Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		**2225556**

Towards Prosperity: Revealing the Secrets Behind the Market

Summary

Financial forecasting deals with dynamically evolving phenomena through time. Accurate price prediction based on historical data is a challenging task in this field, because a large degree of uncertainty governs price evolution.

In this project, our goal is to make decisions that maximize profitability based on known data. To achieve this goal, we have built three models, the main one is our Investment Strategy Model, in addition to two other models to assist the decision making. Our model will give a specific amount of daily transactions and a prediction of future risk.

For model I, its task is to predict the future direction of prices based on the available price data.Considered the prediction accuracy and time efficiency, we can not choose neural network. With this in mind, we use an ***ARIMA*** model, an autoregressive model, which can quickly compute accurate price predictions as our price prediction model.

For Model II, the task is to make a prediction of future market risk. Considering the high profitability and high risk of long-term investments, we need to develop a model that could learn the riskiness of price fluctuations. Here we want to imitate human learning behavior to learn price data, so we defined ***CNN-LSTM*** neural network as our prediction model.

For model III, the task is to give a daily specific investment strategy. We believe that investments should conform to exponential growth, so the core formula of our investment model is an exponential formula. We refer to the principles of the ***Apriori*** algorithm, and we customize an investment strategy formula. Based on the predicted price of model I, we check the current price characteristics with the previous data to do the analysis, and then decide the exact amount of daily investment. Also, when considering that the risk becomes greater when investing for a long period of time, we also make an investment strategy in combination with the risk prediction from model II.

In terms of the process, first of all, we will input the once price into the ***ARIMA*** price prediction model to get the price data in the future period; then, we will call the risk prognosis model to estimate the risk based on the predicted future data, and also call the investment decision model to calculate the current day's investment amount. Finally, we make an investment plan based on a combination of the future risk estimate and the current day's investment amount calculated by the investment Strategy model.

In summary, our model takes into account the timeliness, accuracy, risk and profit maximization properties.

Keywords: ARIMA prediction, Statistics of ups and downs, Apriori, Price trend forecasting, CNN-LSTM.

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1 Introduction

1.1 Background

Gold and Bitcoin price volatilities have a significant impact on many financial activities of the world. The development of a reliable prediction model could offer insights in gold and Bitcoin price fluctuations, behavior and dynamics and ultimately could provide the opportunity of gaining significant profits.

As popular investment products, both gold and Bitcoin frequent price changes naturally raise a question: whether the price of gold and Bitcoin can be predicted. This is a significant question, especially given gold and Bitcoin's short history and the fact that its price is easily influenced by the attitudes of governments around the world,as shown in Figure 1.

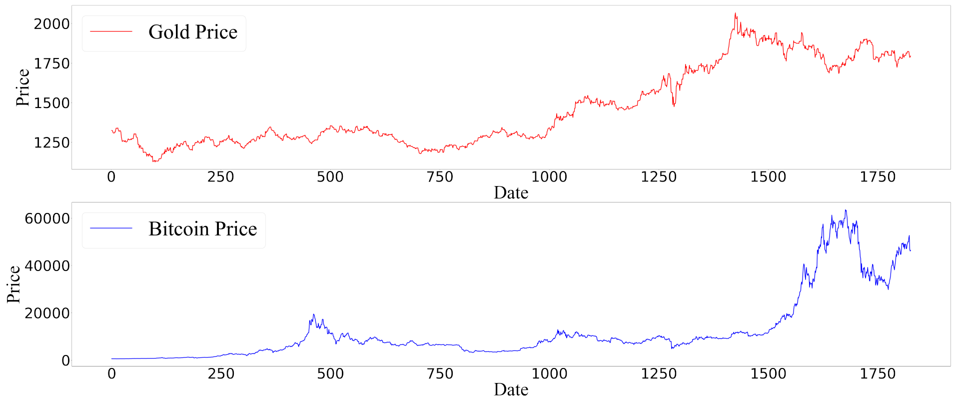

Figure 1: Gold and Bitcoin Price Charts

1.2 Restatement of the Problem

We were required to use the data given in the problem about the daily exchange rate between bitcoin and gold to make a set of decisions. Then we are expected to propose the best daily trading strategy for traders. In this case, we were given an initial amount of $1000; during the transaction, we could only use the exchange rate before that day to assist us in our decision.In addition, We need to use rigorous proofs to confirm that our model can maximize assets on 9/10/2021.

Combining the requirements of the topic, our model should consider the following points:

- **Our models need to ensure accuracy and efficiency.**
- **Our optimization direction is to maximize the benefits.**
- **We need to take into account both long-term and short-term investments to make better investment decisions.**
- **We need the model should have some robustness.**

1.3 Our Work

In order to solve those problems, we will proceed as follows:

- **Stating assumptions**. By stating our assumptions, we will narrow the focus of our approach towards the problems and provide some insight into trading strategies issues.
- **Making notations**. We will give some notations which are important for us to clarify our models.
- **Presenting our model**. In order to investigate the problem deeper, we divide our model into two sub-models. One is a ARIMA Prices Prediction model. The other one is investment strategy model.The third one is Risk Prognosis Model.

  1) **Model one: Prices Prediction Model.** For gold/Bitcoin price forecasting, considering the timeliness efficiency and prediction accuracy, we used the ***ARIMA*** forecasting model as our prices forecasting tool.
  2) **Model two: Investment Strategy Model.** We defined an investment plan, calculated by this formula, we will decide each day to invest to make a decision to invest or not, and if we invest, we will give a specific amount of money, this formula follows the law of exponential increment, which follows the principle of profit maximization, and also takes into account of the stability and security of the investment.
  3) **Model three: Risk Prognosis Model.** We can't always invest in a short term, considering that it causes a lot of unnecessary waste such as commissions and missed opportunities to make better money. However, the ***ARIMA*** exchange rate forecasting model produces relatively large inaccuracies in long-term forecasting. To reduce the risk, we trained a risk assessment model based on ***long-short memory neural network(LSTM)*** and ***convolutional neural network(CNN)***, which will make reliable judgments on the probability of future exchange rate fluctuations based on various types of economic indicators.

- **Integrated investment strategy VS. short-term investment strategy** Here we will evaluate our integrated investment strategy which takes into account risk prognosis, and compare it with the short and medium term investment models.
- **Proof of model optimality**.
- **Robustness and Sensitivity of model.** Here, we test the sensitivity and robustness of the model. We add noise to the price to test the robustness of the model; and change the trading commission to verify the sensitivity of the model
- **Further discussion.** We discuss about different ways to Improving the performance of the model in terms of reliability and profitability. Then we improve our model to apply them in reality.
- **Evaluating the model**. We will discuss the highlights and weaknesses of our model as follows:

  1) **Prices Prediction**
  2) **Risk Prognosis**

3) **Investment Strategy**
4) **Robustness and Sensitivity**

2 Assumptions and Justification

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **Commission costs are deducted directly at the time of the transaction.**
- **Set the default value of the provided data to 0, which does not affect the final investment strategy.**
- **Assets in gold are not tradable on days when the market is not open.**
- **Gold and Bitcoin price trends are not affected by malicious human disturbances.**
- **The price data provided can be treated as the opening or closing price,when calculating financial indicators(e.g. MACD, SMA, RSI $\cdots$).**

3 Model Overview

The contribution of this research is the development of a prediction model for forecasting the gold price and movement utilizing and exploiting advanced deep learning techniques. Convolutional layers are characterized by their ability to extract useful knowledge and learn the internal representation of time-series data, while LSTM networks are effective for identifying short-term and long-term dependencies. The principle idea of our proposed model is to efficiently combine the advantages of these deep leaning techniques. To this end, our proposed model, named CNN-LSTM, consists of two main components: The first component consists of convolutional and pooling layers in which complicated mathematical operations are performed to develop features of the input data, while the second component exploits the generated features by LSTM and dense layers. In the sequel, we present a brief description of the convolutional, pooling and LSTM layers which constitute the core of the proposed model.

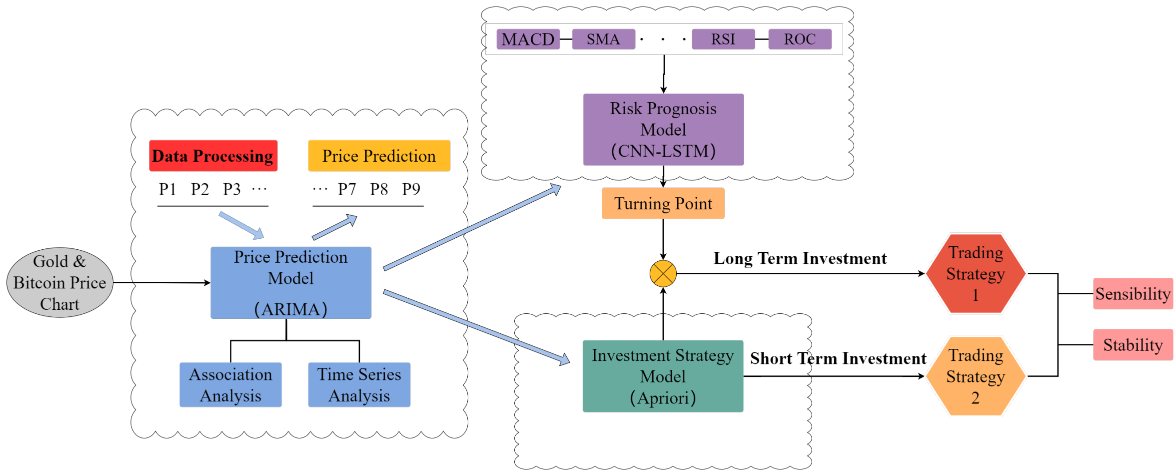

Figure 2: Model Framework

4 Sub-model I : Prices Prediction Model

We hope to make a judgment on the future price direction based on accurate data every day, however, we also need to consider the time cost, because in the actual problem, we cannot train a huge neural network model every day, so by comprehensive consideration, we choose ARIMA as our forecasting model. ARIMA based on statistical principles to predict the price curve, and it has superior time efficiency and accuracy.

4.1 PRELIMINARIES

Before we can make predictions about the data, we first need to understand some prior knowledge.

- **AutoRegressive Model(AR)**
  The autoregressive model finds the relationship between current values and historical data and makes predictions based on the historical data. However, the autoregressive model must require the data to satisfy the smoothness requirement, which we will analyze specifically in the next section.

  The p-order autoregressive model is calculated based on the following equation:

  $$y_t = \mu + \sum_{i=1}^{p} \gamma y_{t-i} + \varepsilon \tag{1}$$

  where y_t represents the data of the current day; γ represents the constant value; y_{t-i} is the data of the previous period; and ε represents the error value.

- **Moving Average Model(MA)**
  The moving average model is an accumulation of the error value of the autoregressive model, and its purpose is to eliminate the random fluctuations of the predictions in the autoregressive model. When AR is combined with MA, the equation is as follows.

  $$y_t = \mu + \sum_{i=1}^{p} \gamma y_{t-i} + \varepsilon + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2}$$

  where y_t represents the data of the current day; γ represents the constant value; y_{t-i} is the data of the previous period; and ε represents the error value. $\sum_{i=1}^{p} \theta_i \varepsilon_{t-i}$ then represents the accumulation of the error value.

- **Difference**
  The difference between the time series at moments t and t-i.

- **Autocorrelation Function**
  Comparing sequential random variables with itself, the autocorrelation function reflects the characteristic correlation of the same sequence at different times. Its calculation formula is as follows:

  $$ACF(k) = \rho = \frac{Cov(y_t - y_{t-k})}{Var(y_t)} \tag{3}$$

  where k represents the lag factor.

- **Partial Autocorrelation Function**
  The ACF correlation coefficient will include the influence of k-1 data between t and t-k, while PACF will eliminate the interference and only calculate the data correlation between X_t and X_{t-k}.

With the above description of the principle of the ARIMA model, we can select the p and q values of ARIMA according to the ACF and PACF data. The formulas are as follows (same as above):

$$y_t = \mu + \sum_{i=1}^{p} \gamma y_{t-i} + \varepsilon + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{4}$$

4.2 Differential Processing

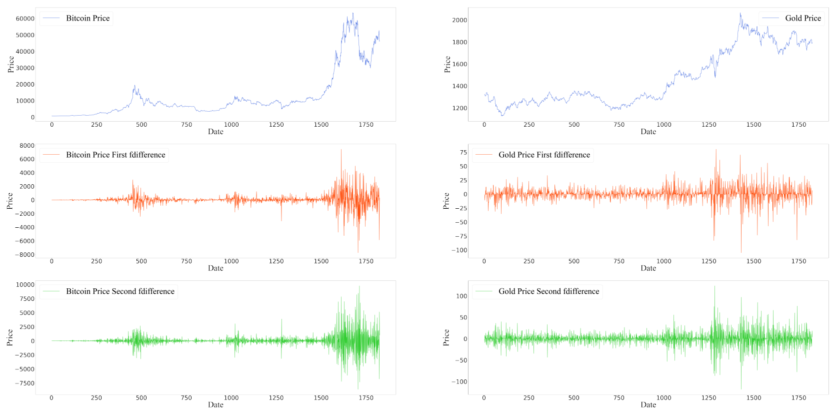

Figure 3: Differential Processing

By plotting the bitcoin and the original price curve, the first-order difference curve, and the second-order difference curve, it can be observed that the graph tends to be smooth in the first order when doing the first-order difference, and passes the unit root test, so the result of the first-order difference can be considered as a smooth non-white noise series.

4.3 ACF and PACF Graph Analysis

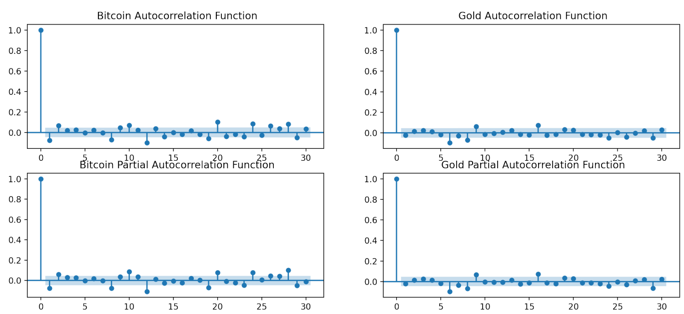

Figure 4: ACF and PACF Graph

- After the differencing process, we have obtained the smooth time series. We need to perform graphical analysis of the autocorrelation coefficient ACF and the partial correlation coefficient PACF on the smooth time series to get the optimal stratum p and order q.

  We can see that the first-order difference series of Bitcoin has ACF at 2 truncated tails and PACF at 2 truncated tails; the first-order difference series of gold has ACF at 1 truncated tails and PACF at 1 truncated tails.

- To get more accurate parameter values, we can use a loop to iterate through all possible parameter values and use a heat map to select the best one.

Figure 5: Gold Heatmap

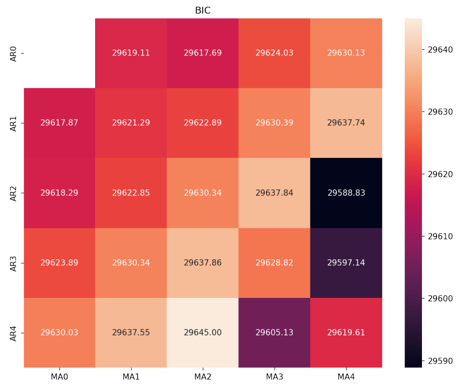

Figure 6: Bitcoin Heatmap

The final parameters are selected as shown in the table below:

Table 1: The Final Parameters

item	p	d	q
Bitcoin	2	1	2
Gold	2	1	2

4.4 Result

Using the parameters obtained above to train the ARIMA model, we obtain the line graphs of predicted and actual prices of bitcoin and gold. And referring to the ADF test, we can see that the prediction model is more reasonable and more in line with the facts.

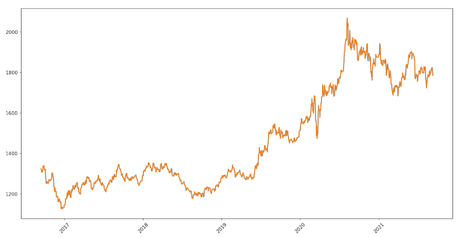

Figure 7: Gold forecast price

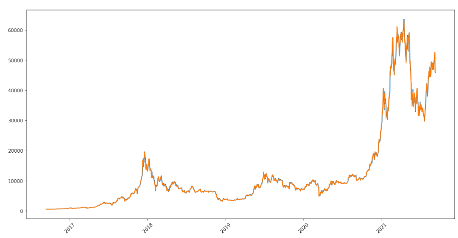

Figure 8: Bitcoin forecast price

5 Sub-model II: Risk Prognosis Model

Our goal is to maximize the assets on 9/10/2021. However, in the actual investment process, our decision is influenced by many factors, such as the relative instability of the ARIMA model in long-term price forecasting and the inability to predict market fluctuations in advance. These uncertainties can cause our investments to fail and reduce profits.

Take an example: the price of bitcoin is rising, yet according to the Trading Strategy model of short term Strategy, we would have sold all of our bitcoins in the first few days of the rise (because of the exponential growth nature of the decision model), which resulted in us missing out on future profits from bitcoin appreciation, as well as consuming a lot of commissions.

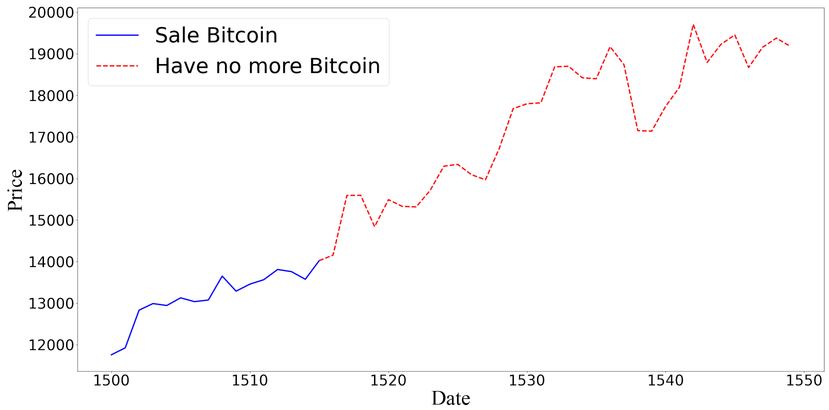

Figure 9: Example of Deficiencies of the short term investment model

Therefore, in order to maximize profits, it is necessary to predict the long-term stability of the gold/Bitcoin price and to assist the Strategy model in making decisions, and eliminating the above problem.

5.1 Definition of Risk in Prediction

First, based on our question, we know that our risk mainly originates from the instability of long-term investments. So we need to make a prediction about the stability of long-term investments.

Before making a prediction of long-term investment stability, we first need to make a few definitions.

Table 2: Definition of Turning Point

Definition 1: Turning Point
Day t is the Turning point, if in the range of $t - 7 < x < t + 7$*, there is* min_{price} *and* max_{price} *exist, and* $max_{price} - min_{price} > \alpha$*,* α *is the threshold value.*

By defining the Turning point, we can surmise that the appearance of an Turning point represents the appearance of a price movement with a consequent increase in risk. Therefore, we can lead to the definition of price stability as follows.

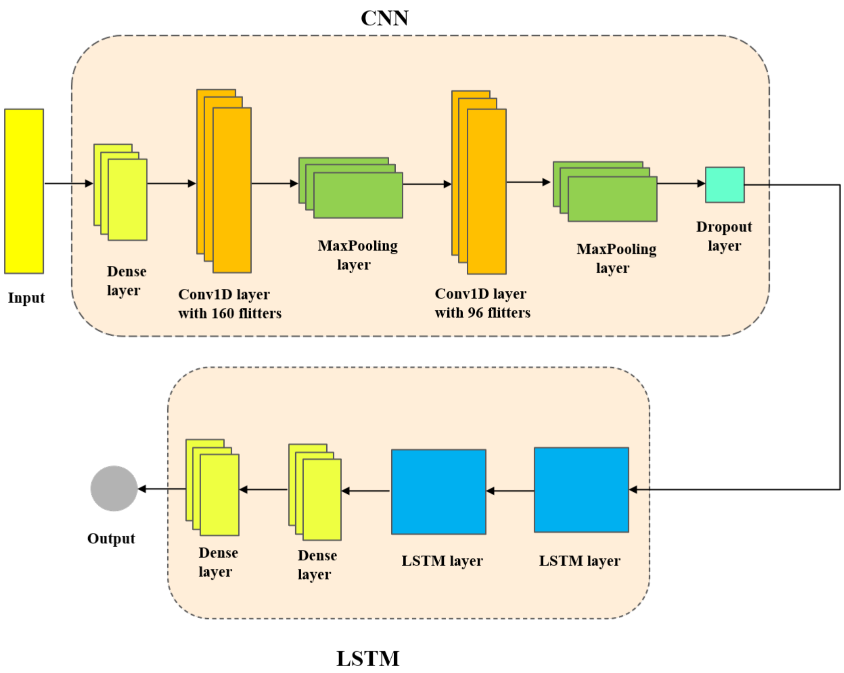

Figure 10: CNN-LSTM Model

Table 3: Definition of Price Stability

Definition 2: Price Stability
The probability of Turning point p<0.3, which is predicted by CNN-LSTM model, means that the price is stable on that day, and our buying and selling should be done before the day which probability of Turning point p>0.7. prepresents the probability of Turning point on that day.

With these two definitions above, we can concretize the abstract risk concept into a specific indicator, that is, a target that can be learned: the probability of the target date is a Turning point. The input features for training will be explained below. With label and feature, we can choose a machine learning model to learn the probability of Turning point, and here we choose the CNN-LSTM model, where the LSTM model is a variant of the RNN model, the valve mechanism of its neurons can make it learn data from serial features, and the CNN can also provide better feature extraction for the LSTM. In the following section, I will briefly introduce the CNN-LSTM model.

5.2 Data Processing

In the risk assessment model, we consider that neural networks are similar to human behavior, and humans consider many professional indicators in the financial industry, such as MACD, SMA, ROC, ROCR, RSI, MOM, etc., when speculating about the future stock market. We speculate that these professional indicators can be used as features and input to machine learning models, so that the models can learn and find out the patterns of these features. Thus, it can assist the decision model (or human) to make long-term investments.

The following table shows the features used in our model, and **because we can only use the given data, we have made some adjustments in the calculation of the following indicators.**

Indicators	Description
$MACD$	Moving average convergence divergence
$MACDHIST$	MACD histgram line
$MACDSIGNAL$	MACD signal line
ROC	Rate of change
$ROCR$	Rate of change rate
$SMA5$	Simple moving average of 5 days
$SMA15$	Simple moving average of 15 days
$SMA30$	Simple moving average of 30 days
RSI	Relative strength index
$PRICE$	Price of bitcoin/gold
MOM	Momentum
EMA	Exponential Moving Average

Table 4: Indicators used in Training Feature(of bitcoin/gold)

- **1) SMA(Sample Moving Average)**. The moving average calculates the average price over a period of time. It is a technical indicator that eliminates short-term fluctuations in financial asset prices and analyzes long-term price trends by smoothing data.

$$SMA\left(N\right) = \frac{1}{N} \textstyle\sum_{i=1}^{N} p_i \tag{5}$$

- **2) EMA(Exponential Moving Average)**. Considering that the closer the time is, the greater the impact of the price, the exponential moving average makes the influence of the price of each period decrease with time by weighting.

$$EMA\left(N\right) = \begin{cases} p_1 & \text{if } N = 1 \\ \frac{2}{N+1} p_N + \frac{N-1}{N+1} EMA\left(N-1\right) & \text{if } N > 1 \end{cases} \tag{6}$$

- **3) MACD(Moving Average Convergence and Divergence)**. Moving Average Convergence and Divergence is a technical indicator for judging the timing of buying and selling financial assets and the trend of price movements. Perform a smoothing operation, where α is the smoothing coefficient. The most commonly used short-term and long-term EMAs are the 12- and 26-day EMAs.

$$\begin{cases} DIF\left(i\right) = EMA\left(N_{fast}\right) - EMA\left(N_{slow}\right) \\ DEA\left(i\right) = \alpha DEA\left(i-1\right) + \left(1-\alpha\right) DIF\left(i\right) \\ MACD\left(i\right) = 2\left(DIF\left(i\right) - DEA\left(i\right)\right) \end{cases} \tag{7}$$

- **4) MOM(Momentum Index)**. Momentum indicator is a technical indicator that studies the fluctuation speed of financial asset prices. The principle is that the increase or decrease of financial asset prices will gradually decrease over time.

$$MOM\,(n) = p_i - p_{i-n} \tag{8}$$

where n is the calculation parameter, in general n=6

- **5) RSI(Relative Strength Index)**. The relative strength indicator is similar to the Williams indicator. It is a technical indicator that studies the fluctuation range of financial asset prices. This technical indicator can be used by investors to judge market changes and predict future financial asset price trends.

$$\begin{cases} RS\,(n) = \frac{Sum\ of\ gains\ over\ n\ days}{Sum\ of\ losses\ over\ n\ days} \\ RSI\,(n) = 100 - \frac{100}{1+RS(n)} \end{cases} \tag{9}$$

- **6) ROC(Rate of Change)**. The rate of change indicator is a technical indicator that studies the dynamics of changes in the price of financial assets. Use this rate of change to analyze the momentum of financial asset prices to determine the trend of price movement.

$$ROC\,(n)_i = \frac{p_i - p_{i-n}}{p_{i-n}} \tag{10}$$

Finally, we normalize the entire feature as follows (MinMaxScaler):

$$X_{std} = \frac{X - X_{min(axis=1)}}{X_{max(axis=1)} - X_{min()axis=1}} \tag{11}$$

$$X_{scaled} = X_{std} \times (max - min) + min \tag{12}$$

Here, $axis = 1$ represents the row axis, $X_{min(axis=1)}$ represents the minimum value of the row axis, $X_{max(axis=1)}$ represents the maximum value of the row axis, and $X_{std(axis=1)}$ represents the standardized data. max, min represent the maximum and minimum values mapped to $[0, 1]$, respectively. X_{scaled} represents the final normalized data

The label is crucial in this risk prognosis model training because its meaning represents the risk. As mentioned in definition1 and definition2 above, here, we set the label of the risk model to whether the day is a Turning point or not, and if so, set it to 1, and if not, set it to 0. In this way, we can get the labels for almost every day for training.

5.3 Structure of Risk Prognosis Model

In this section, I will present the CNN-LSTM structure and its advantages for our risk assessment model.The following figure shows the structure of our model.

We introduce CNN-LSTM instead of applying only LSTM to do prediction with the following considerations:

- Our data has many features, and CNN can extract these features well, especially multi-dimensional features.
- The introduction of CNN can appropriately reduce the number of LSTM neurons and reduce the training time cost
- The combination of CNN and LSTM model has good prediction effect and generalization ability, and is more resistant to interference, which is suitable for learning multiple features.

5.3.1 Pseudocode

Algorithm 1 Investment Joint Model

Input: Gold/Bitcoin price of 1xn (Provided Data)
Output: Current Investment Strategy

for $i = 1$ to l **do**
  $Price_{future} = ARIMA\left(Price_{past}\right)$
  if Long Term Invest **then**
    $Risk_{B/G} = RiskPredictModel_{B/G}\left(Price_{past}\right)$
    $CurrentStrategy = InvestStrategy\left(Risk_{B/G}, Price_{B/G-past}\right)$
  else
    $CurrentStrategy = InvestStrategy\left(ARIMA\left(Price_{past}\right)\right)$
  end if
  if $i\%Y_B == 0$ **then**
    $Retrain\ RiskPredictModel_B$
  end if
  if $i\%Y_G == 0$ **then**
    $Retrain\ RiskPredictModel_G$
  end if
end for

5.3.2 Convolutional Neural Network(CNN) Part

First, we use Convolutional Neura Network to perform feature extraction on our data.

Given a data of X days exchange rate (6 days in training) as:

$$X_{r\times m} = \begin{bmatrix} price_{day1} & macd_{day1} & \cdots & ema_{day1} \\ price_{day2} & macd_{day2} & \cdots & ema_{day2} \\ \vdots & \vdots & \ddots & \vdots \\ price_{day6} & macd_{day6} & \cdots & ema_{day6} \end{bmatrix}$$

Here we input a vector of $r \times m$, where m represents a training group size, and r represents the dimensionality of each data.This matrix can then be used by the CNN to extract features. We use two CNN layers to extract features, the first layer filters = 160 and the second layer filters = 96. RULE activation functions are applied to each layer and MaxPooling is used between the two layers to prevent overfitting. Finally, we added a Dropout layer.

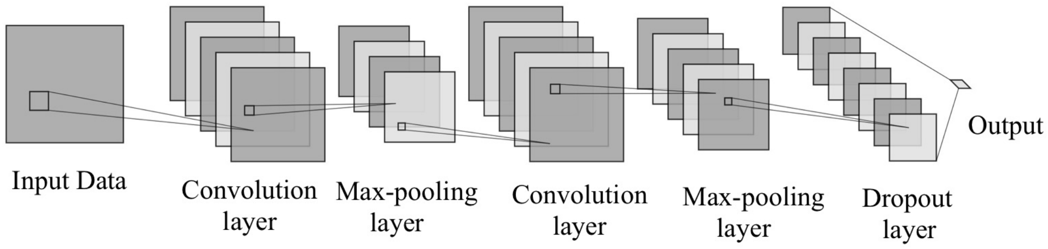

Figure 11: CNN Model

5.3.3 Long Short Term Memory(LSTM) Part

Long short term memory(LSTM) is a deformation structure of RNN by adding memory cell into hidden layer, so as to control the memory information of the time series data. Information is transmitted among different cells of hidden layer through several controllable gates (forget gate, input gate, output gate), thus the memory and forgetting extent of the previous and current information can be controlled.

1) Forget Gate: The purpose of Forget Gate is to determine what information will be discarded. Reading in the output of the previous layer h_{t-1} with the current input x_t, the gate outputs f_t and assigns the current cell C_{t-1}, the calculation formula of f_t is as follows:

$$f_t = \sigma\left(W_f x_t + U_f h_{t-1} + b_f\right) \tag{13}$$

where σ presents the sigmoid function.

2) Input Gate: The role is to update based on existing information. First, run the sigmoid function to get i_t and decide which values to enter. Then, according to the tanh function, a candidate value vector $\widetilde{C}_t$ is obtained, which is multiplied with i_t and added to the state C_t. The formula for this part is as follows:

$$\begin{cases} i_t = \left(W_i x_t + U_i h_{t-1} + b_i\right) \\ \tilde{C}_t = \tanh\left(W_c x_t + U_c h_{t-1} + b_c\right) \\ C_t = f_t C_{t-1} + i_t \widetilde{C}_t \end{cases} \tag{14}$$

3) Output Gate: Output the information of the current point. After running a sigmoid function to get o_t and determining which parts will be output, C_t is processed by the tanh function to obtain a value between -1 and 1. Finally, the value is multiplied with o_t to decide the ultimate output:

$$\begin{cases} o_t = \sigma\left(W_o x_t + U_o h_{t-1} + b_c\right) \\ h_t = o_t \tanh\left(C_t\right) \end{cases} \tag{15}$$

As for our CNN-LSTM model, a LSTM unit is overlaid with another LSTM unit. We feed the hidden state of the bottom LSTM into the upper LSTM again and use the hidden state from the upper state as the input for the output module. In the output module, we simply feed the hidden state of the upper LSTM into a linear layer to make predictions on the feature vector.

5.4 Result

5.4.1 Accuracy in Risk Prognosis

For risk prediction model training, we need to consider the number of Turning points, if the number of Turning points is large, then our training interval should be small. For example, for Bitcoin, the number of Turning points is relatively large, so our training interval should be shortened to 30 days; for gold, the number of Turning points is relatively small, so our training interval can be parented to 50 days.

Each time we train, we will calculate MACD, ROC, MOM and other indicators based on the previous price data, and then input them into the CNN-LSTM model for training; after finishing the training of CNN-LSTM model, we will calculate the future data such as MACD.ROC and other indicators based on ARIMA's prediction of future prices, and then input them into the CNN-LSTM model for prediction.

However, in the actual prediction process, there will be a lot of inaccuracies, for example, when predicting bitcoin, considering the data fluctuation of bitcoin in the first 3 years is very small, if we learn the data of the first 3 years to predict the later data, the error will be very large. So, instead of using all the data from 9/11/2016-current as the training set , we choose the data from the last X days as the training set. Then predict the future Y-day inflection rate. As time progresses, we will retrain the risk assessment model every Y days. The same goes for gold. Here is our test for the risk prediction model.

To better understand our test for the risk prediction model, we need to first specify a definition of the indicator $Rate$:

$$Rate = \frac{X}{X+Y} \tag{16}$$

X is the price data for the last X days, Y is the probability of predicting the inflection point for the next Y days.

In order to find the most appropriate prediction ratio Rate with respect to the training set X size, we have done a lot of experiments. We recorded the inflection point prediction accuracy of the training set at 500, 300, and 150 days with Rate of 1, 0.5, and 0.3 for the bitcoin and gold data.

From the data we collect, the smaller the training set, the higher the accuracy, regardless of whether it is bitcoin or gold, at whatever Rate. Then we applied this model feature to the Investment Strategy model.

6 Sub-model III: Investment Strategy Model

Based on the price prediction model described above, it is possible to predict the likely price for a future trading day from the known bitcoin price and gold price. However, the price can only predict the future trading day trend and cannot reflect a specific trading strategy. Therefore, in order to get a specific trading strategy, you need to count the historical data corresponding to the single day up and down distribution pattern, continuous up and down pattern, in order to get a wise investment strategy.

According to the laws of the financial market, in order to gain income, when the value of the financial product falls, you should buy the right amount; when it rises, you should sell. The specific amount of the transaction requires a study of the product's own regularity to develop a corresponding strategy.

6.1 Notations

Table 5: Notations used in Model II

Symbols	Description	Unit
$M_{0.5}$	Median one-day increase in financial products	USD
$N_{0.5}$	Median one-day decline in financial products	USD
m_k	Number of consecutive up k days	days
n_k	Number of consecutive down k days	days
u_m	Number of days in excess of 90% of all consecutive gains	days
u_n	Number of days over 90% of all consecutive declines	days
$M_{0.9}$	90th percentile of all rises per u_m trading days in the historical data	USD
$N_{0.1}$	The 10th percentile of all declines per un trading day in the historical data	USD

6.2 Statistical Analysis

Using Bitcoin as an example, MATLAB software is used to perform data statistics on financial products as follows:

- **Calculate the actual increase or decrease of the financial product for each trading day d:**

$$r = \frac{(price\,(d) - price\,(d-1))}{price\,(d-1)} \tag{17}$$

- **Calculate the median single-day increase in financial products:**
  First sort the r values of all r positive trading days in ascending order, where p is 0.5.

$$(n-1) * p = i + j \tag{18}$$

  where, n is the number of trading days for which all r is positive, the integer part is i and the fractional part is j. Then the final $M_{0.5}$ is :

$$M_{0.5} = (1-j) * array[i] + j * array[i+1] \tag{19}$$

  where array is the storage location of the ascending sequence.

- **Calculate the median one-day decline in financial products $N_{0.5}$ Calculate as above.**
- **Apriori algorithms are applied to count the number of subsets of financial products with 2, 3, 4, 5 and 6 consecutive price increases m_1,m_2,m_3,m_4,m_5,m_6 and consecutive price decreases n_1,n_2,n_3,n_4,n_5,n_6.**
- **Calculate the number of times all consecutive rises exceed 90%u_m.**

$$\sum_{i=2}^{u_m-1} mi \leq 0.9 \leq \sum_{i=2}^{u_m} mi \tag{20}$$

- **Ditto to calculate the number of times over 90% of all consecutive declinesu_n**
- **Get the 90th percentile of all increases for every u_m increases in the historical data:**

$$r_m = \frac{(price\,(d) - price\,(d - u_n))}{price\,(d - u_n)} \tag{21}$$

- **Reference the formula mentioned above and sort each array(r_m) in ascending order.**

$$(n-1) * 0.9 = i + j \tag{22}$$

Finally, $M_{0.9}$ is calculated:

$$M_{0.9} = (1 - j) * array\,(r_m)\,[i] + j * array\,(r_m)\,[i + 1] \tag{23}$$

- **As above, calculate the 10th percentile$N_{0.1}$ of all the gains obtained from the statistics of each u_n trading days in the historical data.**

6.3 Model Deduction

Using the above statistical laws obtained from the data.

For the purpose of model construction we make the following assumptions.

The number of consecutive rises of the model is u_m, and the maximum cumulative increase is $M_{0.9}$; the number of consecutive falls of the model is u_n, and the maximum cumulative decrease is $N_{0.1}$. Ideally, when going up and down in a row, the magnitude of each change is the same.

An example is given using the add strategy for Bitcoin, and the subtract strategy is similar:

When Bitcoin falls for u_1 days in a row, the cumulative decline happens to be $N_{0.1}$, then buy Bitcoin every time it falls according to the laws of the financial market.

Suppose the next day's price change is $M_{0.5}$, and the first time it goes up, the amount of Bitcoin purchased is P_1.

- **The amount of the second purchase is P_2, and the next day's profit for this operation needs to be satisfied:**

$$p_2 * (1-a) * (1+M_{0.5}) + p_1 * (1-a) * (1+M_{0.5}) * \left(1+\frac{N_{0.1}}{U_n}\right) = p_1 + p_2 \quad (24)$$

where, a is the rate of commission, the left-hand side of the equation represents the funds held on the next day, and the right-hand side of the equation represents the cost of acquiring these financial products.

- **Similarly, the third purchase of P_3 is required to satisfy:**

$$\begin{aligned} p_3 * (1-a) * (1+M_{0.5}) + p_2 * (1-a) * (1+M_{0.5}) * \left(1+\frac{N_{0.1}}{u_n}\right) + \\ p_1 * (1-a) * (1+M_{0.5}) * \left(1+\frac{N_{0.1}}{u_n}\right)^2 = p_1 + p_2 + p_3 \end{aligned} \quad (25)$$

- **The amount of the nth purchase is P_n:**

$$\begin{aligned} p_n * (1-a) * (1+M_{0.5}) + p_{n-1} * (1-a) * (1+M_{0.5}) * \left(1+\frac{N_{0.1}}{u_n}\right) + \dots \\ +p_1 * (1-a) * (1+M_{0.5}) * \left(1+\frac{N_{0.1}}{u_n}\right)^{n-1} = p_1 + p_2 + \dots + p_n \end{aligned} \quad (26)$$

According to the u_n and $N_{0.1}$ obtained in the data, the following table can be obtained:

Table 6: The calculating results

Consecutive days of decline	1	2	3	4
Amount	100	613.95	5082.05	41257.05

The maximum amount of input is constrained by assuming that the amount of Bitcoin and gold can be input is 500 USD respectively, and the data is normalized and fitted by introducing an exponential regression model:

The following regression model is obtained:

$$\begin{cases} x = L * \frac{u_n}{N_{0.1}} \\ y_{add} = 0.1111e^{2.1038x} \end{cases} \quad (27)$$

where y_{add} denotes the amount that should be invested in the purchase of the financial product, and L denotes the cumulative decline in the continuous decline.

Similarly, we can get the strategy of reducing the position, and the strategy of increasing and reducing the position of gold is similar.

$$\begin{cases} x = L * \frac{u_m}{M_{0.9}} \\ y_{sub} = 0.0121e^{2.1259x} \end{cases} \quad (28)$$

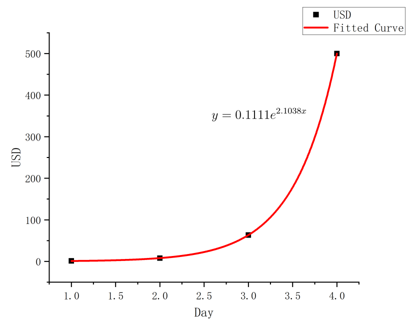

Figure 12: Add fitted

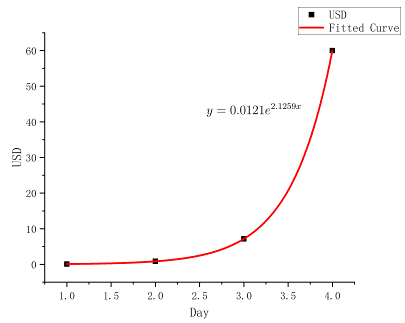

Figure 13: Sub fitted

6.4 Result

Using the regression strategy model described above, the return curve can be plotted as follows:

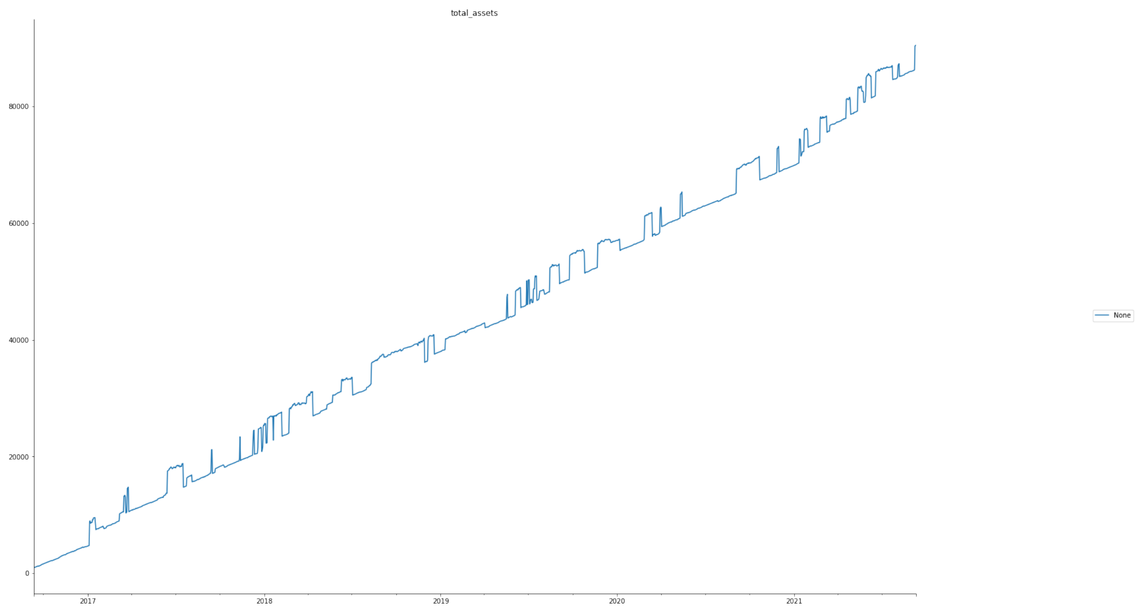

Figure 14: Wealth Accumulation Curve(Short Term Investment)

The analysis can be obtained that the proposed model is stable and the adoption of this strategy in the trading market can be adapted to the short-term financial market fluctuations.

6.5 Model Improvement

Considering the risk assessment model, the following strategy improvements are developed for financial products with long-term value investment to meet the needs of long-term investment:

- The CNN-LSTM model predicts the probability of a possible future price truning point by the price of previous data:

$$rate[0], rate[1]...rate[n] \tag{29}$$

- Define a time interval $span$ that divides the time into multiple $span$ and is used to specify the frequency of executing the trading strategy.
- Count the positions $index$ with the highest probability of turning point in each $span$.
- Calculate the rate of price change of the turning point position from the initial time of $span$ for each $span$:

$$r = \frac{(price[span_{start} + index] - price[span_{start}])}{price[span_{start} + index]} \tag{30}$$

- Rules for transaction amount:
  1. if r < $-M_{0.1}$:Use half of your current USD holdings to purchase this financial product
  2. if r < $M_{0.9}$:Reduce half of your financial holdings to convert to dollars

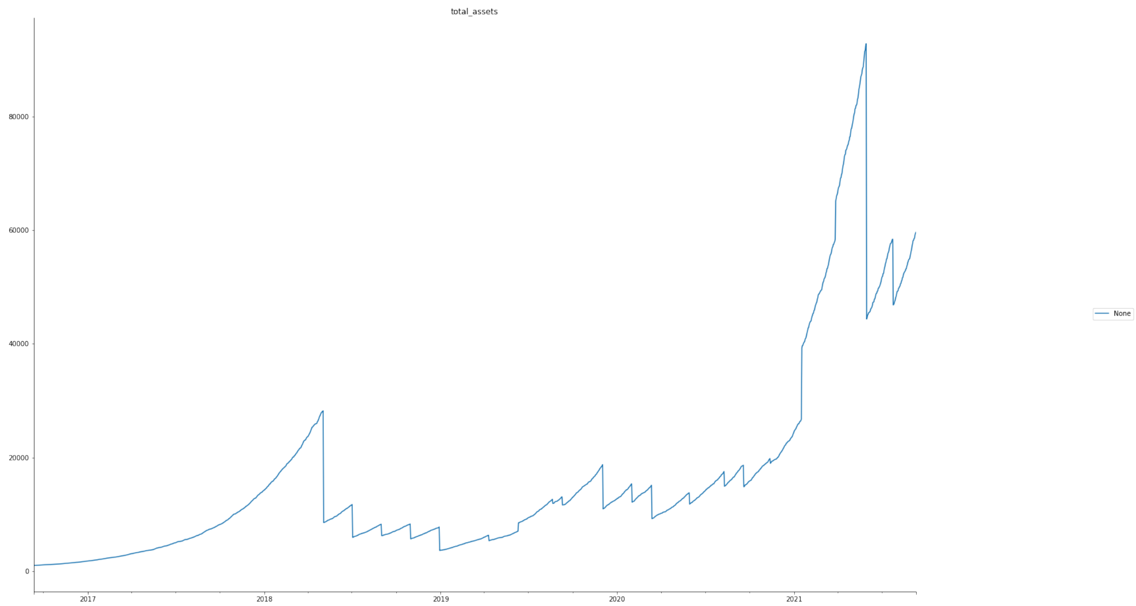

Figure 15: Wealth Accumulation Curve(Long Term Investment)

7 Model Optimality Proof

We consider that the optimality of the model should be measured in the following three dimensions:

- **The prediction accuracy of the model and the time efficiency of training.** For example, neural networks or certain backtracking algorithms may get good prediction results with the increase of training data, however, the time overhead brought by daily repetitive training makes it inevitable to lose time efficiency. In practical applications, it is impossible to achieve the unity of time efficiency and accuracy. We take this into account and use the ARIMA model, an autoregressive model, which can quickly calculate accurate price forecasts.

- **Riskiness of model forecasting.** We know that we cannot learn the risk of the market accurately from a single price data, at least not quantifiable and unexplainable. However, considering the high profitability of long-term investments. We believe we need to develop a model that can learn the riskiness of price movements. Here we simulate human learning behavior, using some price indicators common in the financial industry as features, and using the exchange rate price data fluctuations at as labels to train the CNN-LSTM model, thus allowing the neural network model to assist us in determining the risk of the market and preparing in advance.
- **Profit maximization.** Our ultimate goal is to maximize the total final assets. In order to achieve this, we design an exponential growth investment model, according to which, with the aid of a risk prediction model, we neither sell gold/bitcoin early in a bull market, nor buy gold/bitcoin too early in a bear market. Ultimately we can maximize our profits.

8 Robust and Sensitivity Analysis

In order to verify the robustness of the decision model, it is considered to add random noise from 1 to 100 to the prices and verify the returns, as shown in the following table:

Table 7: The calculating results

Noise	Interests
False	90522.92724120484
True	100579.5296

It can be seen that the model maintains robustness to noise disturbances.

To verify the sensitivity of the model to commission costs, different commission rates are used and the corresponding benefits are verified.

When the transaction costs change, the results obtained due to the use of bitcoin returns to measure the impact are as follows:

Table 8: The calculating results

The Commision Cost	Interests(USD)	y_{add}	y_{sub}
0.00005	116095.1393	$y_{add} = 0.1314e^{2.062x}$	$y_{sub} = 0.015e^{2.0836x}$
0.00010	107920.2675	$y_{add} = 0.1244e^{2.0756}x$	$y_{sub} = 0.0131e^{2.1086x}$
0.00020	90522.9191	$y_{add} = 0.1111e^{2.1038x}$	$y_{sub} = 0.0121e^{2.1259x}$
0.00050	85079.8552	$y_{add} = 0.077e^{2.1953x}$	$y_{sub} = 0.0076e^{2.2185x}$
0.00100	77095.7962	$y_{add} = 0.0372e^{2.3772x}$	$y_{sub} = 0.003e^{2.4021x}$

The above table shows that when the transaction cost changes, the return of investment will also change when using short term investment.

9 Strength and Weakness

9.1 Strength

- Trading strategies are based on financial market laws, extracting the characteristics of short-term financial products, and have the ability to extract characteristics for different financial products.

- The strategy is also based on data analysis of market patterns, and can obtain the optimal strategy to match the specific financial market environment.
- Our model is fairly robust due to our careful corrections in consideration of real-life situations and detailed sensitivity analysis.

9.2 Weakness

- Too few subjects and investment strategies for individual investment products may not be applicable to other investment products.
- Model construction is idealized, and other factors such as environment, policy, and public opinion are not considered when specifying investment strategies.

10 Conclusion

In this project, we explored programs about trading strategies. We proposed models based on price forecasting, risk forecasting, steady investment and actually implemented them.We believe that our models have time efficiency, accuracy, safety and profitability. We also confirm our opinion in the analysis of the results of each sub-model.

Regarding the exploration of model time efficiency, we use a regression model that does not require training, which greatly saves the time needed for daily decision making. At the same time, our risk prediction model will be trained at regular intervals, saving time and making it more feasible for practical application.

Regarding the accuracy of the model, in predicting prices, we will recalculate the future price trends every day to ensure that the daily data are up-to-date, while in risk prediction, we also use a two-layer CNN model, which also changes the extraction of features, and at the same time, each training of Risk Prognosis Model will be conducted 1000 epoch and use the most recent period of data to ensure that the model is adequately and reasonably trained.

Regarding the trading safety of the model, we also evaluate in the robustness and sensitivity of the model, our trading strategy rarely receives noise or commission changes because when investing in the long term, the trading strategy is developed out mainly based on the risk prediction model.For example, the risk estimation model normalizes the 12 financial indicators calculated by the exchange rate and feeds them into the model training, which enhances the robustness while increasing the features.

Regarding the profitability of the model, we also believe that we have achieved a combination of long-term and short-term investments, and each investment is considered to maximize profit and minimize risk. And the accuracy of the prediction model increases with the amount of data.

In summary, we demonstrate the optimality of our model and test the robustness and sensitivity of the model. Although there is still room for improvement in our model, we believe that we have done a good job in fulfilling the requirements of the topic and taking into account many realistic factors.

Memo to traders

Date: February 22, 2022
To: General Traders
From: MCM Team #2225556
Subject: Trading Strategies—How To Invest with Gold and Bitcoin?

Dear general traders,

It is an honor to offer some of our recommendations in your investment process. As we all know, gold and Bitcoin are among the most popular investment products in the world.Perhaps you may now think that gold and bitcoin are too risky to invest in, or you think that you don't have enough money on hand to invest in gold and Bitcoin, that's why we come in!

Now we are glad to bring you good news that whether you are rich or strapped for cash, and regardless of the price movements of gold and Bitcoin, you can earn a substantial income and avoid investment risks to a large extent by using our proposed trading strategy. Let me now present to you our model and the trading strategy obtained from the analysis:

The models we construct extract the characteristics of the price movements of gold and bitcoin from their historical price data and are able to accurately predict the future prices of gold and bitcoin without taking into account environmental, policy and public opinion factors.Here, we assume that the future price predicted from the model is the real future price, whereby we develop well thought out trading strategies based on historical price trends and future prices of gold and bitcoin in an effort to profit from the high risk financial markets.

We constructe Prices Prediction Model with ARIMA, Investment Strategy Model with Apriori and Risk Prognosis Model with CNN-LSTM in general, in which we use ARIMA to predict the future prices of gold and bitcoin, and use CNN-LSTM to evaluate the investment risk, and then use Apriori to make trading strategies for short-term and long-term investments respectively.

1. For currencies with less commissions and financial products with high short-term volatility, but not value investing, short-term investments are preferred.

2. When making short-term investments, it is recommended to use high-frequency trading instruments to stop and liquidate positions in time, thus avoiding the impact of the corresponding uncontrollable factors.

3. For financial products with high commissions and value growth, priority is given to long-term investment models that incorporate the risk analysis specified to obtain greater value investment returns.

Finally, we hope you will adopt our trading strategies and wish you the best of luck in your future investments.

Sincerely yours,

Team #2225556

References

[1] Ioannis E. Livieris, Emmanuel Pintelas, Panagiotis Pintelas, ***"A CNN-LSTM model for gold price time-series forecasting"***, Neural Computing and Applications volume 32, pages17351-17360 (2020).

[2] Zhang Maojun, Rao Huacheng, Nan Jiangxia, Wang Guodong ***"Research on quantitative trading timing strategies based on decision trees"***, [J/OL], Systems Engineering.

[3] Yan Li, Wei Dai, ***"Bitcoin price forecasting method based on CNN-LSTM hybrid neural network model"***, Volume 2020, Issue 13, July 2020, p. 344 – 347, The Journal of Engineering.

[4] Antonio J. Conejo, Miguel A. Plazas, Rosa Espínola, Ana B. Molina, ***"Day-Ahead Electricity Price Forecasting Using the Wavelet Transform and ARIMA Models"***, IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 20, NO. 2, MAY 2005.

[5] Javier Contreras, Member, IEEE, Rosario Espínola, Student Member, IEEE, Francisco J. Nogales, Antonio J. Conejo, Senior Member, IEEE, ***"ARIMA Models to Predict Next-Day Electricity Prices"***, IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 18, NO. 3, AUGUST 2003.

[6] `https://blog.csdn.net/sunnyxidian/article/details/92946542`

[7] `https://www.cnblogs.com/tianqizhi/p/9277376.html`

[8] Timothy J. Kehoe, Pau S. Pujolas, and Jack Rossbach, ***"Quantitative Trade Models:Developments and Challenges"***, Vol. 9:295-325 (Volume publication date August 2017), Annual Review of Economics.

[9] Weipeng Zhang, Lu Wang, Liang Xie, Ke Feng, Xiang Liu, ***"TradeBot: Bandit learning for hyper-parameters optimization of high frequency trading strategy"***, Volume 124, April 2022, 108490, Pattern Recognition.

[10] Gustavo Carvalho Santos, Flavio Barboza, Antônio Cláudio Paschoarelli Veiga, Mateus Ferreira Silva, ***"Forecasting Brazilian Ethanol Spot Prices Using LSTM"***, Energies 2021, 14(23), 7987.

[11] D.H. LEE, D.H. KIM, J.H. YOON, ***"PREDICTION OF U.S. GOLD FUTURES PRICES USING WAVELET ANALYSIS; A STUDY ON DEEP LEARNING MODELS"***, J. Appl. Math. & Informatics Vol. 39(2021), No. 1 - 2, pp. 239 - 249.

[12] Ziaul Haque Munim, Mohammad Hassan Shakil, Ilan Alon, ***"Next-Day Bitcoin Price Forecast"***, J. Risk Financial Manag. 2019, 12(2), 103.