Seminar 11

6.M. $\overline{\underline{II}}$

$$S_7 = \{ r \vee p,\ r \vee \neg p \vee \neg q,\ \neg q \vee \neg r,\ q \}$$

$C_1 = (7)\ r \vee (6)\ p$

$C_2 = (1)\ r \vee (2)\ \neg p \vee (3)\ \neg q$

$C_3 = (5)\ \neg q \vee (4)\ \neg r$

$C_4 = (8)\ q$

$C_5 = \text{Res}^{\text{lock}}_{r}(C_2, C_3)$

$= (2)\neg p \vee (3)\neg q \vee (5)\neg q$

$\equiv (2)\neg p \vee (3)\neg q$

$C_6 = \text{Res}^{\text{lock}}_{p}(C_5, C_1) = (7) r \vee (3)\neg q$

$C_7 = \text{Res}^{\text{lock}}_{q}(C_6, C_4) = (7) r$

$C_8 = \text{Res}^{\text{lock}}_{r}(C_3, C_7) = (5)\neg q$

$C_9 = \ldots = \square \Rightarrow S_7$ - inconsistent

7.7. $S_7 = \{p \vee q, \neg q \vee r, \neg p \vee \neg q \vee \neg r\}$

$C_1 = (1)p \vee (2)q$

$C_2 = (3)\neg q \vee (4)r$

$C_3 = (5)\neg p \vee (6)\neg q \vee (7)\neg r$

- to prove consistency we NEED level saturation

$S^0 = \{C_1, C_2, C_3\} \mid S^k = \{Res(C_i, C_j) \mid C_i \in S^{k-1}, C_j \in S^{\overline{0,k-1}}\}$

$C_4 = Res_p^{LOCK}(C_1, C_3) = {}_{(7)}\neg r \vee (2)q \vee (6)\neg q$ tautology

$S^1 = \{C_4\}$

$C_5 = Res^{LOCK}(C_4, C_2) = {}_7\neg r \vee (4)r \vee (2)q \vee (6)\neg q$ tautology

$S^2 = \{C_5\}$

$S^3 = \emptyset$

II. $C_1 = {}^{(2)}p \vee {}^{(1)}q$

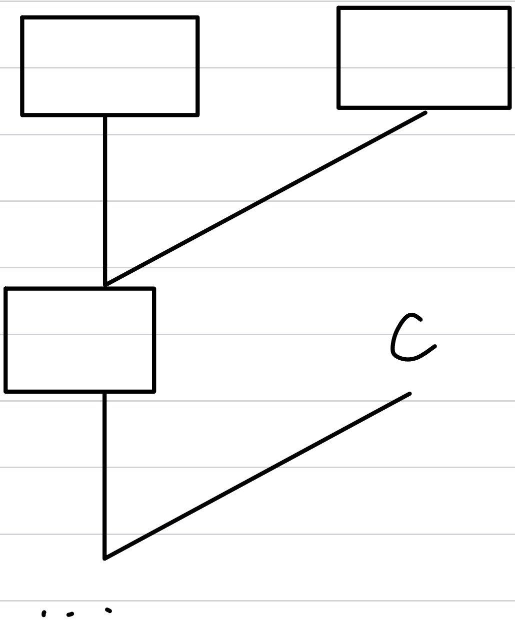
$C_2 = {}^{(4)}\neg q \vee {}^{(3)}r$

$C_3 = {}^{(5)}\neg p \vee {}^{(6)}\neg q \vee {}^{(7)}\neg r$

$S^0 = \{C_1, C_2, C_3\}$

$S^1 = \emptyset \Rightarrow S$ is consistent

LINIAR RESOLUTION



Resolution - Predicate logic

1. Transform the following into prenex, skolem and clausal NF.

$$U_7 = (\forall x)(\forall y)\Big((\exists z) R(z,y) \vee (\exists u)\big(\neg P(x,u) \to (\forall z) \neg Q(y,z)\big)\Big)$$

I: Prenex

1. $A \to B \equiv \neg A \vee B$

$$U_7 = (\forall x)(\forall y)\Big((\exists z) R(z,y) \vee (\exists u)\big(P(x,u) \vee (\forall z) \neg Q(y,z)\big)\Big)$$

Rename bound variables

$$U_7 \quad (\forall x)(\forall y)\Big((\exists z) R(z,y) \vee (\exists u)\big(P(x,u) \vee (\forall t) \neg Q(y,t)\big)\Big)$$

3. Applying De Morgan

$$U_4 = (\forall x)(\forall y)\Big((\exists z)R(z,y) \vee (\exists u)\big(P(x,u) \vee (\forall t)\neg Q(y,t)\big)\Big)$$

4. Extract the quantifiers

$$U_4 = (\forall x)(\forall y)\Big((\exists z)R(z,y) \vee (\exists u)(\forall t)\big(P(x,u) \vee \neg Q(y,t)\big)\Big)$$

these 2 can be moved

$$U_{4_1} = (\forall x)(\forall y)(\exists z)(\exists u)(\forall t)\big(R(z,y) \vee P(x,u) \vee \neg Q(y,t)\big)$$

But NOT these 2

$$U_{4_2} = (\forall x)(\forall y)(\exists u)(\forall t)(\exists z)\big(R(z,y) \vee P(x,u) \vee \neg Q(y,t)\big)$$

We can get more prenex forms, but for each group we must keep the order in which we extract the quantifiers.

II Skolem

- replace existential quantifiers w/ constants or functions^n

n - the number of universal quantifiers IN FRONT

$$U_{\mathcal{M}_1}^{S} = (\forall x)(\forall y)\underset{\exists z,\,\exists u}{}(\forall t)\Big(R(\underbrace{f(x,y)}_{z}, y) \vee P(x, \overbrace{g(x,y)}^{u}) \vee \neg Q(y,t)\Big)$$

For exam strategy
f - binary skolem f.
$z \leftarrow f(x,y,t)\ldots$ etc

$$U_{\mathcal{M}_2}^{S} = (\forall x)(\forall y)(\forall t)\Big(R(\underbrace{f(x,y,t)}_{z}, y) \vee P(x, \overbrace{g(x,y)}^{u}) \vee \neg Q(y,t)\Big)$$

! Try to put the exist. as close to the beginning as possible

III Clausal NF

$$U_{\mathcal{M}_1}^{C} = R(f(x,y), y) \vee P(x, g(x,y)) \vee \neg Q(y,t)$$

$$U_{\mathcal{M}_2}^{C} = R(f(x,y,t), y) \vee P(x, g(x,y)) \vee \neg Q(y,t)$$

2.7. unifiable? if yes, find the mgu

a) $P(a, x, g(f(y)))$

$P(f(y), z, x)$

We cannot unify a and $f(y)$ since neither is a VARIABLE!

(R-VALUES :))))

VANCEA

b) $l_1 = P(x, a, g(h))$

$l_2 = P(f(y), f(y), g(y))$

$\cancel{x \leftarrow f(x)}$

↳ Can't be applied

$\theta = \varepsilon$ (Empty substitution)

0: $\theta(l_1) = P(x, a, g(h))$, $\theta(l_2) = (f(y), f(y), g(x))$

1: $\lambda = [x \leftarrow f(y)]$

$\theta = \theta\lambda = P(f(y), a, g(h))$, $\theta(l_2) = P(f(y), f(y), g(x))$

We cannot unify a with $f(y)$

e) $l_1 = P(h(x,z), f(z), y)$

0. $\theta(l_1) = P(h(x,z), f(z), y)$

1: $\lambda = \{x \leftarrow f(y)\}$

$\theta := \theta\lambda(l_1) = P(h(f(y),z), f(z), y)$

$l_2 = P(h(f(y),x), f(x), a)$

$\theta(l_2) = P(h(f(y),x), f(x), a)$

$\theta(l_2) = P(h(f(y),x), f(x), a)$