Seminar 9

Semantic Tableaux – cont.

Ex 5

H1 : All hummingbirds are richly colored

H2 : No large birds live on honey

H3 : Birds that do not live on honey are dull in color

C : All hummingbirds small

$Hb(x) : D \to \{T, F\}$, "x is a hummingbird"

$RC(x) : D \to \{T, F\}$, "x is richly colored"

D - domain of birds

$Lh(x)$:

"

. . .

α rules

$$\begin{array}{c} A \land B \\ | \\ A \\ | \\ B \end{array} \qquad \begin{array}{c} \neg(A \lor B) \\ | \\ \neg A \\ | \\ \neg B \end{array} \qquad \begin{array}{c} \neg(A \to B) \\ | \\ A \\ | \\ \neg B \end{array}$$

β rules

$$\begin{array}{c} A \lor B \\ \diagup \quad \diagdown \\ A \qquad B \end{array} \qquad \begin{array}{c} \neg(A \land B) \\ \diagup \quad \diagdown \\ \neg A \qquad \neg B \end{array} \qquad \begin{array}{c} A \to B \\ \diagup \quad \diagdown \\ \neg A \qquad B \end{array}$$

δ -rules

$$\begin{array}{c} (\exists x) A(x) \\ | \\ A(c) \end{array} \qquad \begin{array}{c} \neg(\forall x)\, A(x) \\ | \\ \neg A(c) \end{array}$$

c = new constant

γ-rules

$(\forall x) A(x)$

$|$

$A(c_1)$

$|$

$A(c_2)$

$|$

$\vdots$

$|$

$A(c_n)$

$|$

$(\forall) A(x)$ — copy of the formula

$\neg(\exists x) A(x)$

$|$

$\neg A(c_1)$

$|$

$\vdots$

$|$

$\neg A(c_n)$

$|$

$\neg(\exists x) A(x)$

$H_1 : (\forall x)(Ml(x) \rightarrow nc(x)) = V_1$

$H_2) : \neg(\exists x)(\neg dl(x) \wedge Cl(x)) \equiv (\forall x)(\neg dl(x) \rightarrow \neg Cl(x))$
$= V_2$

$H_3 : (\forall x)(\neg Cl(x) \rightarrow nc(x)) = V_3$

$C : (\forall x)(Ml(x) \rightarrow dl(x)) = V$

$V_1, V_2, \ldots, V_n \vdash V$
iff $V_1 \wedge V_2 \wedge \ldots \wedge \neg V$
has a closed ST associated with it

$H_1 \wedge H_2 \wedge H_3 \wedge \neg C \; (1) =$

$\mid \not\forall (1)$

$(\forall x)(hh(x) \rightarrow nc(x))$ (2)

$\mid$

$(\forall x)(\neg oh(x) \rightarrow \neg lh(x))$ (3)

$\mid$

$(\forall x)(\neg lh(x) \rightarrow \neg nc(x))$ (4)

$\mid$

$\neg (\forall x)(hh(x) \rightarrow oh(x))$ (5)

$\mid \not\exists$ (5), Picky a new const

$\neg(hh(Picky) \rightarrow oh(Picky))$ (6)

$\mid \alpha$ (6)

$hh(Picky)$

¬sle(Pily)

γ (2), Pidy used for instantiation

le(Pidy) → te(Pidy) (7)

contr at (2)

β(7)

¬le(Pidy)   te(Pidy)

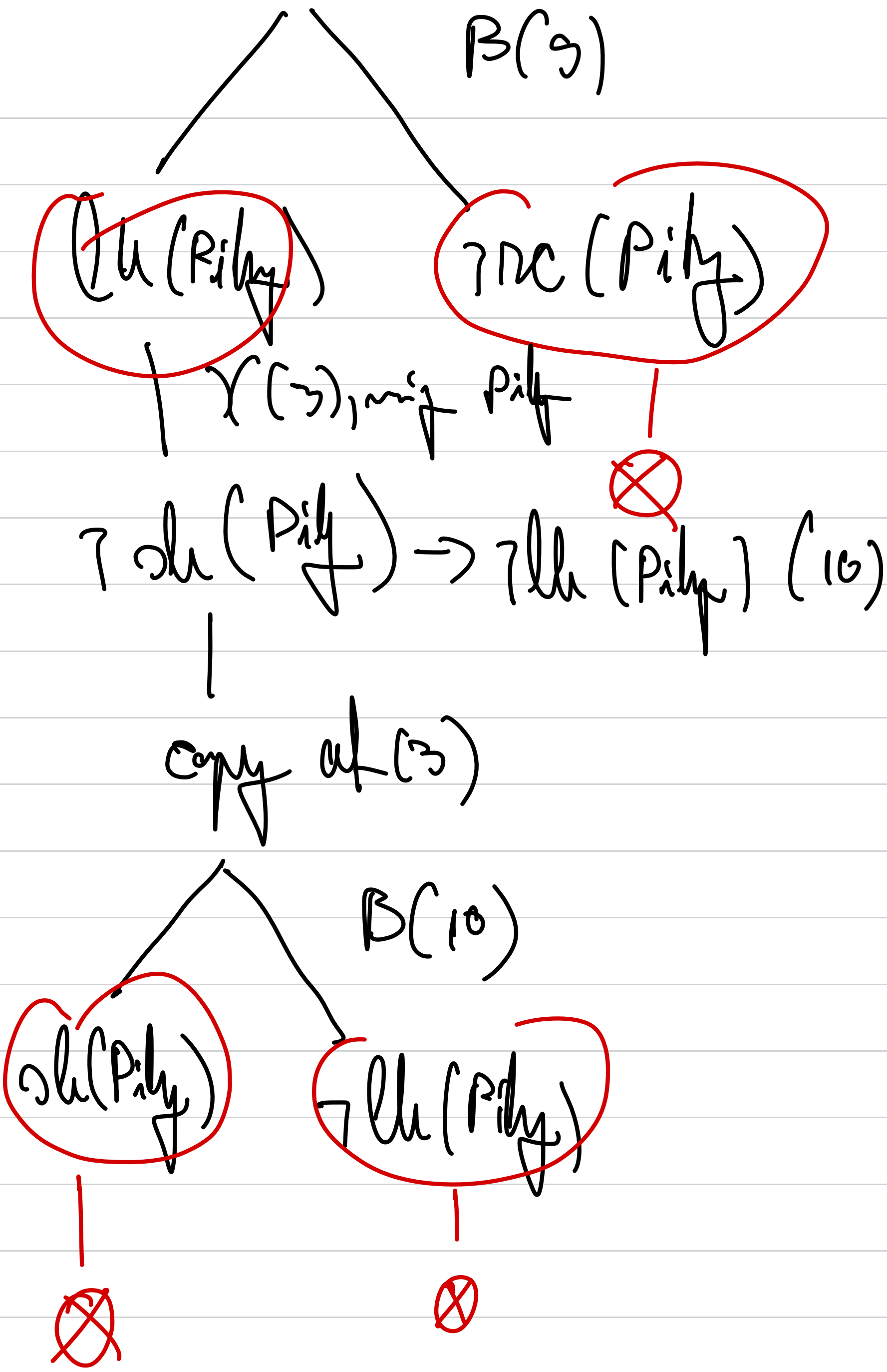
⊗

sc(Piky) (8)

γ (4), using Piky

¬lh(Piky) → ¬sc(Piky) (9)

compr


B(9)
copy of (3)
B(10)

6) Ex 6: Using the ST method, mark:

$$\vDash (\forall x)(A(x) \wedge B(x)) \leftrightarrow (\forall x) A(x) \wedge (\forall x) B(x)$$

TSC of ST

$\vDash U$ (tautology) iff $\neg U$ has a closed ST

$$\neg ((\forall x)(A(x) \wedge B(x)) \leftrightarrow (\forall x) A(x) \wedge (\forall x) B(x))$$

$$\equiv \neg (\forall x)(A(x) \wedge B(x)) \to ((\forall x) A(x) \wedge (\forall x) B(x))$$

α (1)

$(\forall x)(A(x) \wedge B(x))$ (2)

$\neg ((\forall x) A(x) \wedge (\forall x) B(x))$ (3)

$\equiv \neg(\forall x)A(x) \vee \neg(\forall x)B(x)$

Left branch:

$\neg(\forall x)A(x)$ (4)

δ (4)

$\neg A(c)$

c new constant

$\gamma(2)$, using c

$A(c) \wedge B(c)$ (6)

conj of (2)

$A(c)$ (7)

$B(c)$ (8)

$\otimes$

Right branch:

$\neg(\forall x)B(x)$ (5)

δ (5)

$\neg B(c_2)$

c_2, new constant

$\gamma(2)$

$A(c_2) \wedge B(c_2)$

conj of (2)

$A(c_2)$

$B(c_2)$

$\otimes$

$\rightarrow$ tautology

Now the reverse

$\models \neg (\forall x) A(x) \wedge (\forall x) B(x) \to (\forall x)(A(x) \wedge B(x))$

$$\boxed{p \leftrightarrow q = (p \to q) \wedge (q \to p)}$$

Resolution

1. Using general resolution prove that U_7 is a theorem

$$U_7 = (A \to B) \to ((\neg A \to C) \to (\neg B \to C))$$

U_7 is a theorem iff $CNF(\neg U_7) \vdash_{Res} \square$

Semantic methods: - Truth table
- Tableaux

| Syntactic: - Theorem of d.

$$\neg U_7 = \neg((A \to B) \xrightarrow{(1)} ((\neg A \xrightarrow{(2)} C) \xrightarrow{(3)} (\neg B \to C)))$$

Resolution

$C_1 = k \vee l$

$C_2 = g \vee \neg l$

$C_3 = Res_l(C_1, C_2) = k \vee g$

$C_1 = l$

$C_2 = \neg l \Rightarrow C_3 = Res_l(C_1, C_2) = \square$

↳ Empty clause

$$\neg V_M \underset{(2)}{\equiv} \neg(\neg(A \overset{1}{\to} B) \vee ((\neg A \overset{2}{\to} C) \overset{3}{\to} (\neg B \overset{4}{\to} C$$

$$\equiv \neg(\neg(\neg A \vee B)) \vee \neg((A \vee C) \vee (B \vee C))$$

$$\equiv \neg(A \wedge \neg B) \vee ((\neg A \wedge \neg C) \vee B \vee C)$$

$$\equiv (\neg A \vee B) \wedge \neg(\neg A \wedge \neg C) \wedge \neg B \wedge \neg C$$

$$\equiv (\neg A \vee B) \wedge (A \vee C) \wedge \neg B \wedge \neg C$$

$C_1 = \neg A \vee B$

$C_2 = A \vee C$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \underset{\substack{\uparrow \\ C_5}}{\mathrm{Res}_A(C_1, C_2)} = B \vee C =$

$C_3 = \neg B$

$C_3 = \neg B$

$C_4 = \neg C$

$C_6 = \mathrm{Res}_B(C_5, C_3) = C$

$C_4 = \neg C$

$C_7 = \mathrm{Res}_C(C_6, C_4) = \square$

$\Rightarrow CNF(\neg U_7) \vdash_{\mathrm{res}} \square \Rightarrow U_7$ is a theorem

Ex.2:

H1: $\underbrace{\text{Mary will go to London}}_{\text{M}}$ if $\overset{K}{\text{Kent}}$

$S \quad go$

$\tilde{S}$

H2: If K passes EN she will go to London

H3: K not EN

H4: This means S will go to London

C: Mary will go to London //

M: "Mary will go to London"

K: "Kate will go to London"

S: "Susan will go to London"

KE: "Kate takes the E.E."

H_1: $K \wedge S \rightarrow M \equiv \neg K \vee \neg S \vee M$

H_2: $KE \rightarrow K \equiv \neg KE \vee K$ ✗ subsumed

H_3: $\neg KE$ ✗ pure literal

H_4: S

H_5: $\neg M$

$U_1 \ldots U_n \vdash V$ iff. $U_1 \wedge U_2 \wedge \ldots \wedge \neg V \vdash$ ☐

Davis Putmann Procedure

ELIMINATE
→ clauses that are tautologies
→ clauses subsumed by others

$C_1 = P \vee q$ | $C_2 = P \vee q \cancel{\vee \neg R \vee S}$

↳ we keep the shortest

→ always should contain a pure literal