LAPORAN METODA NUMERIK

KELOMPOK 3 METODA NUMERIK C

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02. Dasar Teori

03. Soal 1

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Batasan Masalah

Batasan Masalah

1

$$p(s) = \frac{1}{\sqrt{2\pi}} \int_{-s}^{s} exp\left(-\frac{z^2}{2}\right) dz \qquad (s > 0)$$

Membuat program untuk menghitung p(s) dengan menggunakan kuadratur Gauss komposit dengan m panel dan n titik (*node*) per panel untuk menghitung integral dengan variabel s diinput oleh pengguna. Kemudian hitung integral tersebut dengan aturan trapesium dan Simpson dengan banyaknya titik k = 2, 3, 4, $\cdots$, 31 untuk aturan trapesium dan k = 3, 5, 7, $\cdots$, 31 untuk aturan Simpson. Gambarkan :

1. Garis lurus yang merupakan nilai dari integral (1) dengan menggunakan kuadratur Gauss komposit.
2. Ke-30 nilai integral (1) yang diperoleh dari aturan trapesium.
3. Ke-15 nilai integral (1) yang diperoleh dari aturan Simpson.

Batasan Masalah

2

$$I = \int_0^{\infty} \frac{2}{1+x^2} dx$$

Menghitung dan menjelaskan integral tak wajar secara numerik dengan nilai eksak

$$I = \pi$$

Dasar Teori

Kuadratur Gauss Komposit

$$I \cong \int_a^b f(x)dx$$

Diubah menjadi

$$I \cong \int_{-1}^{1} f(t)dt = c_1 f(x_1) + c_2 f(x_2) + \cdots + c_n f(x_n)$$

dengan

$$x = \frac{(a+b) + (b-a)t}{2}$$

$$dx = \frac{(b-a)}{2} dt$$

Dasar Teori

Kuadratur Gauss Komposit

Metode Gauss-Legendre n-titik

$$\int_{-1}^{1} f(x)dt \approx c_1 f(x_1) + c_2 f(x_2) + \ldots + c_n f(x_n)$$

n	Faktor bobot	Argumen fungsi	Galat pemotongan
2	$c_1 = 1.000000000$ $c_2 = 1.000000000$	$x_1 = -0.577350269$ $x_2 = 0.577350269$	$\approx f^{(4)}(c)$
3	$c_1 = 0.555555556$ $c_2 = 0.888888889$ $c_3 = 0.555555556$	$x_1 = -0.774596669$ $x_2 = 0$ $x_1 = 0.774596669$	$\approx f^{(6)}(c)$
4	$c_1 = 0.347854845$ $c_2 = 0.652145155$ $c_3 = 0.652145155$ $c_3 = 0.347854845$	$x_1 = -0.861136312$ $x_2 = -0.339981044$ $x_3 = 0.339981044$ $x_4 = 0.861136312$	$\approx f^{(8)}(c)$
5	$c_1 = 0.236926885$ $c_2 = 0.478628670$ $c_3 = 0.568888889$ $c_4 = 0.478628670$ $c_5 = 0.236926885$	$x_1 = -0.906179846$ $x_2 = -0.538469310$ $x_3 = 0$ $x_4 = 0.538469310$ $x_5 = 0.906179846$	$\approx f^{(10)}(c)$
6	$c_1 = 0.171324492$ $c_2 = 0.360761573$ $c_3 = 0.467913935$ $c_4 = 0.467913935$ $c_5 = 0.360761573$ $c_6 = 0.171324492$	$x_1 = -0.932469514$ $x_2 = -0.661209386$ $x_3 = -0.238619186$ $x_4 = 0.238619186$ $x_5 = 0.661209386$ $x_6 = 0.932469514$	$\approx f^{(12)}(c)$

Aturan Trapesium

Aturan trapesium 1 segmen

$$I \cong \int_a^b f_1(x)dx = h\left(\frac{f(a)+f(b)}{2}\right)$$

dengan h = (b-a)/1

Aturan trapesium n segmen

$$I = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \cdots + \int_{x_{n-1}}^{x_n} f(x)dx$$

$$I \cong h\frac{f(x_0)+f(x_1)}{2} + h\frac{f(x_1)+f(x_2)}{2} + \cdots + h\frac{f(x_{n-1})+f(x_n)}{2}$$

$$I \cong \frac{h}{2}[f(x_0)+f(x_1)+f(x_1)+f(x_2)+\cdots+f(x_{n-1})+f(x_n)]$$

$$I \cong \frac{h}{2}\left[f(x_0)+\left(2\sum_{i=1}^{n-1} f(x_i)\right)+f(x_n)\right]$$

dengan h = (b-a)/n

Aturan Simpson's 1/3

Aturan simpson's ⅓ 1 segmen

$$I \cong \int_a^b f_2(x)dx = h\frac{f(x_0) + 4f(x_1) + f(x_2)}{3}$$

dengan h = (b-a)/2

Aturan simpson's ⅓ n segmen

$$I = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \cdots + \int_{x_{n-2}}^{x_n} f(x)dx$$

$$I \cong h\frac{f(x_0) + 4f(x_1) + f(x_2)}{3} + h\frac{f(x_2) + 4f(x_3) + f(x_4)}{3} + \cdots + h\frac{f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)}{3}$$

$$I \cong \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n)]$$

$$I = \frac{h}{3}\left[f(x_0) + \left(4\sum_{i=2k-1}^{n-1} f(x_i)\right) + \left(2\sum_{j=2k}^{n-2} f(x_j)\right) + f(x_n)\right]$$

dengan h = (b-a)/n

Integral Tak Wajar

Identitas Integral Tak Wajar

$$\int_a^b f(x)\,dx = \int_{\frac{1}{b}}^{\frac{1}{a}} \frac{1}{t^2} f\left(\frac{1}{t}\right) dt \qquad , ab > 0$$

$$\int_{-\infty}^{b} f(x)dx = \int_{-\infty}^{-A} f(x)dx + \int_{-A}^{b} f(x)dx$$

Extended Midpoint Rule

$$\int_{x_0}^{x_n} f(x)dx = h\left[\frac{3}{2}f(x_1) + \sum_{i=2}^{n-2} f(x_i) + \frac{3}{2}f(x_{n-1})\right]$$

$$\int_{x_0}^{x_n} f(x)dx = h\left[\frac{3}{2}f(x_1) + \sum_{i=2}^{n-1} f(x_i) + \frac{3}{2}f(x_n)\right]$$

$$\int_{x_0}^{x_n} f(x)dx = h\left[f\left(x_{1/2}\right) + f\left(x_{3/2}\right) + \cdots + f\left(x_{(n-3)/2}\right) + f\left(x_{(n-1)/2}\right)\right]$$

Galat (*Error*)

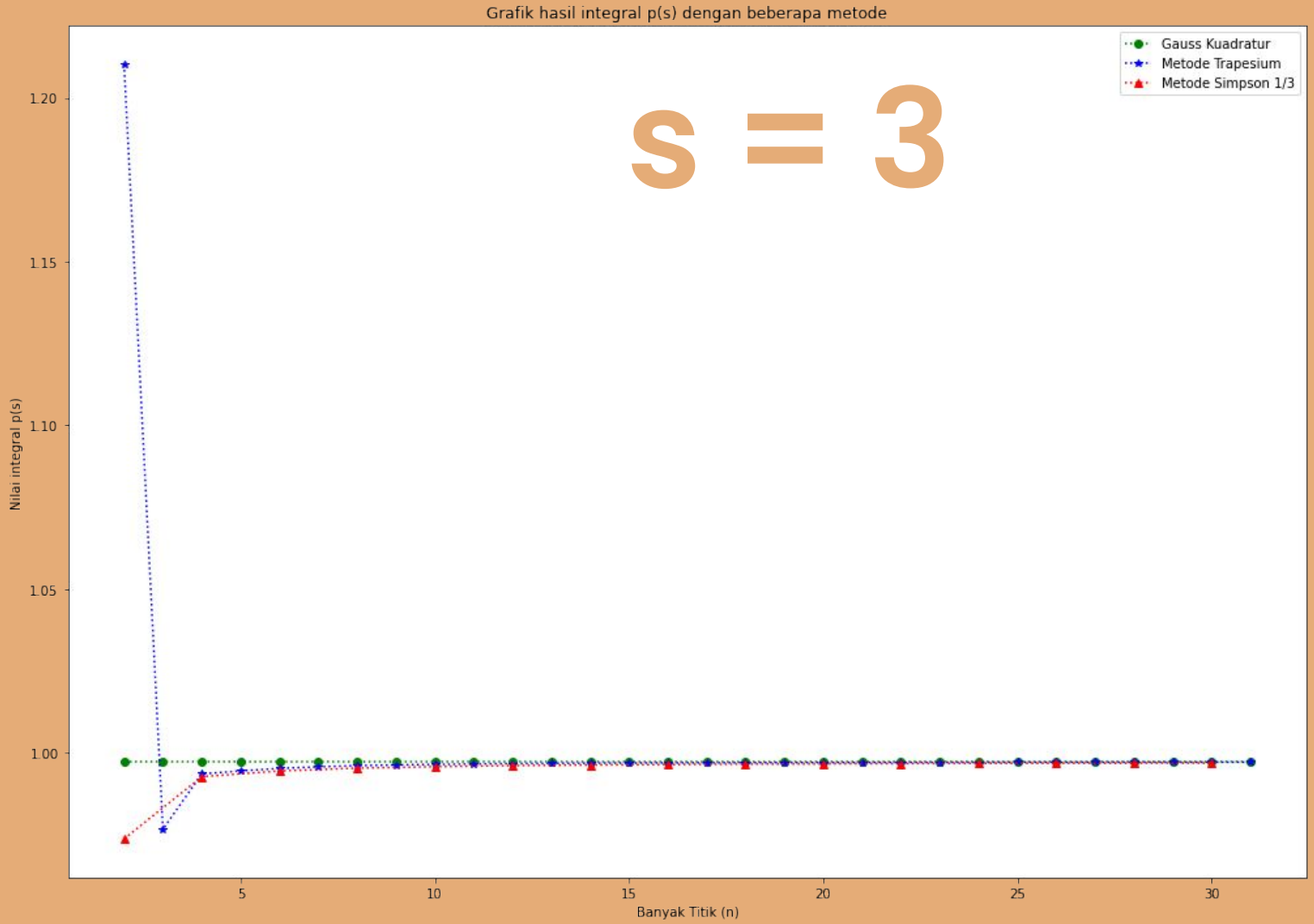
$$\varepsilon_t = \left| \frac{nilai\ sebenarnya - nilai\ aproksimasi}{nilai\ sebenarnya} \right| \times 100\%$$

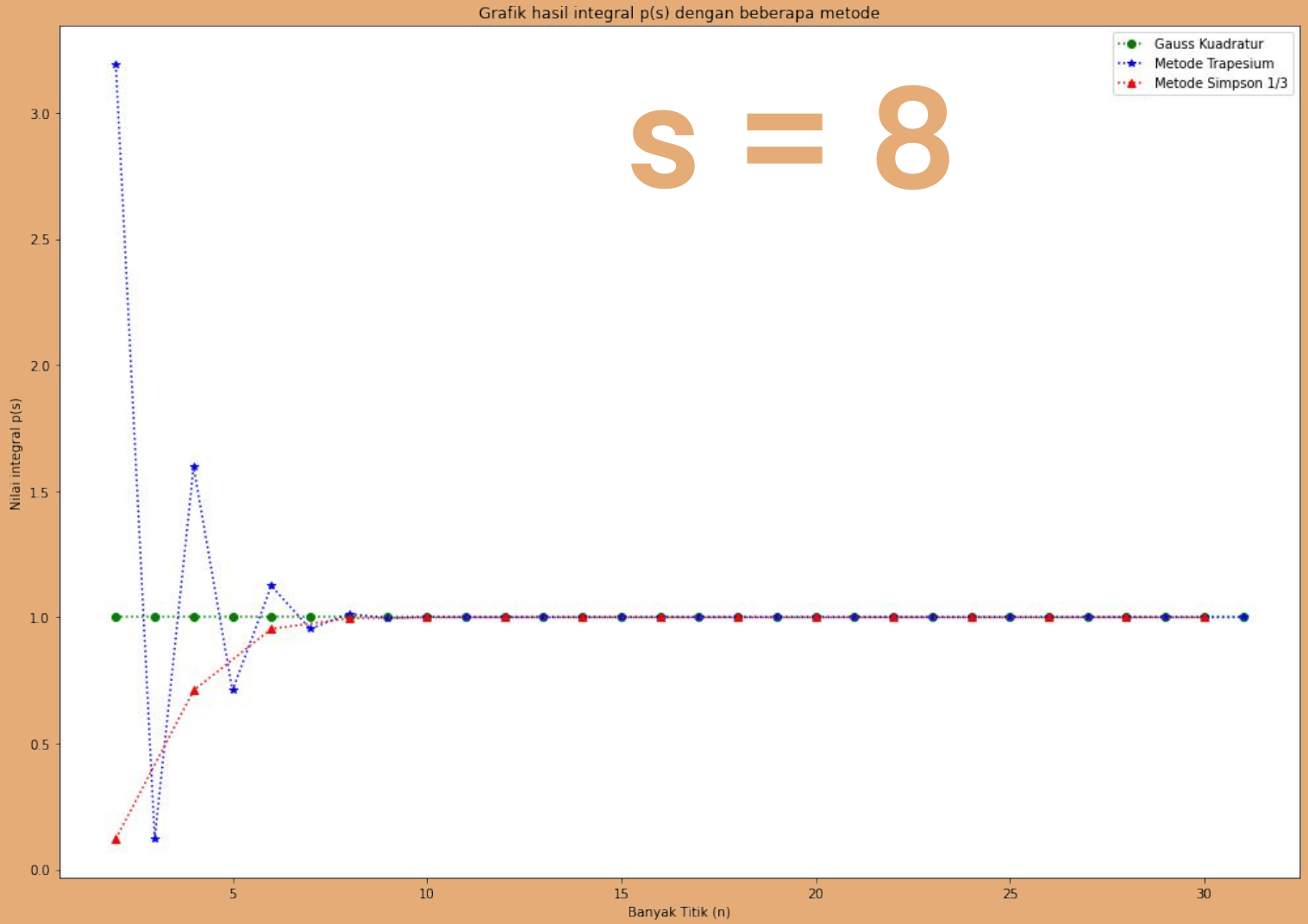
SOAL 1

Tabel perhitungan untuk s=3					
n	Gauss	Trapesium	Galat Relatif Trapesium (%)	Simpson 1/3	Galat Relatif Simpson ⅓ (%)
2	0,997300203936741	1,210122386440112	21,339831		
3	0,997300203936741	0,9767465949004495	2,060925	0,9737920293	2,357181
4	0,997300203936741	0,9936139802177313	0,369620		
5	0,997300203936741	0,9945376449567906	0,277004	0,9927649056	0,454758
6	0,997300203936741	0,9952975108780335	0,200811		
7	0,997300203936741	0,9957888011035815	0,151549	0,9945225587	0,278516
8	0,997300203936741	0,9961226160945735	0,118078		
9	0,997300203936741	0,9963585390727517	0,094421	0,9953736839	0,193174
10	0,997300203936741	0,9965308930529221	0,077139		
11	0,997300203936741	0,9966603734041711	0,064156	0,9958545828	0,144953
12	0,997300203936741	0,9967599783627766	0,054169		

Tabel perhitungan untuk s=3					
n	Gauss	Trapesium	Galat Relatif Trapesium (%)	Simpson 1/3	Galat Relatif Simpson ⅓ (%)
13	0,997300203936741	0,9968381707491409	0,046328	0,9961563479	0,114695
14	0,997300203936741	0,9969006370242968	0,040065		
15	0,997300203936741	0,9969513052871393	0,034984	0,9963603922	0,094236
16	0,997300203936741	0,9969929558490305	0,030808		
17	0,997300203936741	0,9970275994120714	0,027334	0,9965062055	0,079615
18	0,997300203936741	0,9970567181604074	0,024414		
19	0,997300203936741	0,9970814234880235	0,021937	0,9966149131	0,068715
20	0,997300203936741	0,9971025616184731	0,019818		
21	0,997300203936741	0,9971207858323173	0,017990	0,996698705	0,060313
22	0,997300203936741	0,9971366068073556	0,016404		
23	0,997300203936741	0,9971504283153513	0,015018	0,9967650502	0,053660

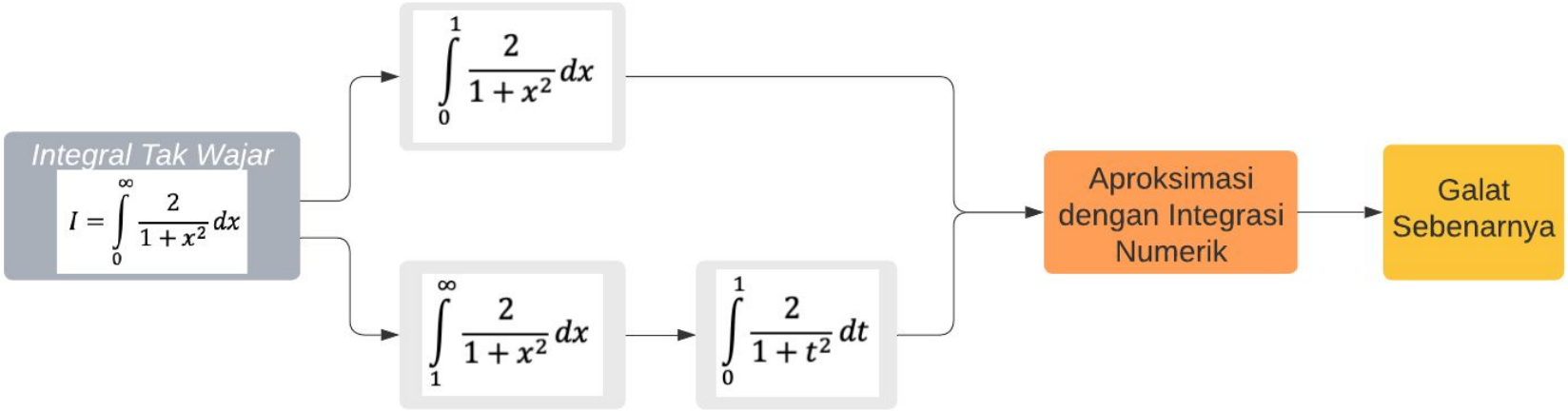
Tabel perhitungan untuk s=3					
n	Gauss	Trapesium	Galat Relatif Trapesium (%)	Simpson 1/3	Galat Relatif Simpson ⅓ (%)
25	0,997300203936741	0,9971733008525546	0,012725	0,996818753	0,048275
26	0,997300203936741	0,9971828237623732	0,011770		
27	0,997300203936741	0,9971913153151121	0,010918	0,9968630302	0,043836
28	0,997300203936741	0,9971989190180939	0,010156		
29	0,997300203936741	0,9972057542949423	0,009471	0,9969001096	0,040118
30	0,997300203936741	0,9972119211712503	0,008852		
31	0,997300203936741	0,9972175039315448	0,008292	0,9969315782	0,036962


Grafik hasil integral p(s) dengan beberapa metode
Gauss Kuadratur
Metode Trapesium
Metode Simpson 1/3
s = 3
Nilai integral p(s)
1.20
1.15
1.10
1.05
1.00
5
10
15
20
25
30
Banyak Titik (n)


Grafik hasil integral p(s) dengan beberapa metode
Gauss Kuadratur
Metode Trapesium
Metode Simpson 1/3
s = 8
Nilai integral p(s)
Banyak Titik (n)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
5
10
15
20
25
30

SOAL 2

Algoritma Aproksimasi Integral Tak Wajar



Aproksimasi Integral *I*

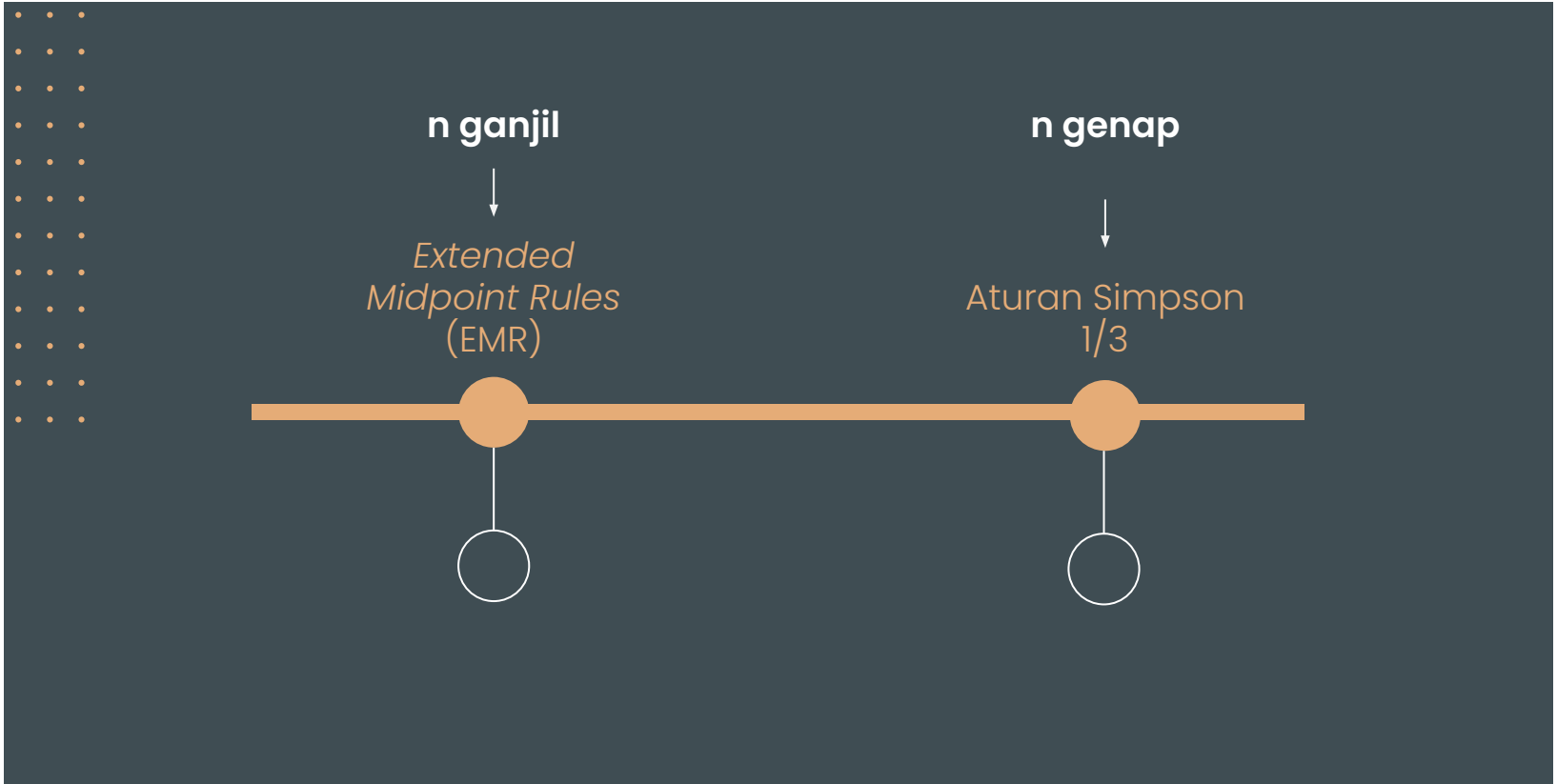
Perbandingan Hasil Aproksimasi

n	Hasil Aproksimasi		
	Aturan Trapesium	Aturan Simpson 1/3	EMR
2	3,10000	3,13333	3,16235
3	3,12308	-	3,15085
4	3,13188	3,14157	3,14680
5	3,13493	-	3,14493
10	3,13493	3,14159	3,14243
15	3,14085	-	3,14196
20	3,14118	3,14159	3,14180
25	3,14133	-	3,14178
50	3,14153	3,14159	3,14163
101	3,14158	-	3,14160
1000	3,14159	3,14159	3,14590

Perbandingan Hasil Galat *(Error)*

n	Hasil Galat *(Error)*		
	Aturan Trapesium	Aturan Simpson 1/3	EMR
2	1,32394%	0,26290%	0,66082%
3	0,58937%	-	0,29465%
4	0,33156%	0,00076%	0,16577%
5	0,21220%	-	0,1061%
10	0,05305%	0,00000%	0,2653%
15	0,02358%	-	0,01179%
20	0,01326%	0,00000%	0,00663%
25	0,00849%	-	0,00424%
50	0,00212%	0,00000%	0,00106%
101	0,00052%	-	0,00026%
1000	0,00001%	0,00000%	0,00000%

Kesimpulan


n ganjil
Extended Midpoint Rules (EMR)
n genap
Aturan Simpson 1/3

PYTHON

TERIMA KASIH