Zhiqiang Tian Fintech 545 09/23/2023

Week04 Project

Problem 1:

I set a uniform parameter in all types of return calculations to make it easier to compare results. The parameters are as follows:

P0	100
sigma	0.1
t	10
repeat time(n)	10000

For the Classical Brownian Motion:

$$P_{t+1} = P_t + r_t$$

$$P_{t+t} \sim N(100,\ t * sigma^2)$$

Therefore, the theoretical mean and standard deviation for Classical Brownian Motion is:

$$Mean = 100,\ Standard\ Deviation\ =\ 0.333$$

which is very close to the experimental result:

```
Mean of Classical Brownian Motionis 100.001
Standard deviation of Classical Brownian Motion is 0.302
```

For the Arithmetic Return System method:

$$P_t = P_{t-1}(1 + r_t)$$

$$P_{1+t} = P_t(1 + r_t)^t$$

$$P_{t+t} \sim N(100, P_0 * t\ * sigma^2)$$

Therefore, the theoretical mean and standard deviation for Arithmetic Return System is:

$$Mean = 100,\ Standard\ Deviation\ =\ 33.3$$

which is very close to the experimental result:

```
Mean of Arithmetic Return Systemis 100.108
Standard deviation of Arithmetic Return System is 30.657
```

For the Geometric Brownian Motion method, as we mentioned on the note:

$$P_{t+t} \sim LN(mean + ln(P_t),\ sigma^2)$$

Therefore, according to the characteristic of log distribution, the theoretical mean and standard deviation for Arithmetic Return System is:

$$Mean = 105.12,\ Standard\ Deviation = 34.09$$

which is very close to the experimental result:

```
Mean of Log Returnis 104.679
Standard deviation of Log Return is 32.136
```

Problem 2

According to the result from my code, the VaR for the 5 models are as follows:

```
VaR for Normal Distribution model is 0.05451621865393625
VaR for normal distribution with exponentially weighted variance is:
0.029777311974946645
VaR for MLE fitted T distribution is: 0.044348003552887996
VaR for AR(1) model is: 0.0013821452738234984
VAR for Historic Simulation is: 0.04202773934104901
```

These differing VaR figures reflect the varied assumptions and data handling in each model, illustrating how the choice of risk modeling approach can significantly impact the perceived risk level of the assets in question. The AR(1) model obtained the lowest VaR value, which means a lower expected maximum loss in “bad days”. The other for models shows very close VaR.

Problem 3

The two models I used in this problem are exponentially weighted covariance with delta normal method and historical simulation. The results are shown below:

```
The current value for A is: 1089316.16
VaR for A using Delta Normal is: 15426.97
VaR for A using Historic Simulation is: 17065.30

The current value for B is: 574542.41
VaR for B using Delta Normal is: 8082.57
VaR for B using Historical Simulation is: 12115.66

The current value for C is: 1387409.51
VaR for C using Delta Normal is: 18163.29
VaR for C using Historic Simulation is: 22187.13

The current value for total is: 3051268.07
VaR for total using Delta Normal is: 38941.38
VaR for total using Historical Simulation is: 46871.90
```

The delta-normal and historical simulation methods provide different approaches to estimating VaR. delta-normal relies on normal distribution assumptions and linear sensitivities (deltas), which simplifies the calculations but may not effectively capture extreme events. On the other hand, the historical simulation method, which makes no parameter assumptions and relies on actual historical data, can better capture extreme events, but this method may be less robust if future market conditions change significantly from the historical data.

The results show that the historical simulation method estimates a higher VaR than the Delta-Normal method in each case. This may indicate that the tails of the real return distribution are larger than the tails of the normal distribution, or that there are nonlinear relationships in the portfolio that are captured by the historical simulation method but not by the Delta-Normal method.