OPTIONAL HOMEWORK
COMPUTATIONAL LOGIC

FARKAS TIBERIA - GIULIA
GROUP : 913

[2] Compose and solve 5 exercices of reasoning modelling using propositional logic.

1) Proof method used: truth table

Premises:

H_1: If it is snowing, then Mary goes skiing.
H_2: If Mary goes skiing, then she needs skis.
H_3: If it is not snowing, then Adam stays home.
H_4: If Adam stays home, then Mary does not go skiing.

Conclusion

C: Either Mary does not go skiing or she needs skis.

Propositional logic form:

S - it is snowing
MS - Mary goes skiing
NS - Mary needs skis
AH - Adam stays home

Propositional formulas

H_1: $S \to MS$

H_2: $MS \to NS$

H_3: $\neg S \to AH$

H_4: $AH \to \neg MS$

C: $\neg MS \vee NS$

TRUTH TABLE

	S	MS	NS	AH	S→MS	MS→NS	¬S→AH	AH→¬MS	¬MS∨NS
i1	0	0	0	0	1	1	0	1	1
i2	0	0	0	1	1	1	1	1	1
i3	0	0	1	0	1	1	0	0	1
i4	0	0	1	1	1	1	1	1	1
i5	0	1	0	0	1	0	0	1	0
i6	0	1	0	1	1	0	1	0	0
i7	0	1	1	0	1	1	0	1	1
i8	0	1	1	1	1	1	1	0	1
i9	1	0	0	0	0	1	1	1	1
i10	1	0	0	1	0	1	1	1	1
i11	1	0	1	0	0	1	1	0	1
i12	1	0	1	1	0	1	1	1	1
i13	1	1	0	0	1	0	1	1	0
i14	1	1	0	1	1	0	1	0	0
i15	1	1	1	0	1	1	1	1	1
i16	1	1	1	1	1	1	1	0	1

To validate the reasoning, we check whether all rows where the premises H_1, H_2, H_3, H_4 are true, also satisfy the conclusion.

As we can see from the Truth table, where all the premises are true, the conclusion also holds $\Rightarrow$

$$\Rightarrow H_1, H_2, H_3, H_4 \vdash C$$

2) Proof method used - definition of deduction

H_1: Tea will do her homework if Maria will do her homework, but Ana won't.

H_2: If John has free time, then he will help Maria do her homework.

H_3: If it is weekend, then John has free time.

H_4: Ana is sick, so she can't do her homework.
H_5: Tomorrow John will go to school again because is Monda
C: Will Tea do her homework?

Notations:

T - Tea will do her homework
M - Maria will do her homework
A - Ana will do her homework
FT - John has free time
W - it is weekend

Propositional formulas

H_1: $M \land \neg A \to T$. (f_1)
H_2: $FT \to M$ (f_2)
H_3: $W \to FT$ (f_3)
H_4: $\neg A$ (f_4)
H_5: W (f_5)
C: T

$U, U \rightarrow V \vdash_{mp} V$

The deduction process:

$f_5, f_3 \vdash_{mp} \neg T$ (f_6)

$f_6, f_2 \vdash_{mp} M$ (f_7)

$f_4, f_7 \vdash \neg A \wedge M$ (conjunction in conclusions)(f_8)

$f_8, f_1 \vdash_{mp} T$ ($f_9 = C$)

The sequence of formulas($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9$) is the deduction of conclusion from the hypotheses therefore, based on the hypotheses, Tea will do her homework.

3) Proof method used: semantic tableaux

Premises

H_1: Tiberia will go to Costinești this summer if both her friends Paula and Karina go.

H_2: If Paula will pass all her exams, then she will go to Costinești.

H_3: Paula hasn't prepared enough for her exams and she failed the Anatomy exam.

H_4: Karina has already booked a vacation in Costinești this summer.

C: Will Tiberia go to Costinești this summer?

Notations:

T - Tiberia will go to Costinesti
P - Paula will go to Costinesti
K - Karina will go to Costinesti
PE - Paula passed all her exams

Formulas

$H_1 : P \wedge K \rightarrow T$

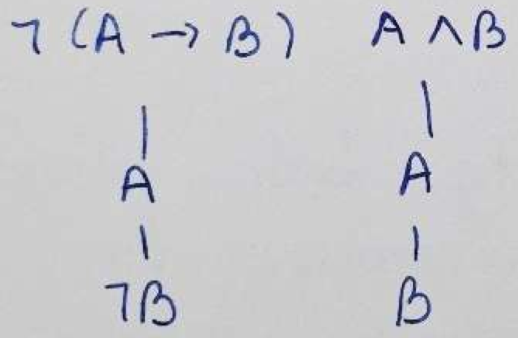
$H_2 : PE \rightarrow P$

$H_3 : \neg PE$

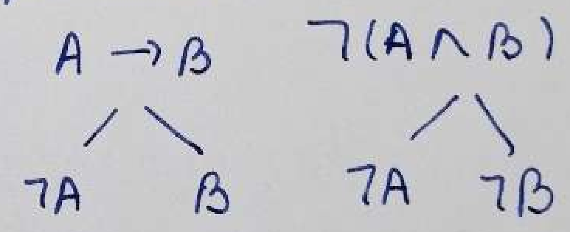
$H_4 : K$

$C : T$

α rule

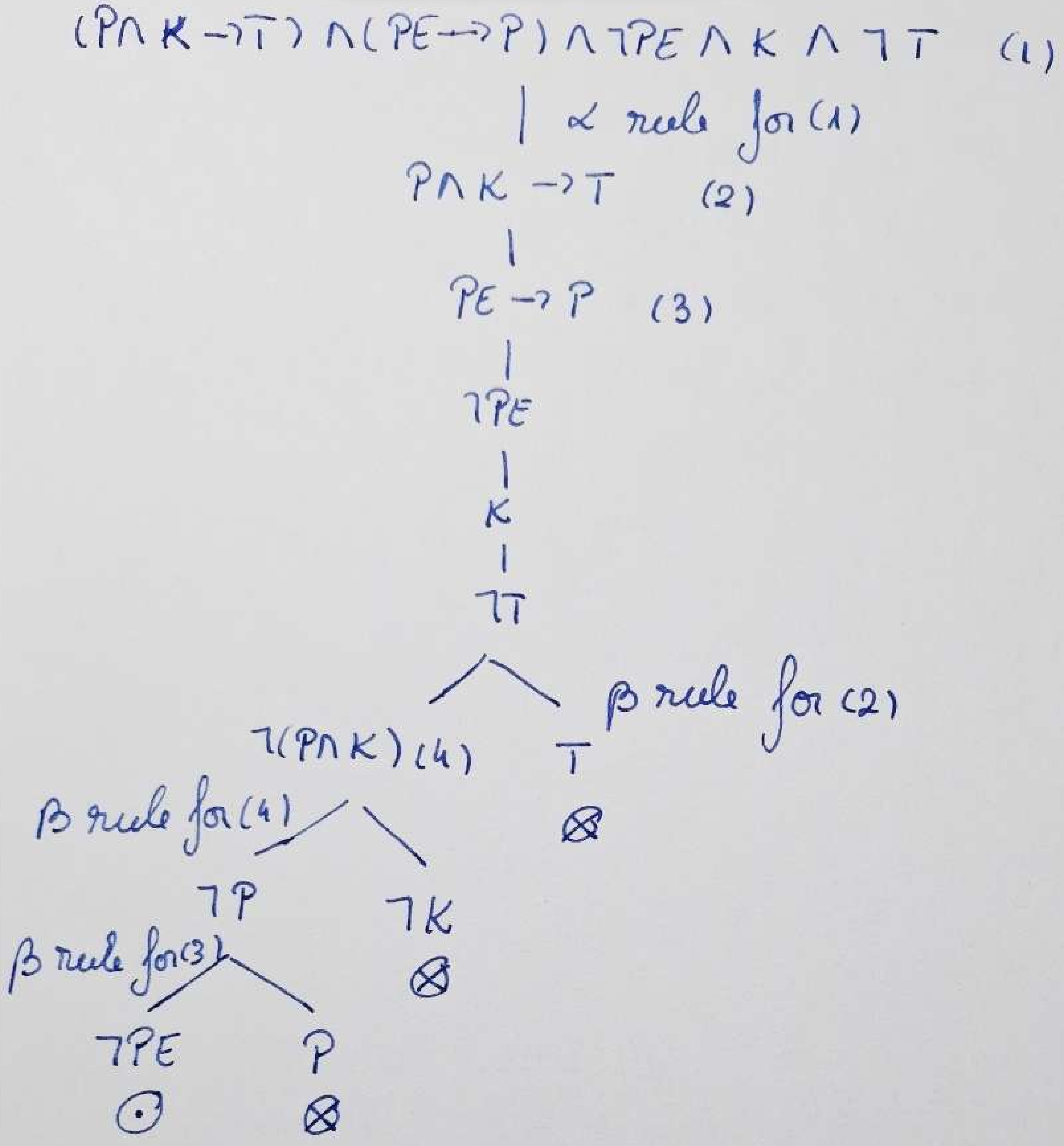


β rule



Because the semantic tableaux method is a refutation proof method, we need to negate the conclusion.

$H_1, H_2, H_3, H_4 \models C$ iff $H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C$ has a closed semantic tableaux



We have obtained a complete and open tableau with one open branch and three closed branches containing the following pairs of literals : $(\neg T, T), (\neg K, K), (\neg P, P)$

Therefore $H_1, H_2, H_3, H_4 \nvDash C$ and based on the hypothesis we can't conclude that Tiberia will go to Costinești this summ

4) Proof method used : resolution(general resolution)

H_1: If it is warm outside, Ted and Barny go jogging.

H_2: Robin goes jogging on Wednesdays.

H_3: It was warm last Wednesday.

C: Did Ted and Barny met Robin when they were jogging last Wednesday?

Notations

T - goes jogging

B - goes jogging

R - goes jogging

W - it is warm

Wed - it is Wednesday

Formulas

H_1: $W \to T \land B \equiv \neg W \lor (T \land B) \equiv (\neg W \lor T) \land (\neg W \lor B) : C_1 \land C$

H_2: $Wed \to R \equiv \neg Wed \lor R : C_3$

H_3: $W \land Wed : C_4 \land C_5$

C: $Wed \land T \land B \land R$

$\neg C$: $\neg(Wed \land B \land T \land R) \equiv \neg Wed \lor \neg T \lor \neg B \lor \neg R : C_6$

$X = \{C_1, C_2, C_3, C_4, C_5, C_6\}$

$$CNF(H_1 \land H_2 \land H_3 \land \neg C) \vdash_{Res} ? \square$$

$C_1 = \neg W \lor T$

$C_2 = \neg W \lor B$

$C_3 = \neg Wed \vee R$

$C_4 = W$

$C_5 = Wed$

$C_6 = \neg Wed \vee \neg T \vee \neg B \vee \neg R$

$C_7 = Res_{Wed}(C_5, C_6) = \neg T \vee \neg B \vee \neg R$

$C_8 = Res_{R}(C_7, C_3) = \neg T \vee \neg B \vee \neg Wed$

$C_9 = Res_{Wed}(C_8, C_5) = \neg T \vee \neg B$

$C_{10} = Res_{T,B}(C_9, C_1) = \neg W$

$C_{11} = Res_{W}(C_{11}, C_4) = \square$

$CNF(H_1 \wedge H_2 \wedge H_3 \wedge \neg C) \vdash_{Res} \square$, so C is deductible from the hypotheses, therefore "Ted and Barny met Robin when they were jogging last Wednesday".

5) Proof method used: resolution (level saturation strategy)

H_1: If it is raining, Rihanna takes her umbrella.

H_2: If Rihanna takes her umbrella, then Tom will perform a magical show.

H_3: If Tom performs a magical show, then he will have a lot of fans.

H_4: It is raining.

C: Tom will have a lot of fans.

The sequence $S^0, S^1, S^2, \ldots$ represents levels of resolvents.

$$S^k = \{ Res(C_i, C_j) / C_i \in S^{k-1}, C_j \in S^0 \cup S^1 \cup \ldots \cup S^{k-1} \}, k = 1, 2, \ldots$$

Notations:

R- it is raining
U- Rihanna takes the umbrella
T- Tom performes a magical show
F- Tom has a lot of fans

Formulas

$H_1 : R \to U \equiv \neg R \vee U : C_1$

$H_2 : U \to T \equiv \neg U \vee T : C_2$

$H_3 : T \to F \equiv \neg T \vee F : C_3$

$H_4 : R : C_4$

$C : F$

$\neg C : \neg F : C_5$

Initial level :

$$S^0 = S = \{ C_1 = \neg R \vee U, C_2 = \neg U \vee T, C_3 = \neg T \vee F, C_4 = R, C_5 = F \}$$

First level

$C_6 = Res_U (C_1, C_2) = \neg R \vee T$

$C_7 = Res_R (C_1, C_4) = U$

$C_8 = Res_T (C_2, C_3) = \neg U \vee F$

$C_9 = Res_F (C_3, C_5) = \neg T$

$$S^1 = \{ C_6 = \neg R \vee T, C_7 = U, C_8 = \neg U \vee F, C_9 = \neg T \}$$

Second level

$C_{10} = Res_T(C_6, C_3) = \neg R \vee F$

$C_{11} = Res_R(C_6, C_4) = T$

$S^2 = \{C_{10}, C_{11}, C_{12}, C_{13}\}$

$C_{12} = Res_U(C_8, C_1) = \neg R \vee F$

$C_{13} = Res_F(C_8, C_5) = \neg U$

Third level

$C_{14} = Res_U(C_{13}, C_7) = \square$

$\Rightarrow CNF(C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5) \vdash_{Res} \square$, so C is deductible from the hypotheses, therefore "Tom will have a lot of fans".

Optional Homework

Farkas Tiberia-Giulia

Group 913

$\boxed{I}$ Solve 5 exercises chosen from the list below using predicate logic.

1) H_1: All hounds howl at night.

H_2: Anyone who has any cats will not have any mice.

H_3: Light sleepers do not have anything which howls at night.

H_4: John has either a cat or a hound.

C: If John is a light sleeper, then John does not have any mice

$$H_1, H_2, H_3, H_4 \vdash C$$

I will use the following predicate symbols:

D - is the domain (the universe of hounds)

John - is a constant of the universe

$H: D \to \{T, F\}$, $H(x) = T$ if x is a hound

$N: D \to \{T, F\}$, $N(x) = T$ if x howls at night

$Cat: D \to \{T, F\}$, $Cat(x) = T$ if x is a cat

$M: D \to \{T, F\}$, $M(x) = T$ if x is a mice

$L: D \to \{T, F\}$, $L(x) = T$ if x is a light sleeper

$Owns: D \times D \to \{T, F\}$, $Own(x, y) = T$ if x owns y

First-order (predicate) formulas:

H_1 : $(\forall x)(H(x) \to N(x))$

H_2 : $(\forall x)(\forall y)[(Owns(y,x) \land Cat(x)) \to (\forall z)(\neg Owns(y,z) \land$
$\land M(z))]$

$H_3: (\forall x)(\forall y)(L(x) \to (\neg \text{Owns}(x,y) \wedge N(y)))$

$H_4: (\forall x)(\text{Owns}(John,x) \wedge Cat(x)) \vee (\text{Owns}(John,x) \wedge H(x))$

$C: L(John) \to ((\forall x)\, M(x) \wedge \neg \text{Owns}(John, x))$

$\neg C: \neg(L(John) \to ((\forall x) M(x) \wedge \neg \text{Owns}(John, x))$

$H_1^C: H(x) \to N(x) \equiv \neg H(x) \vee N(x) \quad : C_1$

$H_2^C: \neg((\forall x)(\text{Owns}(y,x) \wedge Cat(x)) \vee (\forall z) \neg \text{Owns}(y,z) \wedge M(z)) \equiv$

$\equiv (\forall x)(\neg \text{Owns}(y,x) \vee Cat(x)) \vee (\forall z)(\neg \text{Owns}(y,z) \wedge M(z))$

$\equiv (\forall x)(\forall z)(\underbrace{\neg \text{Owns}(John, x) \vee Cat(x) \vee M(z))}_{C_2}) \wedge$

$\underbrace{\wedge(\neg \text{Owns}(John, x) \wedge Cat(x) \wedge \neg \text{Owns}(John, z)))}_{C_3} \quad : C_2 \wedge C_3$

$H_3^C: (\forall x)(\forall y) \neg(L(x) \to (\neg \text{Owns}(x,y) \wedge N(y))) \equiv$

$\equiv (\forall x)(\forall y)(\neg L(x) \vee N(y) \wedge (\neg L(x) \vee \neg \text{Owns}(x,y))) \equiv$

$\equiv (\forall x)(\forall y)(\underbrace{\neg L(John) \vee N(y))}_{C_4} \wedge (\underbrace{\neg L(John) \vee \neg \text{Owns}(John, y)}_{C_5}))$

$H_4^C: (\forall x) \neg(\text{Own}(John,x) \wedge Cat(x)) \vee (\text{Owns}(John,x) \wedge H(x)) \equiv$

$\equiv (\forall x)(\neg \text{Owns}(John,x) \vee \neg Cad(x) \vee \neg H(x)) \quad : C_6$

$S = \{C_1, C_2, C_3, C_4, C_5, C_6\}$

We can eliminate the clauses with pure literals: $\neg H(x)$, $\neg Cat(x)$, and $\neg L(x)$ and obtain S'.

$S' = \emptyset \Rightarrow$ all clauses from S contains pure literals $\Rightarrow$

$\Rightarrow$ The conclusion holds

$H_1, H_2, H_3, H_4 \vdash C$

$\boxed{6}$ Proof using the semantic tableaux

H_1 : Every coyote chases some roadrunner.

H_2 : Every roadrunner who says "beep-beep" is smart.

H_3 : No coyote catches any smart roadrunner.

H_4 : Any coyote who chases some roadrunner but does not catch it is frustrated.

C : If all roadrunners say "beep-beep", then all coyotes are frustrated.

I will use the following predicate symbols :

$Ct : D \times D \to \{T, F\}$, $Ct(x,y) = T$ if x catches y

$Ch : D \times D \to \{T, F\}$, $Ch(x,y) = T$ if x chases y

$Co : D \to \{T, F\}$, $Co(x) = T$ if x is a coyote

$Rd : D \to \{T, F\}$, $Rd(x) = T$ if x is a roadrunner

$Bp : D \to \{T, F\}$, $Bp(x) = T$ if x says "beep-beep"

$Fr : D \to \{T, F\}$, $Fr(x) = T$ if x is frustrated

$S : D \to \{T, F\}$, $S(x) = T$ if x is smart

H_1: $(\forall x)\,(Co(x) \to (\exists y)\,(Rd(y) \wedge Ch(x,y)))$

H_2: $(\forall x)\,((Rd(x) \wedge Bp(x)) \to S(x))$

H_3: $(\forall x)(\forall y)\,((Co(x) \wedge Rd(y) \wedge S(y)) \to \neg Ct(x,y))$

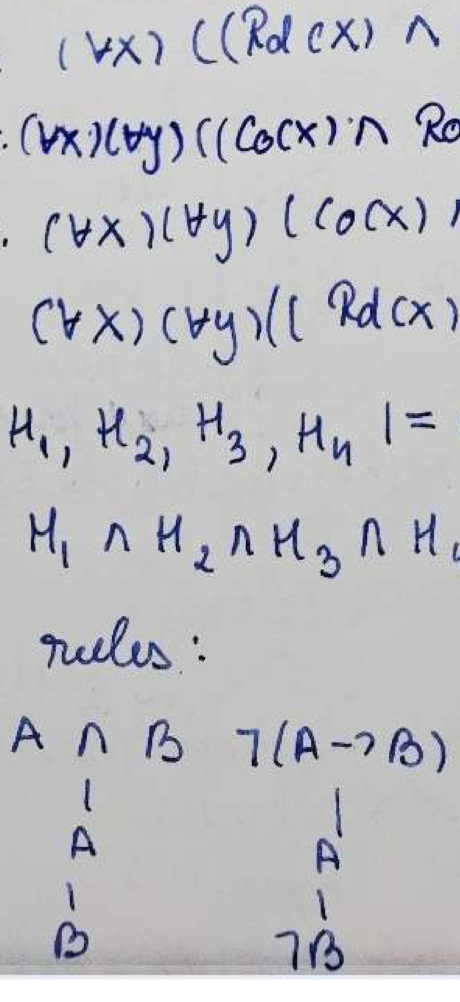
H_4: $(\forall x)(\forall y)\,(Co(x) \wedge Rd(y) \wedge Ch(x,y) \wedge \neg Ct(x,y)) \to Fr(x))$

C: $(\forall x)(\forall y)((Rd(x) \wedge Bp(x)) \to (Co(y) \wedge Fr(y)))$

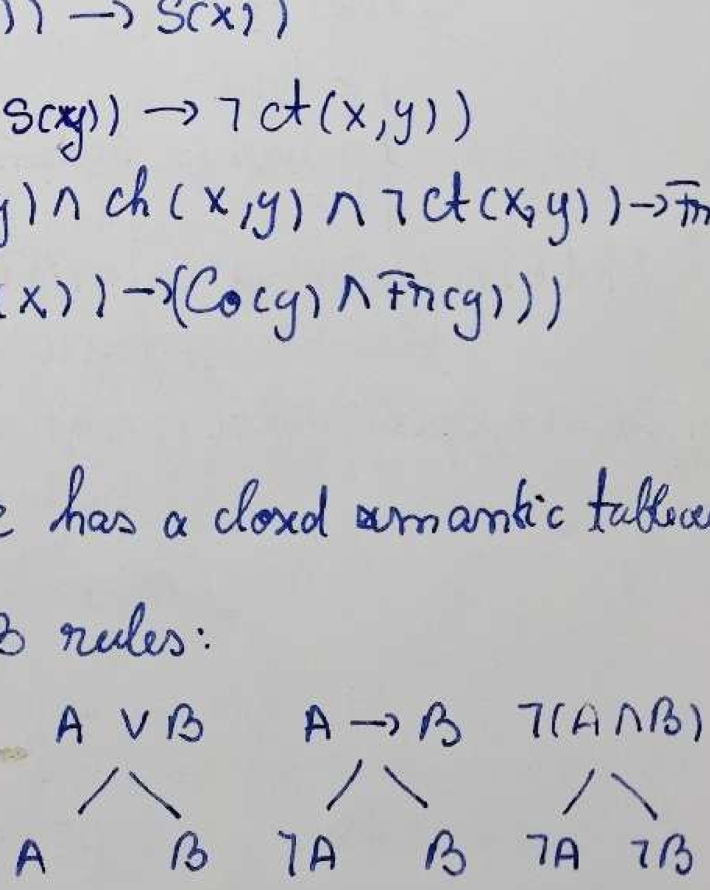
$H_1, H_2, H_3, H_4 \models C$ iff

$H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C$ has a closed semantic tableaux

α rules:



β rules:



δ rule

$(\exists x)\ A(x)$ $\quad \neg(\forall x)$

|

$A(a)$ $\quad \neg A(a)$

a new constant

γ rule

$(\forall x)A(x)$

|

$A(a_1)$

|

$A(a_m)$

|

$(\forall)A(x)$

a_1, a_m are all the constants on the branch

$H_1 \wedge H_2 \wedge H_3 \wedge \neg C$ (1) ✓

| α rule (1)

$H_1 : (\forall x)(Co(x) \to (\exists y)(Rd(y) \wedge Ch(x,y)))$ (2) ✓

|

$H_2 : (\forall x)((Rd(x) \wedge Bp(x)) \to S(x))$ (3) ✓

|

$H_3 : (\forall x)(\forall y)((Co(x) \wedge Rd(y) \wedge S(y)) \to \neg Ct(x,y))$ (4) ✓

|

$H_4 : (\forall x)(\forall y)(Co(x) \wedge Rd(y) \wedge Ch(x,y) \wedge \neg Ct(x,y)) - Fr(x)$ (5)

|

$\neg C : (\forall x)(\forall y)((Rd(x) \wedge Bp(x)) \to (Co(y) \wedge Fr(y))$ (6) ✓

$\equiv (\forall x)(Rd(x) \to Bp(x)) \wedge \neg(\forall x)(Co(x) \to Fr(y))$

$\equiv (\forall x)(Rd(x) \to Bp(x)) \wedge (\exists x)(Co(x) \wedge \neg Fr(x))$

| α for (6)

$(\forall x)(Rd(x) \to Bp(x)$ (7) ✓

|

$(\exists x)(Co(x) \wedge \neg Fr(x))$ (8) ✓

| γ rule for (6)
| b used for instantiation

$Rd(b) \to Bp(b)$ (9) ✓

| δ rule for (8)
a new constant

$Co(a) \wedge \neg Fr(a)$ (10) ✓

| α rule for (10)

$Co(a)$

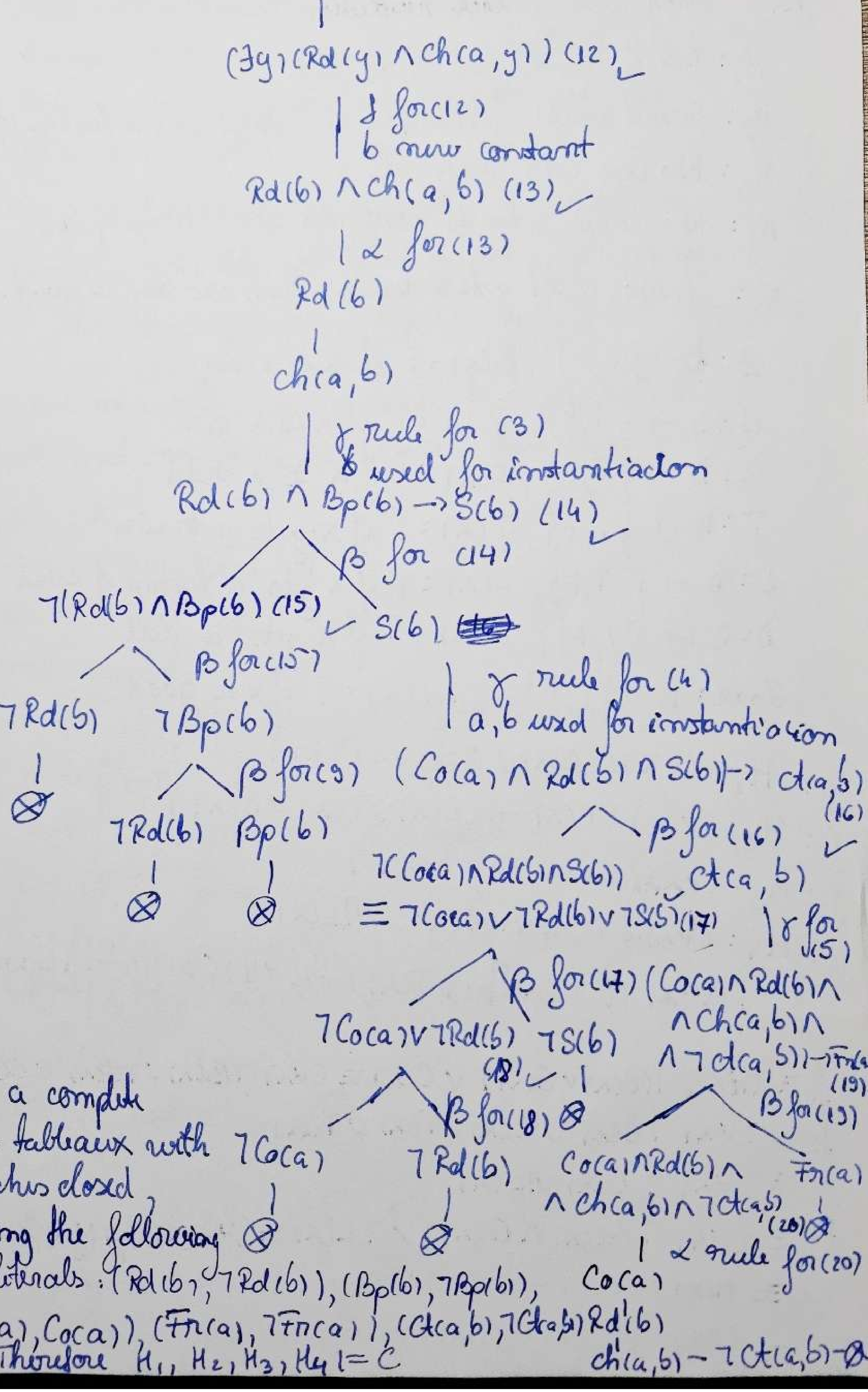
|

$\neg Fr(a)$

| γ for (2)
| a used for instantiation

$Co(a) \to \exists y(Rd(y) \wedge Ch(a,y)$ (11) ✓

| Co(a) is true, simplify



we have a complete semantic tableaux with all branches closed, containing the following pairs of literals: $(Rd(b), \neg Rd(b))$, $(Bp(b), \neg Bp(b))$, $(\neg Co(a), Co(a))$, $(Fr(a), \neg Fr(a))$, $(Ct(a,b), \neg Ct(a,b))$

⑤ Therefore $H_1, H_2, H_3, H_4 \models C$

5) Proof using the deduction method

H_1: Anyone whom Mary loves is a football star.

H_2: Any student who does not pass, does not play.

H_3: John is a student.

H_4: Any student who does not study, does not pass.

H_5: Anyone who does not play is not a football star.

C: If John does not study, then Mary does not love John.

We use variables and constants to transform the hypothesis and conclusion into predicate logic form.

variables: x

constants: Mary, John

domain: D (all people)

predicate symbols:

loves: $D \times D \rightarrow \{T, F\}$, loves$(x,y) = T$ iff x loves y

stud: $D \rightarrow \{T, F\}$, stud$(x) = T$ iff x is a student

pass: $D \rightarrow \{T, T\}$, pass$(x) = T$ iff x passed

play: $D \rightarrow \{T, F\}$, play$(x) = T$ iff x plays

fstar: $D \rightarrow \{T, F\}$, fstar$(x) = T$ iff x is a football star

study: $D \rightarrow \{T, F\}$, study$(x) = T$ iff x studies

$H_1 : (\forall x)(\text{loves}(\text{Mary}, x) \rightarrow \text{fstar}(x))$

$H_2 : (\forall x)((\text{stud}(x) \wedge \neg \text{pass}(x)) \rightarrow \neg \text{play}(x))$

$H_3 : \text{Stud}(\text{John})$

$H_4 : (\forall x)((\text{stud}(x) \wedge \neg \text{study}(x)) \rightarrow \neg \text{pass}(x))$

$H_5 : (\forall x)(\neg \text{play}(x) \rightarrow \neg \text{fstar}(x))$

$C : \neg \text{study}(\text{John}) \rightarrow \neg \text{loves}(\text{Mary}, \text{John})$

$H_2 \equiv (\forall x)(\neg(stud(x) \wedge \neg pass(x)) \vee \neg play(x))$

$\equiv (\forall x)(\neg stud(x) \vee pass(x) \vee \neg play(x))$

$H_4 \equiv (\forall x)(\neg(stud(x) \wedge \neg study(x)) \vee \neg pass(x))$

$\equiv (\forall x)(\neg stud(x) \vee study(x) \vee \neg pass(x))$

$f_6 \overset{H_2 \vee H_4}{\vdash} (\forall x)(\neg stud(x) \vee study(x) \vee \neg play(x))$

$H_1 \equiv (\forall x)(\neg loves(Mary, x) \vee fstar(x))$

$H_5 \equiv (\forall x)(play(x) \vee \neg fstar(x))$

$f_7 \overset{H_1 \vee H_5}{\vdash} (\forall x)(\neg loves(Mary, x) \vee play(x))$

$f_8 \overset{f_6 \vee f_7}{\vdash} (\forall x)(\neg stud(x) \vee study(x) \vee \neg loves(Mary, x))$

$f_8 \equiv (\forall x)(stud(x) \to \neg study(x) \to \neg loves(Mary, x))$

$f_9 \underset{x \leftarrow John}{\overset{univ.\ inst.}{\vdash}} stud(John) \to \neg study(John) \to \neg loves(Mary, John)$

$f_{10} \underset{mp}{\overset{f_9, H_3}{\vdash}} \neg study(John) \to \neg loves(Mary, John) \equiv C$

The sequance $(H_1, H_2, \dots, f_9, f_{10} = C)$ is the deduction of the hypothesis $(H_1, \dots H_5)$.

=) The conclusion holds

13. Proof using lock resolution method

H_1: Every boy or girl is a child

H_2: Every child gets a doll or a train or a lump of coal

H_3: No boy gets any doll

H_4: No child who is good gets any lump of coal

C: If no child gets a train, then no boy is good.

$B : D \to \{T, F\}$, $B(x) = T$ if x is a boy

$G : D \to \{T, F\}$, $G(x) = T$ if x is a girl

$C : D \to \{T, F\}$, $C(x) = T$ if x is a child

$T : D \to \{T, F\}$, $T(x) = T$ if x gets a train

$L : D \to \{T, F\}$, $L(x) = T$ if x gets a lump of coal

$D : D \to \{T, F\}$, $D(x) = T$ if x gets a doll

$Good : D \to \{T, F\}$, $Good(x) = T$ if x is good

f_1: $H_1 : (\forall x)((B(x) \vee G(x) \to C(x))$

f_2: $H_2 : (\forall x)(C(x) \to (L(x) \vee T(x) \vee D(x))$

f_3: $H_3 : (\forall x)(B(x) \to \neg D(x))$

f_4: $H_4 : (\forall x)(C(x) \wedge G(x) \to \neg L(x))$

$C : [\forall x)((C(x) \overset{(2)}{\to} \neg T(x)) \overset{(1)}{\to} (\forall y)(B(y) \overset{(3)}{\to} \neg Good(g)))$

$f_1 \equiv (\forall x) \neg(B(x) \vee G(x)) \vee C(x) \equiv (\forall x) \neg B(x) \vee \neg G(x) \vee C(x)$

$f_2 \equiv (\forall x) \neg C(x) \vee L(x) \vee T(x) \vee D(x)$

$f_3 \equiv (\forall x) \neg B(x) \vee \neg D(x)$

$f_4 \equiv (\forall x) \neg(C(x) \wedge G(x)) \vee \neg L(x) \equiv (\forall x) \neg C(x) \vee \neg G(x) \vee \neg L(x)$

$C \equiv (\forall x) \neg((C(x) \to \neg T(x)) \vee (\forall y)(B(y) \to \neg Good(y))$

$\equiv (\forall x) \neg(\neg C(x) \vee \neg T(x)) \vee (\forall y) \neg B(y) \vee \neg Good(y)$

$C \equiv (\forall x)(C(x) \wedge T(x)) \vee (\forall y)(\neg B(y) \vee \neg Good(y))$

$\therefore \neg C : \neg((\forall x)(C(x) \wedge T(x)) \vee (\forall y)(\neg B(y) \vee \neg Good(y))) \equiv$

$\equiv \neg(\forall x) \quad (C(x) \wedge T(x)) \wedge \neg(\forall y)(\neg B(y) \vee \neg Good(y))$

$\equiv (\forall x)(\neg C(x) \vee \neg T(x)) \wedge (\exists y)(B(y) \wedge Good(y))$

$y \leftarrow a$ (a is a Skolemn constant)

$\equiv \underbrace{(\forall x)(\neg C(x) \vee \neg T(x))}_{f_5} \wedge (\underbrace{B(a)}_{f_6} \wedge \underbrace{Good(a)}_{f_7})$

Convert to Clausal Normal form

$f_1 \equiv \neg B(x) \vee \neg G(x) \vee C(x)$

$f_2 \equiv \neg C(x) \vee L(x) \vee T(x) \vee D(x)$

$f_3 \equiv \neg B(x) \vee \neg D(x)$

$f_4 \equiv \neg C(x) \vee \neg G(x) \vee \neg L(x)$

$f_5 \equiv \neg C(x) \vee \neg T(x)$

$f_6 \equiv B(a)$

$f_7 \equiv Good(a)$

$f_8 = Res^{lock}_{\theta_1}(f_6, f_3) = \neg D(a) \qquad \theta_1 = [x \leftarrow a)$

$f_9 = Res^{lock}_{\theta_2}(f_8, f_2) = \neg C(a) \vee L(a) \vee T(a) \qquad \theta_2 = [x \leftarrow a]$

$f_{10} = Res^{lock}_{\theta_3}(f_9, f_5) = \neg C(a) \vee L(a) \qquad \theta_3 = [x \leftarrow a]$

$f_{11} = Res^{lock}_{\theta_4}(f_4, f_7) = \neg C(a) \vee \neg L(a) \qquad \theta_4 = [x \leftarrow a]$

$f_{12} = Res^{lock}(f_{10}, f_{11}) = \neg C(a)$

$f_{13} = Res^{lock}_{\theta_5}(f_1, f_{12}) = \neg B(a) \vee \neg Good(a) \qquad \theta_5 = [x \leftarrow a]$

$f_{14} = Res^{lock}(f_{13}, f_6) = \neg Good(a)$

$f_{15} = Res^{lock}(f_{14}, f_7) = \square$ The empty clause confirms that the negated conclusion is inconsistent with the premises => the conclusion is valid

17.

H_1 : Everyone who is ace at any final exam studies or is brilliant or is lucky.

H_2 : Everyone who makes an A is an ace at some final exam.

H_3 : No CS major is lucky.

H_4 : Anyone who drinks beer does not study.

C : If every CS major makes an A, then every CS major who drinks beer is brilliant.

Domain : D

variables : x, y

predicate symbols:

$Ace : D \times D \to \{T, F\}$, $Ace(x,y) = T$ if x aced on exam y

$S : D \to \{T, F\}$, T if x studies

$B : D \to \{T, F\}$, T if x is brilliant

$L : D \to \{T, F\}$, T if x is lucky

$MA : D \to \{T, F\}$, T if x makes an A at some final exam

$Beer : D \to \{T, F\}$, T if x drinks beer

$CS : D \to \{T, F\}$, T if x is a CS major

Formulas

$H_1 : (\forall x)(\forall y)(\,Ace(x,y) \to (S(x) \lor B(x) \lor L(x)) \equiv$

$\equiv (\forall x)(\forall y)\,(\neg Ace(x,y) \lor S(x) \lor B(x) \lor L(x))$ f_1

$H_2 : (\forall x)(\exists y)\,(MA(x) \to Ace(x,y)) \equiv$

$\equiv (\forall x)(\exists y)\,(\,\neg MA(x) \lor Ace(x,y))$ f_2

$H_3 : (\forall x)\,(\,CS(x) \to \neg L(x)) \equiv (\forall x)(\,\neg CS(x) \lor \neg L(x))$ f_3

$H_4 : (\forall x)(Beer(x) \rightarrow \neg S(x)) \equiv (\forall x)(\neg Bear(x) \vee \neg S(x))$ f_4

$C : (\forall x)(\forall y)((CS(x) \wedge MA(x)) \rightarrow ((CS(y) \wedge Beer(y)) \rightarrow B(y)) \equiv$

$\equiv (\forall x)(\forall y)\ (\neg CS(x) \vee \neg MA(x) \vee \neg CS(y) \vee \neg Beer(y) \vee B(y))$

$f_5 \ \overset{H_1 \vee H_3}{\vdash} (\forall x)(\forall y)(\neg Ace(x,y) \vee \neg CS(x) \vee S(x) \vee B(x))$

$f_6 \ \overset{f_5 \vee H_2}{\vdash} (\forall x)(\forall y)(\neg MA(x) \vee \neg CS(x) \vee S(x) \vee B(x))$

$f_7 \ \overset{f_6 \vee H_4}{\vdash} (\forall x)(\neg MA(x) \vee \neg CS(x) \vee \neg Beer(x) \vee B(x))$

$f_7 \equiv (\forall x)\ (CS(x) \wedge MA(x) \wedge Beer(x)) \rightarrow B(x)$

f_7: Every CS major who makes an A at some final exam and drink beer is brilliant.

However, from the premises, it results that no CS major is lucky and if a CS major drinks beer, then he doesn't study. That means that the only possibility which remains valid is that the CS majors who makes an A at some final exam are brilliant, but the conclusion states that every CS major who drinks beer is brilliant

$\Rightarrow$ Conclusion does not hold