# Seminar 4

! T: 10.7 (Exercises propositional logic | Files | Teams) (PE FOAIE)
(până sept vânt) — nume + prenume + grupă

Pt redactare: Partea teoretică + Partea de rezolvare
(Model pe teams)

Seminar 5: Prop. logic
Seminar 6: Midterm: 1h
Prop. logic: 1h

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## Subunitary representations

$x = 0,42$

$y = 0,71$

$n = 8$

$[x]_{dir, inv, comp}$

$[y]_{dir, inv, comp}$

$[-x]_{\dots}$

$[-y]_{\dots}$

$[x+y]_{comp}$

$[y-x]_{comp}$

$[x-y]_{comp}$

| | S | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|
| $[x]_{bin,c}$ | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| $[-x]_{din}$ | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| $[-x]_i$ | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| $[-x]_c$ | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |

$0.42_{10} \to ?_2$

$0.42_{(10)} \to \quad_{(8)} \to \quad_{(2)}$

$0,42 \cdot 8 = 3,36$

$0,36 \cdot 8 = 2,88$

$0,88 \cdot 8 = 7,04$

$\Rightarrow 0,42_{(10)} = 0,327_{(8)} = 0,\overbrace{011}^{3}\underbrace{010}_{2}\overbrace{111}^{7}$

$0,71_{(10)} \rightarrow {}_{(8)} \rightarrow {}_{(2)}$

$0,71 \cdot 8 = 5,68$

$0,68 \cdot 8 = 5,44$

$0,44 \cdot 8 = 3,52$

$\rightarrow 0,71_{(10)} = 0,553_{(8)} = 0,\overbrace{101}^{5}\underbrace{101}_{5}\overbrace{011}^{3}$

| | S | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|
| $[y]_{dic}$ | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| $[-y]_d$ | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| $[-y]_i$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| $[-y]_c$ | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |

$[x+y]_c = [x]_c \oplus [y]_c$

$[x-y] = [x]_c \oplus [-y]_c$

| | S | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|
| $[x]_c$ | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| $[y]_c$ | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| $[x+y]_c$ | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

OVERFLOW, x, y are POSITIVE
(Rule I)

| | S | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|
| $[y]_c$ | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| $[-x]_c$ | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| $[y-x]_c^{*}$ | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |

$$[y-x]_c = [y]_c \oplus [-x]_c$$

$$\downarrow$$

$$1 \cdot 2^{-2} + 1 \cdot 2^{-5} + 1 \cdot 2^{-7}$$

$$= 0,25 + 0,03 + 0,007$$

$$\approx 0,287$$

$$\begin{array}{r} 0,71- \\ 0,42 \\ \hline 25 \end{array}$$

$[x-y]_c = [x]_c \oplus [-y]_c$

| | S, | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|
| $[x]_c$ | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| $[-y]_c$ | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| $[x-y]_c$ | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |

Verificare : $[1\,1\,0\,1\,1\,0\,1\,1]_c = \overset{S}{0}|0\,1\,0\,0\,1\,0\,1$

$= 1 \cdot 2^{-2} + 1 \cdot 2^{-5} + 0 \cdot 2^{-7}$

$\approx 0,284$

## ② Representing fixed-point notation

$n = 32$ bits

$i = 14$ bits $\Rightarrow F = 32 - 14 - \underbrace{1}_{\text{Sign}} = 17$

$4183{,}44_{(10)} =$

$4183 = 4096 + 64 + 16 + 4 + 2 + 1$

$= 2^{12} + 2^{6} + 2^{4} + 2^{2} + 2^{1} + 2^{0}$

$= 1000001010111$

$0{,}44 \cdot 8 = 3{,}52$
$0{,}52 \cdot 8 = 4{,}16$
$0{,}16 \cdot 8 = 1{,}28$
$0{,}28 \cdot 8 = 2{,}24$
$0{,}24 \cdot 8 = 1{,}92$
$0{,}92 \cdot 8 = 7{,}36$

$\rightarrow 0{,}44_{(10)} = 0{,}341217_{(8)} =$

$= 0{,}\overbrace{011}^{3}\underbrace{100}_{4}\overbrace{001}^{1}\underbrace{010}_{2}\overbrace{001}^{1}\underbrace{111}_{7}$

F

| S | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | |

$4183{,}44 =$ 2 0 A E E 1 4 7

## (3) Floating point

Single precision (SP) → 32 bits
- 1 bit → sign
- 8 bits → exponent
- 23 bits → M

mantissa < 1

4183,44

$$\boxed{\pm 0,m \cdot b^{e}}$$ e → exponent, b → 2

4183,44 = 1000001010111, 0111000010100011 1

= 0, 1000001010111 0111000010100011 1 $\cdot 2^{13}$ → expont

| S | E | | | | | | | | M | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | | | | 6 | | | | 4 | | | | 1 | | | | 5 | | | | D | | | | C | | | | 2 | | |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |

c = e + d → DP =

SP = 127

M

c = 13 + 127 = 140 = 128 + 8 + 4 = $2^7 + 2^3 + 2^2$

= 10001100

Floating point

SP.

normalized mantissa

$-0,062$

$0,062 \cdot 8 = 0,496$

$0,496 \cdot 8 = 3.968$

$0.968 \cdot 8 = 7.744$

$0,744 \cdot 8 = 5.952$

$0,952 \cdot 8 = 7.616$

$0,616 \cdot 8 = 4.928$

$= 7.424$

$0,424 \cdot 8 = 3,392$

$0,062_{(10)} = 0,03757473_{(8)}$

$= 0,000\,011\,111\,101\,111\,100\,111\,011_{(2)} =$

$= 0,11\,111\,101\,111\,100\,111\,011_{(2)} \cdot 2^{-4}$

$C = \text{exp } C \Rightarrow -4 + 127 = 123 = 64 + 32 + 16 + 8 + 2 + 1$

$= 2^6 + 2^5 + 2^4 + 2^3 + 2^1 + 2^0$

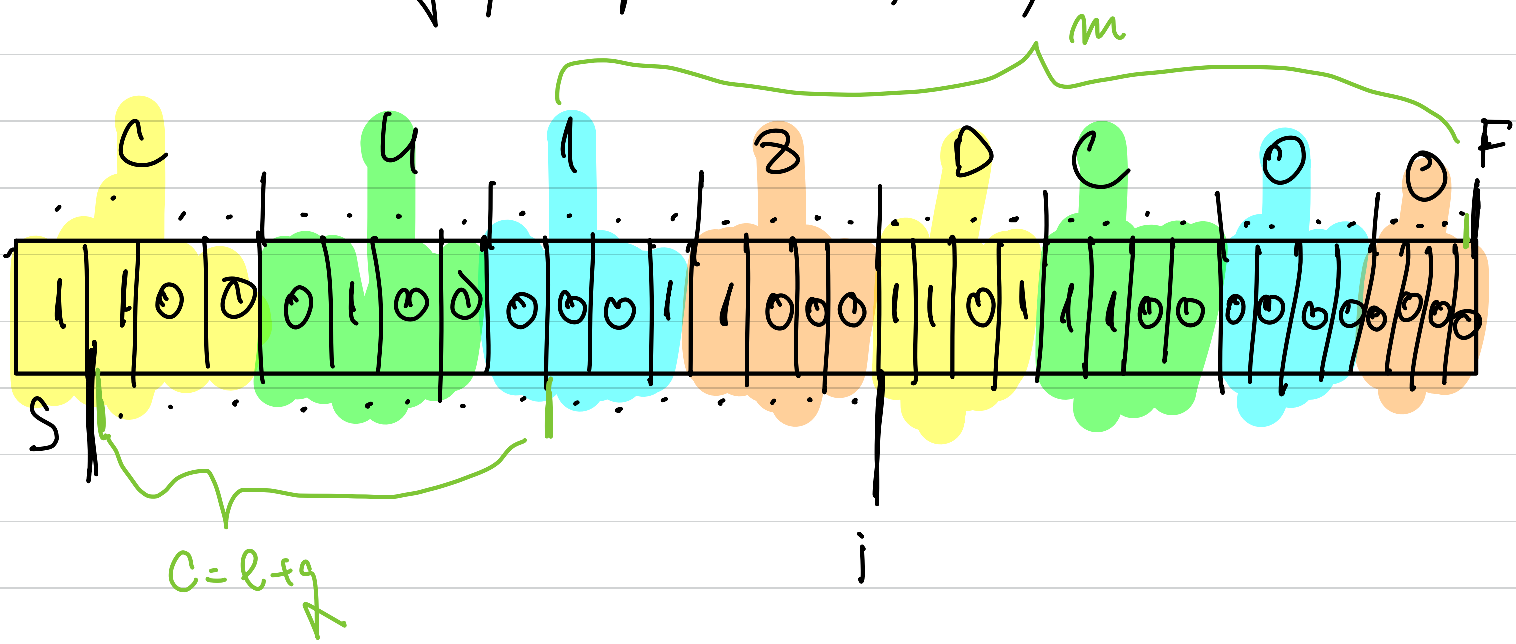
$= 1111011_{(2)}$

Ex 2:

C418DC00

Find the real number x that has:

- Fixed Point (i = 15 bits)
- Floating point representation (SP, m > 1)



# Homework solution

10.7

**Exercise 10**

Prove the following theorems using the theorem of deduction and its reverse.

1. $\vdash p \vee (q \rightarrow r) \rightarrow ((p \vee q) \rightarrow (p \vee r))$;
2. $\vdash (p \rightarrow (\neg r \rightarrow q)) \rightarrow (r \vee \neg p \vee q)$;
3. $\vdash (p \rightarrow (q \rightarrow r)) \rightarrow (p \wedge q \rightarrow r)$;
4. $\vdash (p \wedge q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$;
5. $\vdash (p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow r))$;
6. $\vdash (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$;
7. $\vdash (p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$;

7. $\vdash (p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$

- Theorem of deduction

  If $U_1, \dots, U_{m-1}, U_m \vdash V$, then
  $U_1, \dots, U_{m-1} \vdash U_m \rightarrow V$

- Reverse of the theorem of deduction

  If $U_1, \dots, U_{m-1} \vdash U_m \rightarrow V$, then
  $U_1, \dots, U_{m-1}, U_m \vdash V$

I: First we apply the reverse of the theorem to obtain the initial deduction.

if $\vdash (p \to q) \wedge (p \wedge q \to r) \to (p \to r)$ then

$(p \to q) \wedge (p \wedge q \to r) \vdash p \to r$ then

$(p \to q) \wedge (p \wedge q \to r), p \vdash r$

II: We prove the deduction obtained at I

$(p \to q) \wedge (p \wedge q \to r), p \vdash r$

by building a sequence of formulas

$f_1$: $p$ - premise

$f_2$: $p \to q$ - premise

$f_1, f_2 \vdash_{mp} q$

$A_3: q$ - premise

$A_4: A_1 \wedge A_3 = p \wedge q$ (conjunction at conclusion)

$A_5: p \wedge q \to r$ - premise

$A_4, A_5 \vdash_{mp} r$

$A_6: A_2 \wedge A_5 = (p \to q) \wedge (p \wedge q \to r)$ (conjunction at conclusion)

$A_1, A_6 \vdash_{mp} r$

$A_7: r$

The sequence $(A_1, A_2, A_3, A_4, A_5, A_6)$ is the deduction of $r$ from the premises $(p \to q) \wedge (p \wedge q \to r), p$

III : We begin with the deduction

$(p \to q) \wedge (p \wedge q \to r), p \vdash r$ proved

of II.

We now apply the theorem of deduction three times.

There are $2! = 2$ possibilities, but we only have to prove one.

if $(p \to q) \wedge (p \wedge q \to r), p \vdash r$ then

$(p \to q) \wedge (p \wedge q \to r) \vdash p \to r$ then

$\vdash T_1 = (p \to q) \wedge (p \wedge q \to r) \to (p \to r)$ –

the initial theorem

CNF

8.2 : $U_2 = (q \vee R \overset{1}{\to} P) \overset{2}{\to} (p \overset{3}{\to} r) \wedge q$

We replace 2 using $U \to V = \neg U \vee V$

$U_2 = \neg(q \vee R \overset{1}{\to} P) \vee (p \overset{3}{\to} r) \wedge q$

We replace 1, 3 using $U \to V = \neg U \vee V$

$U_3 = \neg(\neg(q \vee R) \vee P) \vee (\neg P \vee R) \wedge q$

We apply de Morgan's law

$U_3 = ((q \vee R) \wedge \neg P) \vee (\neg P \vee R) \wedge q$

Apply the distributivity law

$U_3 = \big((q \vee R) \vee (\neg P \vee R)\big) \wedge \big(\neg P \vee (\neg P \vee R)\big) \wedge q$

$R \vee R = R$ $\quad$ $\neg P \vee \neg P = \neg P$

$= (q \vee R \vee \neg P \vee R) \wedge (\neg P \vee \neg P \vee R) \wedge q$

$= (q \vee R \vee \neg P) \wedge (\neg P \vee R) \wedge q$ - CNF

**Exercise 9**

Using the definition of deduction, prove the following deductions:

1. $p \to q, r \to t, p \vee r, \neg q \vdash t$;
2. $p \to r, p \vee r \to q, r \vdash q$;
3. $q \to p, t \to r, q \vee t, \neg p \vdash r$;
4. $p \vee (q \to r), p \vee q, \neg p \vdash r$;
5. $\neg p \vee \neg q \vee r, q, p \vdash r$;
6. $p \to \neg q \vee r, p \wedge q, p \vdash r$;
7. $r \vee (q \to p), r \vee q, \neg r \vdash p$;
8. $p \to q, q \to r, r \to t, p \vdash t$.

9.7. $R \vee (q \to P), R \vee q, \neg R \vdash P$

$F_1: \neg R$

$F_2: R \vee q \equiv \neg R \to q$ (using $U \to V \equiv \neg U \vee V$)

$F_1, F_2 \vdash_{MP} q$ | $F_3: q$

$F_4: R \vee (q \to P) \equiv \neg R \to (q \to P)$

$F_1, F_4 \vdash_{MP} (q \to P)$

$F_5: q \to P$

$F_3, F_5 \vdash_{MP} P$

$F_6: P$

$t_6$ was proved from $(t_1, t_2, t_4)$ using modus ponens

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- pt teorie cred că trb. definiția deducerii + maybe modus ponens $(p, p \to q \vdash_{mp} q)$

- ideea basic e să îți definești funcții astfel încât să ai: $\begin{cases} t_i \to t_j \text{ (sau invers)}, \forall i,j \\ \text{ultimul } t_i\ (t_n) \text{ să } \to P \end{cases}$ => practic definiția deducerii

- basically transformi totul în implicare utilizând:

$U \vee P \cong \neg U \to P$ + modus ponens după $(P, P \to q \vdash_{mp} q)$ + maybe de Morgan etc.

(mai îs formule în cursu 3, but those are black magic fuckery)