

Seminar 6



From last time

[...]

$$\equiv (P \wedge \neg Q \vee (P \wedge \neg R) \vee \neg P \vee (Q \wedge R))$$

DNF w/ 4 cubes \rightarrow CNF?

$$\begin{aligned} \equiv & \overbrace{(P \vee P \vee \neg P \vee \neg P)}^T \wedge \overbrace{(\neg P \vee P \vee P \vee \neg P)}^T \\ & \wedge \underbrace{(\neg Q \vee Q)}_T \wedge \underbrace{(\neg R \vee R)}_T \\ & \wedge \underbrace{(\neg P \vee P)}_T \wedge \underbrace{(\neg P \vee P)}_T \\ & \wedge \underbrace{(\neg Q \vee Q)}_T \wedge \underbrace{(\neg R \vee R)}_T \end{aligned}$$

CNF \rightarrow a formula in CNF is valid iff all its clauses are tautologies

6) Using the appropriate normal form find all the models (DNF - gives all the models)
(CNF - tells us if it's a tautology)

$$\begin{aligned}
 U_7 &= (q \vee r \rightarrow p) \rightarrow (p \rightarrow r) \wedge q \\
 &= \neg(\neg(q \vee r) \vee p) \vee (\neg p \vee r) \wedge q \\
 &= ((q \vee r) \wedge \neg p) \vee ((\neg p \wedge q) \vee (r \wedge q)) \\
 &= (q \wedge \neg p) \vee (r \wedge \neg p) \vee (\neg p \wedge q) \vee (r \wedge q) \\
 &= (q \wedge \neg p) \vee (r \wedge \neg p) \vee (r \wedge q) \\
 &\Rightarrow \text{DNF w/ 3 clauses}
 \end{aligned}$$

I: $(q \wedge \neg p)$ - True

$$q = \text{True}$$

$$\neg p = \text{True} \Rightarrow p = \text{False}$$

$$\begin{array}{l|l}
 i_1: q = \text{True}, p = \text{False}, r = \text{False} \\
 i_2: q = \text{True}, p = \text{False}, r = \text{True}
 \end{array}$$

II: $(\neg P) - \text{True}$

$$\begin{array}{l} \neg P = \overline{1} \\ P = 1 \end{array} \quad \Rightarrow \quad \begin{array}{l} i_3: \neg = F, P = \overline{1}, \neg = \overline{1} \\ i_4: \neg = T, P = \overline{1}, \neg = \overline{1} \end{array}$$

III: $(\neg Q) - \text{True}$

$$\begin{array}{l} \neg Q = \overline{1} \\ Q = 1 \end{array} \quad \Rightarrow \quad \begin{array}{l} i_5: \neg = F, P = F, \neg = \overline{1} \\ i_6: \neg = \overline{1}, P = \overline{1}, \neg = T \end{array}$$

Q

Models: $\{i_1, i_2, i_3, i_5\}$

8. Antimodels with CNF
(since $\overline{1} \wedge F = F$)

! DNF-models
- CNF-antimodels

$$\begin{aligned}
4) \quad U_{\text{K}} & \left(P \rightarrow (q \rightarrow r) \right) \wedge \neg \left(P \rightarrow (q \rightarrow r) \right) \\
& = \left(P \rightarrow (\neg q \vee r) \right) \wedge \neg \left(P \rightarrow (\neg q \vee r) \right) \\
& = \left(\neg P \vee \neg q \vee r \right) \wedge \neg \left(\neg P \vee \neg q \vee r \right) \\
& = \left(\neg P \vee \neg q \vee r \right) \wedge \left(P \wedge q \wedge \neg r \right) \\
& = \left(\neg P \wedge P \wedge q \wedge \neg r \right) \vee \left(\neg q \wedge P \wedge q \wedge \neg r \right) \\
& \quad \vee \left(r \wedge P \wedge q \wedge \neg r \right) \\
& \Rightarrow F
\end{aligned}$$