Spatial Tessellations

Second Edition

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Spatial Tessellations: Concepts and Applications of Voronoi Diagrams

Second Edition

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Foreword to the First Edition

I was delighted to be asked to write a preface to this beautiful and outstandingly original book. It is the unique treatise on its subject, it fills a serious gap in the literature and it covers the theory and the huge range of applications in a masterly way.

The authors are right to distinguish Voronoi *diagrams* and Delone *tessellations*. The Delone construction decomposes a Euclidean space of m dimensions, containing a given set of points, into non-overlapping space-filling *simplexes* (not, of course, all of the same shape and size), so that it tessellates the space using tiles that are identical with one another up to linear transformations. The Voronoi construction also splits up the space into polyhedral cells, but now they are much less uniform in character – the number of faces will vary from one cell to another, so that it would be wrong to call the result a tessellation.

These mutually dual procedures give fascinating but different insights into the structure of a set of points in m dimensions, and they have found numerous applications. At the time of writing there is a new application on the largest of all possible scales which throws light on the structure of the Universe as we see it. This will be seen as a particularly interesting development when one recalls that most of the earlier applications (for example, to the study of the structure of metallic composites, and other such aggregates) were on the microscopic scale. The reader of this book is strongly urged to look at a review paper just published by Icke and van de Weygaert (*Quarterly Journal, Royal Astronomical Society*, **32**, 85–112). There it is shown that the Voronoi construction not only gives insight into the distribution of galaxies, but also permits a new approach to the dynamics that mould the shape of the universe we live in.

My own contributions have been in the Delone tradition, and are concerned (for example) with the way in which high-dimensional Delone simplexes pack together around a common vertex. Thus in 15 dimensions the number of such locally associated simplexes turns out to be of the order of 44 million million. This implies a related statement about the Voronoi polyhedra, and there tells us something about the number of faces of an individual cell. It seems likely that the huge number of Delone simplexes in such a local 'fan' can be roughly partitioned into a moderate number of 'chunky' simplexes (substantial faces in the Voronoi case), and a vast number of 'needle-like' ones (tiny faces), but we have no precise information on this matter at the moment.

It is a great pleasure to welcome this book to the Wiley series.

David Kendall

Preface to the Second Edition

The First Edition of this book was published in 1992. In 1995, it was reprinted. At that time, we suggested to the publishers that, given the continuing interest in Voronoi diagrams in so many quarters, rather than consider further reprints they allow us to prepare a new, revised, Second Edition. We were pleased to receive a positive reply and so this volume was born.

While this edition maintains the overall structure of the first, there are substantial changes in the content. In particular, on-going growth in research relating to Voronoi diagrams is reflected in the addition of much new material to this volume. Although such additions occur throughout the book, they are most visible in new generalizations of the ordinary Voronoi diagram, new and revised results relating the Poisson Voronoi diagram, and new applications of all forms of Voronoi diagrams. The growth in Voronoi diagram research is also manifest in several other ways. One is the presence of a fourth author, Sung Nok Chiu, without whose contribution the original three authors would probably still be labouring over the revisions. Another is the increase in the number of references from 677 in the First Edition to 1680, 523 of which have appeared since the First Edition was published.

In order to accommodate the new developments we have omitted some material from the First Edition. This is most obvious in the mathematical preliminaries in Chapter 1 where we have omitted the sections relating to matrices, derivatives, integration and probability.

This book is accompanied by a World Wide Web site (`http://okabe.t.u-tokyo.ac.jp/okabelab/Voronoi/index.html`) which provides additional material such as pointers to available Voronoi diagrams and related geometric software and other Web sites featuring Voronoi diagrams. Our WWW page can also be used to notify us of any errors. Although the text has been proofread many times by ourselves and others, it is inevitable that some logical and typographical errors will not have been detected. We will correct any errors we become aware of and provide an Errata list on our WWW page.

December 1998 The authors

Acknowledgments (First Edition)

So many people helped in so many ways during the preparation of this book that it is only possible to acknowledge a few of them individually. First, we are deeply grateful to D.G. Kendall, who read through the draft and encouraged its publication; and to Y. Asami, C.M. Hoffmann, M. Iri, K. Murota and A. Suzuki, who suggested or commented on parts of the draft. Our special thanks also go to D.A. Aboav, F. Aurenhammer, H. Edelsbrunner, S. Egginton, J.D. Embury, M.F. Goodchild, M. Hori, H.-C. Imhof, G. Le Caër, U. Lorz, J. Mecke, R.E. Miles, J. Møller, L. Muche, Y. Ohsawa, N. Rivier, Y.M. Seoung, D. Stoyan, T. Suzuki, M. Tanemura, G. Toussaint, D.S. Wilkinson, H. Yomono and L. Zaninetti, who provided material. We must also express our debt to A. Dawkins, S. Henry H. Honkers, J. Horton, T. Kaneko, O. Kurita, R. Metcalfe, P. Schaus, M. Stone and T. Yoshikawa, among others, who assisted in production. For the help that they have given us, we are indebted to the staff of the publisher, in particular, C. Farmer. S. Gale, J. Narain and H. Ramsey. We should also acknowledge the award of a Book Preparation Grant from Wilfrid Laurier University which helped meet costs incurred during the preparation of the manuscript. Finally we are grateful for academic e-mail networks which made us feel as if we had been working in the same office.

Acknowledgments (Second Edition)

As with the First Edition, so many people helped us in different ways in the preparation of this edition that it is impossible to acknowledge all of them individually. However, we are especially indebted to two individuals who exposed us to significant applications which were either overlooked or received only passing reference in the first edition. Initially by means of a footnote in Oden *et al.* (1993) and later by direct communication, H. Goebl revealed the use of Voronoi diagrams in linguistics, while E. Agrell, by way of his book (Agrell, 1997), showed us how much we had missed on the use of Voronoi diagrams in coding. For their comments and suggestions on the First Edition or drafts of this edition we would like to thank H. Imai, K. Imai, R. Klein, R.C. Lindenbergh, U. Lorz, M. McAllister, L. Muche, T. Roos, M. Schlather, N. Shiode, and C.A. Wang. We would also like to thank those who generously shared their unpublished research or other material with us, C. Gold, U. Lorz, K. McLeod, C. Moukarzel, L. Muche, K. Ohnishi, M. Schlather, D. Stoyan, and D. Watson. Thanks are also due to those who assisted in the production of this edition, especially P. Churcher, S. Horiike, J. Horton, C. Kanasaki, T. Kuroiwa, M. Lefebvre, H. Rayner, P. Schaus, and C. Yoshimoto. Finally, it is again a pleasure to acknowledge the help and guidance we have received from the staff of the publisher, in particular, S. Clutton, S. Corney, and H. Ramsey, who dealt with the idiosyncrasies of four authors scattered around the globe with both patience and good humour.