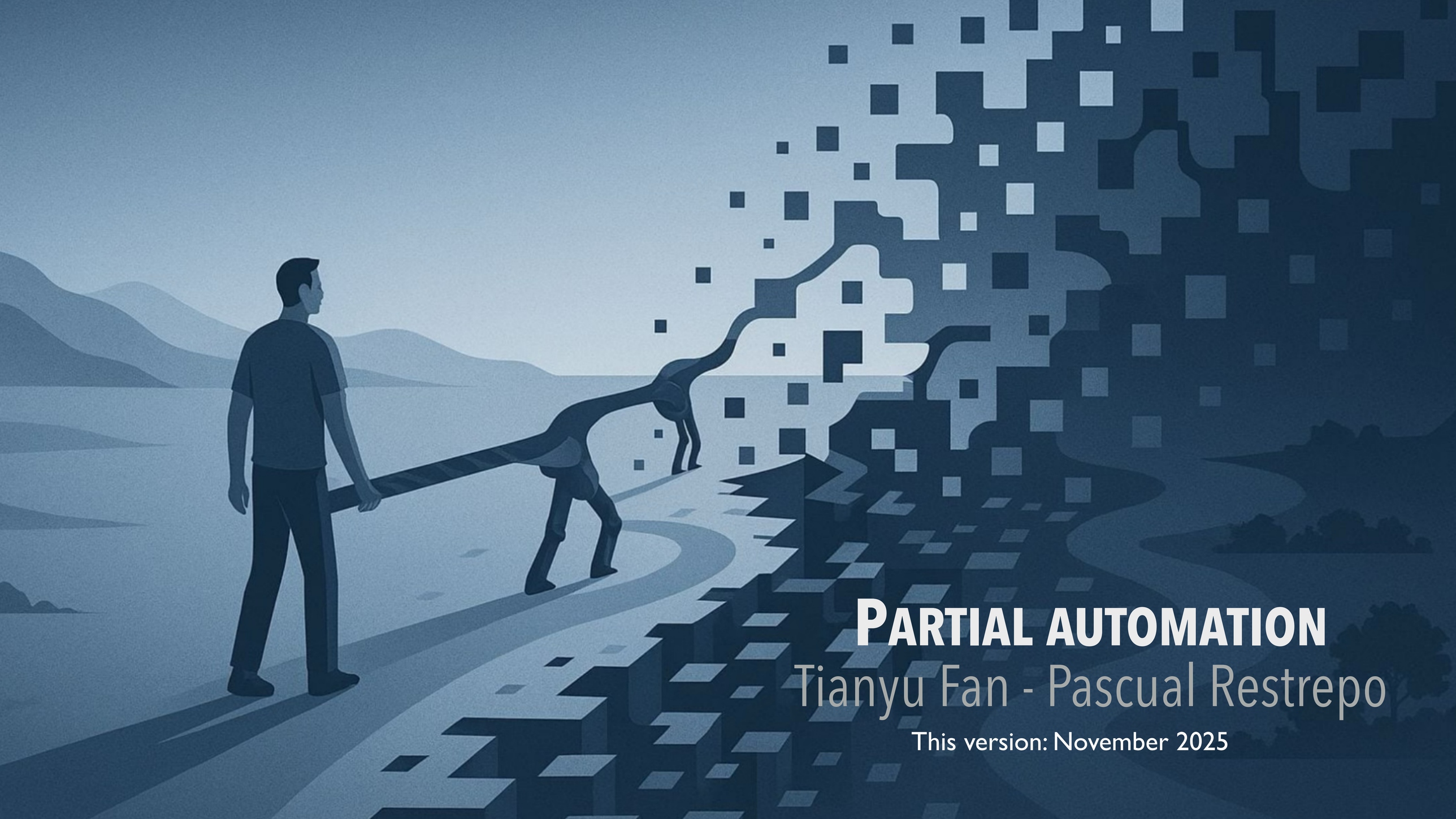


Partial automation

Tianyu Fan - Pascual Restrepo

This version: November 2025

MODEL– BEFORE PARTIAL AUTOMATION

- Jobs $j \in \mathcal{J}$ combine to produce output

$$Y = \left(\sum_j \alpha_j^{1/\sigma} \, Y_j^{1-1/\sigma} \right)^{\sigma/(\sigma-1)}$$

- Each job requires worker to complete tasks / components $x \in \mathcal{T}_j$

- Workers of skill $(a_1, a_2, \ldots, a_J)$ with pdf $f(a_1, a_2, \ldots, a_J)$

  Productivity $z_j(x, a_j)$ in component x of job j

  Output $Y_j(a_j; h) = G\left(\{h(x) z_j(x, a_j)\}_{x \in \mathcal{T}_j} \right)$ with $\int_{\mathcal{T}_j} h(x) dx = 1$ → All tasks in a job produced by same worker (communication costs high)

  Total output Y_j aggregates $Y_j(a_j; h)$ over all workers selecting j

CORE VS PERIPHERAL TASKS

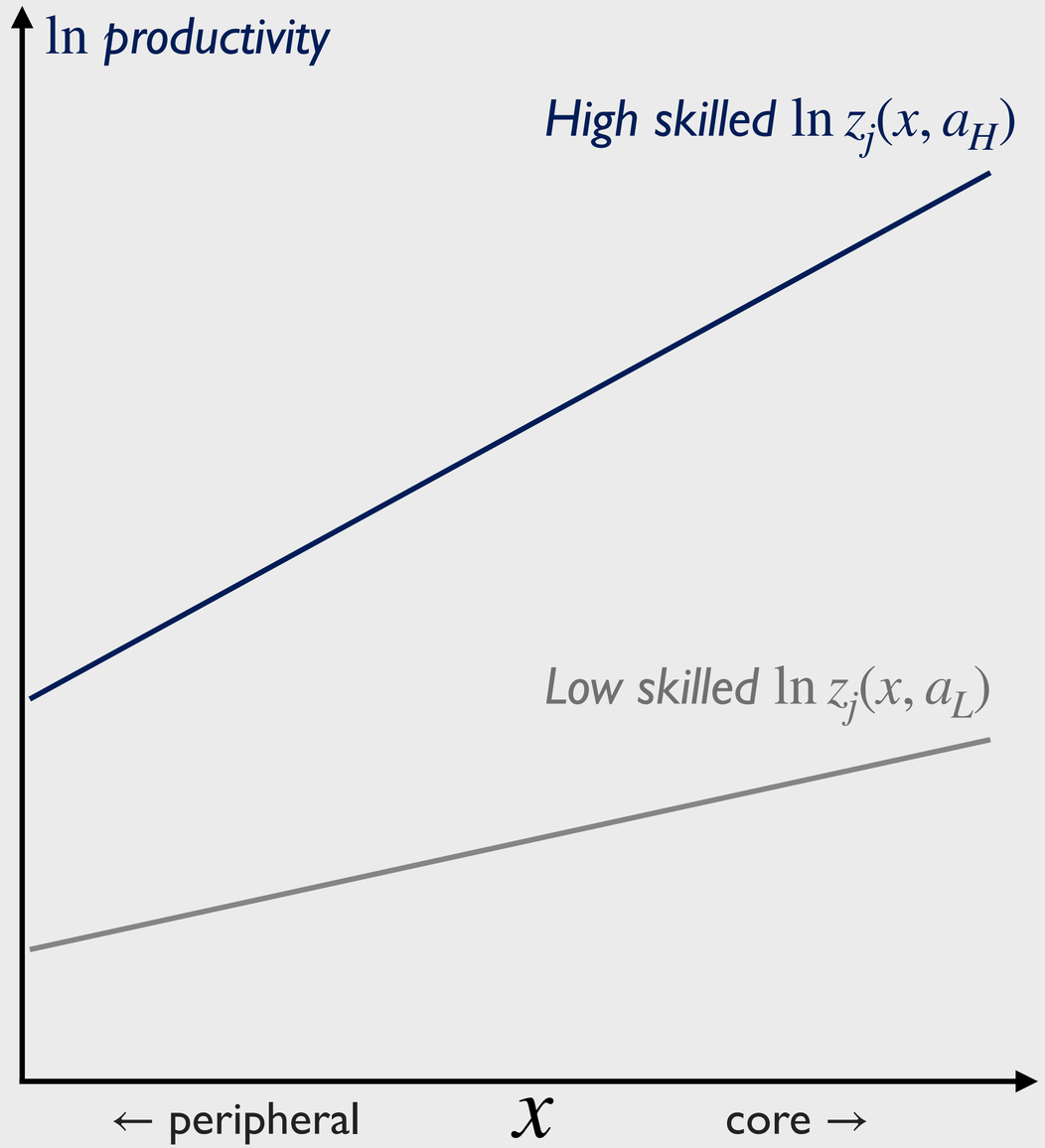
Assumption: $z_j(x, a_j)$ is log super modular in x, a_j and increasing in a_j

- High x tasks in $\mathcal{I}_j$ are "core" component of job— the defining features of job

  being a *good economists* means being good at *core task* of research

  being a *good welder* means being good at *core task* of welding parts together

- Low x tasks in $\mathcal{I}_j$ are "peripheral"— components of job that the *best workers* would outsource if you could



EQUILIBRIUM– BEFORE PARTIAL AUTOMATION

- Job prices P_j, output Y, allocations $\mathcal{S}_j$ such that
  - Income adds up (i.e., ideal price index)

$$\sum_j \alpha_j P_j^{1-\sigma} = 1$$

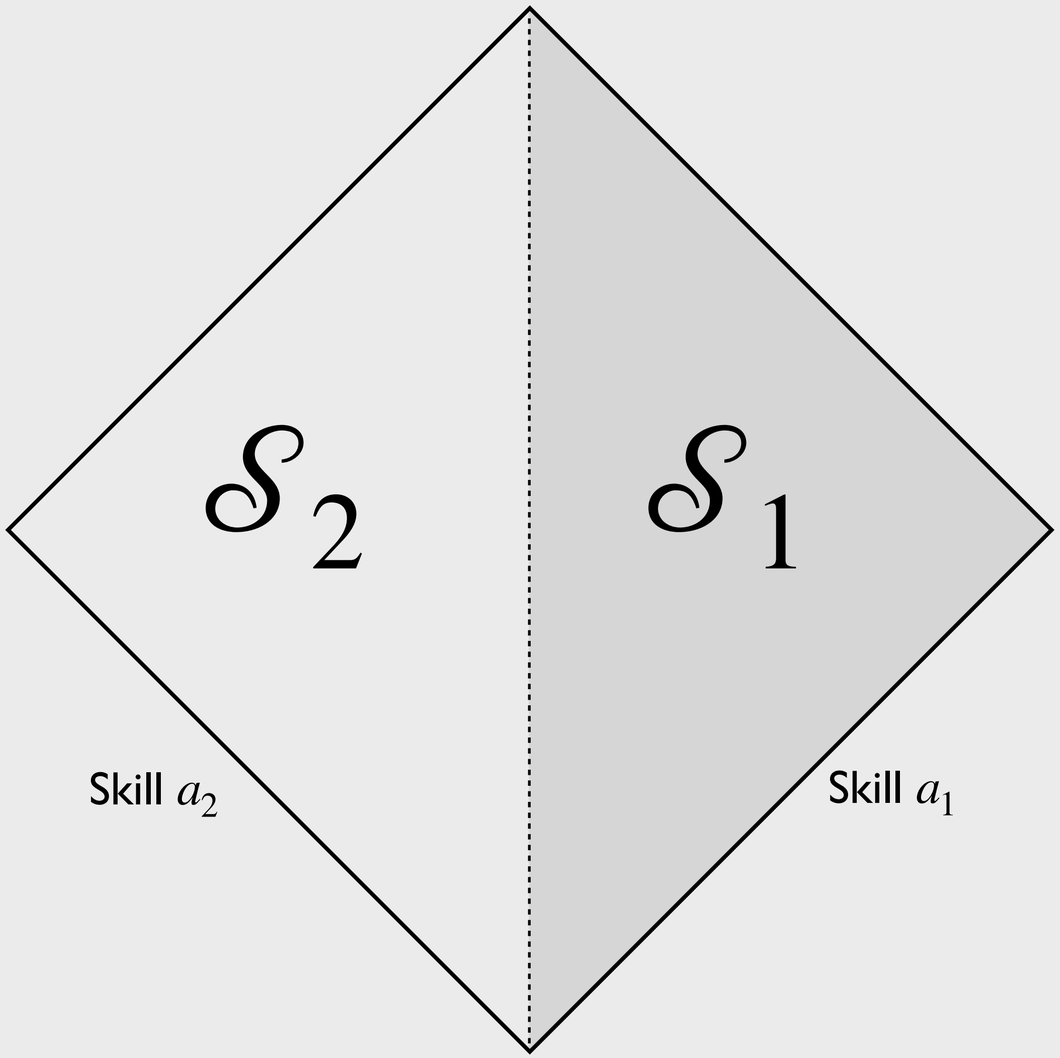
  - Jobs organized optimally to max worker output

$$M_j(a_j) \equiv \max_h Y_j(a_j; h)$$

  - Market for job j clears

$$\alpha_j Y P_j^{-\sigma} = \int_{\boldsymbol{a} \in \mathcal{S}_j} M_j(a_j) f(\boldsymbol{a}) d\boldsymbol{a}$$

$a \in \mathcal{S}_j$ if $W_j(a) \geq W_k(a)$ for all $k \in \mathcal{J}$, where $W_j(a) = P_j M_j(a_j)$



Model– Introducing Automated Systems

- Jobs $j \in \mathcal{J}$ combine to produce output

$$Y = \left(\sum_j \alpha_j^{1/\sigma} \, Y_j^{1-1/\sigma} \right)^{\sigma/(\sigma-1)}$$

- Each job requires worker to complete tasks / components $x \in \mathcal{T}_j$

- Workers of skill $(a_1, a_2, \ldots, a_J)$ with pdf $f(a_1, a_2, \ldots, a_J)$

  Productivity $z_j(x, a_j)$ in component x of job j

  Output $Y_j(a_j; h, k) = G\left(\{ h(x) z_j(x, a_j) \; + \; k(x) \psi_j z_j(x) \}_{x \in \mathcal{T}_j} \right) - \kappa$ ⟶ Worker | Firm can automate some components of the jobs

$$\left(\int_{\mathcal{T}_j} h(x) dx = 1 \text{ and } \; \kappa = \int_{\mathcal{T}_j} k(x) \, dx \text{ is cost of running system } \right)$$

  Total output Y_j aggregates $Y_j(a_j; h)$ over all workers selecting j

Equilibrium– With Automated Systems

- Job prices P_j, output Y, allocations $\mathcal{S}_j$ such that
  - Income adds up (i.e., ideal price index)

$$\sum_j \alpha_j P_j^{1-\sigma} = 1$$

  - Jobs **re-organized optimally** to max worker output

$$M_j^A(a_j) \equiv \max_{h,k} Y_j(a_j; h, k)$$

Only difference is here: we go from $Y_j(a_j; h)$ to $Y_j(a_j; h, k)$

  - Market for job j clears

$$\alpha_j Y P_j^{-\sigma} = \int_{\boldsymbol{a} \in \mathcal{S}_j} M_j^A(a_j) f(\boldsymbol{a}) d\boldsymbol{a}$$

$a \in \mathcal{S}_j$ if $W_j(a) \geq W_k(a)$ for all $k \in \mathcal{J}$, where $W_j(a) = P_j M_j^A(a_j)$

One Example

- Suppose G is CES with EoS γ across tasks:
- Before $Y_j(a_j; h) = \left(\int_{\mathcal{T}_j} \left[h(x)\, z_j(x, a_j) \right]^{1-1/\gamma} dx \right)^{\gamma/(\gamma-1)}$
  - Optimal job **organization**:

$$h(x) = z_j(x, a_j)^{\gamma-1} \Big/ \int_{\mathcal{T}_j} z_j(s, a_j)^{\gamma-1}\, ds \text{ and } M_j(a_j) = \left(\int_{\mathcal{T}_j} z_j(x, a_j)^{\gamma-1}\, dx \right)^{1/(\gamma-1)}$$

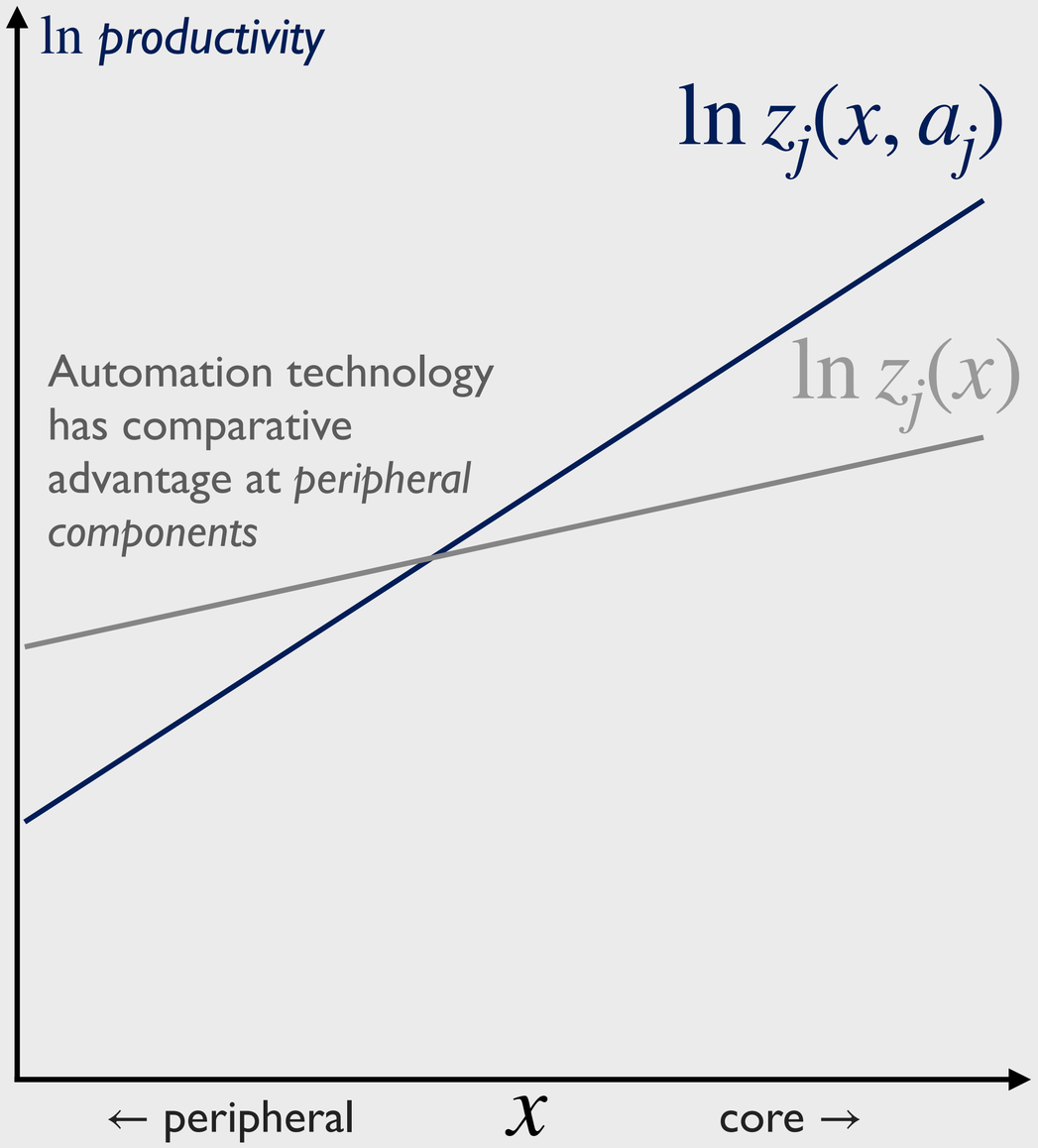
- After $Y_j(a_j; h, k) = \left(\int_{\mathcal{T}_j} \left[h(x)\, z_j(x, a_j) \;+\; k(x)\, \psi_j z_j(x) \right]^{1-1/\gamma} dx \right)^{\gamma/(\gamma-1)} - \kappa$
  - Optimal job **re-organization**:

assign tasks with $\dfrac{z_j(x, a)}{\psi_i z_j(x)} \leq \lambda_j(a)$ to automated system

DEFINITION– PERIPHERAL AUTOMATION

Definition: A *peripheral automation system* is one with $z_j(x, a_j)/z_j(x)$ increasing in x for all a_j.

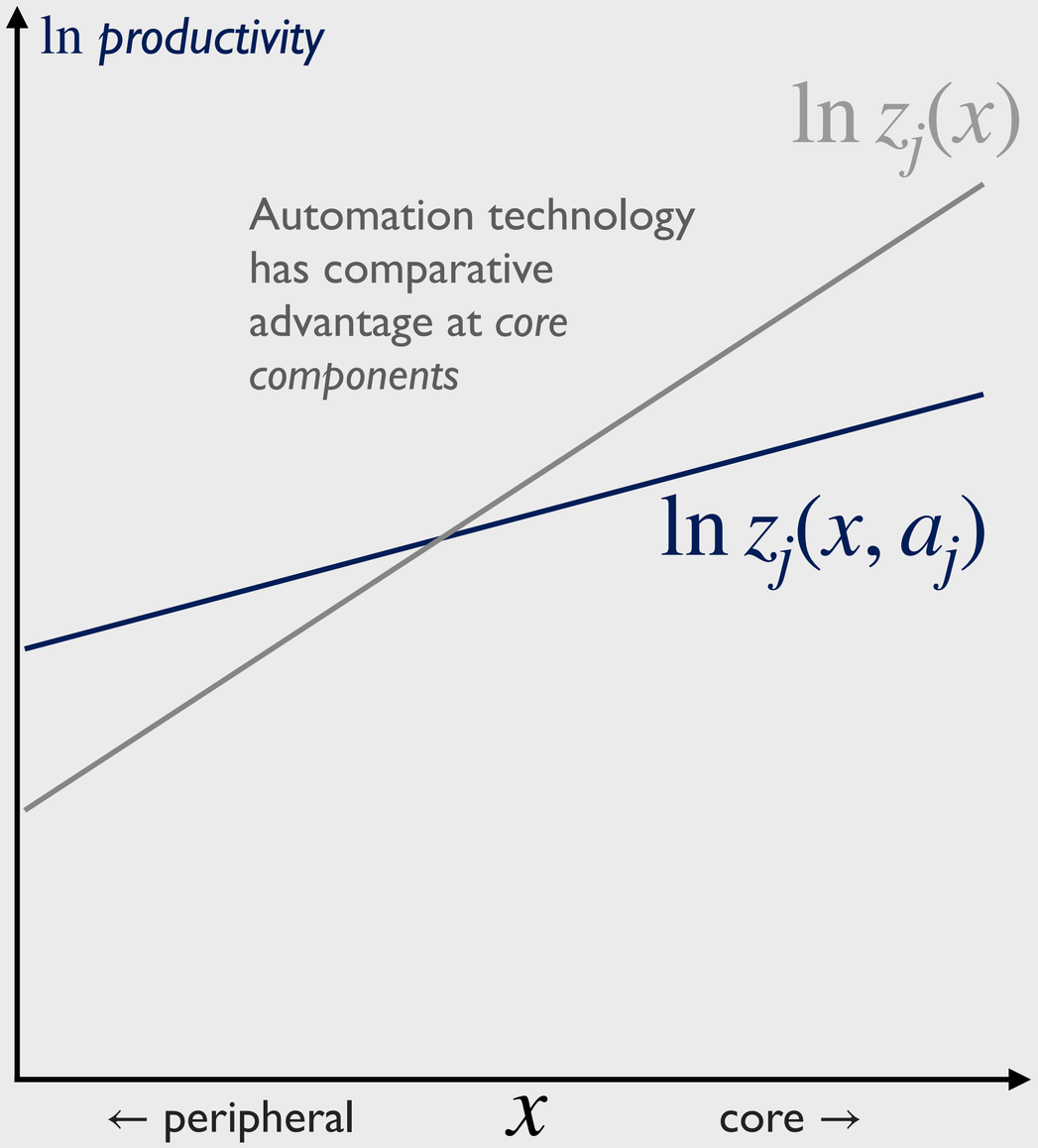
- Entails automating all components below $\underline{x}(a)$ and focus worker effort on core ones.



Definition– Core Automation

Definition: A *core automation system* is one with $z_j(x, a_j)/z_j(x)$ decreasing in x for all a_j.

- Entails automating all components above $\bar{x}(a)$ and focus worker effort on peripheral ones

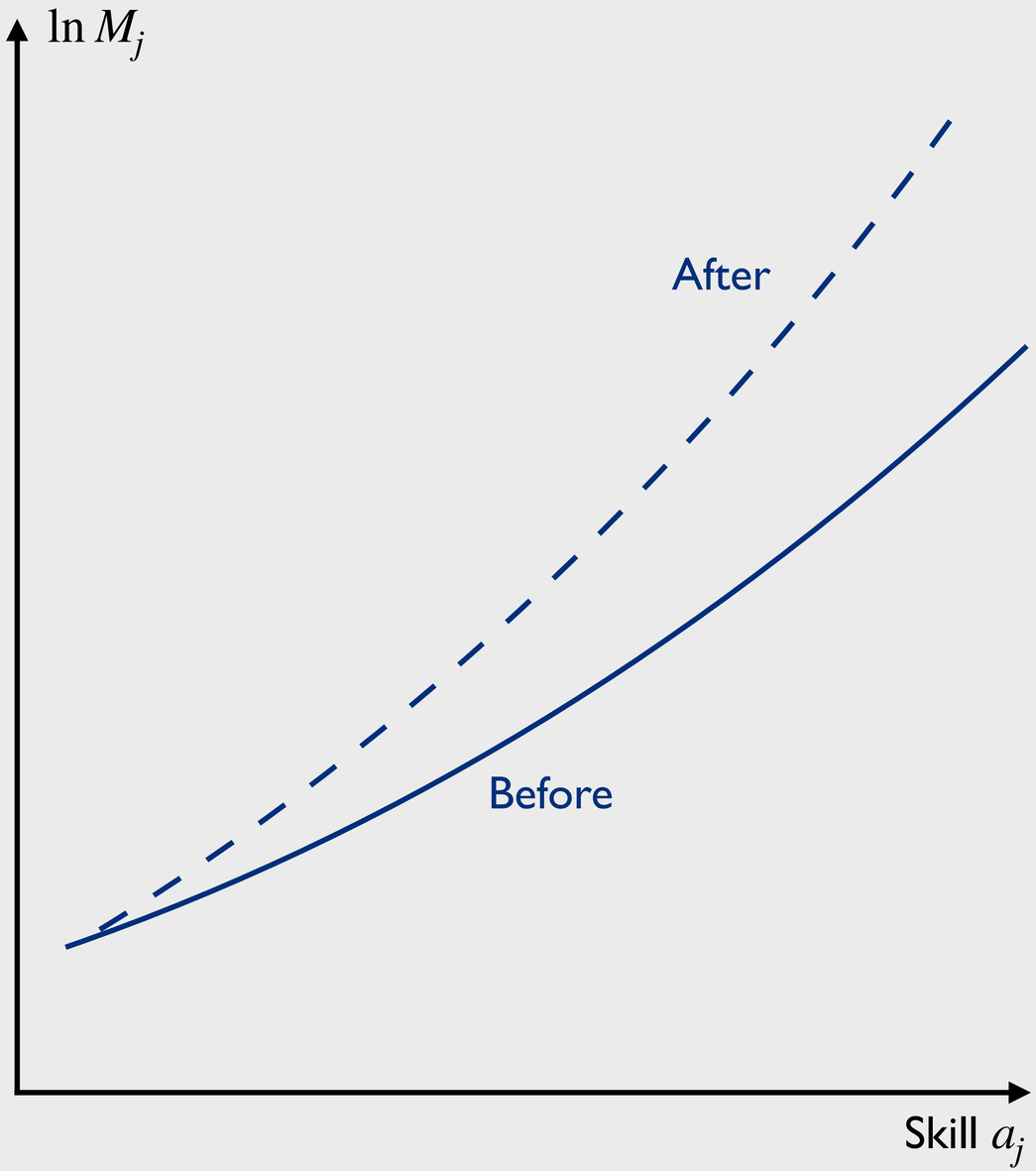


Equilibrium– Peripheral Automation

Proposition: For *Peripheral automation*:

- High a_j workers adopt the technology more intensively (in more tasks)
- Net worker output (their "MPL") increases and gets *convexified* in a_j

  e.g., the increase in net worker output $\Pi_j(a_j) \equiv \ln M_j^A(a_j) - \ln M_j(a_j)$ rises in a_j

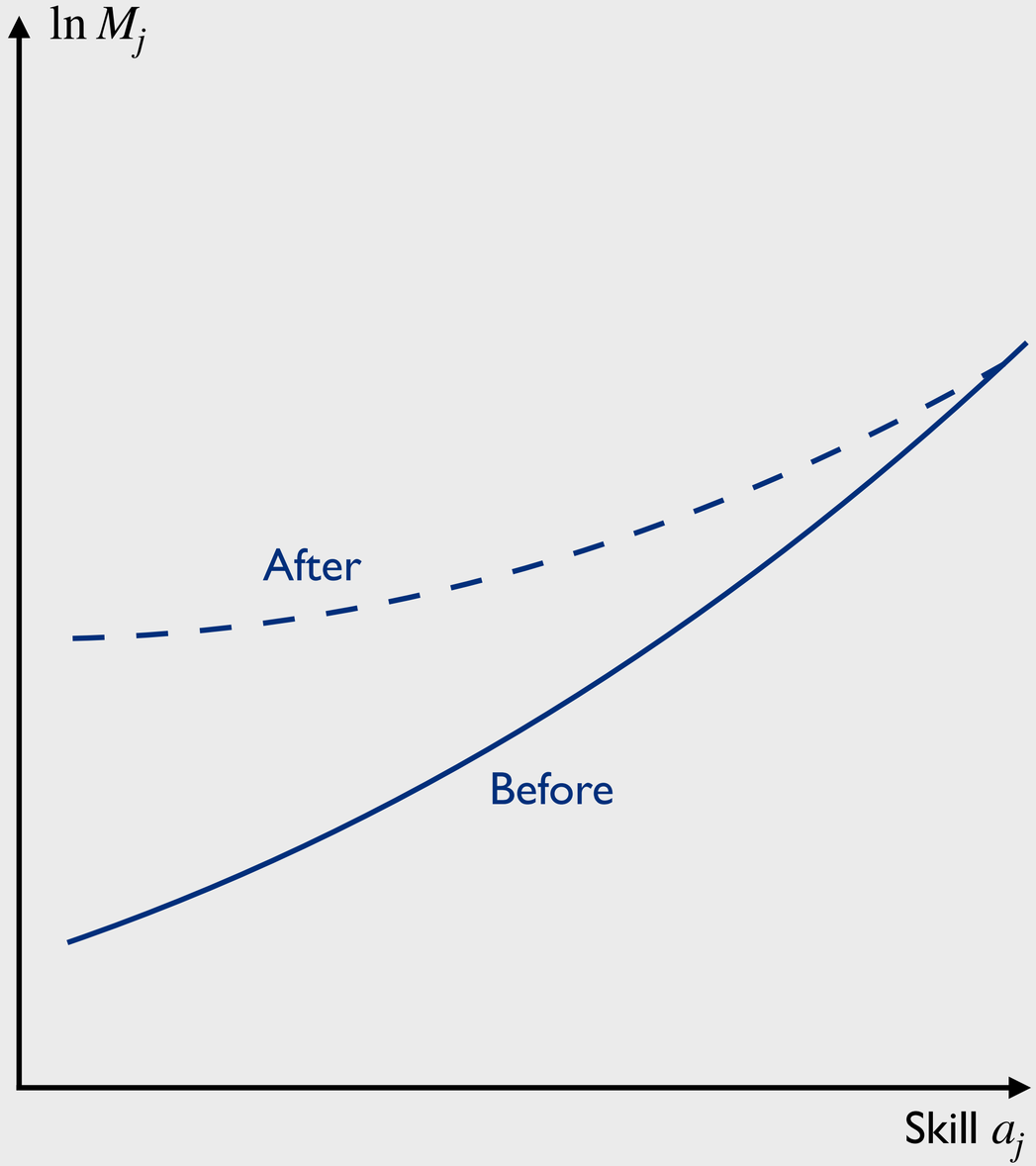


Equilibrium– Core Automation

Proposition: For *Core automation*:

- Low a_j workers adopt the technology more intensively (in more tasks)
- Net worker output (their "MPL") increases and gets *compressed* in a_j

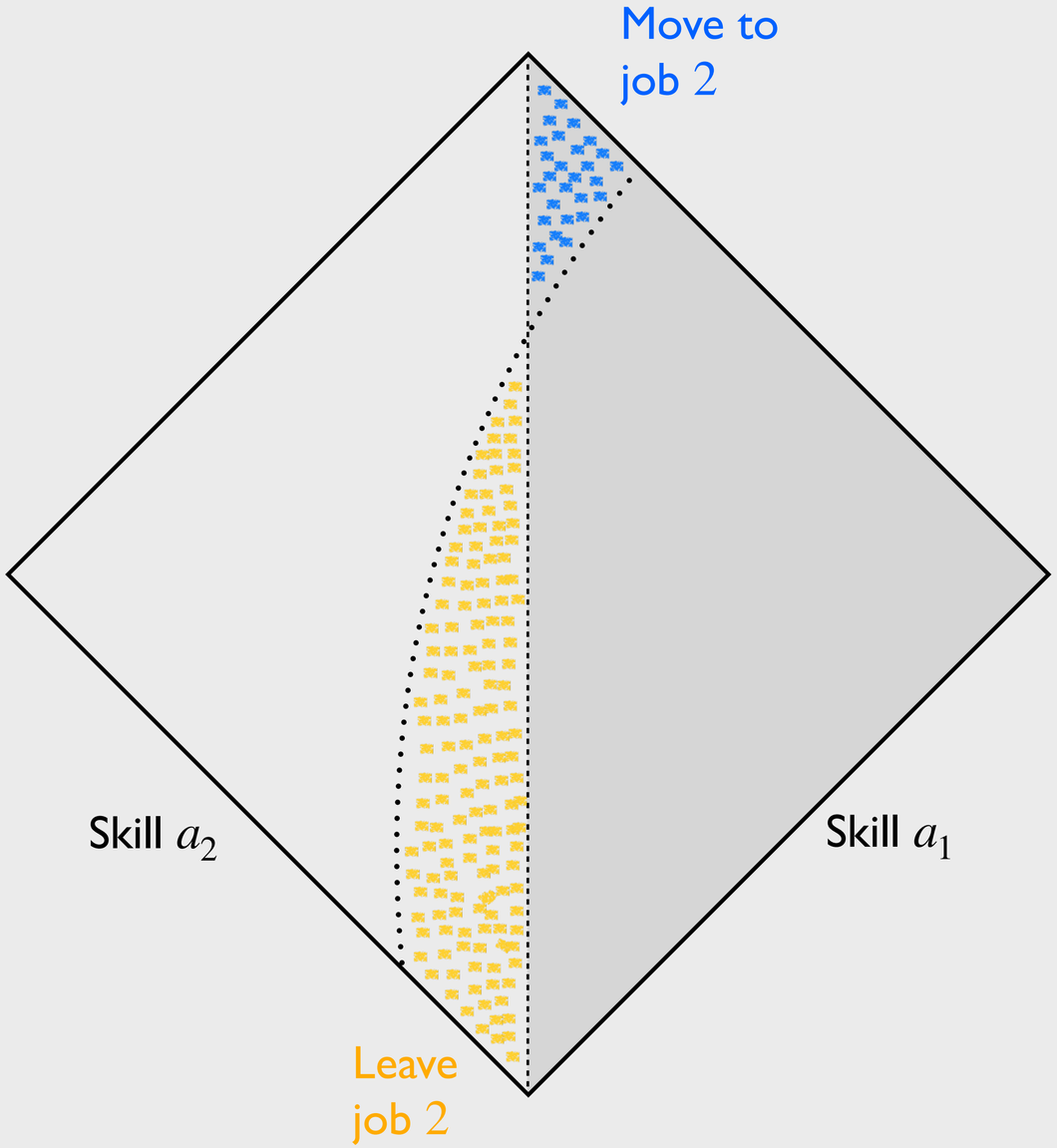
  e.g., the increase in net worker output $\Pi_j(a_j) \equiv \ln M_j^A(a_j) - \ln M_j(a_j)$ decreases in a_j



General equilibrium effects of Peripheral Automation

- Suppose $\sigma \geq 1$ (ie job demand elastic)
- Assume technology adopted by *some but not all* workers in $\mathcal{S}_j$

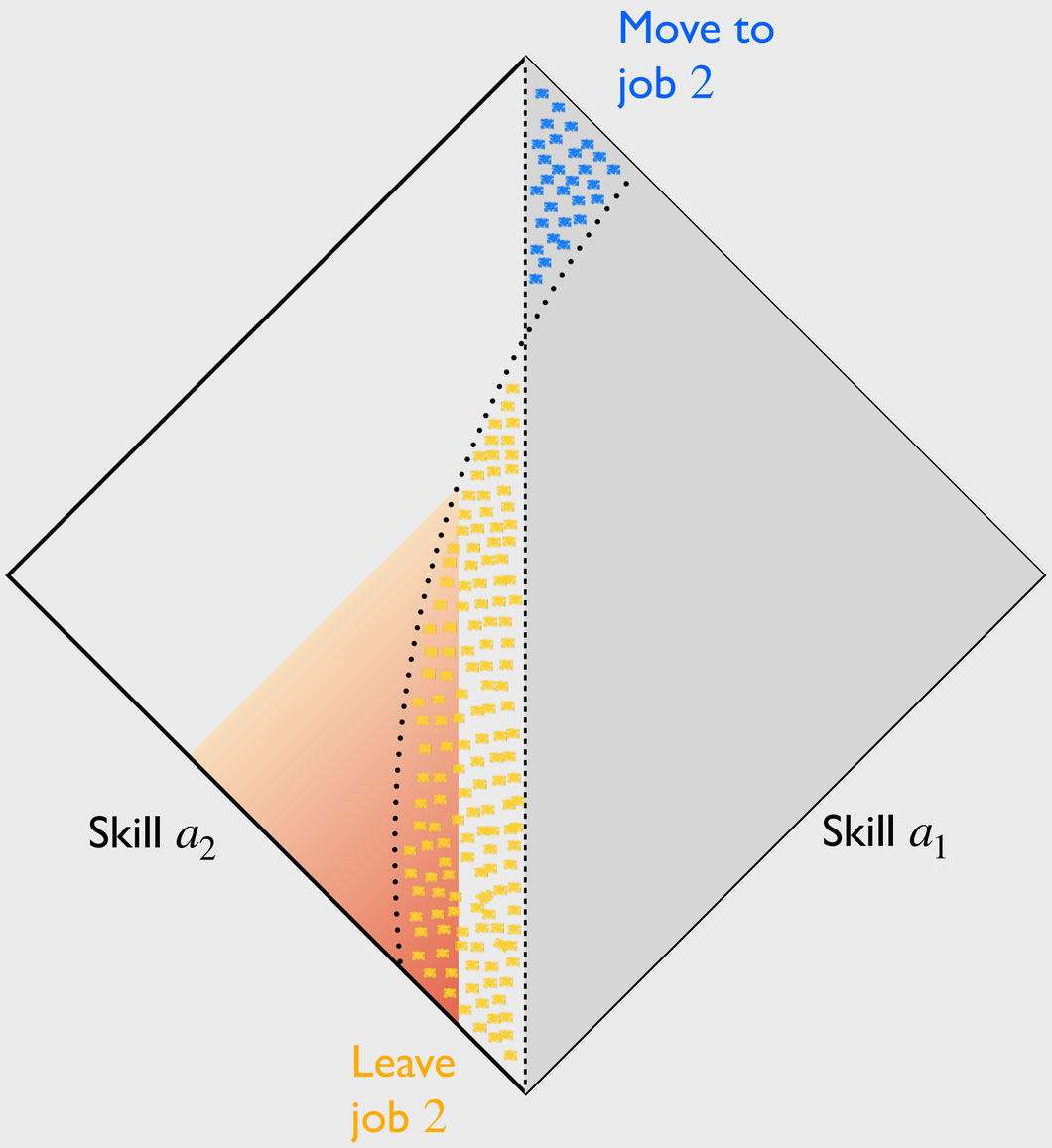
Proposition: Low skill marginal workers leave job j and high skill marginal workers move in.



General equilibrium effects of Peripheral Automation

- Suppose $\sigma \geq 1$ (ie job demand elastic)
- Assume technology adopted by *some but not all* workers in $\mathcal{S}_j$

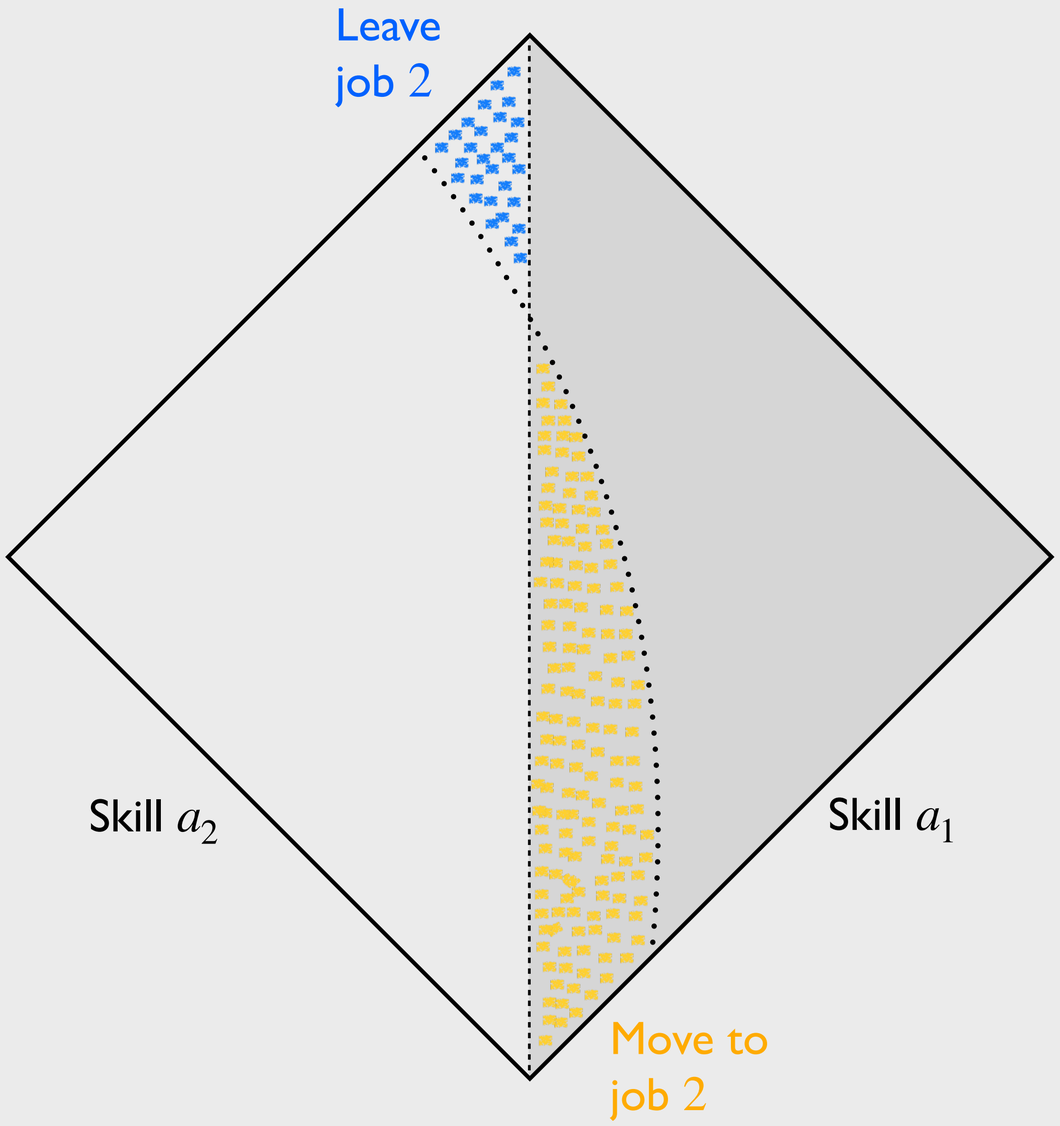
Proposition: Stayers (and marginal movers) with $a_j < \underline{a}_j$ see real wage decline. All other workers benefit.



General equilibrium effects of Core Automation

- Suppose $\sigma \geq 1$ (ie job demand elastic)
- Assume technology adopted by *some but not all* workers in $\mathcal{S}_j$

Proposition: High skill marginal workers leave job j and low skill marginal workers move in.



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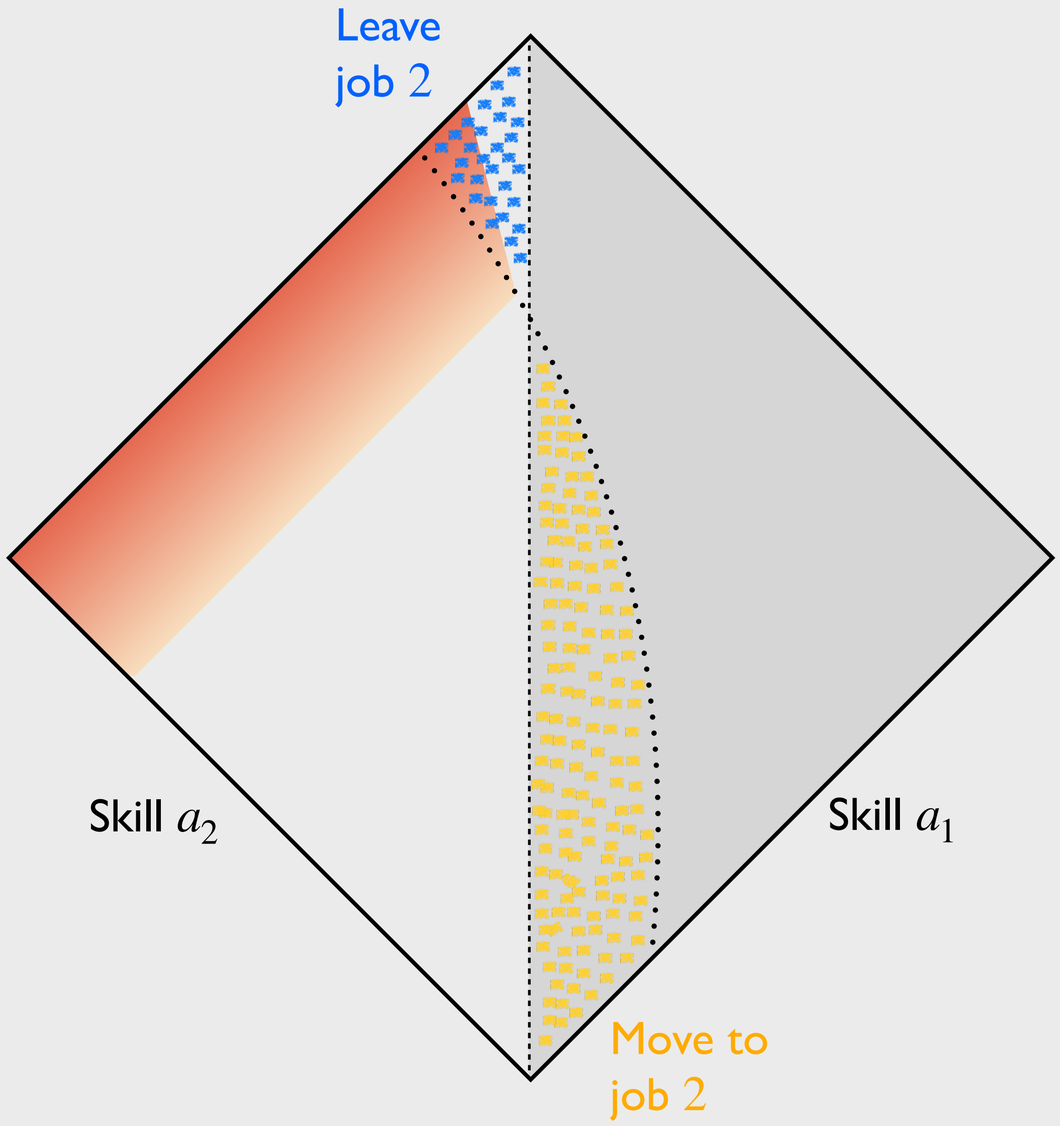
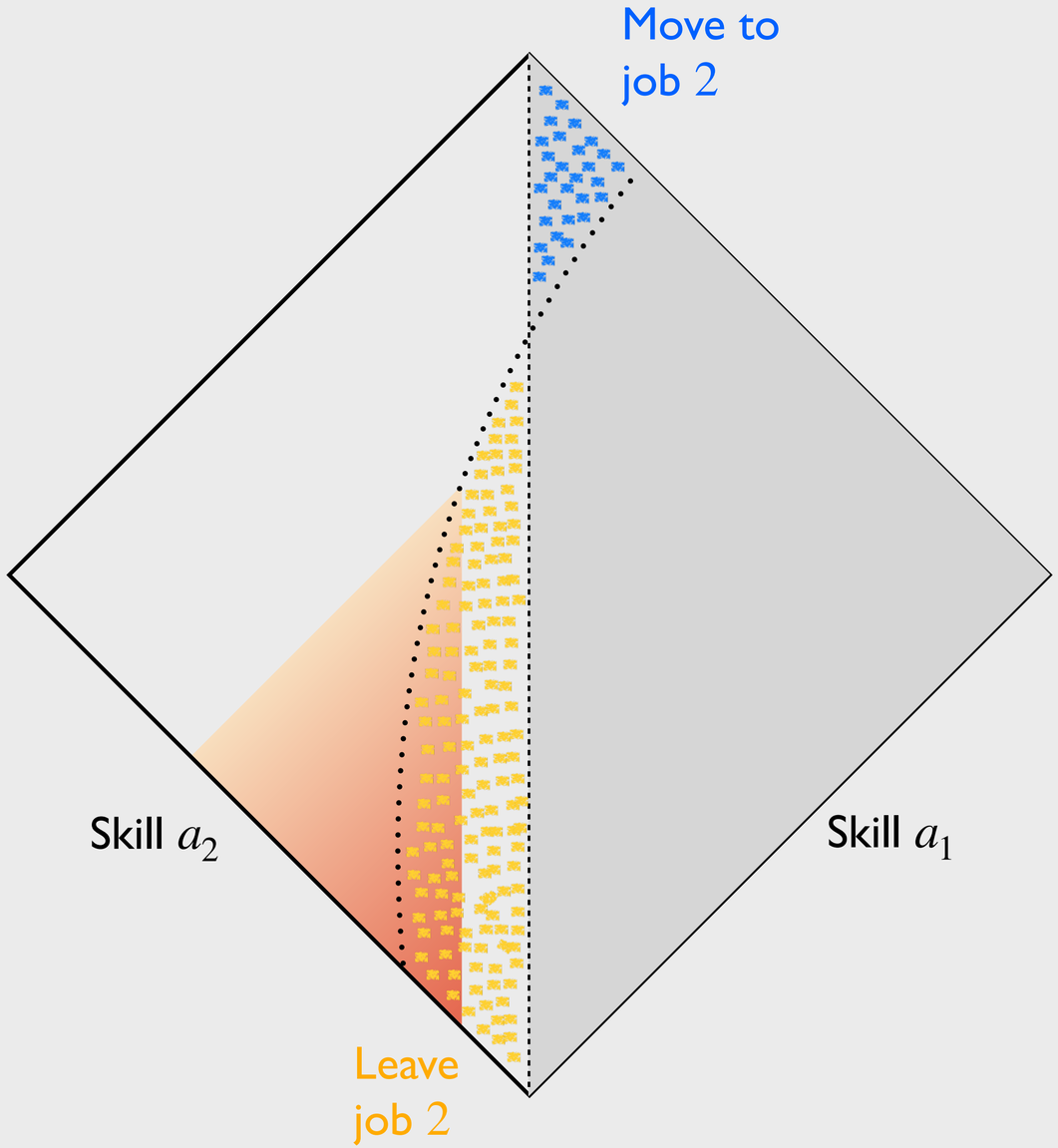
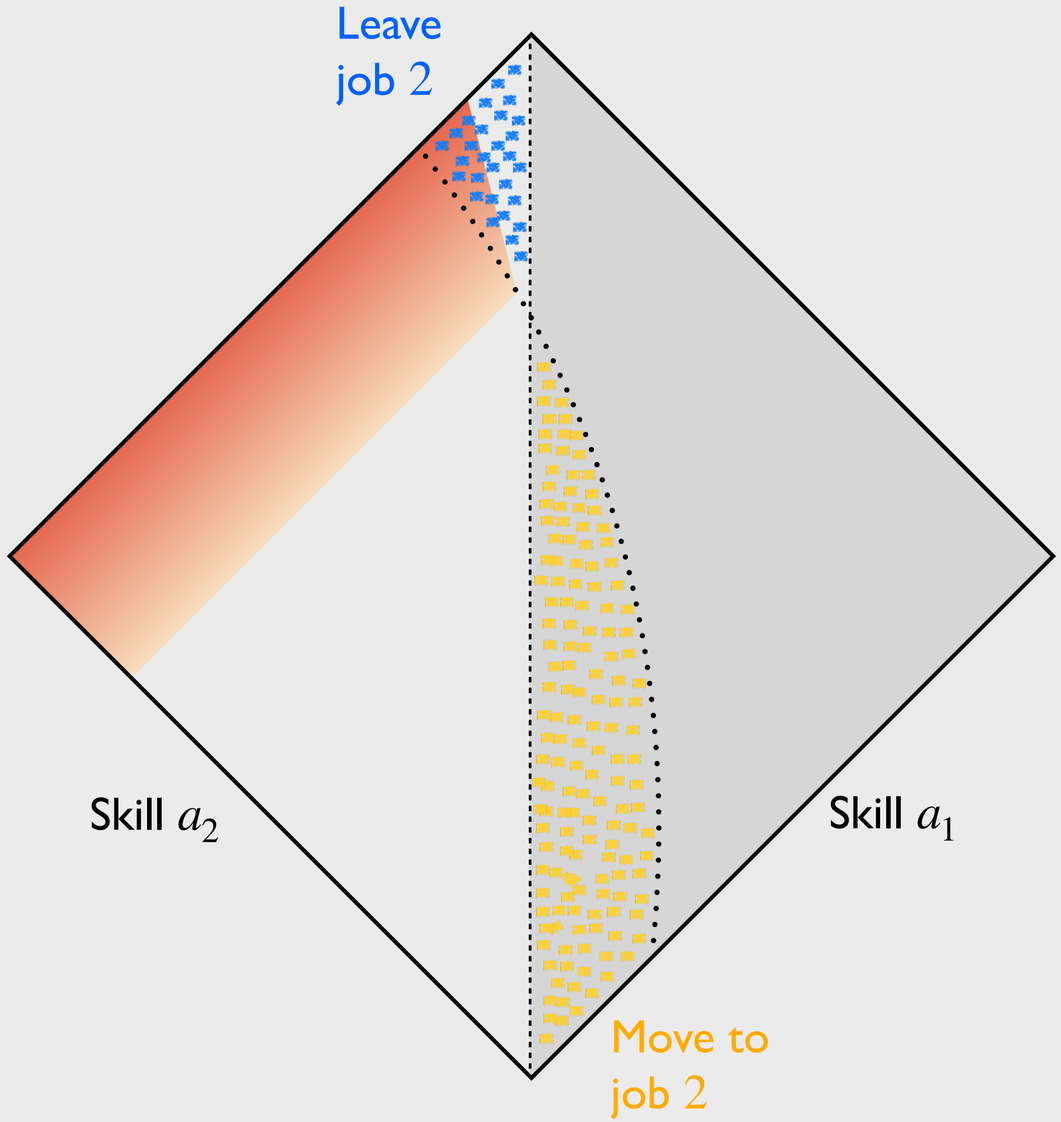


General equilibrium effects: average wages and employment

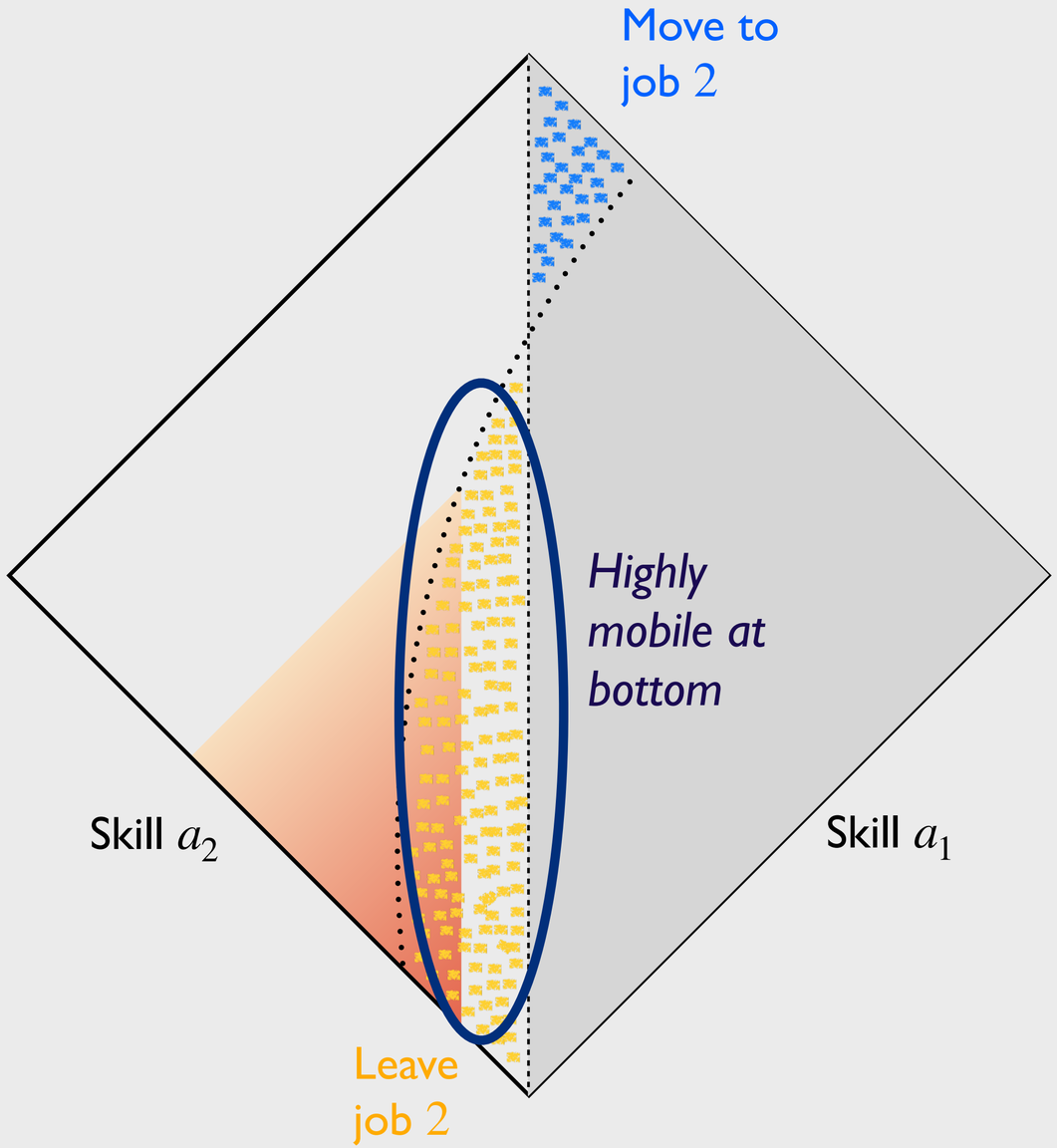


Peripheral Automation of Job 2

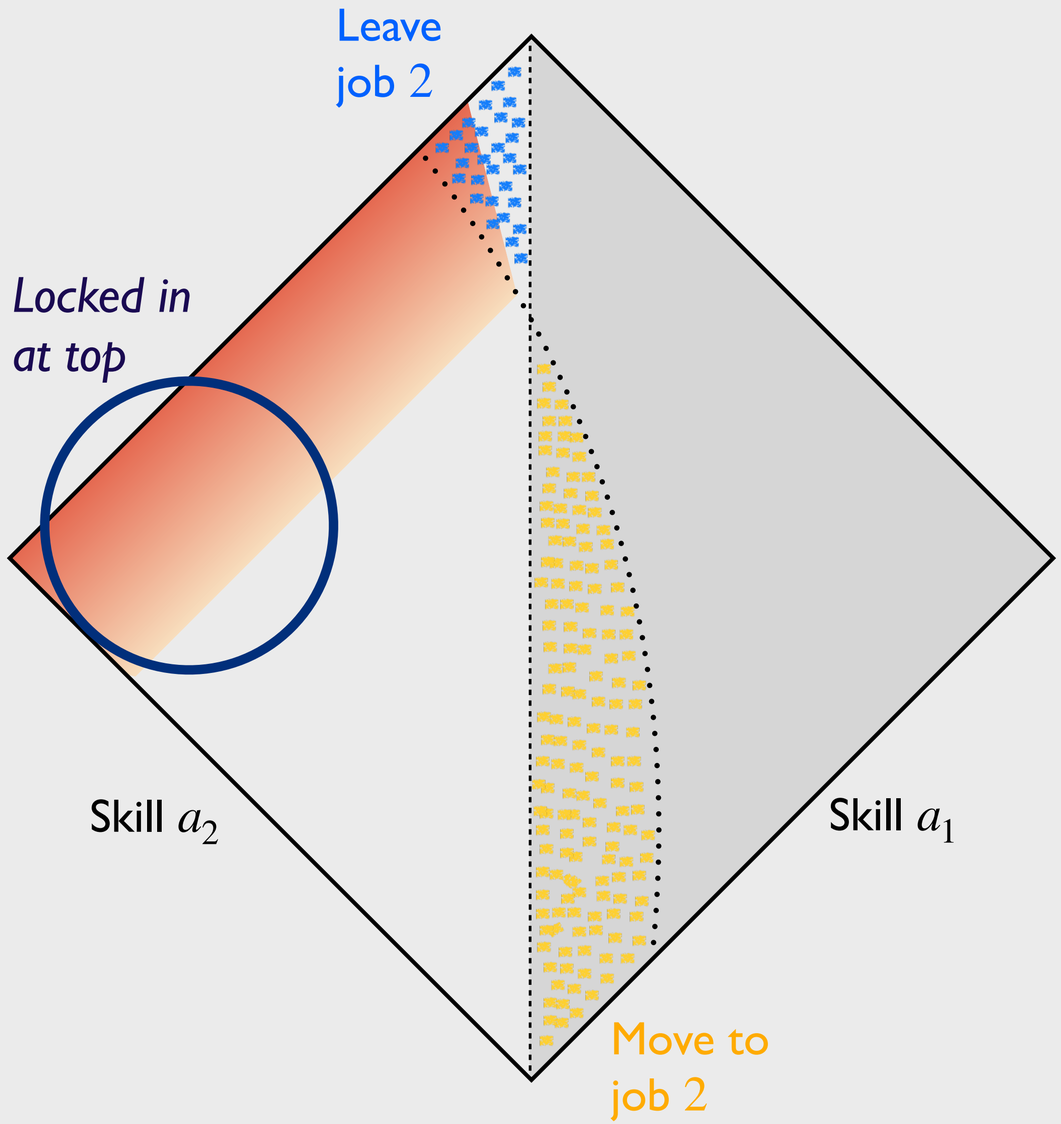


Core Automation of Job 2

General equilibrium effects: average wages and employment



Peripheral Automation of Job 2



Core Automation of Job 2

General equilibrium effects of Core | Peripheral Automation

- Approximate equilibrium responses using **Copulas** to model skill distribution such that:
  - High a level $\Rightarrow$ low correlation across a_js
  - Low a level $\Rightarrow$ high correlation across a_js
- Full analytical solution for **special case**:
  - Mass $M \in (0,1)$ of workers are *generalists* with $a_1 = a_2 = \ldots = a_J = \underline{a}$
  - Mass $\alpha_j\,(1 - M)$ are *specialists* with $a_{-j} = 0$ and CDF $a_j \sim F_j(a_j)$ with range $[\underline{a}, \infty)$
  - Assume M is large enough so that all jobs employ positive mass of generalists

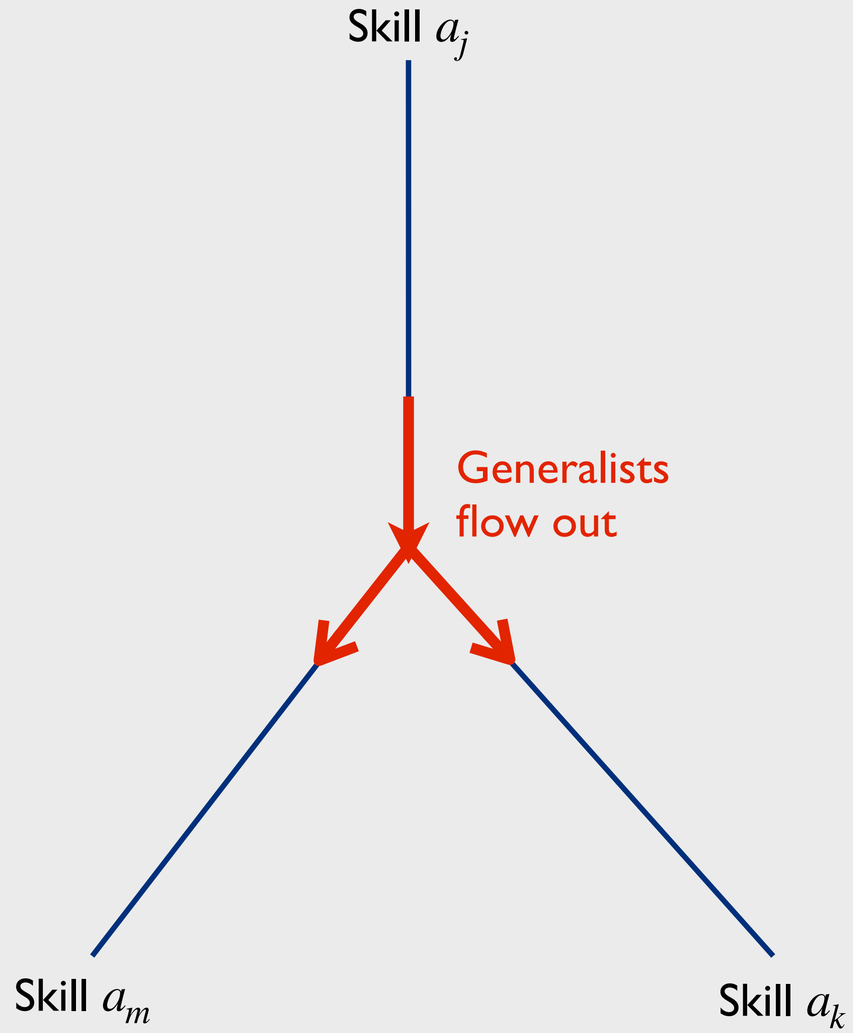
General equilibrium effects of Core | Peripheral Automation

Proposition: *Peripheral automation* in job j has the following GE effects:

- Employment in j contracts due to outflow of generalists
- The real wage of job j specialists *increases* by

$$\Pi_j(a) = \ln M_j^A(a) - \ln M_j(a) \geq 0$$

- The real wage of all other workers remains unchanged (adjustment via quantities)



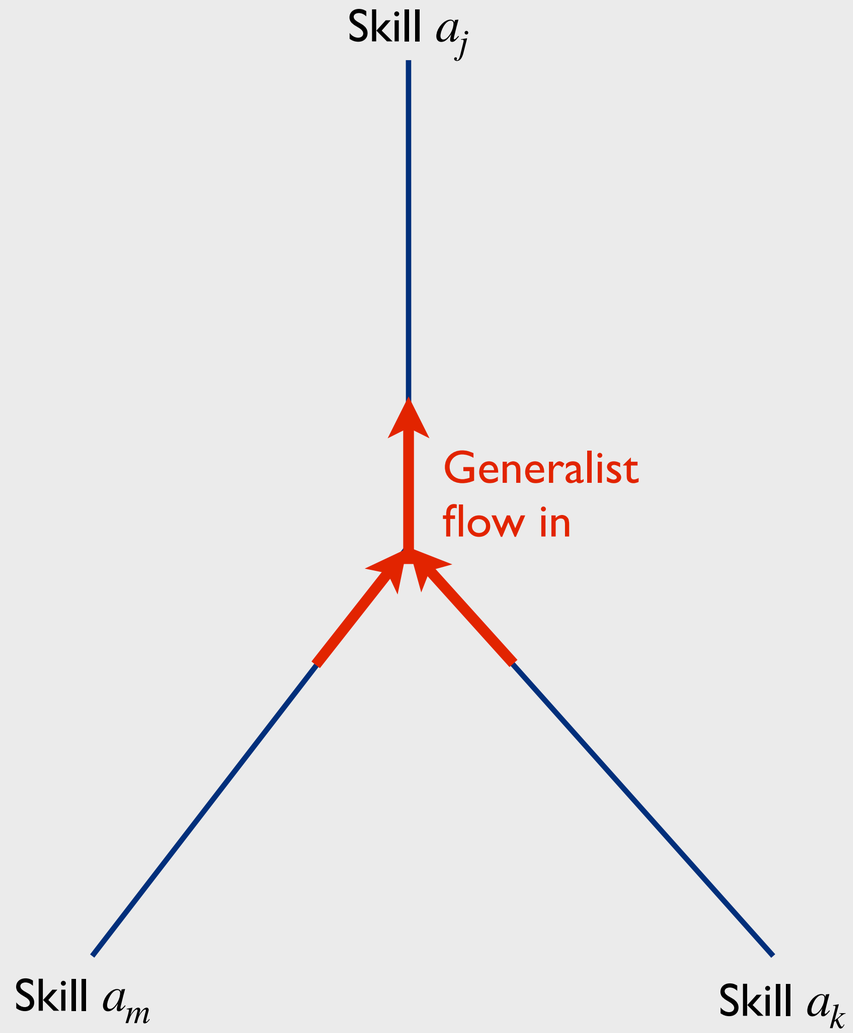
General equilibrium effects of Core | Peripheral Automation

Proposition: *Core automation* in job j has the following GE effects:

- Employment in j expands due to inflow of generalists
- Let $\Pi_j(a) = \ln M_j^A(a) - \ln M_j(a) \geq 0$. The real wage of job j specialists *changes* by

$$\Pi_j(a_j) - \Pi_j(\underline{a}) \; + \; s_j \, \Pi_j(\underline{a})$$

  and *decreases* at top
- The real wage of all other workers rises by $s_j \, \Pi_j(\underline{a})$



General equilibrium effects of Core | Peripheral Automation

- Assume correlation of skills summarized by Copula $C(u_1, \ldots, u_J)$
  - $\Pr(F_1(a_1) \leq u_1, \ldots, F_J(a_J) \leq u_J) \;=\; C(u_1, \ldots, u_J)$
  - "probability that for skill j one is among bottom u_j"
  - Define $M_j(u) \equiv$ net output of worker of skill $F(a_j) = u$
- Equilibrium conditions can be written as
  - $\sum_j \alpha_j \, P_j^{1-\sigma} \;=\; 1$
  - $\alpha_j \, Y \, P_j^{-\sigma} \;=\; \int_0^1 M_j(u) \; C_j\left(\left\{ u_k : P_k \, M_k(u_k) \leq P_j \, M_j(u_j) \right\}_{k \neq j}, u_j \right) du_j$

Probability that all my other skills are low enough, so that I select j given u_j

- Partial automation is a shift up in $M_j(u)$ of $\dfrac{\Delta M_j(u)}{M_j(u)} \;=\; \delta_j(u) \geq 0$

General equilibrium effects of Core | Peripheral Automation

Definition: Define the *bilateral mover elasticity* $\rho_{jk}(u)$ as the elasticity of $C_j\left(\left\{u_k : P_k\, M_k(u_k) \leq P_j\, M_j(u_j)\right\}_{k\neq j}, u_j\right)$ with respect to P_k at $u_j = u$.

Assumption: Bilateral mover elasticity $\rho_{jk}(u)$ decreases with u.

- In example, $\rho_{jk}(u) = \infty$ for $u \leq M$ (*generalists*) and $\rho_{jk}(u) = 0$ for $u > M$ (*specialists*)

General equilibrium effects of Core | Peripheral Automation

Proposition: Suppose $\sigma = 1$ (helpful benchmark). For small $\delta_j(u)$, we have:

- Change in employment in job j is

$$\hat{E}_j = \sum_{k \neq j} \gamma_{jk} \, \mathrm{Cov}_{q_j}(\rho_{jk}(u), \delta_j(u))$$

- Change in average real wage of incumbents is

$$\hat{W}_j^{inc} = s_j \, \Pi_j - \sum_{k \neq j} \gamma_{jk} \, \mathrm{Cov}_{q_j}(\rho_{jk}(u), \delta_j(u))$$

- Here, $\Pi_j = E_{q_j}[\delta_j(u)] > 0$ and $\gamma_{jk} \geq 0$ is elasticity of relative demand curve between j, k

General equilibrium effects of Core | Peripheral Automation

Proposition: Suppose $\sigma = 1$ (helpful benchmark). For small $\delta_j(u)$, we have:

- Change in employment in job j is

$$\hat{E}_j = \sum_{k \neq j} \gamma_{jk} \, \mathrm{Cov}_{q_j}(\rho_{jk}(u), \delta_j(u))$$

Covariance **negative** for peripheral

- Change in average real wage of incumbents is

$$\hat{W}_j^{inc} = s_j \, \Pi_j - \sum_{k \neq j} \gamma_{jk} \, \mathrm{Cov}_{q_j}(\rho_{jk}(u), \delta_j(u))$$

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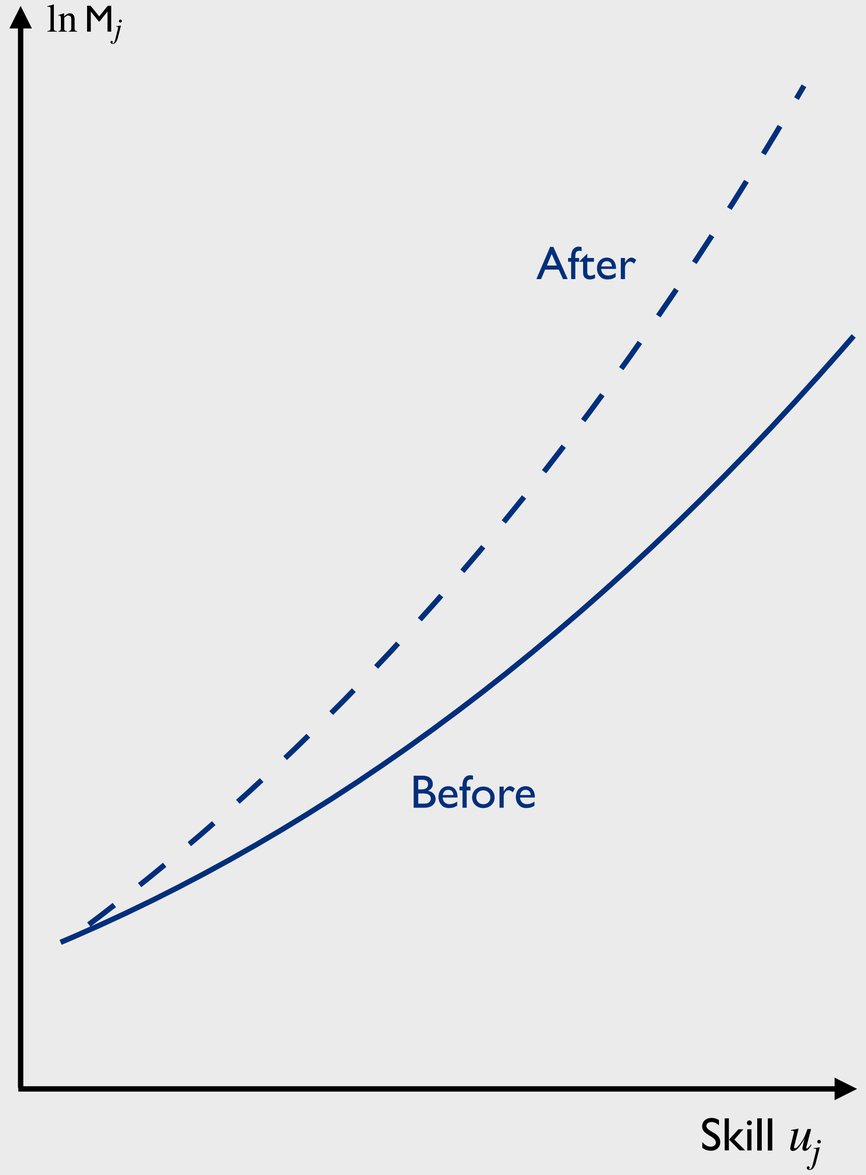

General equilibrium effects of Core | Peripheral Automation

Proposition: Suppose $\sigma = 1$ (helpful benchmark). For small $\delta_j(u)$, we have:

- Change in employment in job j is

$$\hat{E}_j = \sum_{k \neq j} \gamma_{jk} \, \mathrm{Cov}_{q_j}(\rho_{jk}(u), \delta_j(u))$$

Covariance **positive** for core

- Change in average real wage of incumbents is

$$\hat{W}_j^{inc} = s_j \, \Pi_j - \sum_{k \neq j} \gamma_{jk} \, \mathrm{Cov}_{q_j}(\rho_{jk}(u), \delta_j(u))$$

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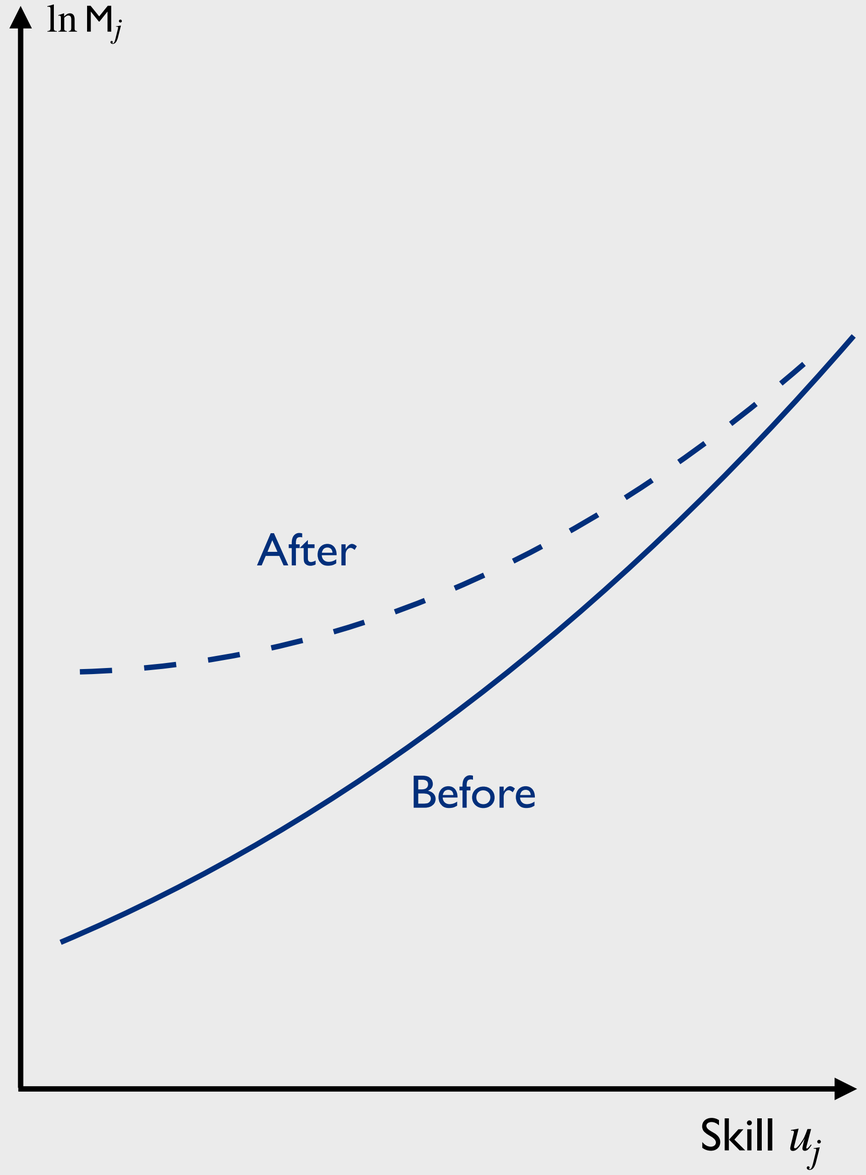


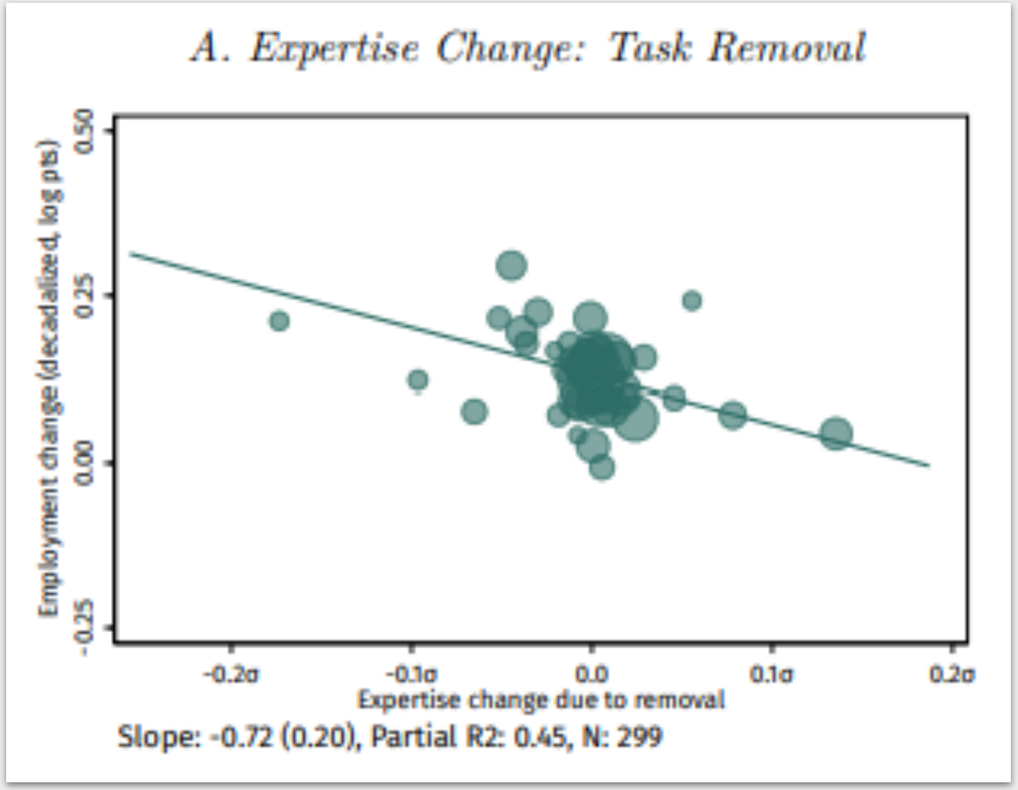
General equilibrium effects of Core | Peripheral Automation

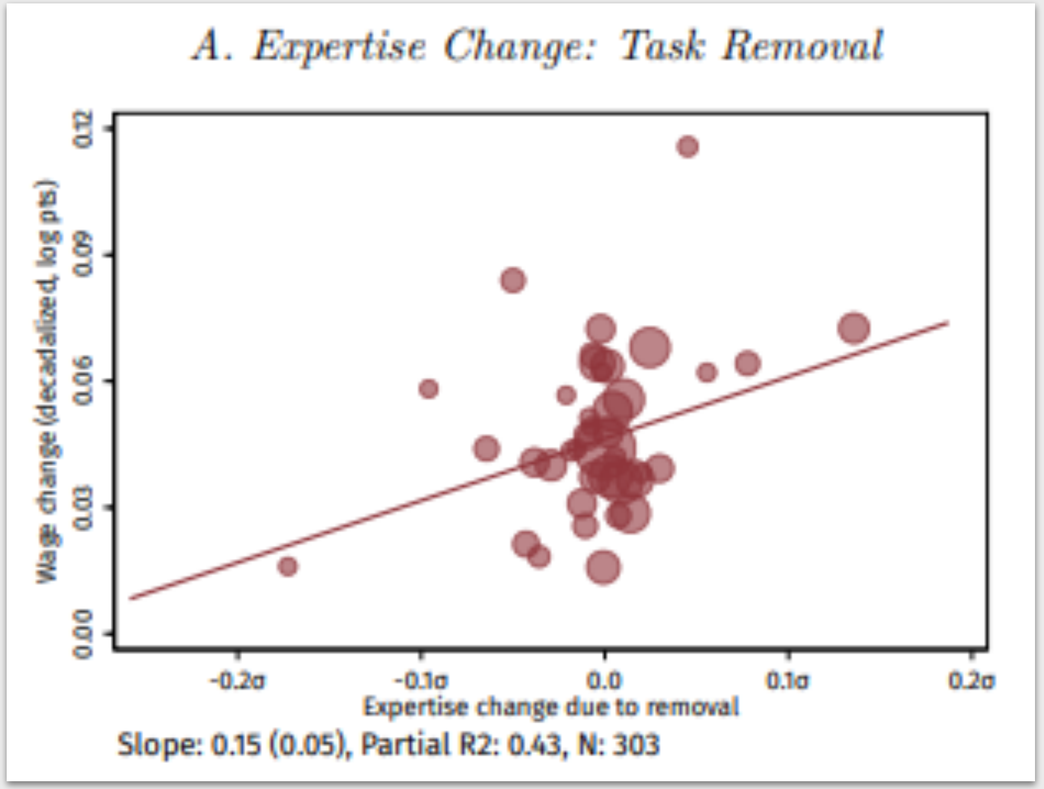
Summary

- Core and peripheral automation have different implications.
  - Core automation reduces real wages of highly skilled incumbents due to competition from generalists.
  - Peripheral automation increases real wages of highly skilled incumbents due to reduced competition from generalists
  - Core automation can bring real wage losses for specialists, since they are locked in. Possibility limited for peripheral, since generalists highly mobile.
  - Core automation expands employment, peripheral contracts it
  - Different implications for between occupation and within occupation inequality
- Aligned with evidence in *Autor-Thompson* and *Eisfeldt-Schubert-Taska-Zhang* (for hiring).

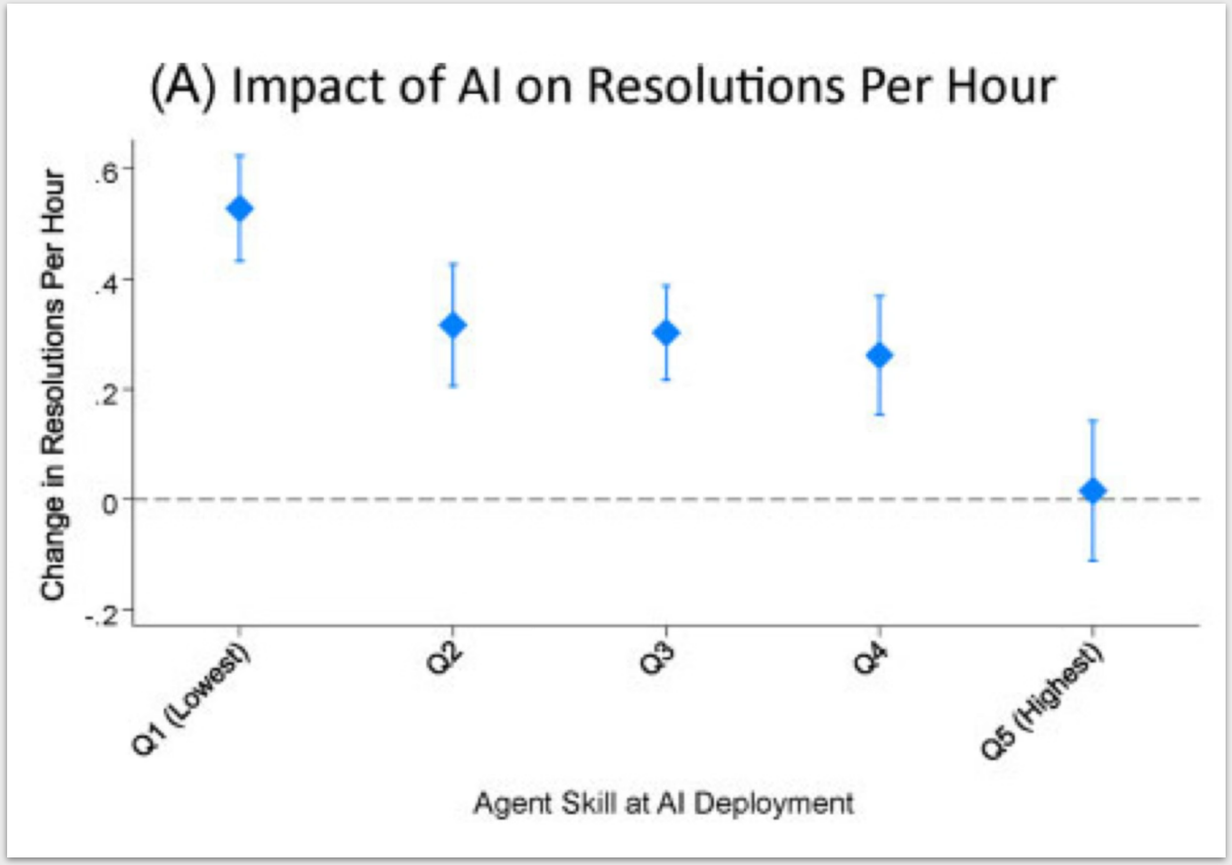
Evidence from Autor-Thompson

- Occupations where "non-expert" tasks removed see rising wages and decreasing employment

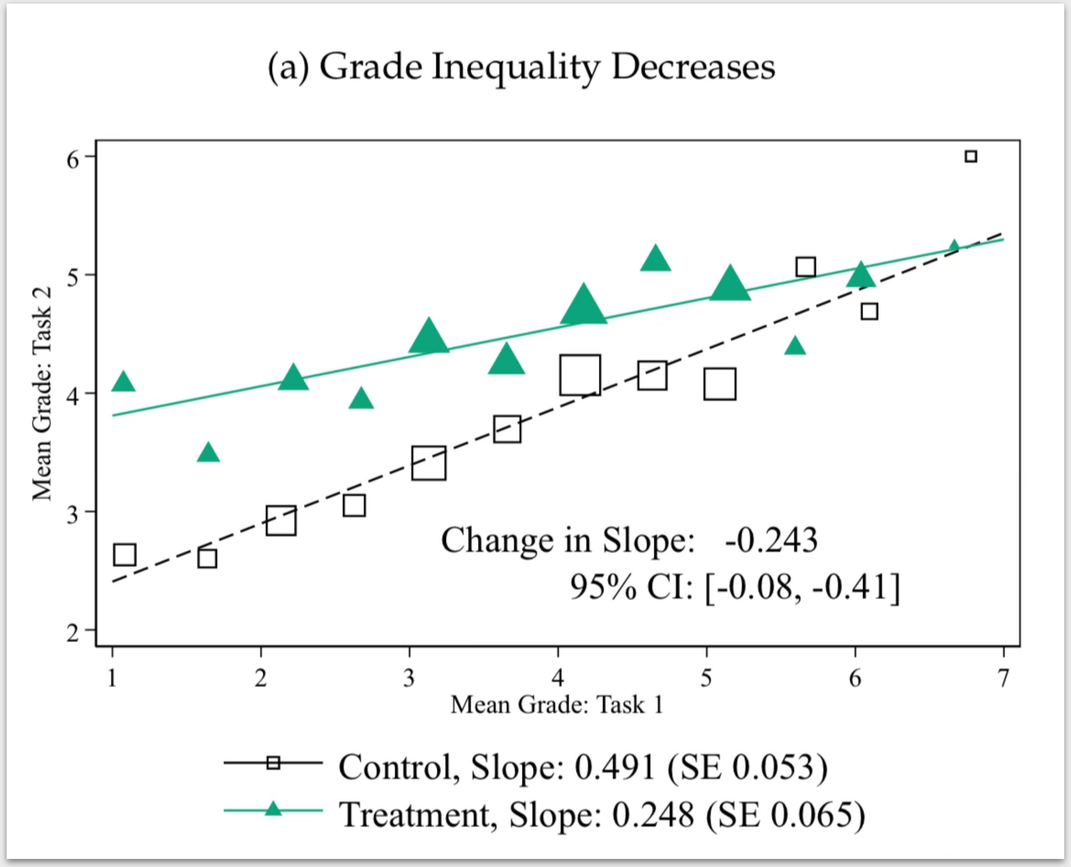




Are LLMs Automating Core or Peripheral Components?



LLM tool used for customer service (Brynjolfsson et al. 2025)



LLM tool for writing (Noy and Zhang 2023)