

Seminar I + II

→ Memorize .bricm (C) able alg. no

→ 11 / 14 outcomes

→ Week 6 : MIDTERM
(MONDAY)

→ 20% of grade

→ questions,
conversions,
representations

→ 5
→ retrograde

→ Seminar activity - 20% (relevant later)
• individual HW
25

→ Final exam - 60%
• don't include seminars (1...6)

+ optional HW + 10%

I. Operations w/ integers in different bases

1.1. Addition

$$A. 5143_{(10)} + 897_{(10)} = 10041_{(10)}$$

$$B. 1154_{(7)}, + 136_{(7)} = 2023_{(7)}$$

$$\begin{array}{r} 1110 \\ 154_{(7)} + \\ 136_{(7)} \\ \hline 2023_{(7)} \end{array}$$

D.

,

+

$$8+3_{16} = 11 \quad \text{DIV } 16 = 0 \\ \text{MOD } 16 = 11$$

$$\begin{array}{r} 0011110 \\ 100100+1 \\ \hline 11100010 \end{array}$$

$$1+1=2 \quad \text{DIV } 2 = 1$$

$$\text{MOD } 2 = 0$$

$$1+1+1=3 \quad /2=1$$

$$1/2=1$$

DESFA^o
 +
 254E3
 ——————
 E1378B

$$8+3=11 \quad /16=0$$

$$\therefore 16=11$$

$$10+14=24 \quad /16=1$$

$$\therefore \therefore 16=2$$

$$15+4+1=23 \quad /16=1$$

$$\therefore 16=4$$

$$1+5+5=15 \quad /16=1$$

$$\therefore 16=3$$

$$14+2+1=17$$

II. Multiplication

$$\begin{array}{r}
 460 \\
 \times 252_{(10)} \\
 \hline
 2064
 \end{array}$$

$$8 \cdot 8 = 64 \quad \text{Div} \ 10 = 6$$

$$\text{Mod} \ 10 = 4$$

$$6+5 \cdot 8 = 46 \quad \text{Div} \ 10 = 4$$

$$\text{Mod} \ 10 = 6$$

$$8 \cdot 2+4 = 20 \quad \text{Div} \ 10 = 2$$

$$\text{Mod} \ 10 = 0$$

$$\begin{array}{r}
 230 \\
 \times 135_{(8)} \\
 \hline
 5(8)
 \end{array}$$

$$5 \cdot 5 = 25 \quad \text{Div} \ 8 = 3$$

$$\text{Mod} \ 8 = 1$$

$$5 \cdot 3+3 = 12 \quad \text{Div} \ 8 = 2$$

$$5 \cdot 1+2 = 7 \quad \text{Div} \ 8 = 1$$

$$\text{Mod} \ 8 = 4$$

$$5 \cdot 4+4 = 36 \quad \text{Div} \ 8 = 4$$

$$\text{Mod} \ 8 = 4$$

$$\text{C. } \begin{array}{r} 122^0 \\ \hline 5 2 3 4 \\ \overline{4(6)} \\ 3424 \\ \overline{4(6)} \end{array}$$

For mid-term: only 1 digit!!

$$4 \cdot 4 = 16 \quad / \quad 6 = 2 \\ \therefore 6 = 4$$

$$4 \cdot 3 + 2 = 14 \quad / \quad 6 = 2 \\ \therefore 6 = 2$$

$$4 \cdot 2 + 2 = 10 \quad / \quad 6 = 4 \\ \therefore 6 = 4$$

$$4 \cdot 5 + 1 = 21 \quad / \quad 6 = 3 \\ \therefore 6 = 3$$

$$\text{D. } \begin{array}{r} 58006^{\frac{1}{16}} \\ 13 \cdot 10 = 130 \text{ DIV } 16 = 8 \\ \text{MOD } 16 = 2 \\ \hline 12468 \\ \overline{16} \end{array}$$

$$13 \cdot 10 = 130 \text{ DIV } 16 = 8 \\ \text{MOD } 16 = 2$$

$$10 \cdot 12 = 140 \text{ DIV } 16 = 8 \\ \text{MOD } 16 = 8$$

$$148 \text{ DIV } 16 = 9 \\ \text{MOD } 16 = 4$$

3. Division

$$\begin{array}{r} 2476_{(10)} \\ \overline{1} \\ \overline{24} \\ \overline{20} \\ \overline{14} \\ \overline{15} \\ \overline{26} \\ \overline{24} \\ \overline{1} \end{array} \quad \left| \begin{array}{r} 5_{(10)} \\ \overline{0455}_{(10)} \\ \overline{11}_{(10)} \end{array} \right.$$

$$24 \text{ DIV } 5 = 4$$

$$\text{MOD } 5 = 4$$

$$47 \text{ DIV } 5 = 9 \\ \text{MOD } 5 = 2$$

...

$$\begin{array}{r}
 4255_8 \\
 \overline{42} \\
 45 \\
 \underline{-25} \\
 1 \\
 1
 \end{array}
 \left| \begin{array}{r}
 5_3 \\
 \hline
 0674 \quad 21
 \end{array} \right.$$

$42_8 = 34_{10}$
 $34 / 5 = 6$
 $\therefore 5 = 4$
 $45_8 = 37$
 $25_8 = 21$

$$\begin{array}{r}
 4338_9 \\
 \overline{1} \\
 \underline{33} \\
 28 \\
 \overline{1}
 \end{array}
 \left| \begin{array}{r}
 7_3 \\
 \hline
 1043, 26
 \end{array} \right.$$

$33_9 = 24 + 3 = 30$
 $30 / 7 = 4$
 $\therefore 7 = 2$
 $28_9 = 18 + 2 = 26$
 $26 \text{ Div } 7 = 3$
 $\text{MOD } 7 = 2$

$$\begin{array}{r}
 4255_{(8)} \\
 \overline{42} \\
 45 \\
 2 \\
 \hline
 \end{array}
 \left| \begin{array}{r}
 5(8) \\
 \hline
 067
 \end{array} \right.$$

$42_8 = 34_{10}$
 $34 / 5 = 6$
 $\therefore 5 = 4$
 $45_8 = 32 + 5 = 37$
 $37 \text{ Div } 5 =$

$$\begin{array}{r}
 D. DA 15_{(10)} \\
 2A \\
 \hline
 51
 \end{array}
 \quad
 \begin{array}{r}
 \overline{B_{16}} \\
 13D320_{(16)}
 \end{array}
 \quad
 \left\{
 \begin{array}{l}
 2A_{16} = 32 + 10 = 42 \\
 42 \text{ DIV } 11 = 3 \\
 \text{MOD } 11 = 9 \\
 \\
 91_{16} = 9 \cdot 16 + 1 \\
 = 144 + 1 \\
 = 145
 \end{array}
 \right.$$

$$\begin{array}{r}
 A=10 \\
 B=11 \\
 C=12 \\
 D=13 \\
 E=14 \\
 F=15
 \end{array}
 \quad
 \begin{array}{r}
 25 \\
 8
 \end{array}
 \quad
 \begin{array}{l}
 145 \text{ DIV } 11 = \\
 13 \\
 \text{MOD } 11 = 2
 \end{array}$$

4. Subtraction

$$\begin{array}{r}
 -11-1 \\
 6130_{(10)} - \\
 1395 \\
 \hline
 4735
 \end{array}
 \quad
 \begin{array}{l}
 0 < 5 \Rightarrow \text{borrow } 1 \\
 \Rightarrow 10 + 0 - 5 = 5
 \end{array}$$

$3 - 1 < 0 \Rightarrow \text{borrow}$

$$10 + 2 - 5 = 3$$

$1 < 3 \Rightarrow \text{borrow}$

$$10 - 3 = 7$$

$$\begin{array}{r} 1010100 \\ -11011 \\ \hline 111001 \end{array}$$

$$\begin{array}{l} 0<1 \Rightarrow \text{borrow} \\ 2-1=1 \\ 0<1 \Rightarrow \text{borrow} \\ 2-1=1 \end{array}$$

$$\begin{array}{l} 1-0-1=0 \\ 0<1 \Rightarrow \text{borrow} \\ 2-1=1 \end{array}$$

$$\begin{array}{l} 0<1 \Rightarrow \text{borrow} \\ 2-1=1 \end{array}$$

$$\begin{array}{r} 4002 \\ - EA5 \\ \hline 3550 \end{array}$$

$$\begin{array}{l} 2<5 \Rightarrow b \\ 16+2-5=13 \\ -1<4 \Rightarrow b \end{array}$$

$$\begin{array}{l} 15-10=5 \\ 12<14 \Rightarrow b \\ 20-14=6 \end{array}$$

$$2<5 \Rightarrow b$$

$$16+2-5=18-5=13=1D$$

$$0<A \Rightarrow b$$

$$0-16+16=4$$

$$-1=$$

0

$$\begin{array}{r} 1010100 \\ -110011^{(2)} \\ \hline 111001 \end{array}$$

$$2+0-1=1$$

$$2+0-1-1=0$$

$$1-1-0=0$$

II Conversions

1. Rapid Conversions $\rightarrow 2, 4, 8, 16$

$$SFAB, 35_{(16)} = ?_{(8)} = 114653, 152_{(8)}$$

$$001001 \mid 1111 \quad 10101011, \quad 001101010$$

$$2107, 31_{(10)} = ? \cdot (8) = ? \cdot (16)$$

$$|010|010|11, 110|00$$

$$= 223,64_{(8)}$$

$$|1001|0011, 1101| = 53, D_{(16)}$$

2. Substitution method

$$2321, 23_{(10)} = ? \cdot (4)$$

$$\begin{aligned} 2321, 23 &= \left(2 \cdot 4^3 + 3 \cdot 4^2 + 2 \cdot 4^1 + 1 + 2 \cdot 4^{-1} + 3 \cdot 4^{-2} \right. \\ &\quad \left. + (2 \cdot 4^1 \cdot 4^2) 4^0 \right) \\ &= (44_4 \cdot 4_4) = 242_4 + 66_4 + 11_4 + 1 \\ &= 341_4 + 12_4 = 353_4 \end{aligned}$$

$$\begin{array}{r} 2_4 \cdot \\ 4_4 + \\ \hline 1 \end{array} \quad \begin{array}{r} 0 \\ 11_4 \cdot \\ 4_4 \\ \hline \overline{11_4} \end{array}$$

$$\begin{array}{r} 2 \\ 44_4 \cdot \\ 4_4 \\ \hline 242_4 \end{array} \quad \begin{array}{l} 4 \cdot 4 = 16 \text{ Div } 4 = 2 \\ \text{Mod } 4 = 2 \end{array}$$

$$4 \cdot 4 = 16 \text{ Div } 4 = 2$$

$$\text{Mod } 4 = 0$$

1 1

$$\begin{array}{r} 242_4 + \\ 66_4 \\ \hline 34_4 \end{array}$$

$$\begin{array}{l} 6 \cdot 2 = 12 \text{ Div } 4 = 1 \\ \text{Mod } 4 = 1 \\ 4 \cdot 6 + 1 = 11 \text{ Div } 4 = 1 \\ \text{Mod } 4 = 1 \end{array}$$

$$3 \cdot 4^2 = 3 \cdot 4_7 \cdot 4_{14} = 15_7 \cdot 4_7 = 66_7$$

$$\begin{array}{r} 2 \\ 15_7 \cdot \\ \hline 6 \end{array}$$

$5 \cdot 4 = 20 \quad \text{Div } 7 = 2$
 $\text{MOD } 7 = 6$

$4 \cdot 1 + 2 = 6 \quad \text{Div } 7 = 0$
 $\text{MOD } 7 = 6$

$$\begin{array}{r} 2 | \overbrace{4} \\ 20 | 0, \end{array}$$

Convert:

$$1. \quad 245_1012_{(12)} \rightarrow ?_{(8)}$$

$$\begin{array}{r} 245 \\ \hline 1 | 36 \\ 2 | 5 \end{array}$$

$24_{12} = 2 \cdot 12 + 4 = 28_{10}$
 $28 \text{ Div } 8 = 3$
 $\text{MOD } 8 = 4$

$45_{12} = 4 \cdot 12 + 5 = 48 + 5 = 53$

$$525,004_{(8)}$$

$$36_{12} = 3 \cdot 12 + 6 = 36 + 6 = 42$$

$$42 \text{ Div } 8 = 5$$

$$\text{MOD } 8 = 2$$

$$\begin{array}{r} 0,012 \cdot 8_{12} = 0,094 \\ 0,054 \cdot 8_{12} = 0,628 \end{array}$$

$$2 \cdot 8 = 16 \quad \text{Div } 12 = 1$$

$$\text{MOD } 12 = 4$$

$$1 \cdot 8 + 1 = 9 \quad \text{Div} \ 12 = 0 \\ \text{Mod} \dots = 9$$

$$4 \cdot 8 = 32 \quad \text{Div} \ 12 = 2 \\ \text{Mod} \ 12 = 8$$

$$\begin{array}{r} \cancel{4} \cancel{4} \\ 0,627 \end{array} \cdot 8_{12} = 4,194$$

$$2 \cdot 8 + 2 = 14 \quad \text{Div} \ 12 = 6 \\ \text{Mod} \ 12 = 2$$

$$3 \cdot 8 = 64 \quad \text{Div} \ 12 = 5 \\ \text{Mod} \ 12 = 4$$

$$2 \cdot 2 + 5 = 21 \quad \text{Div} \ 12 = 1 \\ \text{Mod} \ 12 = 9$$

$$8 \cdot 6 + 1 = 45 \quad \text{Div} \ 12 = 4 \\ \text{Mod} \ 12 = 1$$

$$\underline{\underline{I}}: \quad 525,004_{(8)} \rightarrow ?_{(12)}$$

$$525,004_{(8)} = 5 \cdot 8_{12}^2 + 2 \cdot 8_{12} + 5 + 0 \cdot 8_{12}^{-1} + 0 \cdot 8_{12}^{-2} + 4 \cdot 8_{12}^{-3}$$

$$\begin{array}{l} 5 \cdot 8 = 40 \quad |_{12} = 3 \\ \times : 4 \end{array} \quad \begin{array}{l} = 34 \cdot 8_{12} + 14_{12} + 5_{12} + 4 / 8_{12} / 8_{12} / 8_{12} \\ = 226_{12} + 13_{12} + 0,011_{12} \\ = 243,011_{12} \end{array}$$

$$\begin{array}{r} \cancel{3} \cancel{4} \cdot \\ 34 \cdot \end{array}$$

$$6 \cdot 8 = 48 / 12 = 2$$

$$6+5=15$$

$$\times : 6$$

$$15 \text{ Div} \ 12 = 1$$

$$\overline{2 \ 2 \ 6}$$

$$8 \cdot 3 + 2 = 26 / 12 = 2$$

$$\text{Mod} \ 12 = 3$$

$$\times : 2 = 2$$

$$\begin{array}{c} 4_{12} \\ 40_{12} \\ \hline 0 \end{array} \left| \begin{array}{r} 8_{12} \\ \hline 0,6 \end{array} \right.$$

$$8 \cdot 4 = 32$$

$$40_{12} = 48$$

$$48 \quad \text{Div} \ 8 = 6$$

$$\begin{array}{r|l}
 0,6 & 8_{12} \\
 60 & \overline{0,0\ 9} \\
 0 &
 \end{array}$$

$$60_{12} = 42$$

$$10_{12} = 12$$

$$12 \text{ DIV } 8 = 1$$

$$\text{MOD } 8 = 4$$

$$\begin{array}{r|l}
 0,09 & 8 \\
 5 & \overline{0,011} \quad \dots \\
 10 & \\
 4 &
 \end{array}$$

Captured

More exercises

• Subtraction

$$3032_5 - 2144_5 =$$

$$\begin{array}{r}
 \cancel{-} \cancel{1} \cancel{1} \cancel{-1} \\
 3032_5 - \\
 2144_5 \\
 \hline
 0333
 \end{array}$$

$$2 < 4 \Rightarrow \text{borrow}$$

$$2+5 -4 = 3$$

$$2 < 4 \Rightarrow \text{borrow}$$

$$7 - 4 = 3$$

$$\begin{array}{r}
 \cancel{1} \cancel{+1} \cancel{+1} \\
 2144_5 + \\
 0333 \\
 \hline
 3032
 \end{array}$$
✓

$$-1 < 1 \Rightarrow \text{borrow}$$

$$5 - 1 - 1 = 3$$

$$\begin{array}{r}
 3+4=7 \quad \text{MOD } 5=2 \\
 \hline
 \text{Div } 5=1
 \end{array}$$

$$4+3+1 = 8 \quad \text{MOD } 5=3$$

$$\text{Div } 5=1$$

$$1+3+1 = 5 \quad \text{MOD } 5=0$$

$$\text{Div } 5=1$$

• Division

$$4338_9 \div 4_5$$

$$\begin{array}{r} 4338 \\ \times 4 \\ \hline 17 \\ \hline 1043 \end{array}$$

$$33_9 = 2 \cdot 9 + 3 = 30_{10}$$

$$30 \bmod 4 = 2$$

$$\text{Div } 4 = 4$$

$$5 = 2^2$$

$$28_5 = 18 + 2 = 26$$

$$26 \bmod 5 = 1$$

$$\text{Div } 5 = 3$$

$$\checkmark: 1043_9 \cdot 4_5 + 2_5 =$$

$$\begin{array}{r} 1043_9 \\ \times 4_5 \\ \hline 4333_5 \end{array}$$

$$3 \cdot 4 = 12 \quad \text{Div } 9 = 2$$

$$\bmod 9 = 3$$

$$4 \cdot 4 + 2 = 18 \quad \text{Div } 5 = 3$$
$$\bmod 5 = 3$$

$$4333_5 + 2_5 = 4338_9 \quad \checkmark$$

Conversion: substitution method

$$0, 23 = 2 \cdot 4^{-1} - 3 \cdot 4^{-2}$$