Addition of 2 numbers in base p

Let $A = (a_m a_{m-1} a_{m-2} \dots a_0)_{(p)}$ w/ $m+1$ digits

$B = (b_n b_{n-1} \dots b_0)_{(p)}$ w/ $n+1$ digits

Calculate $A + B = C = (c_k c_{k-1} c_{k-2} \dots c_0)_{(p)}$ w/ $k+1$ digits

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Algorithm $\left(O(\max(m,n)+1) \right)$

- we start from the units digit $(i=0)$, right $\rightarrow$ left
- we add the digits from the same index while keeping track of any carry digits, in which case they are added
- after performing the current iteration we keep track if there is any carry digit for the next one

For consistency we consider the leading 0's of a number.

Example: $a = 123$, $b = 12345$ $\Rightarrow$ $b = 12345$, $a = 00123$

- the initial carry digit t_0 is equal to 0 in the beginning.

Addition can be described using the following formulas:

$$c'_i = (a'_i + b'_i + t_i) - \left\lfloor \frac{a'_i + b'_i + t_i}{p} \right\rfloor \cdot p$$

$$t_{i+1} = \left\lfloor \frac{a'_i + b'_i + t_i}{p} \right\rfloor$$

Where: $a'_i = (a_i)_p$, $b'_i = (b_i)_p$ | The numbers are converted to base 10

c'_i = the remainder obtained by dividing $a'_i + b'_i + t_i$ by p

$= (c_i)_p$

$t_{i+1} =$ the quotient obtained by dividing $a_i + b_i + t_i$ by p

if $t_{\max(m,n)+1} = 0 \Rightarrow k = \max(m,n)$

AKA, if after the algorithm is done there is no carry digit, the final result has $\max(m,n)$ digits

Else $k = \max(m,n)+1$ AND

$c_k = 1$ ALWAYS

Ex 1: $43012_{(5)} + 3243_{(5)} = ?_{(5)}$

$t_0 = 0 \mid m = 4 \mid n = 3$

$i=0 \mid t_0 = 0 \mid 2_5 + 3_5 + 0_5 = 2+3+0 = 5$

$t_1 = [5/5] = 1 \Rightarrow c_0 = 5-5=0$

$i=1 \mid t_1 = 1 \mid 1_5 + 4_5 + 1_5 = 1+4+1 = 6$

$t_2 = [6/5] = 1 \Rightarrow c_1 = 6 - 1\cdot 5 = 6-5 = 1$

$i=2$ | $t_2=1$ | $0_5 + 2_5 + 1 = 0+2+1=3$
$t_3 = [3/5] = 0 \Rightarrow C_2 = 3-0\cdot 5 = 3$

$i=3$ | $t_3=0$ | $3_5 + 3_5 + 0_5 = 3+3=6$
$t_4 = [6/5] = 1 \Rightarrow C_3 = 6-1\cdot 5 = 1$

$i=4$ | $t_4=1$ | $4_5 + 0_5 + 1_5 = 5$
$t_5 = [5/5] = 1 \Rightarrow C_4 = 5-1\cdot 5 = 0$

$t_5 = 1 \Rightarrow C_5 = 1 \Rightarrow$

$\Rightarrow C = 101310_5$

Ex2: $A5F_{16} + 96BD_{16} = ?_{16}$

$m=2 \quad n=3$

$i=0$ | $t_0=0$ | $F_{16} + D_{16} + 0_{16} = 15+13 = 28$
$t_1 = [28/16] = 1 \Rightarrow C_0 = 28-16 = 12 \Leftrightarrow C_{16}$

$i=1$ | $t_1=1$ | $5_{16} + B_{16} + 1_{16} = 5+11+1 =$
$t_2 = [17/16] = 1 \Rightarrow C_1 = 17-16 \Leftrightarrow 1_{16}$

$i=2$ | $t_2=1$ | $A_{16} + 6_{16} + 1 = 17$
$t_3 = 1 \Rightarrow C_2 = 1_{16}$

$i=3$ $\left| \begin{array}{l} t_3 = 1 \;|\; 0_{16} + 9_{16} + 1_{16} = 10 \\ t_4 = 0 \Rightarrow C_3 = 10_{16} \Rightarrow A_{16} \end{array} \right.$

$C = A11C_{16}$

Ex 3: $111010110 1_2 + 110110011_{(2)} = ?_{(2)}$

$$\begin{array}{r} 1\,1\,1\,0\,1\,1\,1\,1\,1\,1\,0 \\ 1110101101_{(2)} + \\ 0110110011 \\ \hline 10101100000_{(2)} \end{array}$$

Multiplication of a nber by a digit in base P

Let $A = (a_n a_{n-1} \dots a_0)_p$, w/ $n+1$ digits

We calculate the product:

$A \cdot b = C = (c_k c_{k-1} \dots c_0)_p$ w/ $k+1$ digits

The algorithm is similar to the addition one

The process is described using the following formulas:

$$k = m$$

$$c'_i = (a'_i \cdot b' + t_i) - t_{i+1} \cdot p$$

$$t_{i+1} = \left[\frac{a'_i \cdot b' + t_i}{p} \right]$$

If the last carry digit is $\neq 0$, then $k = m+1$ AND $c_k = 1$

Example 1: $2031_{(4)} \cdot 3_{(4)} = ?_{(4)}$

$= 12213_{(4)}$

$$\begin{array}{r} 1\ 0\ 2\ 0\ ^{0} \\ 2\ 0\ 3\ 1_{(4)} \\ 3_{(4)} \\ \hline 1\ 2\ 2\ 1\ 3_{(4)} \end{array} \cdot$$

Ex 2: $2B5F_{(16)} \cdot A_{(16)} = ?_{(16)}$

$= 1B1B6_{(16)}$

$$
\begin{array}{r}
1\ 4\ 3\ 9\ 0 \\
2B5F_{(16)} \cdot \\
A_{(16)} \\
\hline
1B1B6_{(16)}
\end{array}
$$

Division of a number by a digit in base p

Let $A = (a_m a_{m-1} a_{m-2} \dots a_0)_p$

The following formulas are used:

$$c_i' = \left[\frac{t_i \cdot p + a_i'}{b'}\right], \quad i = \overline{m,0}$$

$$t_{i-1} = (t_i \cdot p + a_i') - c_i' \cdot b', \quad i = \overline{m,0}$$

Ex 1:

$$
\begin{array}{l|l}
A5B_{(16)} & 8_{16} \\
\cline{2-2}
8 & 14B_{(16)} \\
\cline{1-1}
25 & \\
1 & \\
\cline{1-1}
5B & \\
1 &
\end{array}
$$

$37 - 32 = 5$

$5B = 80 + 11 = 91$

$91 - 88 = 3$

$\overline{3} = 12$ (remainder)

$\Rightarrow A5B_{(16)} = 8_{(16)} \cdot 14B_{(16)} + 3_{(16)}$

Example 2: $2043_{(8)}$ | $5_{(8)}$

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