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Comp Appendix C Reading

Appendix C: Section 1: Theoretical Background

- Figure 1 is a hypothetical signal which is a function that describes how some quantity varies with time
  - Intensity of a sound wave, displacement of a part of a vibrating string, or voltage at some point in an electronic circuit
  - This signal was constructed by adding 5 individual sine waves
    - Can be "decomposed" into a collection of sine waves
    - Joseph Fourier proposed that any function can be written as the sum of sine waves
      - More complicated sums may involve a large (possibly infinite) number of sine waves
    - Common to refer to y(t) as function in time domain to its transform Y(f) as existing in frequency domain
      - *** This is important because we can break down very complex functions into something that can be more easily understood !!
- A forward transform followed by a backward transform will return the original function
  - $S(e^{i*omega*(t-t')}domega = 2\pi\partial(t-t')$ (C.4)
    - $\partial(t)$ is the dirac delta function
- Fourier sum means that it can be viewed as (or decomposed into) a sum of "pure" tones (or sinusoids)
  - Useful because we can go back and forth in order to see every part of a process
  - Frequencies of these tones are the only frequencies present in the original signal
    - Fourier transform Y(f) gives a direct measure of the frequencies present
      - Useful in real life when you might need to know how much a speaker can handle by playing each frequency individually

Appendix C: Section 2: Discrete Fourier Transform

- Fourier transform is calculated by performing the integral of C.3 which is $Y(f) = S(y(t)e^{2\pi ift})dt = S(y(t)e^{i*omega*t})dt$
  - Numerical work almost never gives the analytic form of the signal
    - There is knowledge of amplitude at certain discrete values of t
  - Defining the discrete Fourier transform
    - $y_m = 1/N \Sigma Y_n*e^{-2\pi imn/N}$ (C.5)
    - $Y_n = \Sigma y_m*e^{2\pi imn/N}$ (C.6)
      - m on y corresponds to discrete times ™ = mΔt
      - n on Y corresponds to discrete frequencies $f_n = n/(N\Delta t)$
      - N is number of data points
    - Forward and inverse discrete Fourier transforms are related via:
      - $\Sigma e^{2\pi in(m-m')/N} = N\partial_{m,m'}$ (C.7)
        - $\partial_{m,m'} = 1$ if m = m'
        - $\partial_{m,m'} = 0$ otherwise

- Kronecker delta function

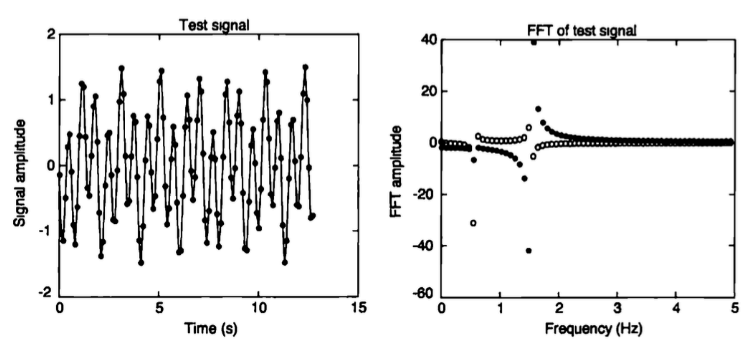
Appendix C: Section 3: Fast Fourier Transform (FFT)

- Exponential terms in the discrete Fourier transforms are multiples of one another
    - Possible to intelligently group terms in the sums so that we can “reuse” many of them in evaluating different fourier components Y_n
        - Possible to evaluate the discrete transform with only of order NlogN operations as opposed to N^2
            - Useful because instead it would take a very long time for even a fast computer for typical values of N
        - The FFT has made important calculations feasible
            - Used for X-ray tomography
            - Improves calculation efficiency from previously impractical tasks
            - Splits the sum into two parts
                - $Y_n = Y_n^e + w^n Y_n^o$ (C.8)
                    - $w = e^{2\pi i/N}$
            - Splitting can be continued one further time depending on the most significant bit of m involved

$$
\begin{aligned}
Y^{ee}_{n_0} &= Y^{eee} + w^{4n_0} Y^{eeo} = y_0 + w^{4n_0} y_4 \\
Y^{eo}_{n_0} &= Y^{eoe} + w^{4n_0} Y^{eoo} = y_2 + w^{4n_0} y_6 \\
Y^{oe}_{n_0} &= Y^{oee} + w^{4n_0} Y^{oeo} = y_1 + w^{4n_0} y_5 \\
Y^{oo}_{n_0} &= Y^{ooe} + w^{4n_0} Y^{ooo} = y_3 + w^{4n_0} y_7 \,.
\end{aligned}
\qquad (C.12)
$$

Appendix C: Section 4: Sampling Interval and Number of Data Points

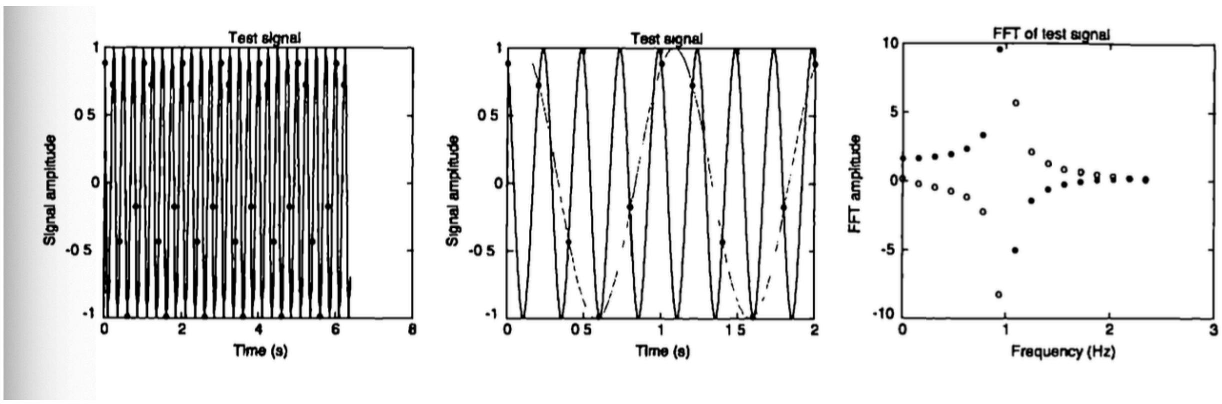
- Two parameters that are under our control in a discrete Fourier transform is the sampling interval and the number of points being sampled
    - Choosing these parameters carefully is important in gaining a useful transformation
        - Sampling interval determines the range of spectral frequencies that can be represented
        - Number of data points determines the amount of detail that can be seen



- If sampling time doesn't match the frequencies of the Fourier components, the FFT will look slightly more complicated
  - If a frequency contained in the signal doesn't coincide with one of the discrete frequencies, the FFT is forced to represent the signal as a sum of components over range of f_j

<u>Appendix C: Section 5: Aliasing</u>

- The sampling theorem states that the FFT will give us a perfect description of the Fourier components as long as the frequencies of these components are below the Nyquist frequency (½Δt)
  - When the frequency of the sine wave is greater than the Nyquist frequency, there are fewer than two sampled points per period
    - Folding back of frequencies above the Nyquist frequency is known as aliasing



<u>Appendix C: Section 6: Power Spectrum</u>

- So far, the real (cosine) and imaginary (sine) parts of the FFT have been displayed separately
  - The advantage is that it contains all of the information in the original signal
  - Essential for using a backward transform to return to the time domain
- Displaying results of an FFT is known as the power spectrum
  - Autocorrelation: measures how well the signal y is correlated across times separated by tau
    - Corr[y](tau) = S y(t)*y(t + tau)dtau (C.13)
  - Power spectrum: useful for measuring the frequency content of stationary signals
    - PS[y](f) = S y(t)*y(t + tau)$e^{2\pi i f * tau}$dtau = $|Y(f)|^2$ (C.14)