Seminar 5 (cont. S4)

Find the real number X having:

C418 DC00

as its:

- fixed point representation, i = 15 bits
- floating point representation, SP, mantissa > 1

S | I

1100	0100	0001	1000	1101	1100	0000	0000
C	4	1	8	D	C	0	0

$$x = -\left(2^3 + 2^4 + 2^{10} + 2^{14} + 2^{-1} + 2^{-2} + 2^{-4} + 2^{-5} + 2^{-6}\right)$$

$$= -(8 + 16 + 1024 + 16384 + 0.5 + 0.25 + 0.06\ldots + 0.003\ldots + 0.01\ldots)$$

$$\begin{array}{r} 1024 \\ \cdot\ 16 \\ \hline 6144 \\ 1024 \\ \hline 16384 \end{array}$$

$$= -(17432 + 0,853\ldots)$$

$$= -17432,853$$

Floating point representation [SP] Mantissa > 1

$c = e + 127$ (E)

S | E | M

S	E								M																						
1	1	0	0	0	1	0	0	0	0	0	1	1	0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0
C				4				1				8				D				C				0				0			

SP => 1 bit = S
8 bits = E
23 bits = M

$\pm 1,m \cdot b^e$

$E = 1000\,1000$
$= 2^3 + 2^7 = 8 + 128 = 136$

$X = -1,0011000110111 \cdot 2^9$
$= -1001100011,0111_{(2)}$
$= \ldots$ (the conversion)

$136 = 127 + e$
$136 = 127 + e$
$e = 9$

Exercise

$\rightarrow 428,56$
SP
$M > 1$

- base 10 -> base 2
- in loc de $0,\ldots$, va fi $1,\ldots \cdot 2^e$

Propositional logic

Ex 1: Check the following properties for '$\downarrow$', '$\uparrow$', '$\oplus$' using the truth table method.

1\. a. $p \uparrow (q \vee r) \equiv (p \uparrow q) \wedge (p \uparrow r)$

Steps to solve

Theoretical part

LOGICAL EQUIVALENCE

$U \equiv V$ if they have identical truth tables

p	q	r	$q \vee r$	$p \uparrow (q \vee r)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	F
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

$$p \uparrow q = \neg(p \wedge q)$$

p	q	r	$p \uparrow q$	$p \uparrow r$	$(p \uparrow q) \wedge (p \uparrow r)$
T	T	T	F	F	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Ex 2 $\quad U_4 = p \to (p \wedge r) \vee q$

p	q	r	$p \wedge r$	$(p \wedge r) \vee q$	U_4
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	F	F	F
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	F	T
F	F	F	F	F	T

~theory

- consistent: has at least 1 model
- contingent: has at least 1 model and 1 antimodel
- inconsistent: all interpretations are anti-models
- valid: — " — are models

MODELS: interpretation $i : \{p_1, \dots, p_n\} \to \{T, F\} : i(U) = T$

— " — $\quad i(U) = F$

i_4 - anti-model
$i_{1,\dots,8} - \{i_4\}$ - models

$\Rightarrow V_7$ is contingent, consistent

Ex 3:

logical consequence

3.7. $p \to q \models (q \to r) \to (p \to q \lor r)$

	p	q	r	$p \to q$	$q \to r$	$q \lor r$	$p \to (q \lor r)$
1	T	T	T	T	T	T	T
2	T	T	F	T	F	T	T
3	T	F	T	F	T	T	T
4	T	F	F	F	T	F	F
5	F	T	T	T	T	T	T
6	F	T	F	T	T	F	T
7	F	F	T	T	T	T	T
8	F	F	F	T	T	F	T

$$(q \to R) \to (P \to q \vee R)$$

T

T

T

F

T

T

T

T

→ only the models matter

We have the logical consequence when all the models of U are models of V

$$(p \to (q \leftrightarrow r)) \to ((p \to q) \leftrightarrow (p \to r))$$

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

p	q	r	q ↔ r	e p → (q ↔ r)	a p → q	b p → r	d a ↔ b	c → d
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	F	F	F	T	F	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	T	T	T	T	T	T

– is a tautology

5. Transform U_7 into CNF and DNF. Use one of the forms to prove U_7 is a tautology

$$U_7 = (p \xrightarrow{1} (q \wedge r)) \xrightarrow{2} ((p \xrightarrow{3} q) \wedge (p \xrightarrow{4} r))$$

$$\equiv \neg(p \to (q \wedge r)) \vee ((p \to q) \wedge (p \to r))$$

$$\equiv \neg(\neg p \vee (q \wedge r)) \vee ((\neg p \vee q) \wedge (\neg p \vee r))$$

$$\equiv (p \wedge (\neg q \vee \neg r)) \vee (\dots)$$

Normalization algo

1) $U \to V \equiv \neg U \vee V$ + $\leftrightarrow$
2) De Morgan's laws
3) Distributivity

$$\equiv ((p \land \neg q) \lor (p \land \neg r)) \lor$$
$$((\neg p \land \neg p) \lor (\neg p \land r) \lor (q \land \neg p) \lor$$
$$(q \land r))$$

$$\equiv (p \land \neg q) \lor (p \land \neg r) \lor (\neg p \land \neg p) \lor$$
$$(\neg p \land r) \lor (q \land \neg p) \lor (q \lor r)$$

$$\equiv (\neg p \lor q) \lor (p \land \neg r) \lor \neg p \lor$$
$$(\neg p \land r) \lor \qquad (q \lor r)$$