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PHYS250 - Chapter 3 Reading

Chapter 3: Section 1: Simple Harmonic Motion

- Example of a simple pendulum is a particle of mass m connected by a massless string to a rigid support
  - Theta is angle that string makes with vertical and assume string is always taut
  - Assume only 2 forces on particle
    - Gravity and tension of string
    - Parallel forces add to zero assuming that string doesn't stretch or break
    - Perpendicular: F_{theta} = -mgsin(theta)
- Newton's second law tells us that this force is equal to the mass times the acceleration of the particle along the circular arc that is the particle's trajectory
  - $F_{theta} = md^2s/dt^2$
    - Where arc is s = l*theta
    - Equation of motion is $(d^2(theta)/dt^2)$ = (-g/l)(theta)
      - Central equation for SHM
    - General solution: theta = $theta_0$(sin(omega*t + phi))
      - Omega = $(g/l)^{½}$ and phi are constants that depend on initial displacement and velocity of the pendulum

Chapter 3: Section 2: Making the Pendulum More Interesting: Adding Dissipation, Nonlinearity, and a Driving Force

- Start by adding damping to simple example in section 1
  - Manner in which friction enters the equation of motion depends on the origin of the friction
    - Sources include effective bearing where the string of the pendulum connects to the support, air resistance, etc
  - Damping force is proportional to velocity
  - Equation of motion for damped pendulum: (d^2theta/dt^2) = (-g/l)(theta)-q(dtheta/dt)
    - Second term on right models friction
    - Still linear, can solve analytically
  - Underdamped solution: theta(t) = $theta_0e^{-qt/2}sin((omega^2-q^2/4)^{1/2}t+phi))$
    - Shows oscillatory behavior with frequency $(omega^2-q^2/4)^{½}$
  - Overdamped solution: theta(t) = $theta_0e^{-(q/2+/-(q\text{^}2/4-omega\text{^}2)\text{^}½)*t}$
  - Critically damped solution: theta(t) = $(theta_0 + Ct)e^{-qt/2}$
- Driven, damped pendulum undergoes a simple harmonic oscillation with angular frequency of driving force

Chapter 3: Section 3: Chaos in the Driven Nonlinear Pendulum

- Do not assume small-angle approximation, and do not expand the sin(theta) term
- Include friction of the form -q(dtheta/dt)
- Add to our model a sinusoidal driving force F_D sin($omega_D$t)
  - Put the three together: (d_2theta/dt^2) = (-g/l)sin(theta)-q(dtheta/dt)+$F_D$sin($omega_D$t)
    - Call this model for nonlinear, damped, driven pendulum (physical pendulum)