

SoftSign non-linear Layer

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1 Introduction

1.1 What is SoftSign

SoftSign is a non-linear function.

- the soft version of $Sign(x)$ function.
- continuously differentiable, so it can be used in neural network, but $Sign(x)$ can not.
- A non-linear function can be used as sigmoid, tanh and so on.
- Output range is $(-1, 1)$

2 Feed Forward

2.1 Formula

$$y = SoftSign(x) = \frac{x}{1 + |x|} \quad (1)$$

$$\mathbf{y} = SoftSign(\mathbf{x}) = \left\{ \frac{\mathbf{x}_0}{1 + |\mathbf{x}_0|}, \frac{\mathbf{x}_1}{1 + |\mathbf{x}_1|}, \dots, \frac{\mathbf{x}_i}{1 + |\mathbf{x}_i|}, \dots, \frac{\mathbf{x}_N}{1 + |\mathbf{x}_N|} \right\}^T \quad (2)$$

$$\mathbf{y}_i = SoftSign(\mathbf{x}_i) = \frac{\mathbf{x}_i}{1 + |\mathbf{x}_i|} \quad (3)$$

2.2 Graph

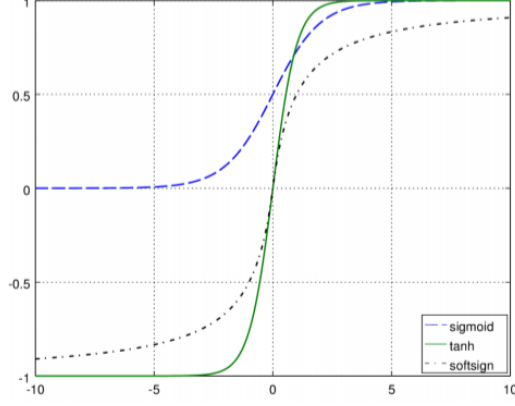


Figure 1: SoftSign compared with tanh and sigmoid

2.3 Some Discuss

- The original $Sign(x)$ is $Sign(x) = \frac{x}{|x|}$. Adding 1 on denominator is soft factor.

3 Back Propagation

$$\begin{aligned}
 & \frac{\alpha(SoftSign(x))}{\alpha(x)} \\
 &= \frac{\alpha(\frac{1}{1+|x|})}{\alpha(x)} = x' * \frac{1}{1+|x|} + x * (\frac{1}{1+|x|})' \\
 &= \frac{1}{1+|x|} + (-x) * \frac{(1+|x|)'}{(1+|x|)^2}
 \end{aligned} \tag{4}$$

$$\frac{\alpha(SoftSign(x))}{\alpha(x)} = \begin{cases} \frac{1}{1+x} + (-x) * \frac{(1+x)'}{(1+x)^2} = \frac{1}{(1+|x|)^2} & x \geq 0 \\ \frac{1}{1-x} + (-x) * \frac{(1-x)'}{(1-x)^2} = \frac{1}{(1+|x|)^2} & x < 0 \end{cases} \tag{5}$$

$$\frac{\alpha(SoftSign(x))}{\alpha(x)} = \frac{1}{(1+|x|)^2} \tag{6}$$