Credibility and Policy Convergence in a Two-Party System with Rational Voters

By Alberto Alesina*

The traditional approach to modeling political parties' behavior, based upon the contribution of Anthony Downs (1957), assumes that the parties' unique objective is to win elections: thus, they maximize their popularity. The crucial implication of this assumption for a two-party system is that if the two parties have the same information about voters' preferences, full convergence of policies results from electoral competition. This is the crucial implication of the "median voter theorem."[1]

More generally, it may be argued that different parties are motivated differently because they represent different constituencies. Parties may not care only about winning elections per se, but also about the quality of the policies resulting from an election. In this case the candidates of the two parties view winning an election not only as a goal per se, but also as a means of implementing a better policy for their respective constituencies.

This paper shows that electoral competitions imply dynamic inconsistency if the voters are modeled as rational and forward-looking agents and parties do not care exclusively about being elected, but also about which policy to implement, once elected. The dynamic inconsistency arises as follows: the parties have an incentive to announce convergent platforms to increase their chances of election. However, if the elected party is not committed to its electoral platform, it has an incentive to follow its most preferred policy rather than the policy announced in its platform. If voters are rational, they account for this incentive. Thus, in general, in a one-shot electoral game the only time-consistent equilibrium is one in which no convergence is possible, the two parties follow their most preferred policies, and the voters rationally expect this outcome. Full convergence of parties' platforms results only as a limiting case when the parties are completely indifferent with respect to the quality of the policies resulting from the election. Thus, these results differ from the existing literature on "ideologically motivated" politicians (for instance, Donald Wittman, 1977, 1983; Randall Calvert, 1985), which implicitly assumes the possibility of binding commitments to electoral platforms.

Complete or partial policy convergence can be the outcome of political competition if the interaction between the parties and the voters is modeled as an infinitely repeated game. In fact, if the candidates have concave objective functions, the welfare-maximizing policy rule implies a complete convergence of parties' policies. However, this cooperative, and agreed-upon policy, may or may not be sustainable as a subgame-perfect equilibrium depending on parameter values; in particular it depends on the discount rates of the two parties, the degree of polarization of their preferences, and the relative popu-

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[1]The result of policy convergence in a two-party system is more general than the "median voter theorem." For discussions of convergence results not at the median, see John Ledyard, 1984; Peter Coughlin, 1984; Coughlin and Shmuel Nitzan, 1981; Melvin Hinich, 1977. For earlier work on spatial competition see Richard McKelvey, 1975; Hinich, Ledyard, and Peter Ordeshook, 1972, 1973, and the references quoted therein. The present paper focuses on the result of convergence rather than on the "median voter theorem" per se.

larity of the two parties. For the case in which the first-best policy is not sustainable, the set of credible policies are also characterized as a function of the same variables.

Thus, this paper shows how the repeated interaction of political parties can reduce the magnitude of policy fluctuations. This smoothing effect can be particularly beneficial if excess volatility, for example in economic policy, is harmful for society as a whole. An example related to monetary policy is developed in Alberto Alesina, 1987, and empirically tested in Alesina and Jeffrey Sachs, 1988, and Alesina, 1988.

I. The Model

Let us consider a political system with two parties, denoted party 1 and party 2. No distinction is made between "candidates" and "parties": a candidate is completely identified by the objective function of his party.[2] The parties are not concerned solely about winning elections but have also preferences defined on policy outcomes, since they represent constituencies with different preferences.

Party 1's objectives are represented by the following unidimensional function

$$(1) \qquad U(l)=\sum_{t=0}^{\infty} q^t u(l) = -\sum_{t=0}^{\infty} \tfrac{1}{2} q^t (l_t - c)^2; \qquad c>0 \qquad 0<q<1.$$

Party 2's objective function is the following:

$$(2) \qquad V(l)=\sum_{t=0}^{\infty} q^t v(l) = -\sum_{t=0}^{\infty} \tfrac{1}{2} q^t l_t^2.$$

[2]Alesina and Stephen Spear, 1987, analyze the relationship between short-lived candidates and the party as a long-lived organization.

The two parties have different bliss points on the policy variable l, which is directly controlled by the party in office. Party 1's bliss point is c and, without loss of generality, party 2's bliss point is normalized at zero. The quadratic specification for (1) and (2) is adopted purely for simplicity; all the results of the paper generalize to any single-peaked, concave objective function. The discount factor (q) is identical for the two parties. Each party attributes a positive utility, denoted by k, to being in office per se, in addition to their objective function defined on l.

The voters have preferences defined on l and they are rational, forward looking, and informed about the objective functions of the two parties (i.e., they know the objective functions (1), (2), and k). Thus, the voting decisions are based upon the rational expectations of the policies that the two parties would follow if elected. Let us indicate these policies with x for party 1 and with y for party 2: if party 1 is elected at time t, it sets $l_t = x_t$, and, if party 2 is elected, it sets $l_t = y_t$. Thus x_t^e and y_t^e are the rational expectations formed before elections of party 1's and 2's policies.

Elections are held at the beginning of the period, say period t. At the end of period $(t-1)$ the parties announce their policies for period t, that is, x_t^a, y_t^a. If the announcements are believed by voters, $x_t^e = x_t^a$ and $y_t^e = y_t^a$; otherwise, $x_t^e = E(x_t/I_{t-1}) \neq x_t^a$ and $y_t^e = E(y_t/I_{t-1}) \neq y_t^a$. $E(\cdot)$ is the mathematical expectation operator and I_{t-1} denotes the information set available to the voters at the end of period $t-1$. Elections take place every period. (Extensions on this point are straightforward.)

The electoral outcomes are uncertain; for any given x_t^e and y_t^e there is an associated probability of party 1's victory, which is indicated by the following function

$$(3) \qquad P_t = P(x_t^e, y_t^e).$$

This function can be derived as follows. Each voter votes for the party which is expected to deliver the policy closest to his own bliss point. However, there is uncertainty about the distribution of voter's bliss points, and in particular, about the position of the bliss point of the median voter. There may also be uncertainty about the number of abstentions, because of uncertainty about the costs of voting as perceived by different voters.[3]

The following assumptions on the function $P_t = P(x_t^e, y_t^e)$ are adopted:

(i) The function $P(x_t^e, y_t^e)$ is time invariant and "common knowledge";

(ii) $0 < P(\cdot) < 1$ for $x_t^e \varepsilon R$; $y_t^e \varepsilon R$;

(iii) $P(x_t^e, y_t^e)$ is continuous and differentiable at least twice;

(iv)

$$\frac{\partial P(x_t^e, y_t^e)}{\partial x_t^e} \equiv P_{x_t^e} \leq (>) 0$$

$$\text{if and only if } x_t^e \geq (<) y_t^e$$

$$\frac{\partial P(x_t^e, y_t^e)}{\partial y_t^e} \equiv P_{y_t^e} \leq (>) 0$$

$$\text{if and only if } x_t^e \geq (<) y_t^e.$$

Assumption (iv) states that if one party converges toward the other, it increases its chances of election by capturing (probabilistically) "middle voters." Note that this assumption is restrictive because it implies a prohibitive barrier to the entry of a third party for any policy followed by the two existing parties. Assumption (iii) is more restrictive than necessary and is adopted for simplicity. In particular, the function $P(\cdot)$ could be discontinuous along the diagonal, that is, for $x_t^e = y_t^e$. All the results of this paper generalize to this case, as shown in Alesina and Alex Cukierman (1987), in which a function such as (3) is explicitly derived from the underlying distribution of preferences of the voters.

II. Electoral Competition as a One-Shot Game

Consider one electoral competition, in isolation. Two cases should be distinguished: 1) the two parties can make binding precommitments to their preelectoral platforms; 2) no precommitments are possible.

The first case has been analyzed by Wittman (1983) and Calvert (1985). With precommitments we can impose $x_t^a = x_t^e = x_t$ and $y_t^a = y_t^e = y_t$; the announced platforms are identical to the policies expected by the voters and implemented by the elected party. The Nash equilibrium of this game can then be found by solving the following problem (the time subscripts are dropped to simplify notation):

$$(4) \quad \max_x w^1 = P(x, y)[u(x) + k] + [1 - P(x, y)]u(y),$$

$$(5) \quad \max_y w^2 = P(x, y)v(x) + [1 - P(x, y)][v(y) + k].$$

Following Wittman (1983) and Calvert (1985) it can be shown that the equilibrium policies ($\hat{x}$ and $\hat{y}$) satisfy the following inequalities

$$(6) \qquad c > \hat{x} > \hat{y} > 0.$$

It can also be shown that the distance between $\hat{x}$ and $\hat{y}$ is inversely related to k.[4] Calvert (1985), however, stresses that the equilibrium policies are in general very close; thus even with ideologically motivated politicians one should observe a high degree of policy convergence.

[3]John Ledyard, 1984, shows how to derive a function analogous to (3) from the underlying uncertainty about the costs of voting which generates an uncertain turnout. An analogous representation of electoral uncertainty can be found in Wittman (1983), and Calvert (1985).

[4]A complete proof of these results is available from the author. A sufficient condition for existence and uniqueness of this equilibrium is that the function $P(x, y)$ is concave in x and convex in y.

Consider now the case in which precommitments are not available. This is the most natural assumption because the policymaker can always change his mind and the law, when in office. Thus when, say, the candidate of party 1 is elected he does not face problem (4) any more, but he can follow his most preferred policy. It is immediate to prove the following result:

PROPOSITION 1: *For any value of k (the utility of winning an election), the time-consistent equilibrium* $(\bar{x}, \bar{y})$ *exists, is unique and it is given by*

$$\bar{x} = c; \qquad \bar{y} = 0. \tag{7}$$

Voters anticipate the behavior of the elected party: thus $x^e = c$ and $y^e = 0$. Then, party 1 is elected with probability $P(c,0)$; for notational simplicity $P(c,0)$ will be referred to as $\bar{P}$ throughout the rest of this paper.

Proposition 1 shows that in a one-shot game without precommitments, there cannot be any policy convergence in equilibrium, regardless of the value of k. This result implies that the convergent equilibria described in the literature are "time inconsistent," or, to put it differently, that they rely upon the assumption of binding precommitments to policy announcements.[5]

III. Electoral Competition as an Infinitely Repeated Game

The infinite repetition of the game allows the two parties to reach better equilibria than the one-shot Nash with no convergence described in Proposition 1, even without binding precommitments.

Let us consider the first-best outcome. For the case of $k = 0$, the efficient frontier of the game can be obtained by solving the following problem (where $0 < \lambda < 1$)[6]

$$\max_{x,y} \lambda\left[P(x,y)u(x) + (1-P(x,y))u(y)\right] + (1-\lambda)\left[P(x,y)v(x) + (1-P(x,y))v(y)\right]. \tag{8}$$

Note that in (8) we imposed $x^e = x$ and $y^e = y$. The solution of this problem yields the following result:

PROPOSITION 2: *The efficient frontier is given by the following policies*:

$$x^* = y^* = \lambda c.$$

The parameter λ represents the weight of party 1. The relevant section of the efficient frontier is the segment in which both parties are better off than in the one-shot Nash equilibrium. This part of the frontier is characterized by the following inequalities, labeled conditions of "individual rationality"

$$1 - \sqrt{1-\bar{P}} \le \lambda \le \sqrt{\bar{P}}. \tag{9}$$

Thus, full convergence of policies is the first-best outcome. This result is due to the concavity of the utility function of the two parties: An appropriate intermediate policy followed by both parties and, therefore, obtained with certainty makes both parties better off than the expected value of the two possible outcomes of Proposition 1 weighted by the probabilities of electoral outcomes.

Naturally, the two parties are not indifferent with respect to the choice of λ, which corresponds to a particular convergent policy. The closer λ is to one, for example, the

[5] In this model the equilibrium of Proposition 1 remains the unique time-consistent equilibrium if this game is finitely repeated. Partial convergence in a finitely repeated game is sustainable with asymmetric information (see Alesina-Cukierman, 1987) or with multiple one-shot Nash equilibria and strategic voting; see, however, fn. 9 on strategic voting. See also Jean Pierre Benoit and Vijay Khrishna (1985) on the possibility of cooperation in finitely repeated games with perfect information.

[6] These results generalize to any value of k. If $k \neq 0$ the two parties would still solve problem (8) to find the "first-best" policies. Then they would split k, namely the benefits of being in office: for example, they would agree to share the term of office. For a formal treatment of this particular problem see Alesina-Spear, 1987.

closer is the chosen policy to the bliss point of party 1 and vice versa; thus, the higher is λ the higher the welfare of party 1. The most standard way of selecting a point on the efficient frontier is the Nash-bargaining solution. To obtain this solution we consider the one-shot Nash as the "disagreement point": in fact, the one-shot Nash can be used as a threat that the parties can make against each other at the "bargaining table."[7] This Nash-bargaining solution implies a choice of λ (say λ^*). The following result characterizes λ^*:

PROPOSITION 3: *If λ^* corresponds to the Nash-bargaining solution, the following holds*:

(*i*) λ^* is a function of $\bar{P}$ and only if $\bar{P}$: $\lambda^*(\bar{P})$; (*ii*) $\partial\lambda^*(\bar{P})/\partial\bar{P} > 0$; (*iii*) $\lambda^*(\frac{1}{2}) = \frac{1}{2}$; (*iv*) $\lambda^*(0) = 0$, $\lambda^*(1) = 1$.

This result is quite intuitive. An increase in $\bar{P}$ makes party 1 stronger at the bargaining table. In fact if $\bar{P}$ is higher party 1 knows that if an agreement is not reached it is more likely to be elected at the "disagreement point"; thus, this party would be better off. It follows that party 1 can impose an agreement closer to its point of view and its utility is increased. Thus, an exogenous increase of the popularity of, say, party 1, implying that $\bar{P}$ becomes higher, makes this party better off in the cooperative regime. It follows that even in a cooperative regime the two parties are not indifferent to their popularity.

An additional implication of Proposition 3 is that, for given preferences of the voters, if a party becomes more radical (in the sense that its bliss point moves away from that of the other party), its bargaining power is reduced, and the Nash-bargaining solution is further away from its most preferred policy. Suppose, for example, that c increases; then, given Proposition 3 and Assumption (*iv*) on the function $P(\cdot)$, one obtains

$$(10)\qquad \frac{\partial\lambda^*(\bar{P})}{\partial c} = \frac{\partial\lambda^*(\bar{P})}{\partial\bar{P}}\frac{\partial\bar{P}}{\partial c} < 0.$$

Note that the qualitative features of Proposition (3) and of (10) are more general than the Nash-bargaining solution. In fact these features derive from the fact that a change in $\bar{P}$ affects the welfare of the two parties at the disagreement point. Any solution concept used to select a point on the efficient frontier is sensitive to such a change (qualitatively) as the Nash-bargaining solution. This is the case, for example, of the solution proposed by Ehud Kalai and Meir Smorodinsky, 1975.

Finally, it should be emphasized that the convergent electoral equilibrium $x^* = y^* = \lambda^* c$ is, in general, different from the equilibrium that would emerge as the one-shot Nash equilibrium of a game between two parties maximizing popularity.

IV. Sustainability of the Efficient Frontier and Credibility

After the election, the party in office has an incentive to follow its most preferred policy, instead of the cooperative policy on the efficient frontier. If binding commitments are unavailable, the cooperative policy is dynamically inconsistent. However, this policy may become credible if reputational considerations make a deviation from it costly enough.

[7] The "disagreement point" $(\bar{w}_1, \bar{w}_2)$ of this problem is

$$\bar{w}^1 = \bar{P}u(c) + (1-\bar{P})u(0) = -\tfrac{1}{2}(1-\bar{P})c^2, \qquad \text{(party 1)}$$

$$\bar{w}^2 = \bar{P}v(c) + (1-\bar{P})v(0) = -\tfrac{1}{2}\bar{P}c^2. \qquad \text{(party 2)}$$

Thus the Nash-bargaining solution can be found by solving

$$\max_{w^1, w^2}\left[w^1 + \tfrac{1}{2}(1-\bar{P})c^2\right]\left[w^2 + \tfrac{1}{2}\bar{P}c^2\right],$$

such that $w^1 + \frac{1}{2}(1-\bar{P})c^2 \geq 0$; $w^2 + \frac{1}{2}\bar{P}c^2 \geq 0$; $w^1 = w^2 + c\sqrt{-2w^2} - \frac{1}{2}c^2$.

The last constraint is the "efficient frontier." The details of the solution are available from the author.

It is assumed that the game between the two parties is played as in Friedman, 1971. The parties "announce" a policy rule before elections. If the elected party deviates from that policy, it loses reputation and it will be expected by everybody to follow in the future its most preferred and noncooperative policy, that is, its bliss point. The other party will not cooperate anymore, since cooperation is broken. The system, therefore, reverts to the one-shot Nash. This reversion constitutes the cost of a deviation. With no loss of generality we shall assume that the reversion to the one-shot Nash lasts forever.[8]

The voters are rational and informed; therefore, they will rationally expect a reversion to the noncooperative Nash equilibrium if they observe a deviation from cooperation and vote accordingly. Thus, voters are fully rational in the sense that they use all the available information (including their knowledge of the parties' strategies) to form their forecasts. However, they are not strategic, in the sense that they do not adopt retrospective strategies, for instance, to punish a cheating party.[9]

An equilibrium is credible, therefore subgame-perfect, if the incentive to deviate from it, the "temptation," is not greater than the cost of a deviation, the "enforcement." A well-known result in the repeated game literature establishes that, if the discount factor is sufficiently close to one, every individually rational point on the efficient frontier can be sustained as a subgame-perfect equilibrium. Thus, sufficiently farsighted parties could always achieve the first-best policies on the efficient frontier.

However, there are many reasons why the discount factor might be quite low in this context. For example, one party may believe that with some probability the leadership of the opponent party may change in the future and that the new leadership may not carry over the threats of the old one. This type of uncertainty has the same effect of a low discount factor. It is easy to show by computing "temptation" and "enforcement" that the equilibrium $x^* = y^* = \lambda^*(\bar{P})c$ is sustainable as a subgame-perfect equilibrium if and only if

$$(11a) \qquad (1-\lambda^*)^2 \leq q(1-\bar{P}),$$

$$(11b) \qquad \lambda^{*2} \leq q\bar{P}.$$

We will next show that Proposition 3 and (11) imply that cooperation is more easily sustainable in a balanced system, that is, in a system in which $\bar{P}$ is close to $1/2$. In fact, if $\bar{P}$ is far from $1/2$, one of the two parties has little bargaining power and the policy which is selected is far from its bliss point. If this party is elected, the incentive to deviate is particularly strong. Given that this incentive is "common knowledge," it follows that cooperation is less easily sustainable. As a corollary, note that if one party adopts more extreme views, for given voters' preferences cooperation becomes more difficult (this follows immediately from (10)).

The qualitative features of these results can be illustrated using the following approximation for the Nash-bargaining solution:

$$(12) \qquad \lambda^*(\bar{P}) = \bar{P}.$$

By substituting (12) into (11) it follows that the policies $x^* = y^* = \lambda^* c$ are sustainable as a subgame-perfect equilibrium if and only if the following holds,

$$(13) \qquad q \geq \bar{P}; \qquad q \geq (1-\bar{P}).$$

[8] With no qualitative changes in the results the model could be solved for an arbitrary length of reversion to noncooperation. The length of the punishment period is completely isomorphic to the discount factor. See Friedman, 1971, on these issues. Dilip Abreu, 1983, has shown that worse "punishments" than the reversion to the one-shot Nash can be credibly threatened by the players of this type of game. However, these more general punishment strategies are not considered in this paper.

[9] In order to sustain credible equilibria by means of strategic voting, one needs to assume a high degree of coordination of strategies. In fact, an atomistic voter knows that his retrospective strategy works only if he knows the strategies of all the other voters. For a discussion along those lines of models based upon strategic behavior of atomistic agents, see for instance, Kenneth Rogoff, 1987.

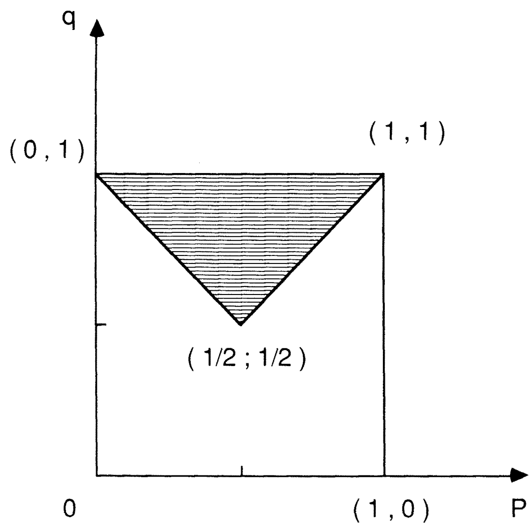

FIGURE 1

In Figure 1 the dashed area represents the locus in which the cooperative rule is sustainable. As expected, it is easiest to sustain cooperation for $\bar{P}=\frac{1}{2}$. This result is not sensitive to variations in the approximation used for λ^*.

Since we have assumed that voters do not behave strategically, the function $P(\cdot)$ does not shift in response to a deviation from cooperation. If, instead, voters used retrospective strategies (see, however, fn. 9) then, we would have a punishment arising directly from the voters. If, say, party 1 breaks the cooperative rule, there might be a shift in the function $P(\cdot)$ such that

$$(14)\quad P^{ch}(x,y)<P(x,y) \quad \text{for any } (x,y),$$

where P^{ch} identifies the probability of reelecting party 1 if this party did not follow the cooperative rule in the past. If there is a loss of popularity because of the deviation from cooperation, then, *ceteris paribus*, it is easier to sustain cooperative rules because the enforcement is stronger.[10]

[10]John Ferejohn, 1986, and Rogoff and Anne Sibert, 1988, consider a similar type of punishment. In their models, however, parties are not "ideological."

V. Incomplete Cooperation

If the first-best outcome is not sustainable because q is too low, the two parties would not, in general, follow the one-shot policies (that is, their bliss points). A certain amount of convergence would still Pareto-improve upon the one-shot Nash. The best credible policies can be obtained as a solution of problem (8) under the conditions of individual rationality and the constraints of subgame perfection. The constraints of subgame perfection can be computed by imposing that for a given pair of policies $(\tilde{x},\tilde{y})$, the "temptation" of deviating from it is not greater than the "enforcement." "Temptation" and "enforcement" for the two parties can be computed following the steps described in Section IV. These constraints are

$$(15)\quad (\tilde{x}-c)^2 \le \frac{q}{1-q}\left[(1-\bar{P})c^2-P(\tilde{x},\tilde{y})(\tilde{x}-c)^2 - [1-P(\tilde{x},\tilde{y})](\tilde{y}-c)^2\right]$$

for party 1 and

$$(16)\quad y^2 \le \frac{q}{1-q}\left[\bar{P}c^2-P(\tilde{x},\tilde{y})\tilde{x}^2 - [1-P(\tilde{x},\tilde{y})]\tilde{y}^2\right]$$

for party 2. In the Appendix the following result is established:

PROPOSITION 4: *If and only if $q=0$ the only credible policies are $x=c$ and $y=0$ (the one-shot Nash). If $q>0$ there exists a nonempty set of policies $\tilde{x}(q,c)$, $\tilde{y}(q,c)$ which Pareto-improve upon the one-shot Nash and which satisfy the following condition*

$$c>\tilde{x}(q,c)>\tilde{y}(q,c)>0.$$

Thus, even though the one-shot Nash still remains an equilibrium of the game, there exist other, partially convergent equilibria,

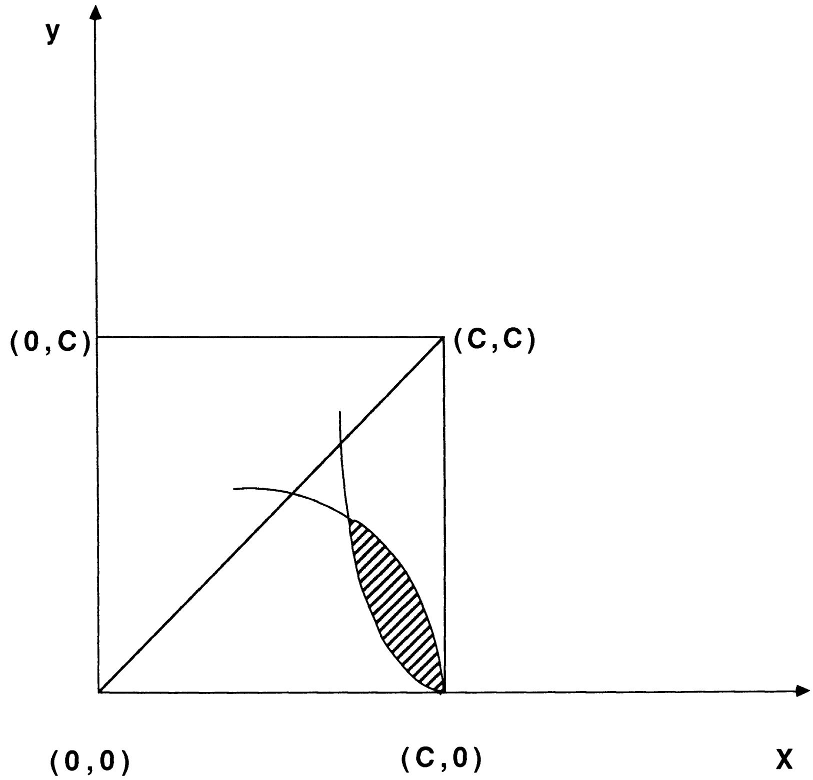

FIGURE A1

which Pareto-improve upon the one-shot Nash.

Note that the policies $(\tilde{x}, \tilde{y})$ described in Proposition 4 and the policies $(\hat{x}, \hat{y})$ of Section II, (i.e., the equilibrium policies of the one-shot game with precommitments to parties' platforms) are in general different and hardly comparable. In particular, the policies $(\hat{x}, \hat{y})$ are unrelated to the discount factor since they are the result of a one-shot game. On the contrary the set of credible policies in the infinitely repeated game $(\tilde{x}, \tilde{y})$ is crucially affected by the value of the discount factor. *Ceteris paribus*, the higher is q the larger is the set of sustainable policies.

APPENDIX: PROOF OF PROPOSITION 4

We have to solve problem (8) under the constraints of individual rationality and of subgame perfection (15) and (16). It is easy to establish that the policies $\tilde{x}$ and $\tilde{y}$ which satisfy the following conditions:

$$\text{(A1)} \qquad (i) \quad \tilde{x} \geq \tilde{y} \geq c \quad \text{or} \quad \tilde{y} > \tilde{x} \geq c.$$

$$\text{(A2)} \qquad (ii) \quad \tilde{y} \leq \tilde{x} \leq 0 \quad \text{or} \quad \tilde{x} < \tilde{y} \leq 0.$$

$$\text{(A3)} \qquad (iii) \quad \tilde{x} > c > 0 > \tilde{y}$$

cannot be a solution of the problem. These policies in fact violate at least one of the constraints of individual rationality, which can be written as

$$\text{(A4)} \quad P(x,y)(x-c)^2 + (1-P(x,y))(y-c)^2 \leq (1-\overline{P})c^2, \quad \text{(party 1)}$$

$$\text{(A5)} \qquad P(x,y)x^2 + (1-P(x,y))y^2 \leq \overline{P}c^2. \quad \text{(party 2)}$$

We now show that a solution $c > \tilde{x} > \tilde{y} > 0$ always exists for any positive q. Consider the two constraints of subgame perfection. By applying the implicit function theorem it can be shown that they cross the 45° line as in Figure A1. In fact, using (15) one obtains

$$\text{(A6)} \qquad \frac{dy}{dx} = -\frac{(x-c)+\dfrac{q}{1-q}[(x-c)P(x,x)]}{\dfrac{q}{1-q}[(x-c)(1-P(x,x))]}.$$

In (A6) we used the fact that $P_x = P_y = 0$ if $x = y$. For $x < c$, (A6) implies $dy/dx < 0$. Analogous argument holds for the other constraint (16). We now show that the two curves always cross for any $q > 0$ so that an area such as the dashed area of Figure A1 always exists. Consider point $N = (c,0)$, the one-shot Nash. Consider a movement in the direction that keeps the function $P(\cdot)$ constant:

$$\text{(A7)} \qquad dP = P_x\,dx + P_y\,dy = 0.$$

Therefore

$$\text{(A8)} \qquad \frac{dy}{dx} = -\frac{P_x}{P_y} < 0.$$

Consider a movement $(c-\delta, \varepsilon)$, such that

$$\text{(A9)} \qquad \varepsilon = \frac{P_x}{P_y}\delta.$$

Define

$$\text{(A10)} \qquad \frac{P_x(0,c)}{P_y(0,c)} \equiv \beta < 0.$$

By rearranging the conditions of subgame perfection (equations (15) and (16)) at the point $(c-\delta, \varepsilon)$, and using (A9) one obtains

$$\text{(A11)} \qquad \delta \le \frac{2\beta cq(1-\bar{P})}{1-q+\bar{P}q+\beta^2 q(1-\bar{P})},$$

$$\text{(A12)} \qquad \delta \le \frac{2c\bar{P}q}{\beta^2(1-\bar{P}q)+\bar{P}q}.$$

Therefore, for all $q > 0$, there exists a $\delta(q)$ such that (A11) and (A12) are satisfied. It is immediately apparent that the movement $(c-\delta, \varepsilon)$, such that $\varepsilon = \beta\delta$, increases the objective function (i.e., equation (8) in the text).

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