

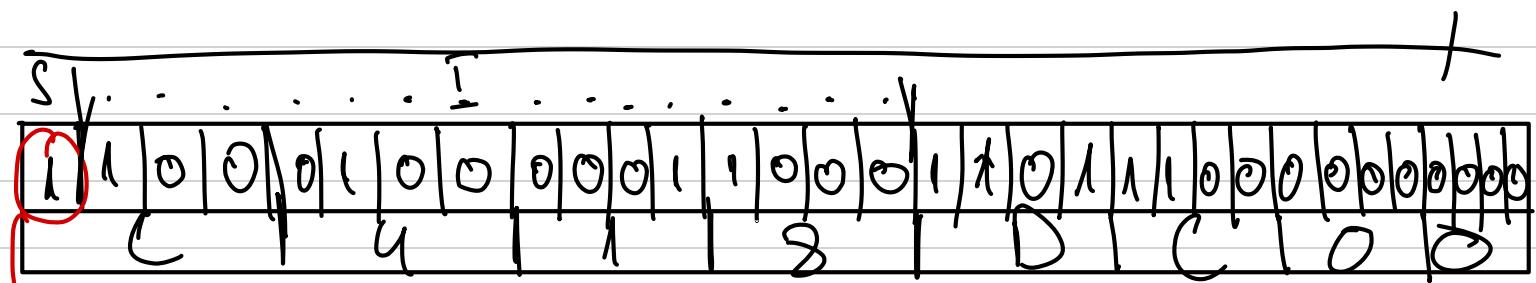
Seminar 5 (cont. Su)

Find the real number x having:

C4 13 DC00

or its:

- fixed point representation, $i=15$ bits
- floating point representation, SP, mantissa > 1



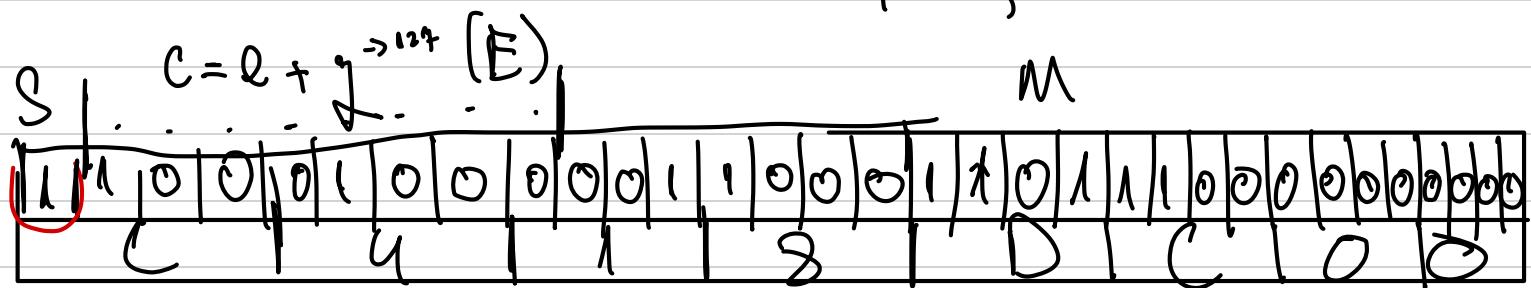
$$x = - \left(2^3 + 2^4 + 2^{10} + 2^{14} + 2^{-1} + 2^{-2} + 2^{-4} + 2^{-5} + 2^{-6} \right)$$

$$= - \left(8 + 16 + 1024 + 16384 + 0.5 + 0.25 + 0.06 \dots + 0.003 \dots + 0.01 \dots \right)$$

$$\begin{aligned} & \left. \begin{aligned} & \frac{1024}{6144} \\ & \frac{1024}{16384} \end{aligned} \right\} = - (14432 + 0.855 \dots) \\ & = - 14432, 855 \end{aligned}$$

Floating point representation

SP | Mantissa > 1



$$\left. \begin{array}{l} SP \Rightarrow 1 \text{ bit} = S \\ 8 \text{ bits} = E \\ 23 \text{ bits} = M \end{array} \right\}$$

$$\pm 1, m \cdot 2^e$$

$$E = 1001000110111$$

$$= 2^3 + 2^4 = 8 + 128 = 136$$

$$\begin{aligned} X &= -1,0001000110111 \cdot 2^9 \\ &= -100100011,0111_{(2)} \\ &= \dots \quad (\text{the conversion}) \end{aligned}$$

$$136 = g + e$$

$$136 = 124 + e$$

$$e = 9$$

Exercise

$$\rightarrow 428, 56$$

$$\begin{array}{c} \text{---} \\ \text{SP} \\ \text{---} \end{array}$$

$$M > 1$$

- base 10 \rightarrow base 2

- in la de $0, \dots, 1, \dots, 2^e$

Propositional logic

Ex₁: Check the following properties for
' \downarrow ', ' \uparrow ', ' \oplus ' using the
truth table method.

$$1. \text{ If } . \quad p \uparrow (q \vee r) \Leftrightarrow (p \uparrow q) \wedge (p \uparrow r)$$

Steps to solve

Theoretical Part

LOGICAL EQUIVALENCE

$U \equiv V$ if they have identical truth tables

$$P \cap J = T(P \cap J)$$

P	$\neg P$	R	$\neg \neg P \wedge R$	$\neg \neg R$	$(\neg \neg P) \wedge (\neg \neg R)$
T	F	T	T	T	T
F	T	F	F	F	F
T	F	F	F	T	F
F	T	F	F	F	F

Ex 2

$$U_g = P \rightarrow (P \wedge R) \vee g$$

P	g	R	$P \wedge R$	$(P \wedge R) \vee g$	U_4
T	T	T	T	T	T
T	F	T	F	F	F
F	T	F	F	F	F
F	F	F	F	F	F

- theory

 - consistent : has at least 1 model
 - contingent : has at least 1 model and 1 anti-model
 - inconsistent : all interpretations are anti-models
 - trivial : $\vdash \perp$ one model

MoDLS: interpretation $i: \{n_1, \dots, n_m\} \rightarrow \{T, F\}$: $i(v) = T$

i_4 - anti-model
 $i_{1,2} \sim \{i_4\}$ - models

$\Rightarrow V_4$ is contingent, consistent

Ex 3.

$P \rightarrow q \vdash (q \rightarrow r) \rightarrow (P \rightarrow q \vee r)$
↑ logical consequence

	P	q	r	$P \rightarrow q$	$q \rightarrow r$	$q \vee r$	$P \rightarrow (q \vee r)$
1	T	T	T	T	T	T	T
2	F	T	F	F	F	T	T
3	T	F	T	F	F	T	T
4	T	F	F	F	F	F	F
5	F	T	T	T	T	T	T
6	F	F	F	T	T	F	T
7	F	F	T	F	T	T	T
8	F	F	F	F	F	F	F

$$(P \rightarrow R) \rightarrow (P \rightarrow P \vee Q)$$

→ only the
models matter

We have the logical
consequence and all the
models at \vee are
models at \vee



$$(P \rightarrow (q \rightarrow r)) \rightarrow f(P \rightarrow q) \rightarrow (P \rightarrow r)$$

$$P \quad \frac{ }{T} \quad P \rightarrow q$$

$$T \quad \frac{ }{T} \quad T$$

$$T \quad F \quad T$$

$$F \quad T \quad F$$

$$P \quad P \quad T$$

P	q	r	$q \rightarrow r$	$P \rightarrow (q \rightarrow r)$	$P \rightarrow q$	$P \rightarrow r$	acsh	$c \rightarrow d$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	F
T	F	T	F	F	F	F	F	F
F	T	F	F	F	F	F	T	T
P	P	T	T	T	T	T	T	T

- is a tautology

5. Transform U_4 into CNF and DNF. Use one of the laws to make U_4 is a tautology.

$$U_4 = (P \rightarrow (q_1 \wedge q_2))^2 \rightarrow ((P \rightarrow q_1) \wedge (P \rightarrow q_2))$$

$$\begin{aligned} &= \neg((P \rightarrow (q_1 \wedge q_2)) \vee (\neg(P \rightarrow q_1) \wedge \neg(P \rightarrow q_2))) \\ &= \neg(\neg P \vee (q_1 \wedge q_2)) \vee (\neg(\neg P \vee q_1) \wedge \neg(\neg P \vee q_2)) \\ &= (P \wedge (\neg q_1 \vee \neg q_2)) \vee (...) \end{aligned}$$

Normalization rules

$$1) U \rightarrow V \equiv \neg U \vee V + \perp \rightarrow$$

2) De Morgan's laws

3) Distributivity

$$= ((\lambda \wedge \gamma_2) \vee (\lambda \wedge \gamma_R)) \vee \\ ((\gamma_P \wedge \gamma_P) \vee (\gamma_P \wedge \gamma_L) \vee (\gamma_J \wedge \gamma_P) \vee \\ (\gamma_2 \wedge \gamma_L))$$

$$\equiv (P \wedge \gamma_2) \vee (\lambda \wedge \gamma_R) \vee (\gamma_P \wedge \gamma_P) \vee \\ (\gamma_P \wedge \gamma_L) \vee (\gamma_J \wedge P) \vee (\gamma_2 \vee \gamma_L)$$

$$= (\gamma_P \vee \gamma_2) \vee (\lambda \wedge \gamma_R) \vee \gamma_P \vee \\ (\gamma_P \wedge \gamma_L) \vee (\gamma_J \vee \gamma_2)$$