Seminar 10

$C_1 = \neg K \vee \neg S \vee M$

$C_4 = S$

$C_5 = \neg M$

Level saturation strategy

- 1st level: the initial set of clauses

$S_0 = \{C_1, C_4, C_5\}$

- $S_k = \{Res(C_i, C_j) \mid C_i \in S_{i-1}, C_j \in \bigcup_{i=0,k-1} S_i\}$

- $S_1 = \{Res(C_i, C_j) \mid C_i \in S_0, C_j \in S_0\}$

$Res_S(C_1, C_4) = \neg K \vee M = C_6$

$Res_M(C_1, C_5) = \neg K \vee \neg S = C_7$

$\Rightarrow S_1 = \{C_6, C_7\}$

$- S_2 = \{ Res(C_i, C_j) \mid C_i \in S_1, C_j \in S_0 \cup S_1 \}$

$Res_m(C_6, C_5) = \neg K = C_7$

$Res_s(C_7, C_4) = \neg K =$

$S_2 = \{ C_8 \}$

$S_3 = \emptyset \Rightarrow$ We couldn't reach the contradiction

$\Rightarrow$ the deduction does not hold

Deletion strategy

- most clauses are tautologies subsumed by another clause

→ Will scende it love

3, H_1: May ... if Lucy ... and not George

$$L \wedge \neg G \to M \equiv \neg(L \wedge \neg G) \vee M$$
$$= \neg L \vee G \vee M$$

H_2: J ... L

$$J \to L = \neg J \vee L$$

H_3: JT ... J

$$JT \to J \equiv \neg JT \vee J$$

H_4: $\neg G$

H_5: JT

C: M

? $H_1, H_2, H_3, H_4, H_5, \neg C \vdash_{Res} \square$?

$C_1 = \neg L \vee G \vee M$

$C_2 = \neg J \vee L$

$C_3 = \neg JT \vee J$

$C_4 = \neg G$

$C_5 = \neg T$

$C_6 = \neg M$

- We apply David Putnam $\Rightarrow$ do not rename anything

$C_7 = Res_{\neg T}(C_5, C_3) = J$

$C_8 = Res_J(C_2, C_7) = L$

$C_9 = Res_L(C_8, C_1) = G \vee M$

$C_{10} = Res_G(C_4, C_9) = M$

$C_{11} = Res_M(C_6, C_{10}) = \square \Rightarrow$ the conclusion holds

Linear reputation

4. $S_4 = \{ \overset{C_1}{p} ;\ \overset{C_2}{q \vee r} ;\ \overset{C_3}{\neg p \vee q \vee \neg r} ;\ \underset{C_4}{\neg p \vee \neg q} \}$

$C_2 = q \vee r$

$C_3 = \neg p \vee r$

$C_4 = \neg p \vee \neg q$

$C_5 = q \vee \neg p$

5.7.

$$S_7 = \{\neg q \vee p,\ q \vee \neg r,\ r \vee \neg p\}$$

$C_1 = \neg q \vee p$

$C_2 = q \vee \neg r$

$C_3 = r \vee \neg p$

→ Linear resolution

$$\begin{tikzcd}
\boxed{C_1 : \neg q \vee p} \arrow[d] & \boxed{C_2 = q \vee \neg r} \arrow[dl, "Res_q(C_1, C_2)"] \\
\boxed{C_4 = \neg r \vee p} \arrow[d] & \boxed{C_3 = r \vee \neg p} \arrow[dl, "Res_p(C_4, C_3)"] \\
\boxed{C_5 = r \vee \neg r} &
\end{tikzcd}$$

, $C_3 = r \vee \neg p$ → side clause

$C_5 = r \vee \neg r$ ⇒ Tautology

$C_1 = \neg q \vee P$ $\qquad$ $C_3 = R \vee \neg P$

$C_4 = R \vee \neg q$ $\qquad$ $C_2 = q \vee \neg R$

$C_5 = R \vee \neg R \Rightarrow$ tautology

Since we have tried all methods, we can conclude that the set of clauses is consistent

Locked Resolution ?

6. $S_7 = \{ R \vee P,\ R \vee \neg P \vee \neg q,\ \neg q \vee \neg R,\ q \}$

$C_1 = {}^{(1)}R \vee {}^{(2)}P$

$C_2 = (3)\, R \vee (4)\, \neg P \vee (5)\, \neg q$

$C_3 = (5)\, \neg q \vee (7)\, \neg R$

$C_4 = (8)\, q$

$S^0 = \{ C_1, C_2, C_3, C_4 \}$. saturation strategy

$C_5 = \mathrm{Res}^{\mathrm{lock}}_{q}(C_3, C_4) = \underline{\neg R\ (7)}$

$S^1 = \{ C_5 \}$

$C_6 = \mathrm{Res}^{\mathrm{lock}}_{R}(C_5, C_1) = \underline{P}\ (2)$

$C_7 = \mathrm{Res}^{\mathrm{lock}}_{R}(C_5, C_2) = \underline{\neg P\,(4) \vee (5)\, \neg q}$

$S^2 = \{C_6, C_7\}$

$C_8 = \mathrm{Res}_p^{\mathrm{Lock}}(C_6, C_7) = \neg q(5)$

$S^3 = \{C_8\}$

$C_9 = \mathrm{Res}_q^{\mathrm{Lock}}(C_8, C_4) = \square$

Second indexing next time