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Candidates with Policy Preferences: A Dynamic Model

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The extensive literature concerned with formal models of political candidate strategies has almost without exception viewed policy as a means to winning.[1] In this paper, I suggest that the reverse is true—that candidates view winning as a means to policy. The paper argues that this perspective is more realistic in its assumptions and more accurate in its predictions. The approach is shown to yield considerable insight into a variety of situations with differing assumptions concerning voters and elections, including a sequential path analysis with no restrictions on the distribution of voter preferences; and a simultaneous presentation of platform positions with certain restrictions on the distribution of preferences.

The path analysis was originally put forward by Downs [3]. The winner in election T (the incumbent) must present the same platform in election $T + 1$. The opposition is unrestricted in his choice of platform. With intransitivity, the incumbent will always lose and there will be a rotation in office. Both Buchanan [2] and Tullock [9] have argued that when the utility functions are type 1 (the indifference curves being concentric circles centered on the most preferred position of the voter, utility being solely a function of the Euclidian distance) the set of platforms chosen will eventually lead to the set of Pareto-optimal alternatives (the convex hull of the most preferred positions). However, McKelvey [6] has shown that majority rule cycles can go almost anywhere in the set of feasible choices and that the path does not converge to the Pareto-optimal set. In fact, it is easy to construct a path that expands ever outward from the Pareto-optimal set by making the winning position slightly closer to a majority of voters' preferred positions and as far away as possible from the preferred positions of a minority of voters, the majority not being identical in each step. If this characterization of the political process is correct, the implications are profound, as there is then no

[1] The main exceptions are Wittman [10–12] and Barry [1], who characterize the electoral process in a different way (for example, only one issue is assumed and an electoral path is not considered). They also use less-formal arguments. The view of winning as the ultimate goal goes back to Smithies [8] and Schumpeter [7], and includes work by Downs [3], Kramer [5], and Hinich, Ledyard, and Ordeshook [4].

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relationship whatsoever between the preferences of the participants and the set of possible outcomes.

Kramer [6] suggested that candidates are not just interested in winning but in maximizing the number of votes. By considering a different hypothesis from Buchanan and Tullock concerning candidate motivation, he is able to show that the path converges to a subset of the Pareto-optimal set. In a remarkable proof, he demonstrated that the path will approach a set N^*. Looking at the intersection of all Pareto-optimal sets composed of V voters ($V = 1, 2, ..., N$), N^* is that nonempty intersection of Pareto-optimal sets with the smallest V. The incumbent will still always lose, but the opposition will tend to move closer to N^*. The underlying structure of the argument can readily be shown when $N = N^*$. Let the incumbent's position be outside the Pareto-optimal set composed of all N voters. There exists a point in the Pareto-optimal set preferred to the incumbent's position by all N voters. The opposition need not choose a position within this Pareto-optimal set, but if he does not, the position he does choose must also be preferred by N voters to the incumbent's position as the candidate maximizes votes. A position is preferred by N voters only if the position is closer to the N voters' Pareto-optimal set. Thus, the opposition will approach the Pareto-optimal set composed of N voters.

It should be noted that closer need not mean convergence to the set. For example, if each candidate in turn moves closer by amount $\epsilon/2^t$ (where t stands for the election, $t = 1, 2, ...$) to the Pareto-optimal set composed of N voters, as $t \to \infty$ the total movement towards this set will only approach ϵ. If this "degenerate" case is disallowed, then the candidates will actually enter or come close to the set N^*.

In this paper it is assumed that given a choice of winning platforms, the candidate will choose that platform which coincides most with his preferences. Certainly, this is a more reasonable assumption than vote maximization, for there is little if any payoff to getting more votes than are necessary for winning the election. In fact, vote maximization has the perverse result that the only participants not interested in policy are the candidates.[2] With imperfect information, a candidate who was interested in policy would again choose the position closest to his own preferred position from a set of posi-

[2] The rationale for vote maximization is due to an incorrect analogy from the economic sphere. Businessmen are not interested in what kind of chair they produce; therefore by analogy candidates should not be concerned with policy. However, this analogy is incorrect for two reasons. In the first place, the consumption of a chair is a private good while government policy is a public good, and therefore the candidate would be concerned with the latter. In the second place, the businessman in not interested in maximizing sales of chairs but rather in maximizing profits. Selling chairs is a means to an end in the same way that gaining office (getting votes) is a means to the end of preferred policy implementation.

tions with equal probability of winning, and would be willing to trade off the probability of winning for a desired platform position if this increased his expected utility (where utility is solely a function of which policy is implemented). Thus, concern for policy is a reasonable assumption under a variety of circumstances.

Furthermore, it has considerable empirical support. President Johnson was willing to undertake an unpopular position concerning the war in Vietnam. In fact, he gave up the chance for reelection rather than give up on the policy which he felt was correct. Both Barry Goldwater and George McGovern were clearly concerned with policy. But even a candidate who is more "politically expedient" still has policy preferences. The candidate is willing to undertake policies which he finds slightly distasteful, if this greatly enhances his chances for election or reelection. On the other hand, the candidate will be less willing to compromise on an issue which is important to him when the compromise will only slightly increase his chance of winning.

An investigation of the alternative model will now be undertaken.

(1) Let M_i be a vector of preferred positions of voter i in the Euclidean K space R^k. $i = 1, 2, \ldots, N + 2$. N is odd. Let $M_x = M_{N+1}$, $M_y = M_{N+2}$ be a vector of preferred positions of candidates X and Y, respectively. The set of feasible points is a compact convex body in R.

(2) Let vR_iw if i prefers v to w or i is indifferent between v and w. R_i is complete. vP_iw if vR_iw and not wR_iv. v, w are K-dimensional vectors.

(3) i has a preference ordering over the points in R^k represented by a type 1 utility function; i.e., $vR_iw \Leftrightarrow \| v - M_i \| \leqslant \| w - M_i \|$, where $\| z \| = (\sum_{i=1}^{K} z_i^2)^{1/2}$.

(4) i votes for X if xP_iy; votes for Y if yP_ix; and votes for the incumbent if yR_ix and xR_iy. For expositional clarity assume that each candidate always votes for himself.

(5) If $X(Y)$ is the winning candidate in election t, then $X_{t+1} = X_t$ ($Y_{t+1} = Y_t$); i.e., the incumbent (winner) from election t presents the same platform in election $t + 1$.

(6) w is a winning position for the opposition if a majority of voters strictly prefer w to the incumbent's position.

(7) Each candidate is assumed to maximize his expected utility in election t. Because a voter votes for X either with probability 1 or 0, this condition is identical to the following: From the set of winning positions the opposition chooses that w' preferred by himself to all other winning positions. If this position is not preferred to the incumbent's position, the opposition chooses his own most preferred position. If there is no unique

best position, he will choose from this subset that position which will lead to the highest preference in the next election; and iterate this process until there is a unique best solution. If there is no unique best position, he will choose from this subset of indifferent positions the "highest ranked" indifferent winning position, the "ranking" being transitive and complete, with all relationships being strict.

(8) Let S_j be a set of $(N+1)/2$ most preferred positions. There are $\binom{N}{(N+1)/2}$ distinct S_j.

(9) S_j is a minimax set for X, S_x if for every set S_j there exists an M_i such that $\| M_i - M_x \| \geqslant \| M_k - M_x \|$ for all $k \in S_x$. S_y, a minimax set for Y, is defined in the same way.

(10) $B_x(B_y)$ is the minimax ball if it is the smallest ball with center $M_x(M_y)$ containing the most preferred points of $S_x(S_y)$ (see Fig. 1).

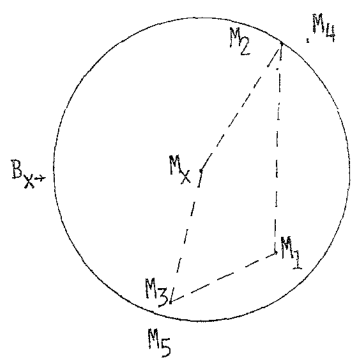

FIG. 1. B_x is the minimax ball. The dotted line is the Pareto-optimal set defined by S_x and M_x.

PROPOSITION 1. *Every $X_t(Y_t)$ $t > 1$ is in the minimax ball $B_x(B_y)$.*[3]

Proof. Assume Y is the incumbent. If $Y_{t-1} = Y_t$ is not an element of the Pareto-optimal set defined by S_x and M_x, then there exists an X' in the Pareto-optimal set strictly preferred by all of the members of the set, including X. Since this set has at least a majority of voters, X' is a winning position over Y_t. If X does not choose a point in this set, by assumption he must choose a winning position X^* that is not farther away from M_x than X' and therefore $X^* \subseteq B_x$. If $Y_{t-1} = Y_t$ is an element of the Pareto-optimal set

[3] Proposition 1 involves assumptions identical to those used by Kramer [5] except for the goal of the candidates. With slight alterations, similar results are obtained if voters vote for the incumbent, with probability P if indifferent. If $P = 0$, then the results are more restrictive.

defined by S_x and M_x and if there exists a point X' strictly preferred by X and a majority of voters to Y_t, then $X' \subseteq B_x$. If a point X' does not exist X will choose M_x.

If the candidates are only interested in winning there is no relationship between preferences of the voters and the set of electoral outcomes. However, when the candidates maximize votes, the preferences of voters do determine the electoral path. If the candidates have policy preferences, the preferences of the voters still influence the outcome, although the preferences of the candidates play a major role. Depending on the particular configuration of voter and candidate preferences the sets B_x and B_y may be smaller or larger than N^*. (However, if "degenerate" paths are considered, B_x and B_y are always more restrictive than the set of limits of vote maximizing paths.)

To best appreciate the difference between vote maximization and maximizing policy outcome, it is useful to look at the *bete noire* of transitivity—income distribution. In this case the indifference curves are no longer type 1. If all participants are purely selfish and each coordinate stands for income for a certain voter (or candidate), then the indifference curves are hyperplane perpendicular to the coordinate for the individual's income. For vote maximizing candidates, N^* is almost every conceivable income distribution where all income is distributed. For the income maximizing (policy preferring) candidates, the results are clear cut. In order to demonstrate this, the following terminology will be introduced.

(11) An equilibrium pair X^*, Y^* is said to exist if $X_t = X^*$, $Y_t = Y^*$ for all $t > 0$.

(12) An electoral path approaches an equilibrium pair X^*, Y^* if for all $\delta > 0$ there exists a t^* such that for all $t > t^*$, $\| X_t - X^* \| + \| Y_t - Y^* \| < \delta$.

(13) Let the amount of income that X offers to i in period $t = X_t^i$, $X_t^i \geqslant 0$.

Proposition 2. *Let the election be solely concerned with income distribution with the amount of income to be distributed being a fixed amount I; if $Y_t^x \neq I$, $X_t^y \neq I$ then the electoral path approaches a unique equilibrium pair X^*, Y^*, where $X^{x*} = I$, $X^{i*} = 0$ for $i \neq x$; $Y^{y*} = I$, $Y^{i*} = 0$ for $i \neq y$.*

Proof. Let Y be the incumbent in election t. Then $X_t^i = Y_t^i + \epsilon$ ($\epsilon > 0$) for the $(N + 1)/2$ smallest Y_t^i not including $Y_t^x < I$. X_t^x will receive the remainder. $X_t^i = 0$ for the remaining $(N - 1)/2$ individuals. In election $t + 1$, Y will offer ϵ to these $(N - 1)/2$ individuals; i.e., $Y_{t+1}^i = \epsilon$ for the $(N - 1)/2$ individuals for whom $X_t^i = 0$. Of the remaining X_t^i ($i \neq y$), Y will offer $Y_{t+1}^i = X_t^i + \epsilon$ to the smallest X_t^i. Y_{t+1}^y will receive the remainder. The other $Y_{t+1}^i = 0$. In election $t + 2$, X will offer ϵ to those $(N - 1)/2$ voters who received $Y_{t+1}^i = 0$, 2ϵ to one of the voters for whom $Y_{t+1}^i = \epsilon$, and the remainder to himself. This process will continue and for all $t > 0$,

$$\| X_{t+3} - X^* \| + \| Y_{t+3} - Y^* \| \leqslant \| X_{t+3}^x - I \| + \sum_{\substack{i=1 \\ i \neq x}}^{N+2} \| X_{t+3}^i - 0 \|$$

$$+ \| Y_{t+3}^y - I \| + \sum_{\substack{i=1 \\ i \neq y}}^{N+2} \| Y_{t+3}^i - 0 \| < \delta$$

for arbitrarily small ϵ ($\delta > NE + 3E$).

Thus almost all of the income goes to the two candidates.[4]

Proposition 2 demonstrates a great difference in form and substance between vote maximization and maximizing the utility from policy outcome. When the issue is income distribution, vote maximization leads to a set only slightly smaller than the feasible set. In contrast, preference for policy leads to a unique equilibrium pair. With regard to substance, vote maximization is typically seen as a method of registering "majority will"—the benevolent invisible hand applied to the political process. If the candidates are vote maximizers, there exists no other position which is preferred by a greater number of voters to the set N^*. However, competitive policy maximizing candidates need not present positions which in any way reflect the "majority will" of the voters. In the income distribution case all but one of the candidates would strongly prefer that the income be divided equally. While the real world is not quite so dramatic, the analysis gives insight into why there is little redistribution of income and why many claim they do not have a real choice in the electoral process.

(14) Let the utility function of X be of the following nature: U_x is solely a function of the winning platform Z. $U_x(\| z - M_x \|)$ is twice differentiable, with

$$\frac{dU_x}{d \| Z - M_x \|} \leqslant 0, \qquad \frac{d^2 U_x}{d \| Z - M_x \|^2} < 0.$$

Let the utility function of Y be a similar form.

PROPOSITION 3. *If for all X_T, Y_T ($X_T \neq Y_T$) the probability that X_T will win the election is less than* 1 *and greater than* 0, *then there always exists a set of platforms X_T', Y_T' such that $X_T' = Y_T'$ and the expected utilities of both X and Y are higher.*

[4] The lower the barriers to entry of a new candidate, the more the results will be mitigated as the candidates will offer income to the potential candidates. It should be noted that all results in the literature on candidate strategies are invalidated when there are low barriers to entry.

The proof is obvious due to the assumption of concavity. It should be noted that this proposition makes no assumptions concerning the indifference maps of the voters nor does it make restrictions on the incumbent's choice of position.

Similarity of the candidates' positions results when the candidates maximize votes and the feasible set is transitive. However, by Proposition 3, only identical positions are (*ex ante*) Pareto optimal for candidates with policy preferences. Where competition would lead the candidates to dissimilar platforms, it pays candidates with policy preferences to collude and present similar platforms. Therefore similarity of the candidates' positions cannot be used as proof of competition, vote maximization, or transitivity. Proposition 3 has implications somewhat similar to those of Proposition 2 (and thereby shows the robustness of the results). The presence of two candidates does not guarantee a competitive outcome. Consequently, the candidates need not respond to the interests of the voters. For example, in an election that is only concerned with income distribution, the candidates might collude to split all the income among themselves. Bipartisanship in the United States and restricted electoral competition among the oligarchy in Central America may be examples of this phenomenon.

Of course, there may be other reasons why the candidates present identical platforms, as Propositions 4 and 5 will show. These two propositions consider interesting "limit" cases. In Proposition 4, the candidates' preferences are diametrically "opposed" to those of the voters, while in the first part of Proposition 5 the candidates' positions are in diametric opposition to each other. It is no longer assumed that $X_t = X_{t-1}$ if X is the incumbent in election t, nor in Proposition 4 that the voters' preference functions are type 1.

PROPOSITION 4. *If $M_x = M_y$ and the candidates present their positions simultaneously, then $X_t = Y_t = M_x$ is stable equilibrium point.*

The proof is obvious.

PROPOSITION 5. *If the indifference curves are type* 1, *there exists a point M_m which is the most preferred position of voter m, and all the remaining voters can be put in pairs such that for each pair the indifference curves through M_m are separated by a hyperplane through M_m, then the following is true.*

(A) *If the indifference curves through M_m of X and Y are separated by a hyperplane through M_m, and M_x, $M_y \neq M_m$, then M_m is a minimax equilibrium if randomized strategies are not allowed.*[5]

[5] In the context of a political campaign, it is not reasonable to assume randomized strategies. For example, one of the candidates (most likely, the incumbent) may have to announce his (nonrandom) position first. A platform which involves lotteries is also ruled out if the voters' utility functions are concave.

(B) *If the indifference curves through M_m of X and Y are on the same side of a hyperplane through M_m, then $M_x(M_y)$ is a minimax position of $X(Y)$ if $M_x(M_y)$ is closer to M_m than $M_y(M_x)$.*

It should be noted that for type 1 indifference curves, the conditions in the proposition are both necessary and sufficient for a majority winner. Minimax means the worst the first candidate can do given that the second candidate maximizes his utility given the first candidate's position.

Proof of A. (See Fig. 2.)

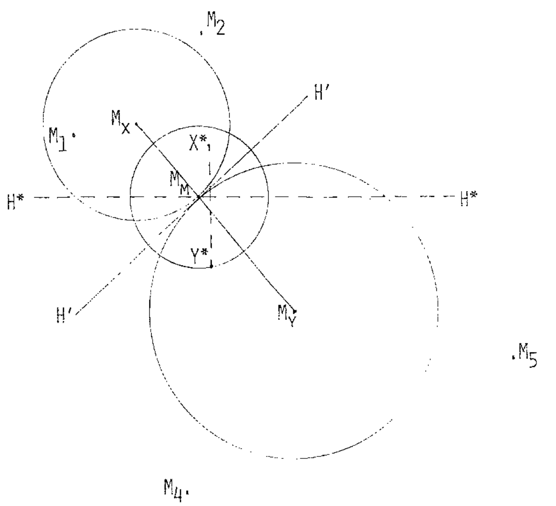

FIG. 2. If $Y^* \neq M_m$, then there exists an X^* preferred by X and a majority of voters to Y^*. X's and Y's indifference curves through M_m are separated by the hyperplane H'. The indifference curves of voters 1 and 5 (2 and 3) through M_m separated by another hyperplane (not drawn). X^* is within voter m's indifference curve through Y^*. H^* is the hyperplane through M_m perpendicular to the line X^*Y^*.

Let Y choose a point Y^* ($Y^* \neq M_m$); then there always exists an arbitrarily small open ball centered on M_m such that every point within the ball is strictly preferred by a majority of voters.

Let $\| Y^* - M_m \| = C$; let X^* be a point such that $\| X^* - M_m \| < C$. Define a hyperplane H^* through M_m which is perpendicular to the line connecting X^* and Y^*. If X^* and Y^* are in the same half-space, then a majority of voters in the other half-space strictly prefer X^* to Y^* as X^* is closer to every voter in this half-space, and every half-space through M_m contains a majority of voters by assumption. If X^* and Y^* are not in the same half-space, then all of the voters in the half-space which includes X^* strictly prefer X^* to Y^*. If Y^* is in the indifference curve of Y through M_m

(or more generally in the half-space H' which includes M_y and is defined by the hyperplane through M_m and perpendicular to the line connecting M_x, M_y, and M_m), then there exists a point X^* strictly preferred by X to M_m and Y^* and strictly preferred by a majority of voters to Y^*. Y strictly prefers M_m to this point. If Y^* is not in H', it is obvious that Y strictly prefers M_m to Y^* and also that Y prefers M_m to X's best choice against Y^*. Since M_m is strictly preferred by a majority of voters to any other point, M_m is a minimax position.

Proof of (B). (See Fig. 3.)

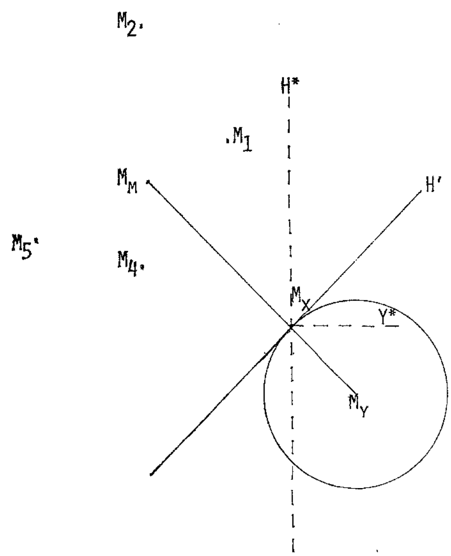

FIG. 3. If M_x is closer to M_m than M_y is to M_m, then M_x is a minimax position for X. Y prefers Y^* to M_x, but a majority of voters prefer M_x to Y^*.

Assume $\| M_x - M_m \| < \| M_y - M_m \|$. By assumption, M_x, M_y, and M_m are on a line. Let Y^* be a point closer to M_y than M_x is to M_y. Then Y^* is in the half-space H' not including M_m defined by the hyperplane through M_x and perpendicular to the line connecting M_x and M_y. The hyperplane through M_x and perpendicular to the line connecting M_x and Y^* creates another half-space H^* which includes M_m. All the voters in this half-space prefer M_x to Y^*, and there are at least a majority of voters in this half-space.

We may alter the rules so that Y will also choose M_x in this case; i.e., if there is no winning position preferred to the other candidate's minimax position, then the candidate will choose the other candidate's minimax position so that he will still be considered a viable candidate. (Both candidates will have a chance of winning if indifferent voters vote for a candidate with probability greater than 0 but less than 1.) Both candidates will then look alike, although they are not at the median voter's preferred position. This may explain those cases where both candidates are to "the left" or "right" of the majority.

CONCLUSION

This paper has shown the usefulness of assuming that candidates have policy goals. It suggests that in order to predict which government policies will be implemented, it is not only necessary to know the voters' preferences, but the candidates' preferences as well.

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