Implementation of Various Methods to Predict Call and Put Option Prices in NASDAQ 100

Petra Abdi Paskalisa[1,a)] and Tiara Yosianti Solekhah[2,b)]
[1-2]Statistics Study Program, Faculty of Mathematics and Natural Sciences, Gadjah Mada University, Sekip Utara BLS 21 Yogyakarta, Indonesia

a) petra.abdi.paskalisa@mail.ugm.ac.id, b) tiara.y@mail.ugm.ac.id

Abstract. Options are still one of the most important financial instruments in risk management despite the recent global financial crisis. An option is the right to buy and sell a security or asset at an agreed value at a certain time within a contract period. The options themselves are divided into two namely put options and call options. An investor must be aware of the fair price of the option when buying or selling to hedge risks using options. In order to help the buyer determine the fairness of the option's price, the simulation of various pricing models for an option can be used. Some of these methods are the Black Scholes method, the Black Scholes Gram Charlier Expansion, Variance Gamma, and Variate Antithetic Reduction Variance Gamma. The shares chosen in this research are shares of Nasdaq 100 namely Advanced Micro Devices, Inc. (AMD), Amazon.com, Inc. (AMZN), Intel Corporation (INTC), Lucid Group, Inc. (LCID), and NVIDIA Corporation (NVDA). As the result, the Black Scholes model with Gram Charlier expansion gives better result for normally distributed data. As for variance gamma distributed return, the gamma variance method provides a smaller MAE compared to the gamma variance with AVR (Antithetic Variance Reduction). Overall for the call and put option, the prediction shows that the price is still relatively reasonable.

Keywords: Option, Black Scholes, Black Scholes with Gram Charlier Expansion, Variance Gamma, Variate Antithetic Reduction Variance Gamma

1. Introduction

1.1. Background Study

Options are still one of the most important financial instruments in risk management despite the recent global financial crisis. An option is the right to buy and sell a security or asset at an agreed value at a certain time within a contract period (Smith, [1]). Options are divided into two, namely put options which are the right to sell securities or assets and call options which are the right to buy securities or assets. Options gave attractive chances for investors who want to be more careful in their investments. Options can give investors the right to buy securities or assets at a relatively cheap price (below the price traded in the market). Meanwhile, the common type of option chosen among investors is stock options.

An investor must be aware of the fair price of the option when buying or selling to hedge risks using options. In order to help the buyer determine the fairness of the option's price, the simulation of various pricing models for an option can be used. Some of these methods are the Black Scholes method, the Black Scholes Gram Charlier Expansion, Variance Gamma, and Variate Antithetic Reduction Variance Gamma.

The stock's return was tested for normality and then the option prices were simulated using the Black Scholes and the Black Scholes Gram Charlier Expansion. The results will be compared to determine whether there is a difference between the two methods of predicting option prices and conclude which method is the best for normally distributed return. As for the non-normally distributed returns, Variance Gamma distribution will be carried out before using the Variance Gamma and Variate Antithetic Reduction Variance Gamma simulation to determine option prices. Based on the results obtained, an assessment of which method provides the best performance in option price prediction will be carried out.

1.2. **Research Objectives**

1) Investigating the reasonability of the options price in the market for return that is normally distributed
2) Compare the Black Scholes method and Black Scholes with Gram-Charlier expansion method to find the most efficient methods to predict stock options price for normally distributed return
3) Investigating the reasonability of the options price in the market for return that variance gamma distributed
4) Compare the Variance Gamma method and Antithetic Variate Reduction method to find the most efficient methods to predict stock options price for variance gamma distributed return

2. **Literature Review**

2.1. **Return**

According to Ruppert [2], return is the rate of return or results obtained as a result of investing. There are several types of returns:
1) Net Return, the net profit that can be obtained from an investment. If P_t is the stock price at the time t and P_{t-1} is the stock price at $t-1$ assuming no dividends, then the net return for the period $t-1$ to t can be calculated as follows

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}$$

2) Gross Return, the total return value of an investment before deducting various expenses. In simple terms, gross return can be calculated as follows:

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

3) Log Return or also called continuously compounded return is denoted by r_t and defined as follows:

$$r_t = ln(1 + R_t) = ln\left(\frac{P_t}{P_{t-1}}\right)$$

The log return has a value that is more or less the same as the net return for small returns. This is because $ln(1+x) \approx x$ if $|x| < 0.1$.
In this study, log returns will be used to calculate stock prices

2.2. **Normal Distribution and Bowman Shelton Test**

The normal distribution is a probability function that shows the distribution of variables. The function is generally proven by a symmetrical graph called a bell curve. When indicating an even distribution, the curve will peak in the middle and slope on both sides by equal values.

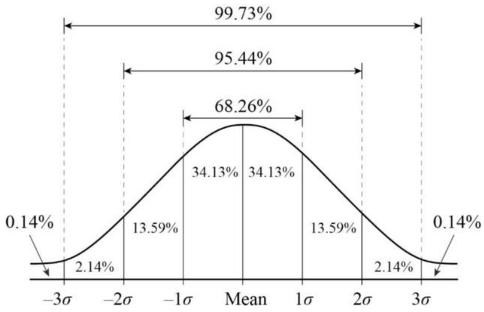

Figure 1. Normal Distribution Characteristics

This distribution theory is also known as the Gaussian Distribution. The term refers to Carl Friedrich Gauss, a German mathematician who developed the theory of distributions containing two-parameter exponential functions in the period 1794-1809. However, the initial theory that became the origin of the distribution function was actually developed by Abraham de Moivre in 1733.

The characteristics of the normal distribution include a bell-shaped distribution curve, symmetrical to μ, the value of the mean, median, and mode are relatively the same, μ (mean) is the center of the bell. The larger σ^2, the more spread the normal distribution is. The probability function of the normal distribution is:

$$P(X = x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

To determine whether the data is normally distributed or not, several techniques can be used, one of which is the Bowman Shelton test. This test is used to test the normality of the data. The Bowman-Shelton test uses measurements of skewness and kurtosis. The Bowman-Shelton statistic follows a chi-square distribution with two degrees of freedom for a large sample. The null hypothesis in this test is that the data follow a normal distribution.

$$BS = n\left[\frac{skewness^2}{6} + \frac{(kurtosis-3)^2}{24}\right]$$

$$Skewness = \frac{\frac{1}{n}\Sigma\left(x_i - \underline{x}\right)^3}{\left(\frac{1}{n}\Sigma\left(x_i - \underline{x}^2\right)^{3/2}\right.}$$

$$Kurtosis = \frac{\frac{1}{n}\Sigma\left(x_i - \underline{x}\right)^4}{\left(\frac{1}{n}\Sigma\left(x_i - \underline{x}^2\right)^2\right.}$$

2.3. **Gamma Distribution**

The Gamma distribution gets its name from the gamma function which is well-known and studied in many fields of mathematics. Gamma distribution is often applied in queuing theory and reliability theory.

$$f(x;\alpha;\beta)\{\frac{1}{\beta^{\alpha}\,\Gamma(\alpha)}\, x^{\alpha-1}e^{-\frac{x}{\beta}},\quad x > 0, 0\ \ otherwise$$

where $\alpha > 0$ and $\beta > 0$

Some of the characteristics of the gamma distribution are the mean $E(X) = \alpha\beta$, the variance which has a value of $Var(X) = \alpha\beta^2$, the moment generating function $Mx(t) = (1 - \beta t)^{-\alpha}$, the characteristics function $Cx(t) = (1 - \beta it)^{-\alpha}$, and the probability generating function $Gx(t) = (1 - \beta \ln t)^{-\alpha}$.

2.4. Black-Scholes Model

The Black-Scholes Model is a model developed by Fischer Black dan Myron Scholes to determine option prices that have been widely accepted by the financial society. The formula for calculating the option price with the Black-Scholes model is as follows:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

It can be further formulated as follow:

$$Call_{BS} = S_0 N\left(\frac{ln\, ln\, \left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) - Ke^{-rt}N\left(\frac{ln\, ln\, \left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

$$Put_{BS} = Ke^{-rt}N\left(-\frac{ln\, ln\, \left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

$$-S_0 N\left(-\frac{ln\, ln\, \left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

where

$S(t)$	: the spot price of the underlying asset at time t
$V(S,t)$	: the option price as a function of the underlying asset S at time t
r	: risk free interest rate
σ	: the return volatility of the underlying asset
T	: time to maturity in years
t	: current time in years
K	: strike price
S_0	: stock's last price

2.5. Black-Scholes Model with Gram-Charlier Expansion

The continuous stock price return for n periods is

$$ln\,(S_{t+n}) = ln\,(S_t) + x_{t+1}^n \qquad \dots \textbf{(1)}$$

where: x_{t+1}^n : sum of stock price returns for n periods
S_t : stock price at time t
S_{t+n} : stock price at time t+n

The initial assumption in determining the price of the European call option is a risk-neutral. This means that the assets owned are not risky, so the price of European call options with a strike price of *K* for n periods is:

$$C_{nt} = E_t\{M_{t,t+n}(S_{t+n} - K)^+\} \quad \cdots \textbf{(2)}$$

where: $M_{t,t+n}$ is the multi-period stochastic discount factor

To form a Gram-Charlier approximation, the first step is to define a standard variable. Let *w* be a standard variable. Using equation (1) where x_{t+1}^n has mean μ_n and standard deviation σ_n, then

$$w = \frac{x_{t+1}^n - \mu_n}{\sigma_n}$$

Using equation (2), the Gram-Charlier expansion which defines a density approximation for *w* is:

$$f(w) = \varphi(w)\{1 + \frac{1}{2}(\mu_2 - 1)H_2 + \frac{1}{6}\mu_3 H_3 + \frac{1}{24}(\mu_4 - 6\mu_2 + 3)H_4 + \cdots\}$$

In this case, it will not only represent the mean and variance but will also show the behavior of skewness dan kurtosis, so the moment used is only the first moments. While the higher moments are ignored. Thus obtained:

$$f(w) = \varphi(w) - \gamma_{1n}\frac{1}{3!}D^3\varphi(w) + \gamma_{2n}\frac{1}{4!}D^4\varphi(w) \qquad \ldots\textbf{(3)}$$

where: $\varphi(w) = \frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}$ is the standard normal density

$D^r\varphi(w) = (-1)^r H_\gamma \varphi(w)$

$K_1 = 0, \mu_2 = K_2 = 1, \mu_3 = K_3, \mu_4 = K_4 + 3K_2^2 = K_4 + 3$

Equation (3) is the density function of the Gram-Charlier expansion which shows that the value of γ_{1n} and γ_{2n} are not zero and affect the density values clearly.

The assumption used in the Black-Scholes model is that stock returns are normally distributed. To calculate the European call price using the Black Scholes formula, it is necessary to have a density function of the return price which is normally distributed. In reality, it is difficult to find stock prices that are normally distributed so the determination of option prices using the Black Scholes model becomes less accurate. Therefore, a new density function is needed which is expanded from the normal density function in which there is a normal density plus a correction factor in the form of skewness and kurtosis elements which is called the **Gram-Charlier** expansion. This is quite reasonable because the Gram Charlier expansion is a description of the Taylor series which uses the first 4 moments. In this expansion, the call option price depends on the value of skewness and kurtosis. Therefore, it is very important to know the behavior of skewness and kurtosis when determining option prices. The call option price based on the Gram-Charlier expansion is

$$C_{GC} = e^{-rT}E[max(S_T - K, 0)]$$

$$= C_{BS} + \frac{\mu_3}{3!}I_1 + \frac{(\mu_4 - 3)}{4!}I_2$$

$$= C_{BS} + \mu_3 Q_3 + (\mu_4 - 3)Q_4$$

where:
$C_{BS} = S_0[N(d_1)] = Ke^{-rT}[N(d_2)]$
μ_3 = skewness
μ_4 = kurtosis
$Q_3 = \frac{1}{3!}S_0\sigma\sqrt{T}(n(d_1)(2\sigma\sqrt{T} - d_1) + \sigma^2 TN(d_1))$
$Q_4 = \frac{1}{3!}S_0\sigma\sqrt{T}(n(d_1)(d_1^2 - 3\sigma\sqrt{T}(d_1 - \sigma\sqrt{T}) - 1) + (\sigma\sqrt{T})^3 N(d_1))$
$d_1 = \frac{ln\frac{S_0}{K} + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$
$d_2 = d_1 - \sigma\sqrt{T}$

2.6. Variance Gamma Model

In order to be able to model the prices of various derivatives, such as options, a stochastic differential equation is needed as a model for the movement of the underlying assets attached to the derivative. There are several models that can be used to describe the movement of underlying assets, such as the Geometric Brownian Motion model, Diffusion Potential model, and Gamma Process.
A Gamma process with mean μ and variance v, written $G(t;\alpha;v)$, is defined as a continuous-time process whose increments are stationary and independent so that for each $dt > 0$,

$$G(t + dt;\mu;v) - G(t;\mu;v) \sim \Gamma(\frac{\mu^2 dt}{v}, \frac{v}{\mu})$$

where $\Gamma\,(.)$ represents the probability density function of the gamma distribution with parameter $\frac{\mu^2 dt}{v}, \frac{v}{\mu}$.
Madan, Carr, and Chang [3] introduced an asymmetric Gamma variance process with three parameters. The first and second parameters can control variance and kurtosis while the third parameter can control the slope (skewness).
Consider Brownian motion with drift e and volatility σ which is expressed by the stochastic differential equation:

$$b(t;\theta;\sigma) = \theta t + \sigma W(t)$$

with $W(t)$ is the standard Brownian Motion.
The Gamma Variance (VG) process can be obtained from Brownian motion by substituting random time change with the gamma process. Using the subordinates of Brownian motion $b(t:\theta;\sigma)$, the VG process can be expressed as a function of the Brownian motion $b(t:\theta;\sigma)$ and the gamma process $G(t;\ 1,v)$:

$$X(t;\sigma,v,\theta) = b(G(t,1,v);\theta;\sigma)$$

Assuming a risk-neutral process, the asset price movement process following the VG process is given by the equation:

$$S(t) = S(0)\ exp\ exp\ (rt + X(t,\sigma,v,theta) + \omega t\ )$$

where $\omega = \frac{1}{v}\ ln\ ln\ (1 - \theta v - 0.5\ \sigma^2 v)$.

2.7. Gamma Variant Model with Antithetic Variate Reduction Simulation

Antithetic variate is a technique for reducing variance in the Monte Carlo method. The principle of this method in reducing variance is to use a pair of random variables that are identical and negatively correlated. Although this method uses a pair of random variables, the random number generation in this method will only be done once. While other random variables can be obtained by utilizing the nature of the distribution of random variables that have been generated. For example, a pair of random variables with uniform distribution $U(0,1)$, then the random variable generated in this method is only X. While random variable Y can be determined by $Y = 1 - X$, because the value of X in in the range (0.1) then Y is also uniformly distributed. So $f(X)$ and $f(Y)$ will be negatively correlated with f being a monotonic function.

$$a_{MC\_AV} - \frac{Z_{\frac{\alpha}{2}} b_{MC-AV}}{\sqrt{M}}, a_{MC_{AV}} + \frac{Z_{\alpha/2} b_{MC-AV}}{\sqrt{M}}$$

3. Results and Discussion

3.1. Data and Variables

The data used in this study is data on stocks and options listed on the NASDAQ 100 and the data carried out from the finance.yahoo.com page. The Nasdaq-100 Index is a collection of the 100 largest and most influential companies listed on the Nasdaq stock exchange.

The stock data used is stock closing price data from December 10, 2021 to December 9, 2022. The options data used are call options and put options and all expire on January 13, 2023.

The stocks chosen are Nvidia (NVDA), Advanced Micro Devices (AMD), Amazon (AMZN), Intel (INTC), and Lucid Group (LCID). The strike price and market price of call and put options are on December 9, 2022. The last price of the option is also on December 9, 2022. The risk-free interest rate (r) used refers to the US Treasury which is 3.57%.

3.2. Bowman Shelton Normality Test

In this section, the normality of the data is investigated using the Bowman-Shelton normality test. The null hypothesis is data that is normally distributed and the alternative hypothesis is that the data is not normally distributed. By taking the significance level at$\alpha = 0.05$, then H_0 will be rejected if p-value < α. From the computational results using R, the following results are obtained:

Table 1. Normality Test Result

Stock	Bowman -Shelton	P-Value	Skewness	Kurtosis	Mean	Variance
AMD	5.284245	0.07120997	-0.0611	3.70024	-0.0028	0.0015268
AMZN	104.1966	2.366e-23	0.003465	6.156418	-0.0026	0.000981
INTC	52.3918	4.2001e-12	0.07207	5.23356	-0.00232	0.0005725

LCID	4.300226	0.116471	-0.23647	3.43301	-0.00585	0.0025776
NVDA	0.654415	0.720934	0.1244767	2.975591	-0.00229	0.001595

Among the 5 stocks, there were 3 stocks that were normally distributed, namely AMD, LCID, and NVDA with p-values greater than 0.05, and proved by skewness and kurtosis which resembled a normal distribution. Meanwhile, the other stocks (AMZN and INTC) are not normally distributed. It can be seen by the very small p-value, the kurtosis value, and the skewness value which deviates from the normal distribution. The Black-Scholes method and the Black-Scholes with Gram-Charlier Expansion will be carried out for the data that are normally distributed.

3.3. **Variance Gamma Test**

Options price where the stock's return is variance gamma distributed will be simulated using Variance Gamma and Antithetic Variate Reduction Variance Gamma. In order to implement those methods, the Variance Gamma distribution test will be carried out for these stocks. The null hypothesis is that the data is gamma variance distributed and the alternative hypothesis is that the data is not distributed with the gamma variance. By taking the significance level at $\alpha = 0.05$, H_0 will experience if p-value < α . From the computational results using R, the following results are obtained:

Table 2. Variance Gamma Test Result

Stock	**Sigma**	**Theta**	**Nu**	**D**	**P-Value**
AMD	0.2895	1.4568	0.1184	0.3992096	0.085842
AMZN	0.030744	0.002107	0.536180	0.0265977	0.085842
INTC	0.023800	-0.00048	0.638978	0.0343689	0.085842
LCID	0.002578	-0.03026	0.11946	0.0287677	0.085842
NVDA	0.03994	0.01408	0.12111	0.0352244	0.085842

Among those 5 stocks that have been tested, it is found that all stocks are gamma variance distributed with a p-value greater than 0.05. From table 1 and table 2, it can be seen that AMD, LCID, and NVDA have a gamma variance distribution as well as a normal distribution.

The simulation of variance gamma and variance gamma with antithetic variate reduction will be carried out for all data with 1,000,000 (1 million) repetitions. It is due to the fact that a higher number of simulations will result in better prediction. Then the simulation results of gamma variance and gamma variance with antithetic variate reduction will compare tp find the most efficient methods to predict stock options.

3.4. Black-Scholes Model and Black-Scholes Model with Gram-Charlier Expansion

The call and put option price for normally distributed return can be estimated using the Black-Scholes model simulation and the Black-Scholes model with Gram-Charlier Expansion simulation. Out of 5 stocks that have been tested for normality, it is found that 3 stocks are normally distributed. The stocks chosen are Advanced Micro Devices (AMD), Lucid Group (LCID), and Nvidia (NVDA).

Table 3. Comparison Between Black Scholes Model and Black-Scholes with Gram-Charlier Expansion Model Result

Stock	Method	Option	S0	K	Simulation Price	Market Price	MAE
AMD	B-S	Call	68.59	65	7.23176822	6.85	0.0557326
		Put	68.59	80	12.890093	11.33	0.1376958
	B-S w/ G-C	Call	68.59	65	7.100853	6.85	0.036621
		Put	68.59	80	12.84416	11.33	0.1336417
LCID	B-S	Call	8.68	9	0.7392332	0.69	0.0713525
		Put	8.68	10	1.7060659	2.03	0.1595734
	B-S w/ G-C	Call	8.68	9	0.7123548	0.69	0.0323983
		Put	8.68	10	0.9972411	2.03	0.5087482
NVDA	B-S	Call	170.01	140	32.9679648	34.05	0.0317778
		Put	170.01	185	22.194691	18.05	0.2296228
	B-S w/ G-C	Call	170.01	140	32.8401	34.05	0.035533
		Put	170.01	185	22.37826	18.05	0.2397928

From table 3 above, it was found that for all shares, the price of stock call options traded in the market did not significantly different from the simulation results indicated by the Mean Absolute Error value which was still relatively small. It can be concluded that the market price of the options was still relatively reasonable.

However, overall, there is no significant difference between the Black-Scholes method with Gram-Charlier Expansion and the Black-Scholes method to predict option prices. It can be seen from the comparison of MAE from these two methods that nothing is smaller or bigger overall. In addition, it is important to keep in mind that the Black-Scholes method with Gram-Charlier expansion provides adjustments for the skewness and kurtosis values hence it can give a better result that is closer to the real market price. Therefore, the Black-Scholes method with Gram-Charlier Expansion is better than the Black-Scholes method.

3.5. Gamma Variance Model dan Gamma Variance with Antithetic Variate Reduction Simulation

The call and put option price for gamma variance distributed return can be estimated using variance gamma simulation and antithetic variate reduction variance gamma simulation).

Out of 5 stocks that have been tested for gamma variance distribution, it is found that all stocks are gamma variance distributed.

Table 4. Gamma Variance Model dan Gamma Variance with Antithetic Variate Reduction Simulation Results

Stock	Method	Option	Simulation Price	Standard Error	Market Price	MAE
AMD	VG	Call	6.419053	0.0145736	6.85	0.06291
		Put	13.41988	0.014624	11.33	0.18445
	VG AVR	Call	5.462215	0.0103106	6.85	0.2026
		Put	12.35083	0.0103049	11.33	0.0901
AMZN	VG	Call	4.383619	0.0008537	7.67	0.42847
		Put	11.56699	0.0008512	12.35	0.063402
	VG AVR	Call	4.379187	0.0006015	7.67	0.42905
		Put	11.56483	0.0006011	12.35	0.06358
INTC	VG	Call	1.332993	0.0002080	2.23	0.40224
		Put	3.650722	0.0002078	3.45	0.05818
	VG AVR	Call	1.33223	0.0001476	2.23	0.40259
		Put	3.650559	0.0001471	3.45	0.05813
LCID	VG	Call	0.0026877	0.0001361	0.69	0.99610
		Put	1.285855	0.0001363	2.03	0.36657
	VG AVR	Call	0.00034768	9.625e-05	0.69	0.9995
		Put	1.285828	9.629e-05	2.03	0.36659
NVDA	VG	Call	30.48429	0.002119	34.05	0.10472
		Put	14.35649	0.0021183	18.05	0.20463
	VG AVR	Call	30.4866	0.001498	34.05	0.10465
		Put	14.35864	0.0014992	18.05	0.20451

The MAE value can be compared from put and call options of variance gamma and variance gamma with antithetic variate reduction with a simulation of 1,000,000 (1 million) times. In overall, the simulation of call options with variance gamma has a smaller MAE value compared to the gamma variance with AVR (Antithetic Variance Reduction). As for the simulation of the put option with AVR has a smaller MAE value compared to the variance gamma.

Based on table 4 above, overall, it can be seen that the variance gamma simulation provides a smaller standard error value (MAE) for the simulation price. The MAE value for the simulation of put and call options price is not significantly different. Hence, the market price of the options was still relatively reasonable.

4. Conclusions and Recommendations

4.1. Conclusions

Options are one of the most important financial instruments in risk management which can give attractive chances for investors who want to be more careful in their investments. Hence, an investor must be aware of the fair price of the option when buying or selling to hedge risks using options.

This research is conducted to see whether the price of options traded in the market is still relatively reasonable or not for returns that are normally distributed and for returns that are variance gamma distributed. Therefore, stock option price prediction will be carried out using the Black Scholes method and Black Scholes with Gram Charlier expansion method for normally distributed return. The normality test used is the Bowman-Shelton Goodness of Fit test. As for the non-normally distributed returns, Variance Gamma distribution will be carried out before using the Variance Gamma and Variate Antithetic Reduction Variance Gamma simulation to determine option prices.

The shares are chosen from NASDAQ 100 which is normally distributed and/or variance gamma distributed. Those shares are Advanced Micro Devices, Inc. (AMD), Amazon.com, Inc. (AMZN), Intel Corporation (INTC), Lucid Group, Inc. (LCID), and NVIDIA Corporation (NVDA). The stock data used is stock closing price data from December 10, 2021 to December 9, 2022. The options data used are call and put options and all expire on January 13, 2023. From the normality test, 3 stocks are normally distributed (AMD, LCID, NVDA). Also, it is found that all stocks are variance gamma distributed (AMD, AMZN, INTC, LCID, NVDA).

For the simulations of the Black Scholes method and Black Scholes with the Gram-Charlier expansion method, it can be concluded that the market price of the options was still relatively reasonable. However, overall, there is no significant difference between the Black-Scholes method with Gram-Charlier expansion and the Black-Scholes method to predict option prices. It can be seen from the comparison of MAE from these two methods that nothing is smaller or bigger overall. In addition, the Black-Scholes method with Gram-Charlier expansion provides adjustments for the skewness and kurtosis values hence it can give a better result that is closer to the real market price. Therefore, the Black-Scholes method with Gram-Charlier Expansion is better than the Black-Scholes method.

For Variance Gamma and Variance Gamma with Antithetic Variate Reduction (AVR) simulations, the higher the number of simulations, the smaller the standard error will be. The price of call and put options traded is still relatively reasonable as indicated by the mean absolute error value which is still relatively small. The simulation of call options with variance gamma has a smaller MAE value compared to the gamma variance with AVR. As for the simulation of the put option with AVR has a smaller MAE value compared to the variance gamma. Overall, the variance gamma simulation provides a smaller standard error value (MAE) for the simulation price. Hence, the variance gamma method gives a better result as the simulation price is closer to its market price compared to the gamma variance with AVR.

Of all these methods used, the Black-Scholes with Gram Charlier Expansion method is the most efficient because it can provide stock options prices that are quite close to market prices while adjusting the difference skewness and kurtosis without having to go through several simulations.

4.2. Recommendations

The developments in the options market nowadays, one is the formation of various pricing models for an option to help the buyer determine the fairness of the price. Out of the 4 methods above (Black-Scholes, Black-Scholes with Gram Charlier Expansion, Variance Gamma and Variance Gamma with Antithetic Variate Reduction) that are implemented in this research, the authors suggest figuring out the others methods to predict the option price.

Acknowledgments. The authors would like thankful to Dr. Abdurakhman for teaching the authors in Introduction to Investment Management so the authors can do this research well. The authors are also thankful to Yahoo finance to provide the data and all friends so the research can run well.

About the Authors

The author is a group of students in the Statistics study program, Faculty of Mathematics and Natural Sciences, Gadjah Mada University. The details of the author's student identity are:

1) Petra Abdi Paskalisa
2) Tiara Yosianti Solekhah

References

[1] Smith, Clifford. (1976). Option pricing: A review. Journal of Financial Economics. 3. 3-51. 10.1016/0304-405X(76)90019-2.
[2] Mooy, M. N., Rusgiyono, A., & Rahmawati, R. (2017). Penentuan Harga Opsi Put Dan Call Tipe Eropa Terhadap Saham Menggunakan Model Black-Scholes. Jurnal Gaussian, 6(3), 407–417.
[3] Madan, D. B., Carr, P. P., Chang, E. C. (1998). The Variance Gamma Process and Option Pricing. European Finance Review 2, 79-105
[4] Park, H., Kim. N., Lee. J. (2014). Parametric models and non-parametric machine learning models for predicting option prices: Empirical comparison study over KOSPI 200 Index options. Expert Systems with Applications (41), 5227-5237
[5] Bodie, Kane, & Marcus. 2006. Investment Investasi Buku 2 Edisi 6. Jakarta: Salemba Empat
[6] Desmon, J. H. 2004. An Introduction to Finance Option Valuation. Cambridge: Cambridge University Press
[7] Bolia, N. and Juneja, S., 2005. Monte Carlo methods for pricing financial options. Sadhana, 30(2-3), pp.347-385.
[8] Haugh, M. (2016). The Black-Scholes Model. Foundation of Financial Engineering. http://www.columbia.edu/~mh2078/FoundationsFE/BlackScholes.pdf [Accessed 20 Nov 2021]

[9] Su, Y. and Fu, M., 2002. Optimal importance sampling in securities pricing. The Journal of Computational Finance, 5(4), pp.27-50.
[10] Yahoo Finance. (2021). Yahoo Finance. https://finance.yahoo.com [Accessed 20 Nov 2021]