into Nonroutine Cognitive, Routine, and Nonroutine Manual groups and estimates relative wage trends accordingly. Instead, we estimate wage trends along a continuous measure of occupational automation exposure for 306 occupations.

the following empirical specification:

$$\text{log hourly wage}_{i,o,t} = \text{Automation}_o \cdot \beta_t + X'_{i,t} \cdot \gamma + \delta_{i,o} + u_{i,o,t}$$

where $\delta_{i,o}$ represents occupation spell fixed effects, controlling for selection based on persistent individual attributes. The vector $X'_{i,t}$ includes year fixed effects and individual characteristics such as marital status, unionization status, and region of residence. The coefficient β_t captures the relative wage growth of occupations with different levels of automation exposure.[21]

A. Wage Effects from PSID

B. Employment Effects from Census

FIGURE 2. Effects of Automation on Wage and Employment

Panel (a) reports the estimated wage effects of automation exposure using PSID data under different fixed effect specifications. Panel (b) is a binscatter plot that presents employment changes across automation exposure levels using Census and ACS data.

Panel (a) of Figure 2 presents the time-trend wage effects of automation under different fixed effect specifications. The solid blue line represents the main specification, which controls for individual job spell fixed effects, showing that occupations most exposed to automation experienced a 40% relative wage decline.[22] The blue dashed line (controlling for both individual and occupational fixed effects) and the green dashed line (controlling for occupation fixed effects only) suggest that failing to account for selection effects underestimates the impact of automation on relative wages.

Panel (b) of Figure 2 plots employment effects using IPUMS Census samples (1980–2000) and ACS samples (2010–2018) for 306 occupations (Ruggles et al. 2024). The binscatter plots log employment share changes against automation exposure, revealing that jobs most exposed to automation experienced a log employment share decline of -1.2 relative to the least exposed occupations. Combining the wage and employment effects implies an

[21] Notably, we do not control for time-varying education effects, as our objective is to capture the various channels through which automation influences occupational wages, including its impact on returns to education.

[22] The estimated related wage trends for jobs with different automation exposures are close to those in Cortes (2016) and Acemoglu and Restrepo (2022).

average labor supply elasticity of 2.85, assuming productivity distributions are independent across occupations.[23] Additional details and further results are provided in Appendix B.5.

Estimating Elasticities. We exploit the heterogeneous employment responses of different demographic groups G (race x gender) to automation shocks to estimate labor supply elasticities. We allow for demographic groups to have distinct labor productivities across occupations, denoted as $\{\boldsymbol{A}_t^g\}_{g\in G}$ while assuming that occupational skill requirements are inherent to jobs. However, we do not distinguish whether differences in comparative advantage stem from innate abilities or labor market discrimination, as this does not affect elasticity estimates given the observed employment distribution.[24] Later, we analyze how group-skill-specific discrimination (Hurst, Rubinstein, and Shimizu 2024) affects the substitution structure.

ASSUMPTION 1. *Workers in different demographic groups have unobserved occupational labor productivity, denoted by* $\{\boldsymbol{A}_t^g\}_{g\in G}$.

Given the relative wage changes $\hat{w}_{o,t+1}^{\text{Auto}}$ induced by occupational demand shifts due to automation exposure, we can derive the corresponding group-specific occupational employment shares implied by automation.

PROPOSITION 5. *Hat-Algebra: Given occupational relative wage changes* $\hat{w}_{o,t+1}^{Auto}$*, the correlation function* F*, and shape parameter* θ*, the observed employment shares* $\{\boldsymbol{\pi}_t^g\}_{g\in G}$ *serve as sufficient statistics for predicting* $\left\{\boldsymbol{\pi}_{t+1}^{g,Auto}\right\}_{g\in G}$*, without information on occupational wage* $\boldsymbol{w}_t$ *or labor productivity* $\{\boldsymbol{A}_t^g\}_{g\in G}$.

PROOF. See Appendix A.1.8 for the proof and algorithm. □

We then estimate the labor supply elasticities $\{\theta, \rho_{\text{Cog}}, \rho_{\text{Man}}, \rho_{\text{Int}}\}$ using changes in employment shares $\{\hat{\boldsymbol{\pi}}_t^g\}_{g\in G}$ in response to the relative wage changes $\hat{w}_{o,t+1}^{\text{Auto}}$, resulting from labor demand shocks due to automation exposure. Formally, we apply the pseudo-Poisson maximum likelihood (PPML) method commonly used in the gravity literature (Fally 2015):

$$\{\theta, \rho_{\text{Cog}}, \rho_{\text{Man}}, \rho_{\text{Int}}\} = \arg\min_{\Theta} \sum_{o\in O, g\in G} d\left(\pi_{o,t+1}^g, \pi_{ot}^g \hat{\pi}_{ot}^{g,\text{Auto}}\right)$$

where $d(x,\hat{x}) \equiv 2[x\ln(x/\hat{x}) - (x-\hat{x})]$. Identification follows from the assumption that automation exposure is orthogonal to labor supply shocks, reflected in the conditional

[23] If productivity is independently distributed across occupations, regressing $d\ln(EmpShare)$ on $d\ln(Wage)$ provides a valid estimate of θ.

[24] Given the correlation structure F and elasticity θ, observed employment shares serve as sufficient statistics for employment adjustments to labor demand shocks.

independence assumption embedded in the Poisson criterion: $E\left[\nu_{ot}^{g} \mid \hat{\boldsymbol{w}}_t, \{\boldsymbol{\pi}_t^{g}\}_{g\in G}\right] = 0$ where $\nu_{ot} = \hat{\pi}_{o,t+1}^{g}/\hat{\pi}_{o,t+1}^{g,\text{Auto}} - 1$. It's worth mentioning that the level of the estimated wage effect only affects the estimation of θ while ρ_s are estimated from the employment changes along the occupational robot exposures across groups.

Table 3 presents the PPML estimation results using two sets of cross-sectional employment shares. The first two columns use data from 1980 and 2000. In the CES specification (first column), we impose $\rho_s = 0$, implying an independent productivity distribution across occupations; this yields an estimated average elasticity of 3.12. In the CNCES specification (second column), where ρ_s is allowed to be nonzero, roughly two-thirds of the variation in the correlation structure is captured. Consequently, θ falls to 1.10, and $\rho_{\text{Cog}} = 0.78$ is the largest—indicating that cognitive skills are the most transferable—while $\rho_{\text{Man}} = 0.48$ is the smallest, suggesting that manual skills are the least portable. These differences are reflected in the elasticities of substitution: cognitive jobs are more substitutable because they rely on highly transferable skills, whereas manual jobs are less substitutable due to their lower skill portability. The subsequent two columns, based on employment shares from 1980 and 2010, yield similar estimates, albeit with slightly larger standard errors.

TABLE 3. PPML Estimation Results of Labor Supply Elasticities

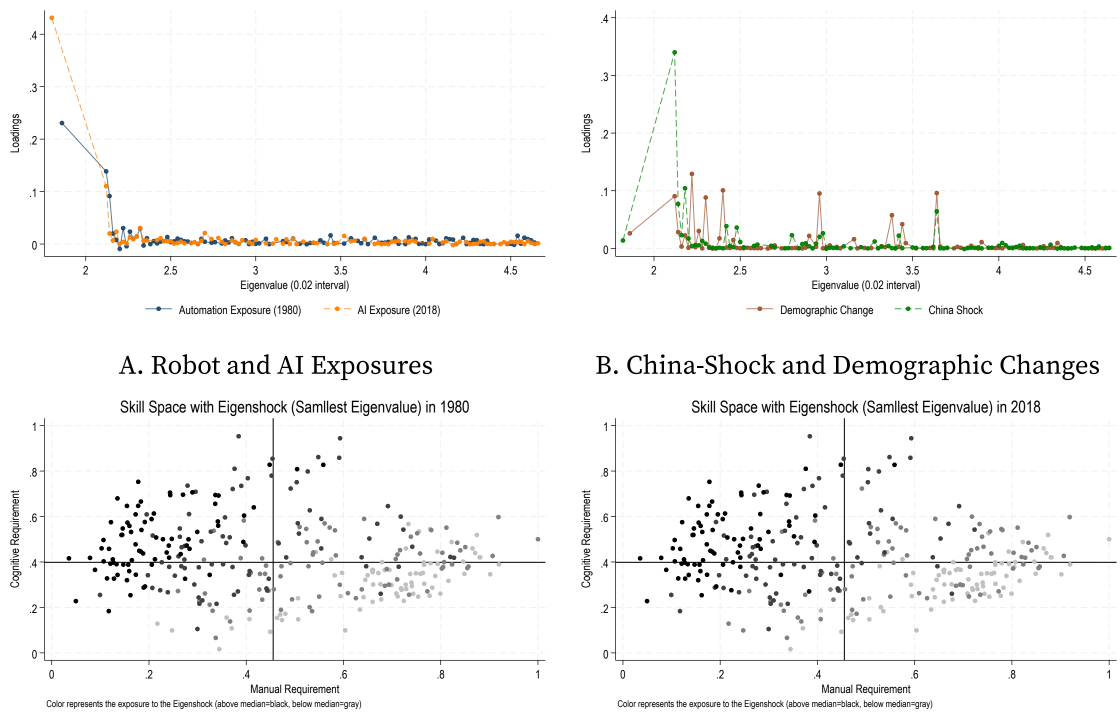
	1980–2000		1980–2010	
	CES	CNCES	CES	CNCES
θ	3.12 (0.20)	1.10 (0.32)	2.85 (0.20)	1.02 (0.30)
ρ_{Cog}	0	0.78 (0.08)	0	0.76 (0.14)
ρ_{Man}	0	0.48 (0.11)	0	0.44 (0.11)
ρ_{Int}	0	0.75 (0.14)	0	0.72 (0.18)

Note: Following the literature, we scale the Poisson deviance by the mean-variance ratio of the data to obtain standard errors. While scaling does not affect the estimates, it aligns the deviance with the data variance. The estimates and standard errors of the CES specification are close to those from a simple OLS regression with predicted wage effect as a regressor.

3.3. The Incidence of Automation and AI

For labor demand elasticity, we use the estimate from Caunedo, Jaume, and Keller (2023) $\sigma = 1.34$, which is based on occupational input responses to labor productivity changes. With the estimated labor supply elasticities, we are now ready to assess the labor market incidence of automation and AI. In the following results, we use the estimates from 1980-2000 data.

Spectral Analysis. We apply the spectral analysis developed in Section 2.3 to examine the local effects of automation and AI shocks. Proposition 1 provides the expression for the matrix of partial elasticities of substitution,[25] allowing us to decompose the variance of automation and AI shocks into eigenshocks for 1980 and 2018.[26]



A. Robot and AI Exposures

B. China-Shock and Demographic Changes

C. Eigenshock with Smallest Eigenvalue in 1980

D. Eigenshock with Smallest Eigenvalue in 2018

FIGURE 3. Spectral Analysis of Occupational Demand Shocks

Notes: Panel A shows the share of variance in automation and AI exposures explained by eigenshocks, while Panel B presents the decomposition for import competition from China and demographic changes. Panels C and D display the distribution of occupations most exposed to the eigenshocks with the smallest eigenvalues in 1980 and 2018, respectively, where darker colors indicate higher exposure.

Figure 3 illustrates the results of this decomposition. Panel A shows the share of variance in empirical automation and AI exposures explained by eigenshocks, ranked by their corresponding eigenvalues. The results indicate that both automation and AI are predominantly explained by eigenshocks associated with the smallest eigenvalues (23% and 44%, respectively). As discussed earlier, smallest eigenvalues imply that these eigenshocks induce minimal employment shifts. Consequently, the labor market has limited capacity to absorb automation and AI shocks, leading to a more unequal distribution of

[25] We first invert the employment shares to obtain the implied $A_{o,t}w_{o,t}^{\theta}$ and then compute the partial elasticities. The inversion is uniquely defined due to the gross-substitute property of the labor supply mapping, which results from the sign-switching property of the correlation function.

[26] Eigenshocks are generally not orthogonal and we allocate the contribution of the covariance equally among them.

wage effects.

Panel B presents the eigenshock decomposition for two other occupational labor demand shocks from the literature: import competition from China and occupational demand shifts due to demographic changes, particularly population aging (Autor et al. 2024). These shocks load more heavily on eigenshocks associated with larger eigenvalues, particularly in the case of demographic changes, suggesting that the labor market can more effectively absorb their impact.

To further examine the structure of eigenshocks, Panels C and D plot the distribution of occupations most exposed to the eigenshocks associated with the smallest eigenvalues in 1980 and 2018, respectively, where darker colors indicate higher exposure. Comparing these distributions to Figure 1, we see that automation and AI exposures almost perfectly overlap with these eigenshocks.[27] These eigenshocks cluster in skill-adjacent occupations, as illustrated in the 2×2 example. This confirms that such clustering hinders workers from transitioning away from affected occupations, exacerbating the labor market disruptions caused by automation and AI.

The Incidence on Wages and Gains from Mobility. To better assess the distributional consequences of automation and AI, we first use the estimated wage effects of automation to invert the model and recover the corresponding relative demand changes and employment effects. For AI, we use the measured distribution of AI exposure and simulate an AI shock that produces the same wage effect as the automation shock.[28]

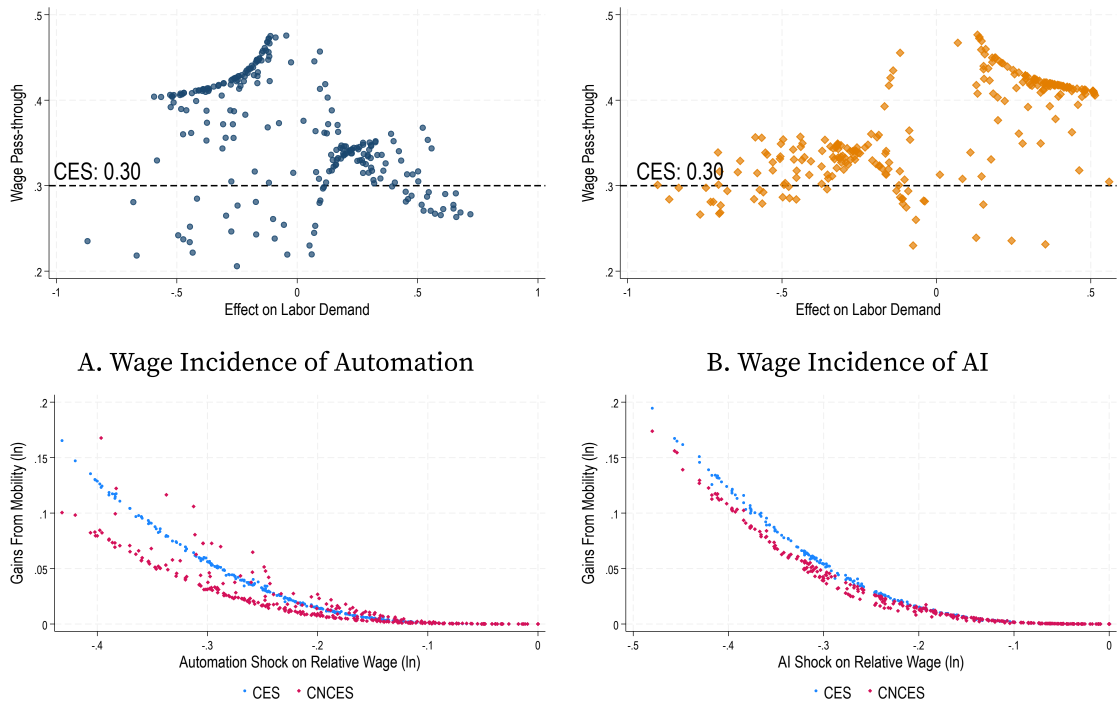
We begin by analyzing how much of the relative demand shock translates into relative wage changes and how much is absorbed through labor market adjustments. Panel A of Figure 4 plots the share of relative demand changes that pass through to wages against the relative demand shifts for occupations with varying levels of automation exposure. The dashed line represents the wage incidence predicted using the estimated average elasticity of substitution, implying that 30% of relative demand changes translate into wage effects. However, the blue dots reveal that the wage incidence is higher for most occupations, particularly for manual occupations (top-left dots), where 45% of relative demand shifts are passed through to wages. This suggests that the clustering of automation exposure lowers the equilibrium wage elasticity from 3.1 to approximately 2.2. Consequently, relying on the average elasticity overestimates worker mobility by 40% and understates wage incidence by more than 10 percentage points (from 40% to 30%).

Panel B plots the wage incidence of AI shocks, showing that the equilibrium elasticity is around 2, with an even larger share of wage effects passing through. This result is con-

[27]For any eigenvector, its negative is also an eigenvector corresponding to the same eigenvalue. Thus, reversing the color scale would yield the same interpretation for robot exposures.

[28]Since we focus on the relative effects of AI exposure, this choice simply facilitates comparison with automation shocks.

sistent with the spectral analysis, which indicated that AI shocks load more variance onto eigenshocks associated with the smallest eigenvalues, reducing labor market absorption capacity. In Appendix B.6, we discussed the corresponding employment effects.



A. Wage Incidence of Automation

B. Wage Incidence of AI

C. Gains from Mobility for Automation

D. Gains from Mobility for AI

FIGURE 4. The Incidence of Automation and AI

Notes: Panels A and B display the share of relative demand shocks that translate into wage changes for automation and AI, respectively. For clarity, we exclude occupations with small relative demand changes or large negative wage effects. Panels C and D illustrate the gains from worker mobility, measured as the share of wage losses recovered (EV) through occupational transitions.

While wage effects reflect the impact on workers who remain in affected occupations, many workers move away from negatively impacted jobs. To quantify the welfare effects of mobility, we compute the equivalent variation (EV), expressed as:

$$\text{EV} \approx \sum_{o'} d\ln\left(\frac{w_{o'}}{w_o}\right) \times \mu_{oo'}$$

where $\mu_{oo'}$ denotes the share of workers transitioning from occupation o to o'. Panel C of Figure 4 plots the average gains from mobility for automation shocks. Workers in negatively impacted occupations recover 20% of their wage loss by transitioning to better off jobs. However, the clustering of automation exposure reduces mobility by about 35% compared to estimates derived from the average elasticity. This discrepancy arises because the CES framework significantly overstates the probability of workers transitioning from the most affected jobs to substantially better off occupations.

Panel D presents the mobility gains for AI shocks, where workers recover approximately 25% of relative wage losses, exceeding the mobility recovery observed for automation shocks. This is because AI primarily affects cognitive-intensive jobs, and cognitive skills are generally more transferable across occupations, leading to greater mobility gains.

3.4. Summary

In this empirical section, we measure occupational skill requirements and use employment responses to automation shocks to estimate labor supply elasticities under DIDES. The clustering of automation and AI shocks significantly reduces worker mobility, with approximately 40% of relative labor demand changes passing through to wages. Meanwhile, workers can recover only about 20% of their relative wage losses through job mobility.

4. Dynamic Model with Transition

Our model retains the Roy structure, enabling a seamless integration of DIDES into related frameworks while incorporating a richer substitution structure — essential for counterfactual analysis. In this quantitative section, we extend our static model into a dynamic discrete choice framework (Artuç, Chaudhuri, and McLaren 2010; Caliendo, Dvorkin, and Parro 2019) to capture gradual labor market transitions. This extension allows us to examine the dynamic labor market incidence of technological adoption in the transition and the long run (Lehr and Restrepo 2022; Adão, Beraja, and Pandalai-Nayar 2024). Different from Dvorkin and Monge-Naranjo (2019), who incorporates skill accumulation into the dynamic occupation choice, we abstract from persistent ex-ante worker heterogeneity while allowing for the flexible substitutability to focus on the incidence.

As established in the static model, the substitution structure plays a central role in determining counterfactual outcomes, with observed job transition flows serving as sufficient statistics. This insight distinguishes our approach from studies that estimate models based on job transition flows with a single elasticity parameter (Traiberman 2019; Grigsby 2022; Bocquet 2024). By contrast, our framework explicitly incorporates correlation structures in the productivity distribution, thereby capturing realistic substitution patterns among jobs.

4.1. Dynamic Discrete Choice with DIDES

Our focus is on studying the dynamic labor market incidence of technological adoption rather than modeling firms' endogenous technology adoption decisions. The production side remains identical to the static framework, while workers make rational, forward-looking occupational choices in response to automation and AI shocks. To model these choices, we adopt a structure similar to Caliendo, Dvorkin, and Parro (2019) with a correlated productivity distribution among jobs.

Workers' Dynamic Decision. In each period, we denote the vector of occupational employment by $\boldsymbol{L}_t$. Workers are assumed to be hand-to-mouth, taking the wage path $\{\boldsymbol{w}_t\}_{t=0}^{\infty}$, as given, and derive utility from consumption and labor supply according to:

$$U\left(\{c_t(i), \ell_t(i)\}_{t=0}^{\infty}\right) = \sum_{t=0}^{\infty} \beta^t \left(\ln c_t(i) - \ln \ell_t(i)\right)$$

At the beginning of each period, workers draw their labor productivity across all occupations from the same distribution as in the static model:[29]

$$\Pr\left[\epsilon_1(i) \leq \epsilon_1, \ldots, \epsilon_O(i) \leq \epsilon_O\right] = \exp\left[-F\left(A_1\epsilon_1^{-\theta}, \ldots, A_O\epsilon_O^{-\theta}\right)\right]$$

After observing their labor productivity, workers choose an occupation, with consumption equal to occupational income, $c_t = w_{o,t}$, and labor supply given by

$$\ln\left(\ell_t\left(i\right)\right) = -\kappa \ln\left(\epsilon_{o,t}\left(i\right)\right)$$

In contrast to the static model, productivity enters the labor supply function with a short-run discounting factor κ which governs short-run labor supply elasticity. Although we could allow workers to redraw their labor productivity with a short-run probability, doing so yields the same sufficient statistics for counterfactual welfare and similar dynamics [30]. However, permitting productivity redraws would imply larger gains from reallocation under the same transition dynamics. In addition, workers incur a job transition cost $\tau_{oo'}$ when switching occupations.

ASSUMPTION 2. *The job transition cost is constant over time $\tau_{oo'}$ and measured in terms of utility.*

Given this economic environment, we formulate workers' decisions recursively via the following Hamilton-Jacobi-Bellman (HJB) equation:

$$v_{o,t}\left(\epsilon_t\right) = \max_{o'}\left\{\ln w_{o'} + \kappa \ln \epsilon_{o',t} + \beta V_{o,t+1} - \tau_{oo'}\right\} \text{ with } V_{o,t+1} = \mathrm{E}_{\boldsymbol{\epsilon}}\left[v_{o,t+1}\left(\boldsymbol{\epsilon}\right)\right]$$

[29] The correlation function in both transition and steady state corresponds to the same empirical elasticities. To see this, if transition probabilities are identical across all origins, then these probabilities correspond to the stationary distribution, implying they share the same substitution structure.

[30] Allowing for a short-run probability of redrawing productivity can also be interpreted as an overlapping generations (OLG) framework.

where $v_{o,t}(\epsilon_t)$ denotes a worker's lifetime utility in occupation o after observing their productivity. This utility comprises current-period benefits, $\ln w_{o'} + \kappa \ln \epsilon_{o',t}$ and the discounted expected future utility, $V_{o,t+1} = \mathrm{E}_{\epsilon}\left[v_{o,t+1}(\epsilon)\right]$, net of the job transition cost $\tau_{oo'}$. Workers will choose the occupation o' to maximize their lifetime utility.

Consequently, we can recursively express the occupational expected utility as

$$V_{o,t} = \ln\left(F\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)^{\frac{\kappa}{\theta}}\right) + \bar{\gamma}\frac{\kappa}{\theta}$$

$$\text{where } Z_{oo',t} = \exp\left(\beta V_{o',t+1} + \ln w_{o',t} - \tau_{oo'}\right)$$

Additionally, the job transition probability can be derived as shown in Appendix A.2.1:

$$\mu_{oo',t} = \frac{A_{o',t}Z_{oo',t}^{\frac{\theta}{\kappa}} \times F_{o'}\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)}{F\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)}$$

The interplay of job transition costs and idiosyncratic productivity shocks generates the slow labor market adjustments appearing in our model. A key distinction of our approach is that it allows for a rich substitution pattern between jobs, as embedded in the correlation function F. When F is additive, our framework reduces to the standard model.

Dynamic Equilibrium. As discussed in the static model, the share of tasks performed by labor, denoted by $\{\boldsymbol{s}_t^{\ell}\}_{t=0}^{\infty}$, characterizes the distributional effects of technological adoption, while aggregate capital productivity, $\left\{a_t^k\right\}_{t=0}^{\infty}$, is Hicks-neutral (See Appendix A.4). All other occupational labor productivity is represented by $\{\boldsymbol{A}_t\}_{t=0}^{\infty}$.

Given the time-varying fundamentals $\{\Psi_t\}_{t=0}^{\infty} = \left\{\boldsymbol{s}_t^{\ell}, a_t^k, \boldsymbol{A}_t\right\}_{t=0}^{\infty}$, we define a dynamic equilibrium under rational expectations. In this equilibrium, there exists a time path of wages, $\{\boldsymbol{w}_t\}_{t=0}^{\infty}$, occupational allocations, $\{\boldsymbol{L}_t\}_{t=0}^{\infty}$, and job transition probabilities, $\{\mu_t\}_{t=0}^{\infty}$ such that

a. The wage vector $\boldsymbol{w}_t = \boldsymbol{w}\left(\boldsymbol{s}_t^{\ell}, a_t^k, \boldsymbol{L}_t\right)$ is the solution to the static production equilibrium.
b. Workers' optimal occupational choices yield job transitions, $\mu_t = \left\{\mu_{oo',t}\right\}_{o=1,o'=1}^{O,O}$.
c. The labor allocation evolves according to $L_{o,t} = \sum_{o'} \mu_{oo',t} L_{o,t-1}$.[31]

[31] We construct job flows from the retrospective responses in the March CPS; consequently, the aggregate flows derived from these responses do not directly match the observed occupational employment levels. To account for this discrepancy, we adjust the evolution of occupational employment as $L_{o,t} = \sum_{o'} \mu_{oo',t} L_{o,t-1} + \Delta L_{o,t}$.

4.2. Dynamic Hat Algbrea with Correlation

In this section, we extend the dynamic hat algebra to incorporate correlated productivity distributions, thereby enabling a richer substitution pattern. The model is designed to address key counterfactual questions. For example, what would have happened to the wage distribution if automation technologies had not been adopted? Alternatively, how much can the labor market absorb the unequal demand shocks caused by AI if AI technologies are adopted to the same extent as automation, but with a much more rapid pace by 2030?

Formally, by counterfactual analysis we study how equilibrium allocations across occupations and over time change relative to a baseline economy when faced with an alternative sequence of fundamentals, which we denote by $\{\Psi_t'\}_{t=1}^{\infty}$. As is standard, we examine the impact of changes in these counterfactual fundamentals on the equilibrium outcomes of interest.

To facilitate the characterization of the dynamic equilibrium, we introduce additional notation. For any scalar or vector x, we denote its proportional change between periods t and $t+1$ as $\dot{x}_{t+1} = x_{t+1}/x_t$. Additionally, we use x_t' to denote the corresponding variable in the counterfactual economy. Lastly, we define $\hat{x}_{t+1} = \dot{x}_{t+1}'/\dot{x}_{t+1}$ which represents the ratio of the time change in the counterfactual equilibrium to that in the initial equilibrium.

Before characterizing counterfactual outcomes, we introduce the correlation-adjusted transition probability:

$$\tilde{\mu}_{oo',t} = A_{o',t} Z_{oo',t}^{\frac{\theta}{\kappa}} / F\left(A_{1,t} Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}}\right)$$

which will serve as a sufficient statistic. Note that when F is additive, $\tilde{\mu}_t$ coincides with μ_t since $\tilde{\mu}_{oo',t} = \mu_{oo',t}/F_{o'}$.

LEMMA 4. *Given the correlation function F, there exists a unique mapping between the occupation transition probability μ_t and the correlation-adjusted transition probability $\tilde{\mu}_t$:*

$$\left\{\mu_{oo',t} = \tilde{\mu}_{oo',t} F_{o'}\left(\tilde{\mu}_{o1,t}, \ldots, \tilde{\mu}_{oO,t}\right)\right\}_{o'=1}^{O}, \quad \forall\, o.$$

With this correlation-adjusted transition probability, we are ready to introduce the dynamic hat algebra with correlation to analyze how economic outcomes change counterfactually. Specifically, we study how allocations and wages across occupations evolve over time in response to alternative sequences of fundamentals, denoted by $\{\hat{\Psi}_t\}_{t=1}^{\infty}$.

PROPOSITION 6. *For the time-varying counterfactual changes in fundamentals $\{\hat{\Psi}_t\}_{t=1}^{\infty} = \left\{\hat{\boldsymbol{s}}_t^{\ell}, \hat{a}_t^{k}, \hat{\boldsymbol{A}}_t\right\}_{t=1}^{\infty}$ with $\lim_{t\to\infty} \hat{\Psi}_t = 1$. The observed allocations and transition probability $\{\boldsymbol{L}_t, \mu_t\}_{t=0}^{\infty}$ are sufficient to characterize the counterfactual changes in allocations, wages, and expected utility ($u_{o,t} = \exp\left(V_{o,t}\right)$). Formally:*

- *The counterfactual changes in wage* $\hat{\boldsymbol{w}}_t = \hat{\boldsymbol{w}}\left(\hat{\boldsymbol{s}}_t^{\ell}, \hat{a}_t^k, \hat{L}_t\right)$ *solves the static production equilibrium.*
- *The counterfactual, correlation-adjusted transition probability is given by*

$$\tilde{\mu}'_{oo',t} = \frac{\tilde{\mu}'_{oo',t-1}\dot{\hat{\mu}}_{oo',t}\hat{A}_{o',t}\hat{u}_{o't+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{\tilde{\mu}'_{oo'',t-1}\dot{\hat{\mu}}_{oo'',t}\hat{A}_{o'',t}\hat{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o'',t}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$

- *The counterfactual change in utility is characterized by*

$$\hat{u}_{o,t+1} = F\left(\left\{\tilde{\mu}'_{oo'',t}\dot{\hat{\mu}}_{oo'',t+1}\hat{A}_{o',t+1}\hat{u}_{o't+2}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o',t+1}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

  with terminal condition $\lim_{t\to\infty} \hat{u}_{o,t} = 1$.
- *The counterfactual occupational allocation evolves according to* $L'_{o',t} = \sum_o \mu'_{oo',t}L'_{o,t-1}$.

PROOF. See Appendix A.7 for proof and for the different expression for time 0 that accounts for unexpected changes in fundamentals. □

Proposition 6 demonstrates the sufficient statistic property of the dynamic hat algebra: the observed allocations and transition probabilities fully characterize the counterfactual outcomes under a new sequence of fundamentals. Moreover, it underscores the critical role of the substitution structure — captured by the function F — which governs the counterfactual implications. In particular, while the observed allocations serve as sufficient statistics, the specific form of F determines how changes in fundamentals translate into counterfactual wages and allocations.[32] Our static results, which show that the clustering nature of technological changes combined with DIDES leads to an unequal labor market incidence, persist in the dynamic framework.

Finally, as derived in the Appendix A.8, the welfare change resulting from a shift in fundamentals—measured in terms of consumption equivalent variation—can be expressed as

$$\mathrm{EV}_{o,t} = \sum_{s=t}^{\infty} \beta^{s-t} \ln\left(\hat{w}_{o,s}/\hat{\tilde{\mu}}_{oo,s}^{\frac{\kappa}{\theta}}\right)$$

Moreover, the changes in the occupation-specific adjusted staying probabilities capture the gains from mobility, echoing the results in Arkolakis, Costinot, and Rodríguez-Clare (2012), once the substitution structure is taken into account.

[32] When F is additive, we are back to the standard dynamic hat algebra approach with independent productivity distribution.

4.3. Data and Estimation

the Euler-Equation Approach. Based on the Euler-equation approach introduced by ACM (Artuç, Chaudhuri, and McLaren 2010), we account for the correlation in the productivity distribution and the corresponding substitution structure. Specifically, we derive the following analogous estimating equation:

$$\ln \frac{\tilde{\mu}_{oo',t}}{\tilde{\mu}_{oo,t}} = \frac{\theta}{\kappa} \ln \frac{w_{o',t}}{w_{o,t}} + \beta \ln \frac{\tilde{\mu}_{oo',t+1}}{\tilde{\mu}_{o'o',t+1}} + (\beta - 1)\,\tau_{oo'} + \nu_t$$

where ν_t is an error term. This expression parallels ACM's formulation, but with correlation-adjusted job transitions. Intuitively, the cross-sectional adjusted job transition flows incorporate information on expected future wages and the option value of job mobility, and the adjusted future job transition flows in this regression serve as sufficient statistics for these option values (see Appendix A.8 for details). The key insight is that, after conditioning on adjusted future values, the coefficient $\frac{\theta}{\kappa}$ [33] represents the elasticity of relative adjusted job transitions with respect to changes in relative wages. As in ACM, the theory implies that lagged values of wages and adjusted job transitions are valid instruments.[34]

Data and Estimation Results. Our estimation strategy requires aggregate job flows across occupations. We construct these measures using individual-level data from the US Census Bureau's March Current Population Survey (CPS). Due to dataset size constraints, we cluster occupations into 15 groups based on their skill requirements using a k-means algorithm. The resulting procedure is intuitive, as occupations with similar skill requirements are grouped together in the skill space. Finally, we compute the annualized job transition probabilities among these 15 clusters, μ_t, for the period 1976–2019. Appendix B.7 provides a detailed discussion of the data construction.

TABLE 4. Estimation of Short-run Elasticity $\frac{\theta}{\kappa}$

	(1)	(2)	(3)	(4)
$\frac{\theta}{\kappa}$	0.063***	0.071***	0.068**	0.080**
	(0.018)	(0.018)	(0.025)	(0.025)
Destination FE			✓	
Origin FE				✓
IV		✓	✓	✓

Notes: Column (1) presents the baseline OLS estimate of the short-run elasticity. Column (2) employs IV estimation using lagged adjusted job transition probabilities and wages as instruments. Columns (3) and (4) add destination and origin fixed effects, respectively.

[33] We cannot separately estimate θ and κ, nor is it necessary to do so, as they enter the equilibrium dynamics and welfare metrics jointly, as demonstrated in Proposition 6.

[34] The exclusion condition is that the error term ν_t is not correlated over time. See ACM for a detailed discussion.

We use $\beta = 0.96$ as the annual discount factor. Table 4 reports the estimation results for the short-run elasticity $\frac{\theta}{\kappa}$. Column (1) presents the OLS estimate, which yields a short-run elasticity of 0.063. Column (2) implements an IV approach using lagged adjusted job transition probabilities and wages as instruments, resulting in an estimate of 0.071. Columns (3) and (4) incorporate destination and origin fixed effects, respectively, yielding estimates of 0.068 and 0.080. Although these estimates are broadly consistent, they are lower than those reported in ACM, primarily due to our use of correlation-adjusted job transition probabilities. As discussed in the static model, this adjustment nets out within-skill substitutability — a major source of variation in the response of job transitions to relative wage changes. Moreover, grouping occupations by similar skill requirements further reduces across-cluster job transition responses, contributing to the smaller elasticity estimates.

4.4. The Dynamic Incidence of Automation and AI

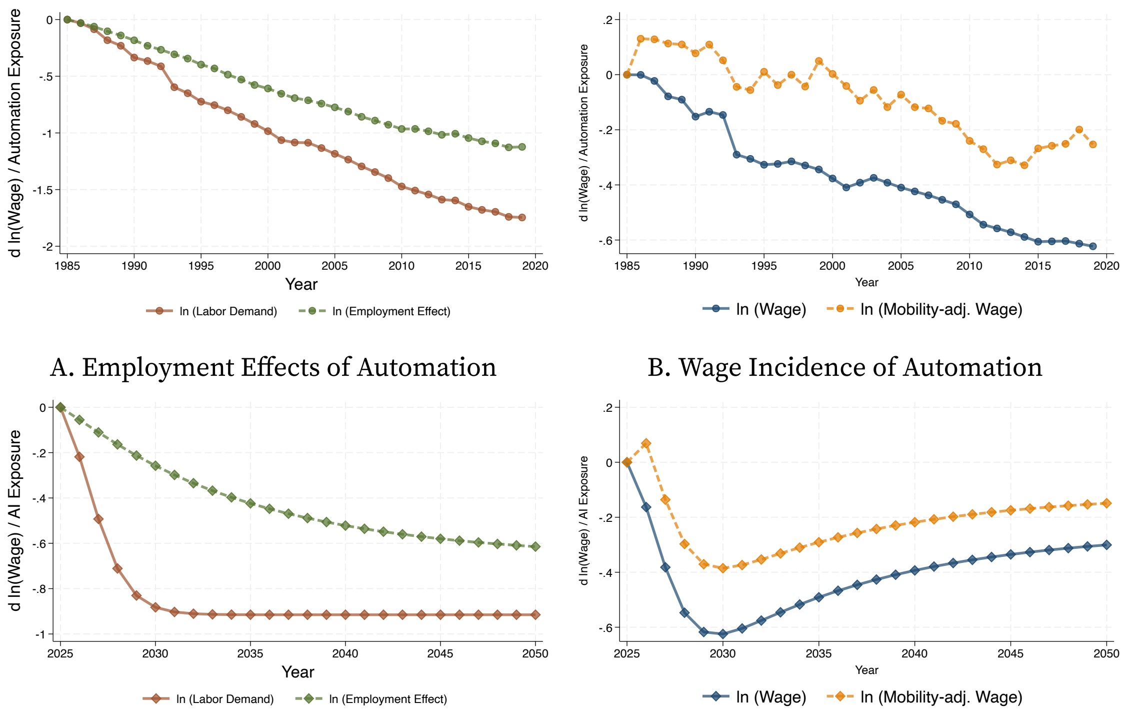
We now assess the distributional effects of automation and AI within a slow-adjustment labor market framework. In our quantitative evaluation, we employ 15 occupation clusters with transition probabilities constructed from CPS data. For our counterfactual applications, we use elasticities from our static estimation, augmented by a short-run labor supply elasticity $\theta/\kappa = 0.07$ estimated via the Euler-equation approach. To maintain clarity, we focus on average effects over time and do not present cross-sectional heterogeneity here, as those results closely mirror those from the static model.

For automation technologies, we obtain an ex-post estimate of their dynamic wage effects, as shown in Panel A of Figure 2. The findings indicate that occupations with higher automation exposure have experienced a gradual relative wage decline since 1985, resulting in up to a 50% difference between occupations where all tasks are exposed and those where none are. To match this observed wage trend, we calibrate the share of tasks performed by labor, $\{s_t^{\ell}\}$. We then implement the following counterfactual scenario: what would have occurred if the task labor shares $\{s_t^{\ell}\}$ had remained unchanged since 1985? [35]

Panel A of Figure 5 illustrates the relative decline in occupational labor demand due to larger automation exposures (dashed line) alongside the share of demand changes absorbed by employment shifts (green line). Employment adjustments mitigate roughly two-thirds of the relative demand changes, with the remaining one-third materializing as relative wage changes (depicted by the blue line in Panel B). In Panel B, the orange line represents the cumulative effect of mobility-adjusted wage changes, given by $\sum_{s=1985}^{t} \ln\left(\hat{w}_{o,s}/\hat{\tilde{\mu}}_{oo,s}^{\frac{\kappa}{\theta}}\right)$, which accounts for worker mobility gains. These gains offset approximately half of the wage loss. Compared to the static model, mobility gains are higher because we allow workers to redraw their productivity, a more realistic setting when

[35] Since we focus on the unequal effects of additional automation exposure, we omit discussions of aggregate gains.

occupational productivity is not permanent in reality.[36] Furthermore, because workers are forward-looking, mobility gains occur early in the adjustment process, as outside options improve immediately for negatively impacted jobs, while wage effects accumulate gradually over time.



A. Employment Effects of Automation

B. Wage Incidence of Automation

C. Employment Effects of AI

D. Wage Incidence of AI

FIGURE 5. The Dynamic Incidence of Automation and AI

Notes: Panels A and B show the employment and wage effects of automation exposure, while Panels C and D depict the projected effects of rapid AI adoption. Dashed lines represent changes in labor demand, green lines indicate employment shifts, and blue lines capture wage incidence. The orange line in Panels B and D accounts for mobility-adjusted wage changes.

The gradual wage impact of automation suggests that its adoption occurred progressively over the past four decades, allowing the labor market to absorb roughly two-thirds of the associated labor demand shifts over time. The gradual adoption of automation makes the labor market adjustment in transition similar to that in the long run. However, if AI advances rapidly — as many practitioners advocate — the labor market may face greater adjustment challenges. To explore this scenario, we consider a counterfactual in which AI adoption reaches the same scale as automation by 2030, allowing us to evaluate labor market responses to a rapid technological transition.

[36]Workers in their current jobs typically have higher occupation-specific productivity due to selection; if productivity were permanent, they would face greater losses when transitioning. Allowing new productivity draws each period provides the same sufficient statistics for mobility gains as an overlapping generations (OLG) model.

Panels C and D of Figure 5 illustrate the dynamic incidence of accelerated AI adoption. Panel C shows that the labor market adjusts sluggishly, absorbing less than one-third of relative demand shifts initially, with another one-third absorbed over the subsequent two decades. In Panel D, occupations highly exposed to AI experience a sharp wage decline as full adoption materializes by 2030, followed by a gradual recovery. Mobility gains offset approximately one-third of the relative wage loss during the transition. Overall, these findings suggest that the slow pace of labor market adjustment severely limits its ability to absorb the impact of rapid AI advancements.

These findings underscore a key insight that extends beyond the static model: the clustering nature of both automation and AI exposure constrains worker mobility, limiting the labor market's capacity to absorb these shocks through occupational transitions in both the short and long run. In the case of automation, this rigidity is most pronounced in the long run, as gradual adoption has contributed to persistent wage disparities across occupations. For a rapid AI expansion, however, the constraints on mobility operate in both the short and long run, amplifying labor market inequality and resulting in a highly uneven distribution of economic incidence.

5. Conclusion

This paper develops a general framework for evaluating the labor market incidence of technological shocks, with a particular focus on automation and AI. At the heart of our analysis is the distance-dependent elasticity of substitution (DIDES), which captures how substitutability between occupations declines with their distance in skill space. This structure provides a unified approach to studying how automation and AI affect wages, employment, and mobility across occupations.

Our static analysis highlights that automation and AI affect distinct sets of occupations but share a common feature: they cluster within skill-adjacent occupations, leading to significant unequal wage changes with limited employment reallocation. This clustering nature significantly reduces labor market adaptability: the equilibrium labor elasticity is one-third lower than under a unitary elasticity assumption. Using spectral analysis, we show that these technological shocks primarily load onto eigenshocks with small eigenvalues, meaning they induce large wage dispersion with minimal employment shifts. Empirically, we estimate that approximately 40% of relative labor demand shocks pass through to wages, while only 20% of wage losses are offset by worker mobility—suggesting that technological shocks create large and persistent distributional effects in the long run. In contrast, a standard framework assuming unit elasticity would understate the wage effects of automation by 30% and overestimate the gains from occupational mobility by up to a half.

To assess the dynamic evolution of these effects, we extend our framework into a slow-adjustment labor market model with occupational transition probabilities and forward-looking worker behavior. Empirical estimates show that automation-driven labor demand shifts since 1985 have led to up to a 50% cumulative wage gap between highly exposed and unaffected occupations. While employment reallocation absorbs two-thirds of these shocks, the remaining one-third passes through to wages, with mobility gains offsetting only half of the wage losses. However, the gradual adoption of automation has allowed the labor market to absorb these unequal effects over time, stabilizing the long-run wage distribution at a persistent but steady disparity across occupations.

For AI, we simulate a counterfactual scenario where adoption reaches automation's historical scale by 2030. The results suggest that less than one-third of labor demand shifts would be absorbed initially, with another third absorbed over two decades. Unlike automation, where gradual adoption facilitated labor market absorption, a rapid AI expansion would generate immediate and severe wage losses, particularly in cognitive-intensive occupations with limited transition pathways. This sluggish labor market response wouldn't mitigate wage inequality in the transition, leading to a more uneven and disruptive adjustment compared to past technological changes.

These findings highlight the need for policies that facilitate transitions across skill boundaries, rather than assuming that market forces alone will smooth out the impact of technological disruptions. One key implication is that skill-based mobility subsidies — which provide targeted support for workers moving into occupations where their existing skills remain valuable — could help mitigate wage losses and improve reallocation efficiency. Additionally, tax incentives for firms that invest in structured, job-specific retraining would encourage greater adaptability among affected workers while reducing labor market frictions. Given the clustering of automation and AI exposure, occupational mobility support — such as relocation assistance or wage insurance for transitioning workers — could also ease the burden on those facing the most severe wage pressures.

While this paper focuses on the short- and medium-run labor market incidence of automation and AI, future research could extend the framework to capture endogenous skill acquisition, firm technology adoption, and global trade dynamics. Additionally, the DIDES framework could be applied to study other forms of labor market shocks, such as trade-induced disruptions (e.g., the China shock) or demographic shifts.

Overall, our findings underscore the importance of considering the substitution structure between jobs, skill-space clustering of technological changes, and dynamic reallocation frictions when assessing the labor market effects of automation and AI. As these technologies continue to reshape the economic landscape, understanding their dynamic and distributional consequences will be critical for designing policies that ensure more equitable labor market outcomes.

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Appendix A. Proof of Results in Main Text

A.1. Results in the Static Framework

A.1.1. Properties of the Correlation Function

The correlation function $F : \mathbb{R}_+^N \to \mathbb{R}_+$ satisfies the following properties:

a. F is homogeneous of degree one.
b. F is unbounded.
c. If the mixed partial derivatives of F exist and are continuous up to order N, then the n-th partial derivative of F with respect to the distinct argument of n is nonnegative if o is odd and non-positive if o is even.

Additionally, the function $C(u_1, \ldots, u_N) = \exp[-F(-\ln u_1, \ldots, -\ln u_N)]$ is a max-stable copula satisfying the property:

$$C(u_1, \ldots, u_N) = C\left(u_1^{1/m}, \ldots, u_N^{1/m}\right)^m, \forall m > 0 \text{ and } (u_1, \ldots, u_N) \in [0,1]^N$$

A.1.2. Proof of Proposition 1

Labor Demand. We begin by deriving the labor demand equation, following the framework established in Acemoglu and Restrepo (2022). The production equilibrium of this economy, with $\boldsymbol{h}$ taken as given, is determined by solving the following optimization problem for the representative firm:

$$\begin{aligned}
&\max_{\{k(x), h_o(x)\}_{x\in\mathcal{T}}} y - \int_{\mathcal{T}} (k(x)) \cdot dx \\
&\text{Subject to: } y = \left(\int_{\mathcal{T}} (y(x))^{\frac{\sigma-1}{\sigma}} \cdot dx\right)^{\frac{\sigma}{\sigma-1}} \\
&y(x) = a(x)k(x) + h_o(x), \forall x \in \mathcal{T}_o, \forall o \\
&L_o = \int_{\mathcal{T}_o} \ell_o(x) dx, \forall o
\end{aligned}$$

Since the objective function is concave and the constraint set is convex, the optimization problem has a unique maximum, provided certain assumptions hold. Specifically, we assume that $\left(\int_{\cup_o \mathcal{T}_o^k} a(x)^{\sigma-1} dx\right)^{\frac{1}{1-\sigma}} > 1$, ruling out cases where capital could self-replicate to infinity. Let w_o denote the Lagrange multiplier associated with the labor supply constraint for occupation o. Task allocation must satisfy: (1):

$$\mathcal{T}_o^\ell = \left\{x : \frac{1}{a(x)} \geq w_o, x \in \mathcal{T}_0\right\} \text{ and } \mathcal{T}_o^k = \left\{x : \frac{1}{a(x)} < w_o, x \in \mathcal{T}_0\right\}$$

Additionally, the demand for task x is given by:

$$y(x) = y \cdot p(x)^{-\sigma}$$

where the equilibrium task price $p(x)$ is

$$p(x) = \begin{cases} a(x)^{-1} & \text{if } x \in \mathcal{T}_o^k \\ w_o & \text{if } x \in \mathcal{T}_o^\ell \end{cases} \tag{A1}$$

This implies that the task-level capital and labor demands are:

$$k(x) = \begin{cases} y \cdot a(x)^{\sigma} & \text{if } x \in \mathcal{T}_o^k, \\ 0 & \text{if } x \notin \mathcal{T}_o^k, \end{cases}$$

$$\ell_o(x) = \begin{cases} y \cdot w_o^{-\sigma} & \text{if } x \in \mathcal{T}_o^\ell, \\ 0 & \text{if } x \notin \mathcal{T}_o^\ell. \end{cases}$$

Integrating labor demand over the set of tasks for occupation o gives:

$$L_o = \int_{x \in \mathcal{T}_o^\ell} \ell_o(x)\, dx = y \cdot M_{\mathcal{T}_o^\ell} w_o^{-\sigma}$$

$$\Longrightarrow w_o = (y \cdot M_{\mathcal{T}_o^\ell} \cdot)^{\frac{1}{\sigma}} \cdot L_o^{-\frac{1}{\sigma}} \tag{A2}$$

Differentiating which yields (3) in vector form:

$$d\ln \boldsymbol{w} = \frac{1}{\sigma} d\ln y \cdot \mathbf{1} - \frac{1}{\sigma} d\ln \Gamma^j - \frac{1}{\sigma} d\ln \boldsymbol{L}$$

The final output can be derived by substituting (A1) into the CES price index $1 = \int_{\mathcal{T}} p(x)^{1-\sigma} dx$:

$$1 = \int_{\cup_o \mathcal{T}_o^k} a(x)^{\sigma-1}\, dx + \sum_o w_o^{1-\sigma} \cdot M_{\mathcal{T}_o^\ell}.$$

Next substituting (A2) gives the expression for final output,

$$1 = \int_{\cup_o \mathcal{T}_o^k} a(x)^{\sigma-1}\, dx + \sum_o \left(\frac{y}{h_o}\right)^{\frac{1-\sigma}{\sigma}} \cdot M_{\mathcal{T}_o^\ell}^{\frac{1}{\sigma}}$$

$$y = \left(1 - \int_{\cup_o \mathcal{T}_o^k} a(x)^{\sigma-1}\, dx\right)^{\frac{\sigma}{1-\sigma}} \cdot \left(\sum_o h_o^{\frac{\sigma-1}{\sigma}} \cdot M_{\mathcal{T}_o^\ell}^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{A3}$$

Lastly, the capital share can be obtained as

$$s^K = \frac{\int_{\cup_o \mathcal{T}_o^k} y \cdot p(x)^{1-\sigma}\, dx}{y} = \int_{\cup_o \mathcal{T}_o^k} a(x)^{\sigma-1}\, dx.$$

Labor Supply. Part of this section follows Lind and Ramondo (2023). The economy comprises a continuum of workers, each indexed by i, with productivity across occupations represented as:

$$\boldsymbol{\epsilon}(i) = \{\epsilon_o(i)\}_{o=1}^{O}$$

where the joint distribution of productivity across occupations is given by:

$$\Pr\left[\epsilon_1(i) \le \epsilon_1, \ldots, \epsilon_O(i) \le \epsilon_O\right] = \exp\left[-F\left(A_1\epsilon_1^{-\theta}, \ldots, A_O\epsilon_O^{-\theta}\right)\right]$$

Workers select occupations to maximize their utility, $o(i) = \text{argmax}_{o'}\, w_{o'}\epsilon_{o'}(i)$, which also follows a Fréchet distribution:

$$\begin{aligned}\Pr\left[\max_o w_o\epsilon_o(i) \le t\right] &= \Pr\left[w_1\epsilon_1(i) \le t, \ldots, w_O\epsilon_O(i) \le t\right] \\ &= \exp\left[-F\left(A_1 w_1^{\theta} t^{-\theta}, \ldots, A_O w_O^{\theta} t^{-\theta}\right)\right] \\ &= \exp\left[-F\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right) t^{-\theta}\right]\end{aligned}$$

the last line follows F is HD1. Furthermore, the principle of maximum stability ensures that the conditional distribution of utility is identical to the unconditional distribution:

$$\Pr\left[w_o\epsilon_o(i) \le t \mid w_o\epsilon_o(i) = \max_{o'} w'_o\epsilon'_o(i)\right] = \exp\left[-F\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right) t^{-\theta}\right]$$

As a result, the utility distribution across occupations is identical. The average utility of workers is then given by:

$$\mathbb{E}[u] = \int_0^{\infty} t \cdot d\Pr\left[\max_o w_o\epsilon_o(i) \le t\right] = \Gamma\left(1 - \frac{1}{\theta}\right) F\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right)^{\frac{1}{\theta}}$$

the wage index can be defined as

$$W = F\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right)^{\frac{1}{\theta}}.$$

Next, the employment share of each occupation is derived as:

$$\pi_o \equiv \Pr\left[w_o\epsilon_o(i) = \max_{o'} w'_o\epsilon'_o(i)\right] = \frac{A_o w_o^{\theta} F_o\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right)}{F\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right)}$$

where $F_o = \frac{\partial F(x_1,\ldots,x_O)}{\partial x_o}$.

PROOF. See Appendix C.1. □

Now, the total occupation labor supply is $L_o = \pi_o L$. The elasticity of labor supply for occupation o with respect to relative wages is derived as

$$\frac{\partial \ln \pi_o}{\partial \ln (w_{o'}/W)} = \frac{\partial \ln \frac{w_o^\theta G_o\left(w_1^\theta,\cdots,w_O^\theta\right)}{G\left(w_1^\theta,\cdots,w_O^\theta\right)}}{\partial \ln (w_{o'}/W)} = \frac{\partial \ln\left(\left(\frac{w_o}{W}\right)^\theta G_o\left(w_1^\theta/W^\theta,\ldots,w_O^\theta/W^\theta\right)\right)}{\partial \ln (w_{o'}/W)}$$

$$= \begin{cases} \theta \frac{w_{o'}^\theta G_{oo'}}{G_o} & \text{if } o' \neq o \\ \theta \frac{w_o^\theta G_{oo}}{G_o} + \theta & \text{if } o' = o \end{cases}$$

the derivation uses F is HD1. Finally, the partial elasticity of h_o with respect to absolute wages is:

$$\begin{aligned}
\frac{\partial \ln \pi_o}{\partial \ln w_{o'}} &= \frac{\partial \ln \pi_o}{\partial \ln (w_{o'}/W)} \cdot \frac{\partial \ln (w_{o'}/W)}{\partial \ln w_{o'}} + \frac{\partial \ln \pi_o}{\partial \ln W} \cdot \frac{\partial \ln W}{\partial \ln w_{o'}} \\
&= \theta \frac{w_{o'}^\theta G_{oo'}}{G_o} - \theta \pi_{o'}^\ell \\
\frac{\partial \ln \pi_o}{\partial \ln w_o} &= \theta + \theta \frac{w_o^\theta G_{oo}}{G_o} - (\theta)\, \pi_o^\ell \\
&= \theta \frac{w_o^\theta G_{oo}}{G_o} + \theta \left(1 - \pi_o^\ell\right)
\end{aligned}$$

This concludes the derivation, as summarized in (5) of Proposition 1. In addition, we have the row sum of Θ equal to 0:

$$\begin{aligned}
\sum_{o'} \Theta_{oo'} &= \sum_{o'} \theta \frac{w_{o'}^\theta G_{oo'}}{G_o} + \theta \left(1 - \pi_o^\ell\right) - \sum_{o' \neq o} s_o \\
&= \frac{\theta}{G_o} \sum_{o'} G_{oo'} = 0
\end{aligned}$$

the last equality comes from G_o is HD0.

A.1.3. Proof of Lemma 1

We solve for labor demand and supply in equilibrium as follows:

$$\left(\mathbf{I} + \frac{1}{\sigma}\Theta\right) \times d\ln \boldsymbol{w} = \frac{1}{\sigma} d\ln y \cdot \mathbf{1} - \frac{1}{\sigma} d\ln \Gamma^j$$
$$\Longrightarrow \ln \boldsymbol{w} = \frac{1}{\sigma} d\ln y \cdot 1 - \left(\mathbf{I} + \frac{1}{\sigma}\Theta\right)^{-1} \frac{d\ln \Gamma^j}{\sigma}$$

Here, we use the fact that the row sums of Θ equal one.

For the wage change $d\ln \boldsymbol{w}$, we denote the share of workers transitioning from occupation o to o' by $\mu_{oo'}$. For an individual worker i making this transition, the equivalent variation (EV) of mobility gains is given by:

$$(\ln w_o + d\ln w_o) + \ln \epsilon_o(i) + \ln(1 + \mathrm{EV}) = (\ln w_{o'} + d\ln w_{o'}) + \ln \epsilon_{o'}$$

Additionally, based on workers' optimal choices, we have:

$$\ln w_o + \ln \epsilon_o \geq \ln w_{o'} + \ln \epsilon_{o'}$$
$$\ln w_o + d\ln w_o \ln \epsilon_o \leq \ln w_{o'} + d\ln w_{o'} + \ln \epsilon_{o'}$$

As $d\ln \boldsymbol{w} \to 0$, it follows that $\ln w_o + \ln \epsilon_o = \ln w_{o'} + \ln \epsilon_{o'}$. Consequently, we can express the equivalent variation as:

$$\mathrm{EV}(i) = d\ln w_{o'} - d\ln w_o \equiv \mathrm{EV}_{oo'}$$

Since workers transition only when the relative wage change is positive, we must have $d\ln w_{o'} - d\ln w_o > 0$.

By the definition of the partial elasticity of substitution, the total change in employment is:

$$d\ln \ell = \sum_{o'} \Theta_{oo'} \times dw_{o'} = \sum_{o'} \Theta_{oo'} (d\ln w_{o'} - \ln w_o)$$

For $d\ln w_{o'} - d\ln w_o > 0$, the share of workers transitioning is given by: $\mu_{oo'} = -\Theta_{oo'}(d\ln w_{o'} - \ln w_o)$. Finally, the average mobility gains are computed as:

$$\mathrm{EV}_o = \sum_{o'} \mu_{oo'} \mathrm{EV}_{oo'} = -\sum_{o'} \Theta_{oo'} (d\ln w_{o'} - d\ln w_o)^2 \mathbf{1}_{d\ln w_{o'} - d\ln w_o > 0}.$$

A.1.4. Proof of Proposition 2

The proposition follows from the eigen-decomposition of the elasticity of substitution matrix

$$\Theta = U\Lambda V \Longrightarrow \Theta = \sum_{n}^{O} \lambda_n \boldsymbol{u}_n \boldsymbol{v}_n'$$

where the equation holds because $\boldsymbol{u}_n \cdot \boldsymbol{v}'_m = 0, \forall m \neq n$ by the construction of eigenvectors. Next, since the set $\{\boldsymbol{u}_0\}$ forms a basis that spans O-dimensional space, we can project the empirical shocks into the basis and the coefficient is given by

$$\frac{d \ln \boldsymbol{\Gamma}}{\sigma} = \sum_{n=1}^{O} \boldsymbol{v}_n \cdot b_n, \text{ where } \boldsymbol{b} = \left(U'U\right)^{-1} U' \cdot \frac{d \ln \boldsymbol{\Gamma}}{\sigma}$$

Finally, using equation (7) , we derive:

$$\begin{aligned}
d \ln \boldsymbol{w} &= \frac{1}{\sigma} y \cdot \mathbf{1} - \left(\mathbf{I} + \frac{1}{\sigma}\Theta\right)^{-1} \cdot \frac{d \ln \boldsymbol{\Gamma}}{\sigma} = \frac{1}{\sigma} y \cdot \mathbf{1} - \left(\mathbf{I} + \frac{1}{\sigma} U \Lambda V\right)^{-1} \cdot \frac{d \ln \boldsymbol{\Gamma}}{\sigma} \\
&= \frac{1}{\sigma} y \cdot \mathbf{1} - V^{-1} \cdot \left(\mathbf{I} + \frac{1}{\sigma}\Lambda\right)^{-1} \cdot U^{-1} \cdot \frac{d \ln \boldsymbol{\Gamma}}{\sigma} = \frac{1}{\sigma} y \cdot \mathbf{1} - U \cdot \left(\mathbf{I} + \frac{1}{\sigma}\Lambda\right)^{-1} \cdot V \cdot \frac{d \ln \boldsymbol{\Gamma}}{\sigma} \\
&= \frac{1}{\sigma} y \cdot \mathbf{1} - \sum_{n=1}^{O} \frac{1}{1 + \frac{\lambda_n}{\sigma}} \boldsymbol{u}_n \boldsymbol{v}'_n \cdot \frac{d \ln \boldsymbol{\Gamma}}{\sigma} = \frac{1}{\sigma} y \cdot \mathbf{1} - \sum_{n=1}^{O} \frac{1}{1 + \frac{\lambda_n}{\sigma}} \boldsymbol{u}_n \boldsymbol{v}'_n \cdot \left(\sum_{n=1}^{O} \boldsymbol{u}_n \cdot b_n\right) \\
&= \frac{1}{\sigma} d \ln y \cdot \mathbf{1} + \sum_{n=1}^{O} \frac{1}{1 + \frac{\lambda_n}{\sigma}} \boldsymbol{u}_n \cdot b_n
\end{aligned}$$

which concludes the proof.

A.1.5. Proof of Lemma 2

We first observe that the correlation function F is homogeneous of degree 1 and satisfies the sign-switching property:

$$\frac{\partial^2 F}{\partial x_i \partial x_j} \leq 0, \forall i \neq j \text{ and } \frac{\partial^2 F}{\partial x_i^2} \geq 0, \forall i$$

. From this, it immediately follows that:

$$\begin{aligned}
\frac{\partial \ln h_o}{\partial \ln w_o} &= \theta \frac{A_o w_o^\theta F_{oo}}{F_o} + (\theta - 1)(1 - s_o) > 0 \\
\frac{\partial \ln h_o}{\partial \ln w_{o'}} &= \theta \frac{A_{o'} w_{o'}^\theta F_{oo'}}{F_o} - (\theta - 1) s_{o'} < 0
\end{aligned}$$

Additionally, the row sum of Θ is zero, as shown below:

$$\sum_{o'} \Theta_{oo'} = \sum_{o'} \theta \frac{A_{o'} w_{o'}^\theta F_{oo'}\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)}{F_o\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)} - \sum_{o' \neq o} (\theta - 1) s_{o'} + (\theta - 1)(1 - s_o) = 0$$

The second equality follows from the homogeneity of degree 1 (HD1) property of the function F. Therefore, Θ is a diagonally dominant matrix, implying that all its eigenvalues are non-negative. Furthermore, the existence of $\lambda_1 = 0$ is guaranteed because the row sum of Θ is zero.

A.1.6. Proof of Proposition 3.

The joint productivity distribution for each skill s is given by:

$$\Pr\left[\epsilon_1^s(i) \le \epsilon_1^s, \ldots, \epsilon_O^s(i) \le \epsilon_O^s\right] = \exp\left[-\left(\sum_{o=1}^{O}\left(\left(\epsilon_o^s\right)^{-\theta}\right)^{\frac{1}{1-\rho_s}}\right)^{1-\rho_s}\right]$$

Using this, we can derive the joint productivity distribution:

$$\begin{aligned}
\Pr\left[\epsilon_1(i) \le \epsilon_1, \ldots, \epsilon_O(i) \le \epsilon_O\right] &= \Pr\left[\max_{s\in\mathcal{S}} A_1^s \cdot \epsilon_1^s(i) \le \epsilon_1, \ldots, \max_{s\in\mathcal{S}} A_O^s \cdot \epsilon_O^s(i) \le \epsilon_O\right] \\
&= \Pi_{s\in\mathcal{S}} \Pr\left[\epsilon_1^s(i) \le \frac{\epsilon_1}{A_1^s}, \ldots, \epsilon_O^s(i) \le \frac{\epsilon_O}{A_O^s}\right] \\
&= \Pi_{s\in\mathcal{S}} \exp\left[-\left(\sum_{o=1}^{O}\left(\left(A_o^s\right)^{\theta}\cdot\left(\epsilon_o\right)^{-\theta}\right)^{\frac{1}{1-\rho_s}}\right)^{1-\rho_s}\right] \\
&= \exp\left[-\sum_{s\in\mathcal{S}}\left(\sum_{o=1}^{O}\left(\left(A_o^s\right)^{\theta}\cdot\left(\epsilon_o\right)^{-\theta}\right)^{\frac{1}{1-\rho_s}}\right)^{1-\rho_s}\right]
\end{aligned}$$

Finally, after defining: $A_o \equiv \sum_{s\in\mathcal{S}}\left(A_o^s\right)^{\theta}$ and $\omega_o^s \equiv \frac{\left(A_o^s\right)^{\theta}}{A_o}$, we can rewrite the last expression as in proposition 3.

A.1.7. Proof of Proposition 4.

From Appendix A.1.2, the employment share is given by:

$$\pi_o = \frac{A_o w_o^{\theta} F_o\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right)}{F\left(A_1 w_1^{\theta}, \ldots, A_O w_O^{\theta}\right)} = \frac{A_o w_o^{\theta} F_o}{\sum_{o'} A_{o'} w_{o'}^{\theta} F_{o'}}$$

and the CNCES parameterization specifies: $F\left(x_1, \ldots, x_O\right) = \sum_{s\in\mathcal{S}}\left[\sum_{o=1}^{O}\left(\omega_o^s x_o\right)^{\frac{1}{1-\rho_s}}\right]^{1-\rho_s}$. Let $G_s = \sum_{o'=1}^{O}\left(\omega_{o'}^s x_{o'}\right)^{\frac{1}{1-\rho_s}}$, The partial derivative of F is then:

$$\frac{\partial F}{\partial x_o} = \sum_{s\in\mathcal{S}} \frac{\partial}{\partial x_o}\left(G_s^{1-\rho_s}\right) = \sum_{s\in\mathcal{S}}\left(1-\rho_s\right) G_s^{-\rho_s} \cdot \frac{\partial G_s}{\partial x_o}$$

$$\text{where } \frac{\partial G_s}{\partial x_o} = \frac{1}{1-\rho_s}\left(\omega_o^s x_o\right)^{\frac{\rho_s}{1-\rho_s}} \omega_o^s$$

Simplifying:

$$\frac{\partial F}{\partial x_o} = \sum_{s \in \mathcal{S}} G_s^{-\rho_s} \left(\omega_o^s x_o\right)^{\frac{\rho_s}{1-p_s}} \omega_o^s$$

Using this result, we find:

$$\frac{x_o F_o}{\sum_{o'} x_{o'} F_{o'}} = \sum_{s \in \mathcal{S}} \left(\frac{\left(\omega_o^s x_o\right)^{\frac{1}{1-\rho_s}}}{G_s} \right) \left(\frac{G_s^{1-\rho_s}}{F} \right)$$

Substituting $x_o = A_o w_o^\theta$, the employment share under CNCES is:

$$\pi_o = \sum_{s \in \mathcal{S}} \underbrace{\frac{\left(\omega_o^s A_o w_o^\theta\right)^{\frac{1}{1-\rho_s}}}{\sum_{o'=1}^{O} \left(\omega_{o'}^s A_{o'} w_{o'}^\theta\right)^{\frac{1}{1-\rho_s}}}}_{\pi_o^{s,W}} \cdot \underbrace{\frac{\left[\sum_{o'=1}^{O} \left(\omega_{o'}^s A_{o'} w_{o'}^\theta\right)^{\frac{1}{1-\rho_s}}\right]^{1-\rho_s}}{\sum_{s' \in \mathcal{S}} \left[\sum_{o'=1}^{O} \left(\omega_{o'}^{s'} A_{o'} w_{o'}^\theta\right)^{\frac{1}{1-\rho_{s'}}}\right]^{1-\rho_{s'}}}}_{\pi^{s,B}}$$

as shown in equation (9). To derive the correlated elasticity, we first calculate $\frac{x_{o'} F_{oo'}}{F_o}$. Define $\Phi_o^s = \left(\omega_o^s x_o\right)^{\frac{1}{1-\rho_s}}$, $\quad \Psi_o^s = \omega_o^s \left(\Phi_o^s\right)^{\rho_s}$. The first derivative is:

$$F_o = \sum_{s \in \mathcal{S}} G_s^{-\rho_s} \Psi_o^s$$

and the second derivative is:

$$F_{oo'} = \frac{\partial^2 F}{\partial x_o \partial x_{o'}} = \sum_{s \in \mathcal{S}} \left[-\rho_s G_s^{-\rho_s - 1} \Psi_o^s \frac{\partial G_s}{\partial x_{o'}} \right]$$

$$\text{where } \frac{\partial G_s}{\partial x_{o'}} = \frac{1}{1-\rho_s} \left(\Phi_{o'}^s\right)^{\rho_s} \omega_{\sigma'}^s$$

Simplifying:

$$F_{oo'} = -\sum_{s \in \mathcal{S}} \frac{\rho_s}{1-\rho_s} G_s^{-\rho_s - 1} \Psi_o^s \left(\Phi_{o'}^s\right)^{\rho_s} \omega_{o'}^s$$

Combining the first and second derivatives:

$$\frac{x_{o'} F_{oo'}}{F_o} = -\frac{\sum_{s \in \mathcal{S}} \frac{\rho_s}{1-\rho_s} G_s^{-\rho_s - 1} \Psi_o^s \Phi_{o'}^s}{\sum_{s \in \mathcal{S}} G_s^{-\rho_s} \Psi_o^s}$$

Now, we introduce share variables: Share of o' in G_s, $\mu^s_{o'} = \frac{\Phi^s_{o'}}{G_s}$, and Weight of s in F_o, $\gamma^s_o = \frac{G_s^{-\rho_s}\Psi^s_o}{F_o}$. We can rewrite:

$$\frac{x_{o'}F_{oo'}}{F_o} = -\sum_{s\in\mathcal{S}} \frac{\rho_s}{1-\rho_s}\mu^s_{o'}\gamma^s_o$$

Substituting $x_o = A_o w_o^\theta$, we obtain:

$$\theta\frac{x_{o'}F_{oo'}}{F_o}\,|_{\left\{x_o = A_o W_o^\theta\right\}} = -\theta\sum_{s\in\mathcal{S}} \frac{\rho_s}{1-\rho_s}\cdot\pi^{s,W}_o\pi^{s,W}_{o'}\cdot\frac{\pi^{s,B}}{\pi_o}$$

as claimed in 4.

A.1.8. Proof of Proposition 5.

Given occupational wage $\boldsymbol{w}_t$ and labor productivity $\left\{\boldsymbol{A}^g_t\right\}_{g\in G}$, the occupational employment shares by group can be expressed as:

$$\pi^g_{o,t} = \frac{A^g_{o,t}w^\theta_{o,t}F_o\left(A^g_{1,t}w^\theta_{1,t},\ldots,A^g_{O,t}w^\theta_{O,t}\right)}{F\left(A^g_{1,t}w^\theta_{1,t},\ldots,A^g_{O,t}w^\theta_{O,t}\right)}.$$

We define the correlation-adjusted employment share as:

$$\tilde{\pi}^g_{o,t} = \frac{A^g_{o,t}w^\theta_{o,t}}{F\left(A^g_{1,t}w^\theta_{1,t},\ldots,A^g_{O,t}w^\theta_{O,t}\right)}.$$

Since F_o is homogeneous of degree zero (HD0), we can express employment shares in terms of correlation-adjusted employment shares:

$$\pi^g_{o,t} = \tilde{\pi}^g_{o,t}F_o\left(\tilde{\pi}^g_{1,t},\ldots,\tilde{\pi}^g_{O,t}\right).$$

This establishes a one-to-one mapping between the correlation-adjusted employment shares $\left\{\tilde{\pi}^g_{o,t}\right\}_{o\in O}$ and observed employment shares $\left\{\pi^g_{o,t}\right\}_{o\in O}$. Given the correlation function F, we can solve for $\left\{\tilde{\pi}^g_{o,t}\right\}_{o\in O}$. Next, for an alternative occupational wage $\boldsymbol{w}_{t+1}$, the correlation-adjusted employment shares evolve as:

$$\frac{\tilde{\pi}^g_{o,t+1}}{\tilde{\pi}^g_{o,t}} = \frac{w^\theta_{o,t+1}/w^\theta_{o,t}}{F\left(A^g_{1,t}w^\theta_{1,t+1},\ldots,A^g_{O,t}w^\theta_{O,t+1}\right)/F\left(A^g_{1,t}w^\theta_{1,t},\ldots,A^g_{O,t}w^\theta_{O,t}\right)}$$

then we can express the denominator as

$$\begin{aligned}\frac{F\left(A^{g}_{1,t}w^{\theta}_{1,t+1},\ldots,A^{g}_{O,t}w^{\theta}_{O,t+1}\right)}{F\left(A^{g}_{1,t}w^{\theta}_{1,t},\ldots,A^{g}_{O,t}w^{\theta}_{O,t}\right)} &= F\left(\left\{\frac{A^{g}_{o,t}w^{g}_{o,t+1}}{F\left(A^{g}_{1,t}w^{\theta}_{1,t},\ldots,A^{g}_{O,t}w^{\theta}_{O,t}\right)}\right\}_{o\in O}\right)\\ &= F\left(\left\{\hat{w}^{\theta}_{t+1}\tilde{\pi}^{g}_{o,t}\right\}_{o\in O}\right)\end{aligned}$$

where $\hat{w}_{o,t+1} = w_{o,t+1}/w_{o,t}$ denotes the relative wage change. Thus, the adjusted employment share can be written as:

$$\tilde{\pi}^{g}_{o,t+1} = \tilde{\pi}^{g}_{o,t}\hat{w}^{\theta}_{o,t}F\left(\left\{\hat{w}^{\theta}_{t+1}\tilde{\pi}^{g}_{o,t}\right\}_{o\in O}\right)$$

Finally, we recover the implied employment shares from the wage change $\hat{\boldsymbol{w}}_t$ using the adjusted employment shares:

$$\pi^{g}_{o,t+1} = \tilde{\pi}^{g}_{o,t+1}F_o\left(\tilde{\pi}^{g}_{1,t+1},\cdots,\tilde{\pi}^{g}_{O,t+1}\right)$$

This concludes the proof and provides a systematic algorithm to compute implied employment shares based on wage changes.

A.2. Results in the Dynamic Model

This appendix formalizes a dynamic discrete choice model of workers choosing occupations, coupled with a production equilibrium. The model captures occupational mobility, productivity shocks, and equilibrium dynamics, enabling counterfactual analysis via "hat algebra" techniques.

A.2.1. Wokers' Optimization

Consider a population of hand-to-mouth workers distributed across O occupations, with $L_{o,t}$ workers in occupation o at time t. Workers have intertemporal preferences over consumption $c_t(i)$ and labor disutility $\ell_t(i)$ for individual i:

$$U\left(\{c_t(i),\ell_t(i)\}_{t=0}^{\infty}\right) = \sum_{t=0}^{\infty}\beta^{t}\left(\ln c_t(i) - \ln \ell_t(i)\right)$$

where $\beta \in (0,1)$ is the discount factor.

At the start of each period t, workers redraw idiosyncratic productivity shocks $\epsilon t(i) = (\epsilon 1, t(i),\ldots,\epsilon_{O,t}(i))$ from a multivariate Fréchet distribution:

$$\Pr\left[\epsilon_{1,t}(i)\le\epsilon_1,\ldots,\epsilon_{O,t}(i)\le\epsilon_O\right] = \exp\left[-F\left(A_{1,t}\epsilon_1^{-\theta},\ldots,A_{O,t}\epsilon_O^{-\theta}\right)\right],$$

where $F(x_1, \ldots, x_O) = \sum_{s=1}^{S} \left[\sum_{o=1}^{O} (\omega_{so} x_o)^{\frac{1}{1-\rho_s}} \right]^{1-\rho_s}$ is as in the static model. Here, $A_{o,t}$ is an occupation-specific productivity parameter, $\theta > 0$ governs the dispersion of shocks, $\omega_{so} \geq 0$ are weights reflecting skill requirements across S nests, and $\rho_s \in (-\infty, 1)$ captures correlation within nest s.

After observing $\epsilon t(i)$, a worker in occupation o can switch to occupation o' by paying a fixed utility cost $\tau oo' \geq 0$. Upon choosing occupation o', the worker's instantaneous utility is:

$$\ln c_t(i) - \ln \ell_t(i) = \ln w_{\sigma',t} + \kappa \ln \epsilon_{\sigma',t}(i),$$

where $w_{o',t}$ is the wage in occupation o', and $\kappa > 0$ scales the impact of productivity shocks, influencing short-run labor supply elasticities.

Define the value function $v_{o,t}(\boldsymbol{\epsilon}_t)$ as the lifetime utility for a worker in occupation o at time t with productivity shocks $\boldsymbol{\epsilon}_t$:

$$v_{o,t}(\boldsymbol{\epsilon}_t) = \max_{\sigma'} \left\{ \ln w_{o',t} + \kappa \ln \epsilon_{o',t} + \beta V_{o',t+1} - \tau_{oo'} \right\},$$

where $V_{o',t+1} = \mathbb{E}\boldsymbol{\epsilon}\left[vo', t+1(\boldsymbol{\epsilon}) \right]$ is the expected utility in occupation o' at $t+1$, taken over future productivity draws.

To compute $V_{o,t} = \mathbb{E}\boldsymbol{\epsilon}\left[vo, t(\boldsymbol{\epsilon}_t) \right]$, define: $Z_{oo',t} = \exp\left(\beta V_{\sigma',t+1} + \ln w_{\sigma',t} - \tau_{oo'} \right)$, so that: $v_{o,t}(\epsilon_t) = \ln\left(\max_{\sigma'} \left\{ Z_{oo',t} \epsilon_{o',t}^{\kappa} \right\} \right)$. Given the Fréchet distribution, the cumulative distribution of the maximum is:

$$\Pr\left[\max_{\sigma'} \left\{ Z_{oo',t} \epsilon_{\sigma',t}^{\kappa} \right\} < z \right] = \exp\left[-F\left(A_{1,t} Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}} \right) z^{-\frac{\theta}{\kappa}} \right]$$

Thus, $\ln\left(\max_{o'} \left\{ Z_{oo',t} \epsilon_{o',t}^{\kappa} \right\} \right)$ follows a Gumbel distribution, and its expectation is:

$$V_{o,t} = \ln\left(F\left(A_{1,t} Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}} \right)^{\frac{\kappa}{\partial}} \right) + \bar{\gamma} \frac{\kappa}{\theta},$$

where $\bar{\gamma}$ is the Euler-Mascheroni constant.

A.3. Occupation Switching Probability

The probability that a worker in occupation o switches to o' at time t is:

$$\mu_{oo',t} = \Pr\left[Z_{oo',t} \epsilon_{d',t}^{\kappa} \geq \max_{d''} Z_{od'',t} \epsilon_{d'',t}^{\kappa} \right].$$

Using properties of the multivariate Fréchet distribution as C.1:

$$\mu_{oo',t} = \frac{A_{o',t} Z_{oo',t}^{\frac{\theta}{\kappa}} F_{o'}\left(A_{1,t} Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}}\right)}{F\left(A_{1,t} Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}}\right)},$$

where $F_{o'} = \partial F / \partial x_{o'}$ is the partial derivative with respect to the o'-th argument.

Define the correlation-adjusted occupational mobility:

$$\tilde{\mu}_{oo',t} = \frac{A_{o,t} Z_{oo',t}^{\frac{\theta}{\frac{\theta}{2}}}}{F\left(A_{1,t} Z_{ol,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}}\right)},$$

so that:

$$\mu_{oo',t} = \tilde{\mu}_{od',t} F_{o'}\left(\tilde{\mu}_{o1,t}, \ldots, \tilde{\mu}_{oO,t}\right).$$

This forms an O-to-O mapping between observed probabilities $\mu_{oo',t}$ and adjusted probabilities $\tilde{\mu}_{oo',t}$. The recursive equation for adjusted relative switching probabilities is:

$$\ln \frac{\tilde{\mu}_{oo',t}}{\tilde{\mu}_{oo,t}} = \frac{\theta}{\kappa} \ln \frac{w_{o',t}}{w_{o,t}} + \beta \ln \frac{\tilde{\mu}_{oo',t+1}}{\tilde{\mu}_{o'o',t+1}} + (\beta - 1)\, \tau_{oo'}$$

A.4. Static Equilibrium

When taking the task assignment as exogenous, the production function is a CES aggregator over capital and labor across occupations:

$$Y_t = \left(\left(s_t^k\right)^{\frac{1}{\sigma}} \left(y_t^k\right)^{\frac{\sigma-1}{\sigma}} + \sum_{o=1}^{O} \left(s_{o,t}^{\ell}\right)^{\frac{1}{\sigma}} \left(y_{o,t}^{\ell}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where $y_t^k = a_t^k k_t$, $y_{o,t}^{\ell} = L_{o,t}$, $\sigma > 0$ is the elasticity of substitution, s_t^k and $s_{o,t}^{\ell}$ are time-varying shares, and a_t^k is capital productivity. Assuming $s_t^k (a_t^k)^{\sigma-1} < 1$ for finite output, capital demand is $k_t = s_t^k (a_t^k)^{\sigma} Y_t$, yielding net output:

$$Y_t = \left[\frac{\sum_{o=1}^{O} \left(s_{o,t}^{\ell}\right)^{\frac{1}{\sigma}} \left(L_{o,t}\right)^{\frac{\sigma-1}{\sigma}}}{1 - s_t^k \left(a_t^k\right)^{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}}$$

Wages are determined by the marginal product of labor:

$$w_{o,t} = \frac{\partial Y_t}{\partial L_{o,t}} = Y_t^{\frac{1}{\sigma}} \left(s_{o,t}^{\ell}\right)^{\frac{1}{\sigma}} L_{o,t}^{-\frac{1}{\sigma}}.$$

A.5. Dynamic Equilibrium

We construct job flows from the retrospective responses in the March CPS; consequently, the aggregate flows derived from these responses do not directly match the observed occupational employment levels. To account for this discrepancy, we adjust the evolution of occupational employment as

$$L_{o,t} = \sum_{\sigma'=1}^{O} \mu_{\sigma'',t} L_{\sigma',t-1} + \Delta L_{o,t},$$

where $\Delta L_{o,t}$ is an exogenous net inflow/outflow, with $\sum_o L_{o,t} = 1$ and $\sum_o \Delta L_{o,t} = 0$. Time-varying fundamentals are $\boldsymbol{A}t = Ao, t$, $\boldsymbol{s}t = so, t^{\ell}$, and a_t^k, while constant parameters include $\tau_{oo'}$, ω_{os}, σ, θ, ρ_s, κ, and β.

A dynamic equilibrium is a sequence $\boldsymbol{L}_t$, $\boldsymbol{w}_t$, μ_t, V_t satisfying:

- Production Equilibrium:

$$w_{o,t} = Y_t^{\frac{1}{\sigma}} \left(s_{o,t}^{\ell}\right)^{\frac{1}{\sigma}} L_{o,t}^{-\frac{1}{\sigma}}$$

$$Y_t = \left[\frac{\sum_o \left(s_{o,t}^{\ell}\right)^{\frac{1}{\sigma}} L_{o,t}^{\frac{\sigma-1}{\sigma}}}{1 - s_t^k \left(a_t^k\right)^{\sigma-1}} \right]^{\frac{\sigma}{\sigma-1}}$$

- Expected Utility: $V_{o,t} = \ln \left(F \left(A_{1,t} Z_{o1,t}{}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}} \right)^{\frac{\kappa}{\theta}} \right) + \bar{\gamma}\frac{\kappa}{\theta}$.
- Switching Probabilities: $\mu_{oo',t} = \dfrac{A_{o',t} Z_{oo',t}^{\frac{\theta}{\kappa}} F_{o'}\left(A_{1,t} Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}} \right)}{F\left(A_{1,t} Z_{o1,t}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t} Z_{oO,t}^{\frac{\theta}{\kappa}} \right)}$.
- Labor Allocation: $L_{o,t} = \sum_{o'} \mu_{o'o,t} L_{o',t-1} + \Delta L_{o,t}$.

A.6. System in Changes

Define the growth factor $\dot{x}t + 1 = xt + 1/x_t$. From the wage equation:

$$\sigma \ln \dot{w}_{o,t+1} + \ln \dot{L}_{o,t+1} = \ln \dot{Y}_{t+1} + \ln \dot{s}_{o,t+1}^{\ell}$$

For the dynamic system, define $u_{o,t} = \exp(V_{o,t})$ and $\dot{u}o, t + 1 = uo, t + 1/u_{o,t}$:

$$\frac{\tilde{\mu}_{oo',t}}{\tilde{\mu}_{oo',t-1}} = \frac{\dot{A}_{o',t} \dot{u}_{o't+1}^{\beta\frac{\theta}{\kappa}} \dot{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{ \tilde{\mu}_{oo'',t-1} \dot{A}_{o'',t} \dot{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}} \dot{w}_{o'',t}^{\frac{\theta}{\kappa}} \right\}_{o''=1}^{O} \right)}$$

$$\dot{u}_{o,t+1} = F\left(\left\{ \tilde{\mu}_{oo'',t} \dot{A}_{o'',t+1} \dot{u}_{o'',t+2}^{\beta\frac{\theta}{\kappa}} \dot{w}_{o'',t+1}^{\frac{\theta}{\kappa}} \right\}_{o''=1}^{O} \right)^{\frac{\kappa}{\theta}}$$

with $\mu_{oo',t} = \tilde{\mu}oo', tFo' \left(\tilde{\mu}o1, t, \ldots, \tilde{\mu}oO, t\right)$ and labor evolution as above. Appendix C.2 provides additional derivation.

A.7. Dynamic Hat Algebra

For counterfactual fundamentals $\hat{\boldsymbol{A}}_t$, $\hat{\boldsymbol{s}}_t$, and $\hat{a}_t^k$, define $\hat{x}_t = (\dot{x}_t'/\dot{x}_t)$, where $\dot{x}t' = x_t'/xt-1'$ is the counterfactual growth rate. From production:

$$\hat{w}_{o,t+1} = \left(\frac{\hat{Y}_{t+1}\hat{s}^{\ell}_{o,t+1}}{\hat{L}_{o,t+1}}\right)^{\frac{1}{\sigma}}$$

Counterfactual switching probabilities evolve as:

$$\tilde{\mu}'_{od',t} = \frac{\tilde{\mu}'_{oo',t-1}\dot{\hat{\mu}}_{oo',t}\hat{A}_{o',t}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o't+1}\hat{w}^{\frac{\theta}{\kappa}}_{o',t}}{F\left(\left\{\tilde{\mu}'_{oo'',t-1}\dot{\hat{\mu}}_{oo'',t}\hat{A}_{o'',t}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o'',t+1}\hat{w}^{\frac{\theta}{\kappa}}_{o'',t}\right\}_{o''=1}^{O}\right)}$$

and counterfactual changes in expected utility:

$$\hat{u}_{o,t+1} = F\left(\left\{\tilde{\mu}'_{oo'',t}\dot{\hat{\mu}}_{oo'',t+1}\hat{A}_{o',t+1}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o't+2}\hat{w}^{\frac{\theta}{\kappa}}_{o',t+1}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}.$$

with $\mu'_{oo',t} = \tilde{\mu}oo', t'Fo' \left(\tilde{\mu}o1, t', \ldots, \tilde{\mu}oO, t'\right)$ and $L'_{o',t} = \sum_o \mu'_{oo',t}L'_{o,t-1} + \Delta L_{o',t}$. Appendix C.2 provides additional derivations.

A.7.1. Initial Dynamics

At $t = 0$, assume $\hat{u}o, 0 = 1$, $\mu oo', 0' = \mu_{oo',0}$, and $L'_{o,0} = L_{o,0}$. For $t = 1$:

$$\tilde{\mu}'_{oo',1} = \frac{\vartheta_{oo',1}\hat{A}_{o',1}\hat{w}^{\frac{\theta}{\kappa}}_{o',1}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o'2}}{F\left(\left\{\vartheta_{oo'',1}\hat{A}_{o'',1}\hat{w}^{\frac{\theta}{\kappa}}_{o'',1}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o'',2}\right\}_{o''=1}^{O}\right)}$$

$$\hat{u}_{o,t+1} = \left(\left\{\vartheta_{oo',1}\hat{A}_{o',1}\hat{w}^{\frac{\theta}{\kappa}}_{o',1}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o'2}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

where $\vartheta_{oo',1} = \tilde{\mu}_{oo',1}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o',1}$.

A.8. Welfare Metrics

Then, we can see the own migration share contains information about the expected life-time return,

$$
\begin{aligned}
V_{o,t} &= \ln\left(w_{o,t}\right) + \beta V_{o,t+1} + \frac{\kappa}{\theta}\ln\left(\frac{F\left(A_{1,t}Z_{o1,t}{}^{\frac{\theta}{\kappa}}, \ldots, A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)}{\exp\left(\beta V_{o,t+1} + \ln w_{o,t}\right)^{\frac{\theta}{\kappa}}}\right) + \bar{\gamma}\frac{\kappa}{\theta} \\
&= \ln\left(A_{o,t}^{\frac{\kappa}{\theta}} w_{o,t}\right) + \beta V_{o,t+1} + \frac{\kappa}{\theta}\ln\left(\frac{1}{\tilde{\mu}_{oo,t}}\right) + \bar{\gamma}\frac{\kappa}{\theta}
\end{aligned}
$$

iterating repeatedly, we can obtain:

$$
V_{o,t} = \sum_{s=t}^{\infty} \beta^{s-t} \ln\left[\left(\frac{A_{o,s}}{\tilde{\mu}_{oo,s}}\right)^{\frac{\kappa}{\theta}} w_{o,s}\right] + \bar{\gamma}\frac{\kappa}{\theta(1-\beta)}.
$$

For counterfactual $V'_{o,t}$, define equivalent variation $\delta_{o,t}$ such that:

$$
V'_{o,t} = V_{o,t} + \sum_{s=t}^{\infty} \beta^{s-t} \ln \delta_{o,t},
$$

yielding:

$$
\delta_{o,t} = (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} \ln \frac{w'_{o,s}}{w_{o,s}} \left(\frac{A'_{o,s}}{\tilde{\mu}'_{oo,s}} / \frac{A_{o,s}}{\tilde{\mu}_{oo,s}}\right)^{\frac{\kappa}{\theta}}
$$

additionally, we can write the welfare metric in terms of hat values (counterfactual ratio of changes),

$$
\begin{aligned}
\delta_{o,t} &= \sum_{s=t}^{\infty} \beta^{s-t} \frac{w'_{o,s}}{w_{o,s}} \left(\frac{A'_{o,s}}{\tilde{\mu}'_{oo,s}} / \frac{A_{o,s}}{\tilde{\mu}_{oo,s}}\right)^{\frac{\kappa}{\theta}} - \sum_{s=t}^{\infty} \beta^{s-t+1} \frac{w'_{o,s}}{w_{o,s}} \left(\frac{A'_{o,s}}{\tilde{\mu}'_{oo,s}} / \frac{A_{o,s}}{\tilde{\mu}_{oo,s}}\right)^{\frac{\kappa}{\theta}} \\
&= \sum_{s=t}^{\infty} \beta^{s} \ln\left(\hat{w}_{o,s} / \hat{\tilde{\mu}}_{oo,s}^{\frac{\kappa}{\theta}}\right)
\end{aligned}
$$

Appendix B. Additional Empirical Results

B.1. Occupation Classification

Our analyses require constructing occupation-level panels for the period 1980–2018. To this end, following Autor et al. (2024), we use a consistent occupation coding scheme (occ1990dd), originally developed by Dorn (2009) and updated through 2018, which yields a balanced panel of 306 consistent, 3-digit occupations. This detailed classification preserves crucial occupational variation and accurately captures the structure of the labor market over the period.

B.2. Occupational Skill Requirements

The O*NET database used in this study is O*NET 28.2, which provides information on skill requirements and task outputs for 873 occupations. ONET ratings are derived from two sources: (i) a survey of workers, who rate their own occupations based on a subset of ONET descriptors, and (ii) a survey of occupational analysts, who assess the remaining descriptors in the O*NET dataset.

The database includes 277 descriptors, each rated by importance, level, relevance, or extent. These descriptors are organized into nine broad categories: skills, abilities, knowledge, work activities, work context, experience/education levels required, job interests, work values, and work styles. For this study, we focus on 218 descriptors from the skills, abilities, knowledge, work activities, and work context categories, as they serve as the empirical counterparts to occupational skill requirements.

In the PCA procedure, we reduce the large set of descriptors to three skill dimensions commonly used in the labor literature: cognitive, manual, and interpersonal skill requirements. These dimensions are identified using the following exclusion restrictions: (i) the mathematics score loads exclusively onto cognitive skill requirements, (ii) the mechanical knowledge score loads exclusively onto manual skill requirements, and (iii) the social perceptiveness score loads exclusively onto interpersonal skill requirements. By construction, the three skill dimensions are orthogonal, aligning with the model assumption that individual productivity distributions across skills are independent. The proportion of variance explained by the three principal components is denoted as Var_s and together they account for 58% of the variation in the original descriptors.

One limitation of the PCA procedure is that it assigns negative loadings to certain principal components, which contradicts the restriction that skill requirements ω_O^s should be non-negative. To address this, we apply a linear transformation to rescale the occupational loadings on the three principal components, ensuring that the resulting skill requirement indices satisfy $r_O^s \in [0, 1]$.Finally, we measure occupational skill requirements based on their relative importance in explaining variation across occupations:

$$\omega_o^s = \frac{r_0^{r_0^s} \times \mathrm{Var}_s}{\sum_{\mathrm{se}\in} r_0^s \times \mathrm{Var}_s}.$$

This formulation aligns with the theoretical definition of skill requirements as the relative contribution of each skill to occupational labor productivity. Lastly, we map the estimated skill requirements to the *occ1990dd* classification.

B.3. Measures of Automation and AI Exposure

The literature has identified several measures of occupational exposure to automation, including occupational routine task intensity (Autor and Dorn 2013), the decline in labor share due to the adoption of industrial robots, machines, and software (Acemoglu and Restrepo 2022), and occupational exposure to automation patents (Autor et al. 2024). These measures share a common characteristic, as highlighted by Autor (2015) in Polanyi's Paradox: jobs that can be codified into well-defined rules or algorithms are more likely to be automated and are typically classified as routine jobs. Moreover, numerous studies have shown that jobs more exposed to automation have experienced slower wage growth over the past four decades.

In contrast, ex-post estimation of occupational exposure to artificial intelligence (AI) is infeasible, as its full impact has yet to materialize. Given this challenge, recent research has explored the use of large language models (LLMs) to assess economic outcomes. Eloundou et al. (2023) evaluate occupational exposure to LLMs by classifying O*NET tasks with both human annotators and GPT-4, developing an exposure rubric to determine whether LLMs can perform or assist specific tasks. Their findings highlight the potential of LLMs as general-purpose technologies. Rather than training a model, I adopt a simpler yet comparable approach, relying directly on ChatGPT queries to estimate AI and automation exposure.

To measure occupational exposure to automation and AI, we employ a structured approach using the O*NET database and GPT-4o assessments. First, we utilize O*NET's dataset, which provides detailed descriptions of 19,200 tasks across 862 occupations. Each task is evaluated to determine whether it can potentially be performed without human intervention.

Our method consists of two distinct assessments using GPT-4o:

- AI Exposure: We query GPT-4o: "Can AGI (e.g., large language models like GPT-4) potentially perform the task without human intervention?" This assessment captures the extent to which occupations are exposed to AI-driven technologies.
- Automation Exposure: We query GPT-4o: "Can industrial robots, machines, and com-

puters (no AI capability) perform the task without human intervention?" This distinguishes tasks that can be automated using conventional, rule-based systems from those requiring more advanced AI capabilities.

Based on these evaluations, GPT-4o estimates that approximately 6,000 tasks (roughly one-third of the total) can be performed by AI without human intervention. This classification provides a detailed perspective on the differing impacts of AI-driven technologies and traditional automation across occupations. Finally, I calculate the share of tasks that can be automated or potentially performed by AI, using these estimates to construct occupational exposure measures for automation and AI.

We compare existing measures of automation exposure to GPT-4o's evaluation, as shown in Figure A1. Panels (a) and (b) compare GPT-4o's estimates with the automation exposure measure from Acemoglu and Restrepo (2022). Since their measure is not defined at the occupational level but rather at the group level (demographic-age-education), we plot exposure against the log median wage in 1980. The two distributions are strikingly similar across income levels. Panel (c) plots occupational routine task intensity against automation exposure, showing a strong correlation. Similarly, Panel (d) plots exposure to automation patents, revealing the same pattern.

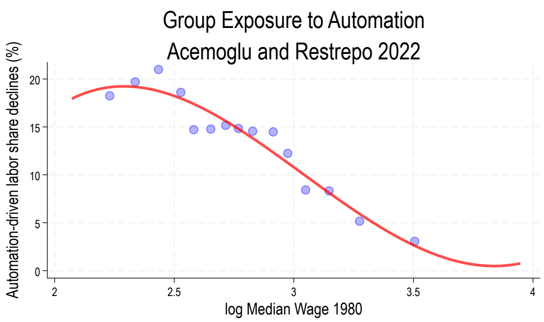
B.4. Technological Exposure across Inter-personal Dimension

Figure A2 illustrates how occupational exposure to automation and AI varies with interpersonal skill requirements. Panel (a) shows that occupations requiring greater interpersonal skills tend to be less exposed to automation, aligning with the intuition that social and emotional intelligence—often critical in managerial, negotiation, and caregiving roles—are difficult to codify into rule-based processes. In contrast, Panel (b) reveals that occupations with higher interpersonal skill requirements tend to be more exposed to AI, though with greater variance. This noisier relationship suggests that while AI can assist or complement interpersonal tasks (e.g., customer support or education), full automation remains limited by the complexity of human interaction.

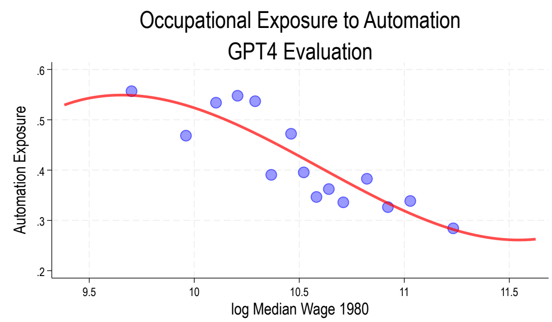
These findings reinforce the distinct nature of AI and automation risks: whereas automation displaces predictable, rule-based tasks, AI is more likely to augment or replace cognitive tasks, including those requiring some degree of human interaction. However, interpersonal-intensive occupations—such as psychologists, teachers, and business executives—still rely on empathy, persuasion, and social nuance, which remain challenging for AI to fully replicate.

B.5. Wage and Employment Effects of Automation

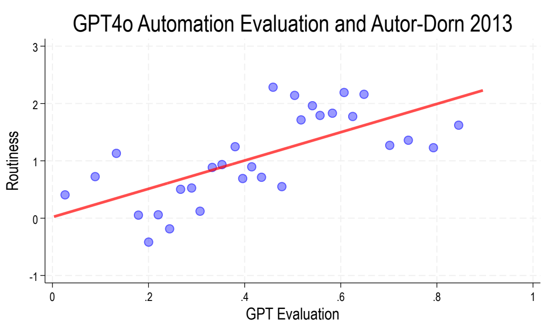
This section provides additional details on the wage and employment effects of automation exposure. The Panel Study of Income Dynamics (PSID) is a widely used longitudinal dataset



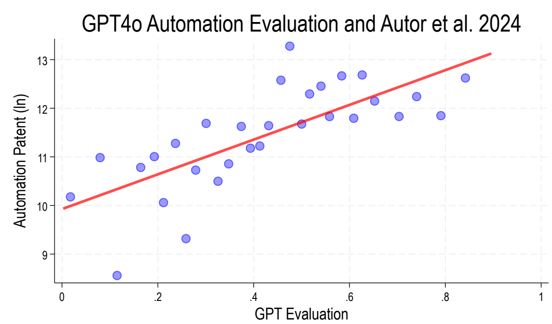
A. Automation Exposure and Wage, AR 2022



B. Automation Exposure and Wage, GPT-4o



C. Routine Task Intensity



D. Automation Patent Exposure

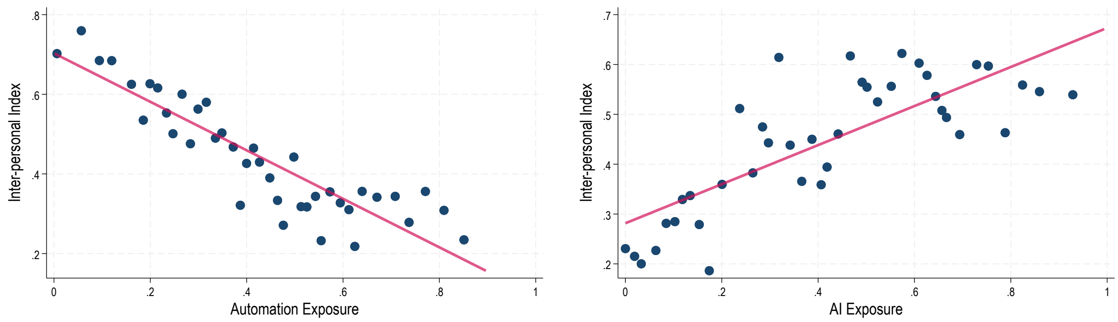
FIGURE A1. Automation Exposures

This figure compares automation exposure as evaluated by **GPT-4o** with existing measures with Binscatter, including the **decline in labor share due to the adoption of industrial robots, machines, and software (panels A and B)**, **routine task intensity (panel V)**, and **occupational exposure to automation patents (panel D)**.

that has tracked nearly 9,200 U.S. families since 1968. We leverage its panel structure to estimate relative wage trends by occupation while controlling for selection effects.

Since the main specifications have already been discussed, we now present additional results in Figure A3, which examines wage effects by gender and under different control specifications. Panel A reports the wage effects of automation separately for men and women, showing that the results are nearly identical, with no statistically significant differences. Panel B introduces additional controls, with the blue line accounting for age and age^2 for the blue line and allows for changing return to education for the green line, while the green line further allows for a changing return to education. The results suggest that changes in the return to education explain about a quarter of the wage effects attributed to automation.

However, when estimating elasticities, we prefer the main specification without controlling for changes in the return to education. From a long-run perspective, new workers may adjust their educational and occupational choices in response to shifts in the skill premium.

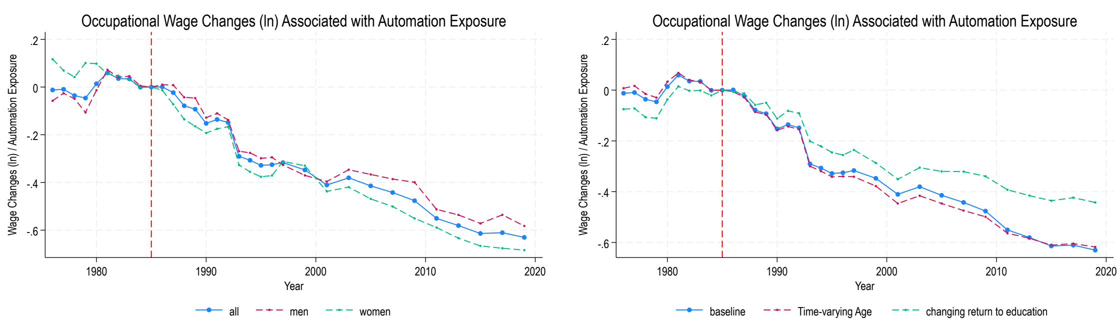


A. Interpersonal vs. Automation Exposure

B. Interpersonal vs. AI Exposure

FIGURE A2. Technological Exposures across Interpersonal Skills

This figure illustrates the relationship between occupational interpersonal skill requirements and exposure to automation (Panel (a)) and AI (Panel (b)).



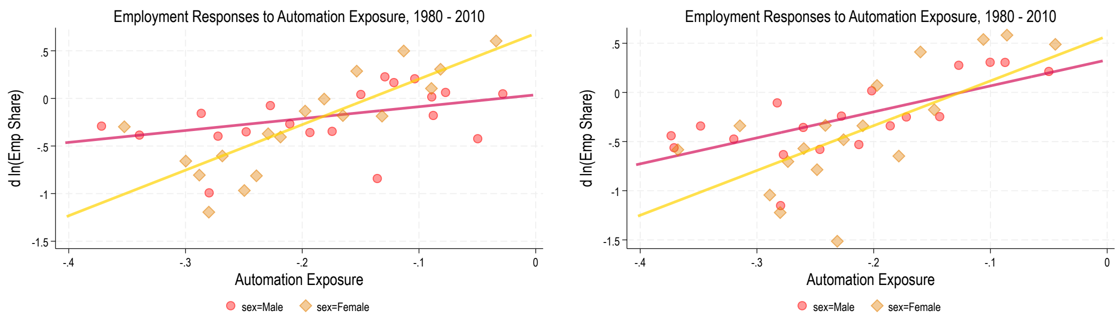
A. Wage Effects by Gender

B. Additional Controls

FIGURE A3. Effects of Automation on Wages

Notes: Panel A presents the wage effects of automation separately for men and women, showing no statistically significant differences. Panel B introduces additional controls, where the blue line includes age and age^2, and the green line further accounts for a changing return to education. The latter explains approximately 25% of the wage effects attributed to automation.

We now present additional results on the heterogeneous employment effects of automation across demographic groups, which are used to estimate correlation structures. Panel A of Figure A4 displays the average change in log employment shares between 1980 and 2010 by gender for white workers, while Panel B presents the corresponding employment effects for Black workers. The results indicate that white men are the least responsive to automation. Based on the data, this group was predominantly employed in occupations requiring more manual skills, which, as shown in our estimation results, are less portable across occupations. This pattern is reflected in our estimation procedure, which captures the variation in occupational transitions. As a result, the estimated correlation parameter for manual skills, ρ_{Man}, is relatively small, indicating lower substitutability of manual-intensive jobs.



A. White Workers

B. Black Workers

FIGURE A4. Heterogeneous Employment Effects by Demographic Groups

Notes: Panel A shows the employment effects of automation for white workers by gender, while Panel B presents the results for Black workers.

The PPML estimator jointly incorporates changes in the employment distribution, naturally weighting employment shares in the estimation process.

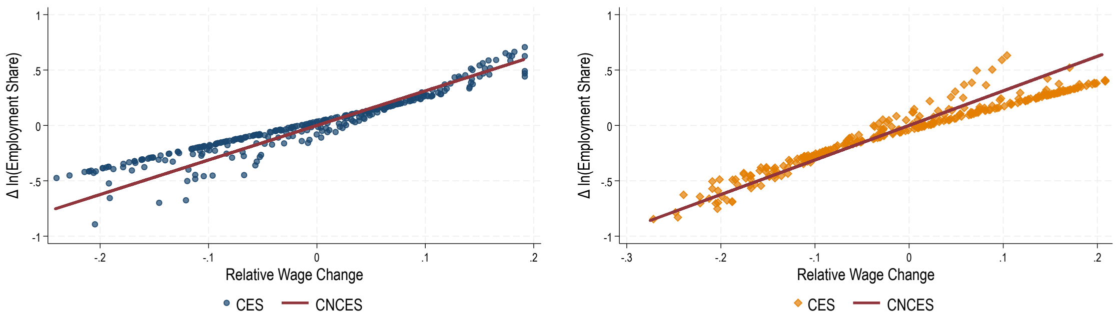
B.6. The Employment Effects of Automation and AI

As discussed in the main text, clustering shocks lead to smaller employment adjustments while exacerbating wage disparities. Panel (a) of Figure A5 illustrates the relationship between changes in log employment shares and relative wage changes for automation. The CES benchmark (dashed line) rotates counterclockwise, overstating employment shifts, particularly for negatively impacted occupations. This suggests that the CES framework underestimates the rigidity in labor reallocation caused by clustering shocks.

Panel (b) presents the same employment effects for AI exposure, revealing a similar pattern. The CES model again overstates employment adjustments, failing to account for the constrained worker mobility induced by the skill-clustering nature of AI-exposed occupations. These findings highlight the importance of incorporating a richer substitution structure, as captured by DIDES, to better reflect labor market frictions in response to technological change.

B.7. Construct Job Transition with CPS

Our estimation strategy hinges on observing aggregate job flows across occupations. To construct our occupation-level panel for the period 1980–2018, we rely on individual-level data from the US Census Bureau's March Current Population Survey (CPS). Each March CPS provides detailed information on respondents' current occupation as well as the occupation in which they spent most of the previous calendar year. We restrict our sample to individuals aged 25–64 who are employed full-time and have worked at least 26 weeks



A. Automation B. AI

FIGURE A5. Employment Effects of Technological Shocks

Notes: This figure compares employment effects of automation (Panel a) and AI (Panel b) against the CES benchmark. The CES framework overestimates employment adjustments, particularly in negatively impacted occupations, due to its failure to account for clustering shocks that restrict labor mobility.

in the preceding year, thereby ensuring the reliability of our occupational transition estimates. We also exclude observations with extreme or inconsistent income values to mitigate measurement error. Using these data, we construct annual job flow rates for occupations.

Employing a consistent occupation coding scheme, we generate a balanced panel of 306 three-digit occupations. Given the sparsity of observed transitions at this detailed level, we further aggregate these occupations into 15 clusters using a k-means algorithm based on occupational skill requirements. This intuitive clustering groups together occupations with similar skill profiles, ensuring robust estimates of aggregate job flows and facilitating subsequent analyses.

Furthermore, as noted by Artuç, Chaudhuri, and McLaren (2010), the retrospective design of the March CPS captures job transitions over a period shorter than a full year—respondents report the longest-held job from the previous calendar year, typically reflecting employment around mid-year. To correct for this timing bias, we annualize the observed job transition probabilities using the transformation [37]

$$\mu_t^{ANN} = \mu_t^2.$$

[37]This approach ensures that no annual job-to-job flows are missing.

Appendix C. Additional Materials

C.1. Derive Employment Shares in A.1.2.

With productivity distribution $\Pr\left[\epsilon_1(i) \le \epsilon_1, \ldots, \epsilon_O(i) \le \epsilon_O\right] = \exp\left[-F\left(A_1\epsilon_1^{-\theta}, \ldots, A_O\epsilon_O^{-\theta}\right)\right]$,

$$
\begin{aligned}
&\Pr\left[w_o\epsilon_o(i) < t \text{ and } w_o\epsilon_o(i) = \max_{o'} w'_o\epsilon'_o(i)\right] \\
&= \Pr\left[w_o\epsilon_o(i) < t \text{ and } w_o\epsilon_o(i) \ge w'_o\epsilon'_o(i), \forall o' \neq o\right] \\
&= \int_0^t \frac{\partial}{\partial z} \Pr\left[w_{o'}\epsilon_{o'} \le t \text{ and } w_o\epsilon_o < z\right]_{t=z} dz \\
&= \int_0^t \frac{\partial}{\partial t_o} \exp\left[-F\left(A_1 w_1^\theta t_1^{-\theta}, \ldots, A_O w_O^\theta t_O^{-\theta}\right)\right] |_{t_1=z,\ldots,t_O=z}\, dz \\
&= \int_0^t \left\{ \begin{array}{c} A_o w_o^\theta F_o\left(A_1 w_1^\theta t_1^{-\theta}, \ldots, A_O w_O^\theta t_O^{-\theta}\right) \times \\ \exp\left[-F\left(A_1 w_1^\theta t_1^{-\theta}, \ldots, A_O w_O^\theta t_O^{-\theta}\right)\right] \theta t_o^{-\theta-1} \end{array} \right\} |_{t_1=z,\ldots,t_O=z}\, dz \\
&= \int_0^t A_o w_o^\theta F_o\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right) \exp\left[-F\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right) z^{-\theta}\right] \theta z^{-\theta-1} dz \\
&= \frac{A_o w_o^\theta F_o\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)}{F\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)} \\
&\quad \times \int_0^t \exp\left[-F\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right) z^{-\theta}\right] F\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right) \theta z^{-\theta-1} dz \\
&= \frac{A_o w_o^\theta F_o\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)}{F\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)} \exp\left[-F\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right) t^{-\theta}\right]
\end{aligned}
$$

let $t \to \infty$, we have

$$
\begin{aligned}
\pi_o &= \Pr\left[w_o\epsilon_o(i) = \max_{o'} w'_o\epsilon'_o(i)\right] \\
&= \lim_{t\to\infty} \Pr\left[w_o\epsilon_o(i) < t \text{ and } w_o\epsilon_o(i) = \max_{\sigma'} w'_o\epsilon'_o(i)\right] \\
&= \frac{A_o w_o^\theta F_o\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)}{F\left(A_1 w_1^\theta, \ldots, A_O w_O^\theta\right)}
\end{aligned}
$$

C.2. Additional Derivation in Dynamic Model

C.2.1. Derivations of System in Changes A.6

Production Equilibrium. The production equilibrium condition relates changes in wages, labor allocations, output, and task shares. The equation is:

$$
\sigma \ln \dot{w}_{o,t+1} + \ln \dot{L}_{o,t+1} = \ln \dot{Y}_{t+1} + \ln\left(\dot{s}^{\ell}_{o,t+1}\right)
$$

This equation implies that wage changes $\dot{\boldsymbol{w}}_{t+1}$ can be solved as a function of labor changes $\dot{\boldsymbol{L}}_{t+1}$ and fundamentals $\dot{\Theta}_{t+1}$ denoted

$$\dot{\boldsymbol{w}}_{t+1} = \dot{w}\left(\dot{\boldsymbol{L}}_{t+1}, \dot{\Theta}_{t+1}\right)$$

Dynamic Equilibrium System. Given a converging sequence $\lim_{t\to\infty} \dot{\Theta}_t = 1$ (indicating fundamentals stabilize) and an initial allocation $(\boldsymbol{L}_0, \mu_0)$, the dynamic equilibrium is characterized by the evolution of adjusted mobility $\tilde{\mu}_{od',t}$, expected utility $\dot{u}_{o,t+1}$, and labor allocations $L_{\alpha',t}$. The adjusted mobility from origin o to o' at time t is $\tilde{\mu}_{oo',t} = \frac{A_{o',t}Z_{oo',t}^{\frac{\theta}{\kappa}}}{F\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}},\ldots,A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)}$.

We derive the change in adjusted mobility:

$$\begin{aligned}
\frac{\tilde{\mu}_{oo',t}}{\tilde{\mu}_{oo',t-1}} &= \frac{\dfrac{A_{d',}Z_{od',t}^{\frac{\theta}{\kappa}}}{F\left(A_{1,t}Z_{o\omega,t}^{\frac{\theta}{\kappa}},\ldots,A_{O,t}Z_{oo,t}^{\frac{\theta}{\sigma}}\right)}}{\dfrac{A_{o',t-1}Z_{od',t-1}^{\frac{\theta}{\kappa}}}{F\left(A_{1,t-1}Z_{ol,-1}^{\frac{\theta}{\kappa}},\ldots,A_{O,t-1}Z_{oO,t-1}^{\frac{\theta}{\kappa}}\right)}} \\
&= \frac{A_{o,t}Z_{oo',t}^{\frac{\theta}{\kappa}}/A_{o,t-1}Z_{oo',t-1}^{\frac{\theta}{\frac{1}{6}}}}{F\left(A_{1,t}Z_{ol,t}^{\frac{\rho}{\alpha}},\ldots,A_{O,t}Z_{oO,t}^{\frac{\theta}{\sigma}}\right)/F\left(A_{1,t-1}Z_{ol,t-1}^{\frac{\theta}{\sigma}},\ldots,A_{O,t-1}Z_{oO,t-1}^{\frac{\theta}{\sigma}}\right)} \\
&= \frac{\dot{A}_{o',t}\exp\left(\beta V_{o',t+1}-\beta V_{o',t}\right)^{\frac{\theta}{\kappa}}\dot{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}}/F_{t-1},\ldots,A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}F_{t-1}\right)} \\
&= \frac{\dot{A}_{o',t}\exp\left(\beta V_{o',t+1}-\beta V_{o',t}\right)^{\frac{\theta}{\kappa}}\dot{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\dfrac{A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}}}{A_{1,t-1}Z_{o1,t-1}^{\frac{\theta}{\kappa}}}\times\dfrac{A_{1,t-1}Z_{o1,t-1}^{\frac{\theta}{\kappa}}}{F_{t-1}},\ldots,\dfrac{A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}}{A_{O,t-1}Z_{oO,t-1}^{\frac{\theta}{\kappa}}}\dfrac{A_{O,t-1}Z_{oO,t-1}^{\frac{\theta}{\kappa}}}{F_{t-1}}\right)} \\
&= \frac{\dot{A}_{o',t}\dot{u}_{o't+1}^{\beta\frac{\theta}{\kappa}}\dot{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{\tilde{\mu}_{oo'',t-1}\dot{A}_{o'',t}\dot{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\dot{w}_{o'',t}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}
\end{aligned}$$

now we derive $\dot{u}_{o,t+1} = \frac{\exp(V_{o,t+1})}{\exp(V_{o,t})}$,

$$V_{o,t+1} - V_{o,t} = \frac{\kappa}{\theta}\ln F\left(A_{1,t+1}Z_{o1,t+1}^{\frac{\theta}{\kappa}},\ldots,A_{O,t+1}Z_{oO,t+1}^{\frac{\theta}{\kappa}}\right) - \frac{\kappa}{\theta}\ln F\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}},\ldots,A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)$$

$$= \frac{\kappa}{\theta} \ln F\left(\left\{\frac{A_{o'',t+1}Z_{o''1,t+1}^{\frac{\theta}{\kappa}}}{F\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}},\ldots,A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)}\right\}_{o''=1}^{O}\right)$$

$$= \frac{\kappa}{\theta} \ln F\left(\left\{\frac{A_{o'',t}Z_{o'',t}^{\frac{\theta}{\kappa}}\dot{A}_{o'',t+1}\dot{Z}_{o''1,t+1}^{\frac{\theta}{\kappa}}}{F\left(A_{1,t}Z_{o1,t}^{\frac{\theta}{\kappa}},\ldots,A_{O,t}Z_{oO,t}^{\frac{\theta}{\kappa}}\right)}\right\}_{o''=1}^{O}\right)$$

$$\frac{\kappa}{\theta} \ln F\left(\left\{\tilde{\mu}_{oo'',t}\dot{A}_{o'',t+1}\dot{u}_{o'',t+2}^{\beta\frac{\theta}{\kappa}}\dot{w}_{o'',t+1}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)$$

Thus:

$$\dot{u}_{o,t+1} = F\left(\left\{\tilde{\mu}_{oo'',t}\dot{A}_{o'',t+1}\dot{u}_{o'',t+2}^{\beta\frac{\theta}{\kappa}}\dot{w}_{o'',t+1}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

the labor allocation follows

$$L_{o',t} = \sum_{o} \mu_{oo',t}L_{o,t-1}$$

where $\mu_{oo',t} = \tilde{\mu}_{oo',t}F_{o'}\left(\tilde{\mu}_{o1,t},\ldots,\tilde{\mu}_{oO,t}\right)$.

C.2.2. Derivations of Dynamic Hat Algebra A.7

Production Equilibrium. The counterfactual change in wages is derived from the production side. For occupation o at time $t+1$:

$$\hat{w}_{o,t+1} = \frac{\dot{w}'_{o,t+1}}{\dot{w}_{o,t+1}} = \left(\frac{\dot{Y}'_{t+1}\dot{s}^{\ell}_{o,t+1}}{\dot{L}'_{o,t+1}} / \frac{\dot{Y}_{t+1}\dot{s}^{\ell}_{o,t+1}}{\dot{L}_{o,t+1}}\right)^{\frac{1}{\sigma}}$$

This defines $\hat{w}_{o,t+1}$ as a function of labor allocations and fundamentals, denoted $\hat{w}_{o,t+1}(\hat{\boldsymbol{L}}_{t+1},\hat{\Psi}_{t+1})$, where $\hat{\Psi}_{t+1}$ encapsulates production-side changes.

Counterfactual Switching Probabilities. The adjusted switching probability $\tilde{\mu}'_{oo',t}$ represents the counterfactual probability of moving from occupation o to o' at time t. Its evolution is:

$$\frac{\tilde{\mu}'_{oo',t}}{\tilde{\mu}'_{oo',t-1}} = \frac{\dot{\tilde{\mu}}_{oo',t}\hat{A}_{o',t}\hat{u}_{o't+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{\tilde{\mu}'_{oo'',t-1}\dot{A}'_{o'',t}\dot{u}'^{\beta\frac{\theta}{\kappa}}_{o'',t+1}\dot{w}'^{\frac{\theta}{\kappa}}_{o'',t}\right\}_{o''=1}^{O}\right)/F\left(\left\{\tilde{\mu}_{oo'',t-1}\dot{A}_{o'',t}\dot{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\dot{w}_{o'',t}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$

$$= \frac{\dot{\tilde{\mu}}_{oo',t}\hat{A}_{o',t}\hat{u}_{o't+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{\frac{\tilde{\mu}'_{oo'',t-1}}{\tilde{\mu}_{oo'',t-1}}\frac{\tilde{\mu}_{oo'',t-1}\dot{A}_{o'',t}\dot{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\dot{w}_{o'',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{\tilde{\mu}_{oo'',t-1}\dot{A}_{o'',t}\dot{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\dot{w}_{o'',t}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}\hat{A}_{o'',t}\hat{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o'',t}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$

$$= \frac{\dot{\tilde{\mu}}_{oo',t}\hat{A}_{o',t}\hat{u}_{o't+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{\frac{\tilde{\mu}'_{oo'',t-1}}{\tilde{\mu}_{oo'',t-1}}\tilde{\mu}_{oo'',t}\hat{A}_{o'',t}\hat{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o'',t}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$

In Recursive Form: Since $\tilde{\mu}'_{oo',t} = \tilde{\mu}'_{oo',t-1} \cdot \frac{\tilde{\mu}'_{oo',t}}{\tilde{\mu}'_{oo',t-1}}$, we obtain:

$$\tilde{\mu}'_{oo',t} = \frac{\tilde{\mu}'_{oo',t-1}\dot{\tilde{\mu}}_{oo',t}\hat{A}_{o',t}\hat{u}_{o't+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o',t}^{\frac{\theta}{\kappa}}}{F\left(\left\{\tilde{\mu}'_{oo'',t-1}\dot{\tilde{\mu}}_{oo'',t}\hat{A}_{o'',t}\hat{u}_{o'',t+1}^{\beta\frac{\theta}{\kappa}}\hat{w}_{o'',t}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$

Counterfactual Expected Utility. we can derive the counterfactual changes of expected welfare,

$$\frac{\dot{u}'_{o,t+1}}{\dot{u}_{o,t+1}} = \frac{F\left(\left\{\tilde{\mu}'_{oo'',t}\dot{A}'_{o'',t+1}\dot{u}'^{\beta\frac{\theta}{\kappa}}_{o'',t+2}\dot{w}'^{\frac{\theta}{\kappa}}_{o'',t+1}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}}{F\left(\left\{\tilde{\mu}_{oo'',t}\dot{A}_{o'',t+1}\dot{u}^{\beta\frac{\theta}{\kappa}}_{o'',t+2}\dot{w}^{\frac{\theta}{\kappa}}_{o'',t+1}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}}$$

$$= F\left(\left\{\frac{\tilde{\mu}'_{oo'',t}}{\tilde{\mu}_{oo'',t}}\frac{\tilde{\mu}_{oo'',t}\dot{A}_{o'',t+1}\dot{u}^{\beta\frac{\theta}{\kappa}}_{o'',t+2}\dot{w}^{\frac{\theta}{\kappa}}_{o'',t+1}}{F\left(\left\{\tilde{\mu}_{oo'',t}\dot{A}_{o'',t+1}\dot{u}^{\beta\frac{\theta}{\kappa}}_{o'',t+2}\dot{w}^{\frac{\theta}{\kappa}}_{o'',t+1}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}}\hat{A}_{o',t+1}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o't+2}\hat{w}^{\frac{\theta}{\kappa}}_{o',t+1}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

$$= F\left(\left\{\tilde{\mu}'_{oo'',t}\dot{\tilde{\mu}}_{oo'',t+1}\hat{A}_{o',t+1}\hat{u}^{\beta\frac{\theta}{\kappa}}_{o't+2}\hat{w}^{\frac{\theta}{\kappa}}_{o',t+1}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

Actual Switching Probabilities. The actual counterfactual switching probability $\mu'_{oo',t}$ adjusts $\tilde{\mu}'_{oo',t}$ for correlation effects:

$$\mu'_{oo',t} = \tilde{\mu}'_{oo',t}F_{o'}\left(\tilde{\mu}'_{o1,t}, \ldots, \tilde{\mu}'_{oO,t}\right)$$

Labor Allocation Evolution. The counterfactual labor allocation evolves as:

$$L'_{o,t} = \sum_{o} \mu'_{oo,t} L'_{o,t-1} + \Delta L_{o,t}$$

Where $\Delta L_{o',t}$ is the exogenous net inflow/outflow of workers.

C.2.3. Intial Dynamics

At $t = 1$, the counterfactual fundamentals change unexpectedly (unknown before $t = 1$). Initial conditions at t = 0 are identical: $\hat{u}_{o,0} = 1$, $\mu'_{oo',0} = \mu_{oo',0}$, $L'_{o,0} = L_{o,0}$.

we first write the expected utility at time $5 = 0$,

$$u_{o,0} = F\left(\left\{A_{o',0} Z_{o',0}^{\frac{\theta}{\kappa}}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}} = F\left(\left\{\frac{A_{o',0}}{A'_{o',0}}\left(\frac{Z_{o',0}}{Z'_{o',0}}\right)^{\beta\frac{\theta}{\kappa}} A'_{o',0} Z'^{\frac{\theta}{\kappa}}_{o',0} \exp\left(\tau_{oo'}\right)^{-\frac{\theta}{\kappa}}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

similarly, we can write the expected utility in the counterfacctual world at time $t = 1$,

$$u'_{o,1} = F\left(\left\{A'_{o',1} Z'^{\frac{\theta}{\kappa}}_{o',1}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

then take the ratio between these two

$$\frac{u'_{o,1}}{u_{o,0}} = \frac{F\left(\left\{A'_{o',1} Z'^{\frac{\theta}{\kappa}}_{o',1}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}}{F\left(\left\{\frac{A_{o',0}}{A'_{o',0}}\left(\frac{Z_{o',0}}{Z'_{o',0}}\right)^{\beta\frac{\theta}{\kappa}} A'_{o',0} Z'^{\frac{\theta}{\kappa}}_{o',0}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}}$$

$$== F\left(\left\{\frac{A'_{o',1} Z'^{\frac{\theta}{\kappa}}_{o',1}}{F\left(\left\{\frac{A_{o',0}}{A'_{o',0}}\left(\frac{Z_{o',0}}{Z'_{o',0}}\right)^{\beta\frac{\theta}{\kappa}} A'_{o',0} Z'^{\frac{\theta}{\kappa}}_{o',0}\right\}_{o'=1}^{O}\right)}\right\}_{o'=1}^{O}\right)$$

additionally, note that

$$\tilde{\mu}_{oo',0} = \frac{A_{o',0} Z_{oo',0}{}^{\frac{\theta}{\kappa}}}{F\left(A_{1,0} Z_{o1,0}{}^{\frac{\theta}{\kappa}}, \ldots, A_{O,0} Z_{oO,0}^{\frac{\theta}{\kappa}}\right)}$$

$$= \frac{\frac{A_{o',o} Z_{oo',o}{}^{\frac{\theta}{\kappa}}}{A'_{o',o} Z'_{oo',o}{}^{\frac{\theta}{\kappa}}} A'_{o',0} Z'_{oo',0}{}^{\frac{\theta}{\kappa}}}{F\left(\left\{\frac{A_{o'',0} Z_{oo'',0}{}^{\frac{\theta}{\kappa}}}{A'_{o'',0} Z'_{oo'',0}{}^{\frac{\theta}{\kappa}}} A'_{o'',0} Z'_{oo'',0}{}^{\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$

combining these two equations gives us

$$\frac{u'_{o,1}}{u_{o,0}} = F\left(\left\{\frac{\frac{A_{o',0}}{A'_{o',0}}\left(\frac{Z_{o',0}}{Z'_{o',0}}\right)^{\beta\frac{\theta}{\kappa}}}{F\left(\left\{\frac{A_{o',1}}{A'_{o',1}}\left(\frac{Z_{o',1}}{Z'_{o',1}}\right)^{\beta\frac{\theta}{\kappa}} A'_{o',1} Z'^{\frac{\theta}{\kappa}}_{o',1}\right\}_{o'=1}^{O}\right)} \frac{A'_{o',1} Z'^{\frac{\theta}{\kappa}}_{o',1}}{\frac{A_{o',0}}{A'_{o',0}}\left(\frac{Z_{o',0}}{Z'_{o',0}}\right)^{\beta\frac{\theta}{\kappa}}}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

$$= F\left(\left\{\frac{\tilde{\mu}_{oo',0}}{A'_{o',0} Z'_{oo',0}{}^{\frac{\theta}{\kappa}}} \frac{A'_{o',1} Z'^{\frac{\theta}{\kappa}}_{o',1}}{\frac{A_{o',0}}{A'_{o',0}}\left(\frac{Z_{o',0}}{Z'_{o',0}}\right)^{\beta\frac{\theta}{\kappa}}}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

$$= F\left(\left\{\frac{\tilde{\mu}_{oo',0}}{\frac{A_{o',0}}{A'_{o',0}}\left(\frac{Z_{o',0}}{Z'_{o',0}}\right)^{\frac{\theta}{\kappa}}} \dot{A}'_{o',1} \dot{Z}'^{\frac{\theta}{\kappa}}_{o',1}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

Then, we use the fact that $u'_{o,0} = u_{o,0}, A'_{o,0} = A_{o,0}, w'_{o,0} = w_{o,0}$, we can write this in dot:

$$\dot{u}'_{o,1} = F\left(\left\{\frac{\tilde{\mu}_{oo',0}}{\frac{u_{o',1}}{u'_{o',1}}\beta\frac{\theta}{\kappa}} \dot{A}'_{o',1} \dot{w}'^{\frac{\theta}{\kappa}}_{o',1} \dot{u}'^{\beta\frac{\theta}{\kappa}}_{o',2}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

remember we have $\dot{u}_{o,1} = F\left(\left\{\mu_{oo'',0} \dot{A}_{o'',1} \dot{w}^{\frac{\theta}{\kappa}}_{o'',1} \dot{u}^{\beta\frac{\theta}{\kappa}}_{o'',2}\right\}_{o''=1}^{O}\right)^{\frac{\kappa}{\theta}}$ and take the ratio of these:

$$\hat{u}_{o,1} = F\left(\left\{\frac{\frac{\dot{A}'_{o',1} \dot{w}'^{\frac{\theta}{\kappa}}_{o',1} \dot{u}'^{\beta\frac{\theta}{\kappa}}_{o',2}}{\dot{A}_{o',1} \dot{w}^{\frac{\theta}{\kappa}}_{o',1} \dot{u}^{\beta\frac{\theta}{\kappa}}_{o',2}}}{\frac{u_{o',1}}{u'_{o',1}}\beta\frac{\theta}{\kappa}} \frac{\tilde{\mu}_{oo',0} \dot{A}_{o',1} \dot{w}^{\frac{\theta}{\kappa}}_{o',1} \dot{u}^{\beta\frac{\theta}{\kappa}}_{o',2}}{F\left(\left\{\mu_{oo'',0} \dot{A}_{o'',1} \dot{w}^{\frac{\theta}{\kappa}}_{o'',1} \dot{u}^{\beta\frac{\theta}{\kappa}}_{o'',2}\right\}_{o''=1}^{O}\right)}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

$$= F\left(\left\{\frac{\tilde{\mu}_{oo',1}}{\frac{u_{o',1}}{u'_{o',1}}^{\beta\frac{\theta}{\kappa}}}\hat{A}_{o',1}\hat{w}_{o',1}^{\frac{\theta}{\kappa}}\hat{u}_{o',2}^{\beta\frac{\theta}{\kappa}}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

$$= F\left(\left\{\tilde{\mu}_{oo',1}\hat{u}_{o',1}^{\beta\frac{\theta}{\kappa}}\hat{A}_{o',1}\hat{w}_{o',1}^{\frac{\theta}{\kappa}}\hat{u}_{o',2}^{\beta\frac{\theta}{\kappa}}\right\}_{o'=1}^{O}\right)^{\frac{\kappa}{\theta}}$$

the next step is to derive the switching probability at $t = 1$:

$$\frac{\tilde{\mu}'_{oo',1}}{\tilde{\mu}_{oo',1}} = \frac{A'_{o',1}Z'^{\frac{\theta}{\kappa}}_{oo',1}/A_{o',1}Z^{\frac{\theta}{\kappa}}_{oo',1}}{F\left(\left\{A'_{o'',1}Z'^{\frac{\theta}{\kappa}}_{oo'',1}\right\}_{o''=1}^{O}\right)/F\left(\left\{A_{o'',1}Z^{\frac{\theta}{\kappa}}_{oo'',1}\right\}_{o''=1}^{O}\right)}$$

$$= \frac{\frac{A'_{o',1}}{A_{o',1}}\left(\frac{w'_{o',1}}{w_{o',1}}\right)^{\frac{\theta}{\kappa}}\left(\frac{u'_{o',2}}{u_{o',2}}\right)^{\beta\frac{\theta}{\kappa}}}{F\left(\left\{\frac{A'_{o'',1}Z'^{\frac{\theta}{\kappa}}_{oo'',1}}{F\left(\left\{A_{o'',1}Z^{\frac{\theta}{\kappa}}_{oo'',1}\right\}_{o''=1}^{O}\right)}\right\}_{o''=1}^{O}\right)}$$

$$= \frac{\frac{A'_{o',1}}{A_{o',1}}\left(\frac{w'_{o',1}}{w_{o',1}}\right)^{\frac{\theta}{\kappa}}\left(\frac{u'_{o',2}}{u_{o',2}}\right)^{\beta\frac{\theta}{\kappa}}}{F\left(\left\{\mu_{oo'',1}\frac{A'_{o'',1}}{A_{o'',1}}\left(\frac{w'_{o'',1}}{w_{o'',1}}\right)^{\frac{\theta}{\kappa}}\left(\frac{u'_{o'',2}}{u_{o'',2}}\right)^{\beta\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$

notice that:

$$\tilde{\mu}_{oo',1}\frac{A'_{o',1}}{A_{o',1}}\left(\frac{w'_{o',1}}{w_{o',1}}\right)^{\frac{\theta}{\kappa}}\left(\frac{u'_{o',2}}{u_{o',2}}\right)^{\beta\frac{\theta}{\kappa}} = \tilde{\mu}_{oo',1}\hat{A}_{o',1}\hat{w}_{o',1}^{\frac{\theta}{\kappa}}\hat{u}_{o'2}^{\beta\frac{\theta}{\kappa}}\left(\frac{u'_{o',2}}{u_{o',2}}/\frac{u'_{o',1}}{u_{o',1}}\right)^{\beta\frac{\theta}{\kappa}}\left(\frac{u'_{o',1}}{u_{o',1}}\right)^{\beta\frac{\theta}{\kappa}}$$

$$= \tilde{\mu}_{oo',1}\hat{u}_{o',1}^{\beta\frac{\theta}{\kappa}}\hat{A}_{o',1}\hat{w}_{o',1}^{\frac{\theta}{\kappa}}\hat{u}_{o',2}^{\beta\frac{\theta}{\kappa}}$$

thus,

$$\tilde{\mu}'_{oo',1} = \frac{\vartheta_{oo',1}\hat{A}_{o',1}\hat{w}_{o',1}^{\frac{\theta}{\kappa}}\hat{u}_{o',2}^{\beta\frac{\theta}{\kappa}}}{F\left(\left\{\vartheta_{oo'',1}\hat{A}_{o'',1}\hat{w}_{o'',1}^{\frac{\theta}{\kappa}}\hat{u}_{o'',2}^{\beta\frac{\theta}{\kappa}}\right\}_{o''=1}^{O}\right)}$$