Seminar 7

6. Write all models using DNF

8. antimodels using CNF

$(\vee \quad \vee) \wedge (\vee \quad \vee) \ldots$

C_F

Ex 8 $(\neg P \vee q) \wedge (\neg r \vee \neg P) \wedge (\neg r \vee \neg q)$

CNF

Clause 1: $(\neg P \vee q)$: F

$\Rightarrow \neg P = F \rightarrow P = T$

$q = F$

i	P	q	r
i_1	T	F	T
i_2	T	F	F

Clause 2: $(\neg r \vee \neg p)^F$

$\neg r$: T, $\neg p$: T

	p	q	r
i_3	T	T	T
i_4	T	F	T

Clause 3: $(\neg r \vee \neg q)^F$

$\neg r$: T, $\neg q$: T

i_5	T	T	T
i_6	F	T	T

$i_1 = i_4$

$i_3 = i_5$

antimodels: i_1, i_2, i_3, i_6

a) Using the definition of deduction prove the following

4. $r \vee (q \to p), r \vee q, \neg r \vdash p$

$f_1: r \vee (q \to p) \equiv \neg r \to (q \to p)$

INFERENCE RULES

$p \to q \equiv \neg p \vee q$

$U, U \to V \vdash_{mp} V$

$U \wedge V \vdash_{simpl} U$

$U \wedge V \vdash_{simpl} V$

$U, V \vdash_{conj} U \wedge V$

$\neg V, U \to V \vdash_{mt} \neg U$

$\equiv \neg q \to r$

$f_2: r \vee q \equiv \neg r \to q$

$f_3: \neg r$

$f_1, f_3 \vdash_{mp} q \to p = f_4$

$f_2, f_3 \vdash_{mp} q = f_5$

$f_4, f_5 \vdash_{mp} p = f$

Conclusion

$(f_1, \ldots, f_6)$ is the deduction of the conclusion from the puisez

1°

12. H_1: $\neg$ sunny $\wedge$ colder

H_2: swim $\rightarrow$ sunny

H_3: $\neg$ swimming $\rightarrow$ canoe

H_4: canoe $\rightarrow$ home

C = home

f_1: $\neg$ sunny $\wedge$ colder

$f_1 \vdash_{simpl}$ $\neg$ sunny $= f_2$
colder $= f_3$

f_4: swim $\rightarrow$ sunny

$f_2, f_4 \vdash_{mt}$ $\neg$ swim $= f_5$

f_6: $\neg$ swim $\rightarrow$ canoe

$f_5, f_6 \vdash_{mp}$ canoe $= f_7$ | f_8: canoe $\rightarrow$ home

$f_7, f_8 \vdash_{mp}$ home

P1: If the dog detects an intruder he will bark

P2: The dog did not bark

Predicate Logic

Ex 1:

4. For any positive integer x, if x is not prime, then there exists a prime y such that y divides x and $x < y$

$pi : \mathbb{N} \to \{T, F\}$, $pi(x) = T$, if x is a positive integer

$pr : \mathbb{N} \to \{T, F\}$, $pr(x) = T$, if x is prime

$dv : \mathbb{N} \to \{T, F\}$, $dv(y, x) = T$ if y divides x

$sm : \mathbb{N} \to \{T, F\}$, $sm(y, x) = T$ if $y < x$

$$\forall(x)\big(\mathrm{Pi}\ (x) \wedge \neg \mathrm{Pr}(x) \rightarrow (\exists y)(\mathrm{Pr}(y) \wedge \mathrm{dr}(y,x) \wedge \mathrm{Sm}(x,y))\big)$$

$$D = \mathbb{Z}^+ = \mathbb{N} \iff$$

$$\iff \big(\underset{D=\mathbb{N}}{\forall(x)}\big)\big(\neg \mathrm{Pr}(x) \rightarrow (\exists y)(\mathrm{Pr}(y) \wedge \mathrm{dr}(y,x) \wedge \mathrm{sm}(x,y))\big)$$

1.5. For every positive integer x, if x is square of an integer then there exists an integer y s.t. $(y+1)\cdot(y-1) = x-1$

- at the exam, write the properties for the function as well

2.

7. Anyone who loves some candy is not a nutrition fanatic.

candy : FD $\to \{T, F\}$, $c(x) = T$ if x is a candy

nut. Fan : P $\to \{T, F\}$, $n(x) = T$ if x is a nutrition fanatic

loves : P $\times$ FD $\to \{T, F\}$,
$l(x, y) = T$ if x loves y

$$(\forall x)_{D=P} (\exists y)_{D=FD} (\text{candy}(y) \wedge l(x, y) \to n(x))$$

14. Wolves do not like to eat horses or giraffes

$w : A \to \{T, F\}$, $w(x) = T$ if x is a wolf

A = animals

P = plants

$fx : A \to \{T, F\}$, $fx(x) = T$ if x is a ~~tea~~

$g : P \to$ --- if $~~x~~$ is a grain

$likeEat(x, y) : A \times P \to \{T, F\}$

$= T$ if x likes to eat y

$$(\forall x)\Big(W(x) \wedge (\forall y)\big(fx(y) \vee g(y) \to \neg likeEat(x, y)\big)\Big)$$

H_1: if x is king and y is his eldest son, then y king

H_2: if x king and y defeats x then y king

H_3: R_3 is king

H_4: H_7 defeated R_3

H_5: H_8 is H_7's eldest son

C: H_8 king?

$K(x)$ "x is king"

$O(x,y)$ "y is eldest son of x"

$D(x,y)$ "y defeats x"

$H_1: (\forall x)(\forall y)(K(x) \wedge O(x,y) \to K(y))$

$H_2 \quad (\forall x)(\forall y)(K(x) \wedge D(x,y) \to K(y))$

$H_3: K(R_3)$

$H_4: D(R_3, H_7)$

$H_5: O(\cancel{H_7}, H_8)$

$C: K(H_8)$

$H_2 \vdash_{\text{universal inst.}} (\forall y)(K(R_3) \wedge D(R_3, y) \to K(y)) = H_6$

$x \leftarrow R_3$

$H_6 \vdash_{\text{universal inst}} K(R_3) \wedge D(R_3, H_7) \to K(H_7) = H_7$

$y \leftarrow H_7$

$$H_3, H_4 \vdash_{\text{conjunction}} K(R_3) \wedge D(R_3, H_7) = H_8$$

$$H_8, H_7 \vdash_{MP} K(H_7)$$