Seminar 6

From last time

$[\ldots]$

$\equiv (p \wedge \neg q \vee (p \wedge \neg r) \vee \neg p \vee (q \wedge r)$

DNF w/ 4 cubes → CNF?

$\equiv (p \vee p \vee \neg p \vee q) \wedge (p \vee p \vee \neg p \vee r)$
$\wedge (p \vee \neg r \vee \neg p \vee q) \wedge (p \vee r \vee \neg p \vee r)$
$\wedge (\neg q \vee p \vee p \vee q) \wedge (\neg q \vee p \vee \neg p \vee r)$
$\wedge (\neg q \vee \neg r \vee \neg p \vee q) \wedge (\neg q \vee p \vee \neg p \vee r)$

CNF – a formula in CNF is valid iff all its clauses are tautologies

6) Using the appropriate normal form find all the models (DNF - gives all the models)
(CNF - tells us if it's a tautology)

$$U_7 = (q \vee r \to p) \to (p \to r) \wedge q$$

$$= \neg(\neg(q \vee r) \vee \neg p) \vee (\neg p \vee r) \wedge q$$

$$= ((q \vee r) \wedge p) \vee ((\neg p \wedge q) \vee (r \wedge q))$$

$$= (q \wedge p) \vee (r \wedge p) \vee (\neg p \wedge q) \vee (r \wedge q)$$

$$= (q \wedge p) \vee (r \wedge p) \vee (r \wedge q)$$

$\Rightarrow$ DNF w/ 3 cubes

I: $(q \wedge \neg p)$ - True

$q = T$

$\neg p = T \Rightarrow p = F$

i_1: $q = T, p = F, r = F$

i_2: $q = T, p = F, r = T$.

II: $(r \land p)$ - True

$r = T$, $p = T$ $\Rightarrow$ i_3: $q = F$, $p = T$, $r = T$

i_4: $q = T$, $p = T$, $r = T$

III: $(r \land q)$ - True

$r = T$, $q = T$ $\Rightarrow$ i_5: $q = F$, $p = F$, $r = T$

i_6: $q = T$, $p = T$, $r = T$

Rt

Models: $\{i_1, i_2, i_3, i_5\}$

8. Antimodels with CNF (since $T \land F = F$)

! DNF - models

- CNF - antimodels

4) $U_M = (P \to (q \to r)) \wedge \neg(P \to (q \to r))$

$= (P \to (\neg q \vee r)) \wedge \neg(P \to (\neg q \vee r))$

$= (\neg P \vee \neg q \vee r) \wedge \neg(\neg P \vee \neg q \vee r)$

$= (\neg P \vee \neg q \vee r) \wedge (P \wedge q \wedge \neg r)$

$= (\neg P \wedge \not{\neg} \wedge q \wedge \neg r) \vee (\neg q \wedge P \wedge q \wedge \neg r)$

$\vee (r \wedge P \wedge q \wedge \neg r)$

$\Rightarrow F$