CS188 HW2

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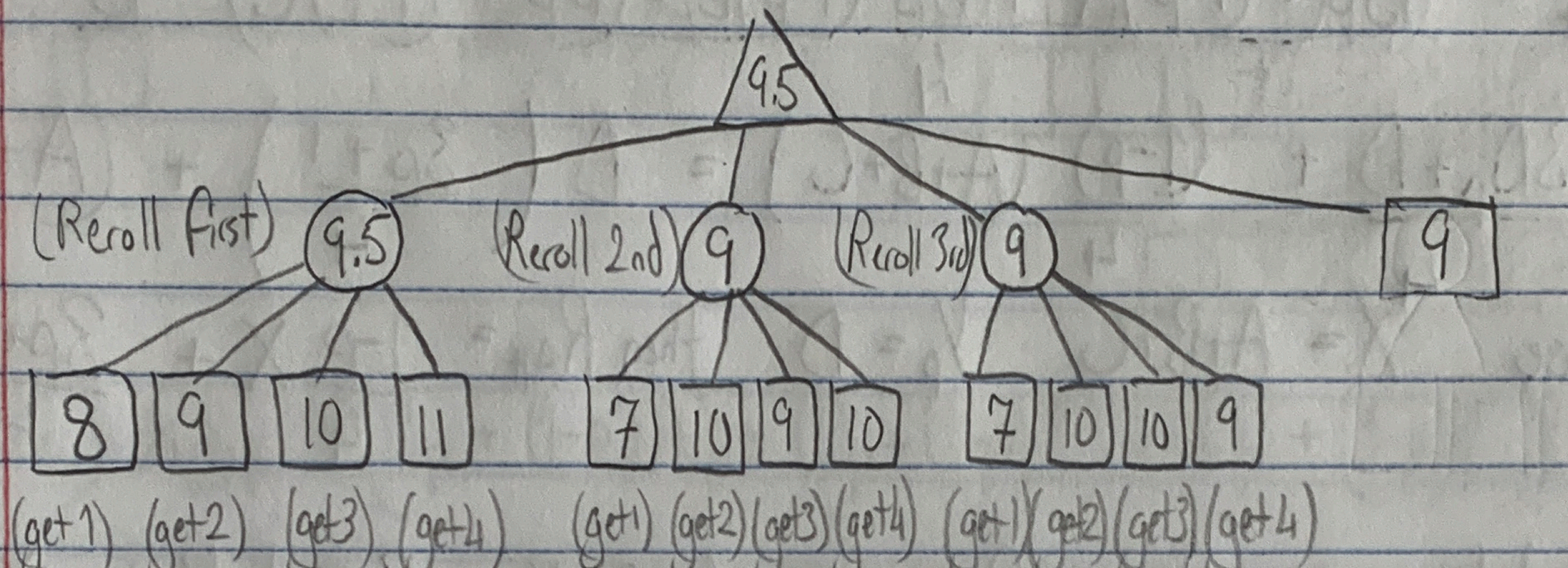
1) a) i) Branching factor from the root is 4

Branching factor from the chance node is 4

One solution; suppose the initial roll is 2,3,4



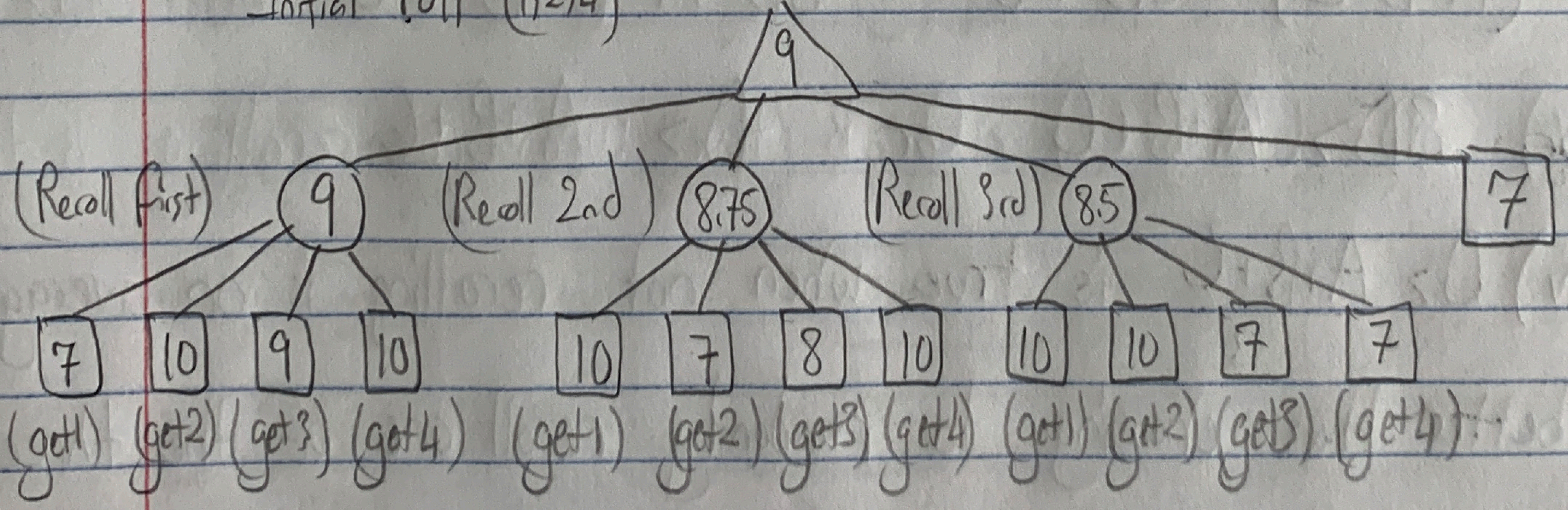
Chance Node 1: 9.5, the expected value to get if choosing to reroll the first die

Chance Node 2: 9, the expected value to get if choosing to reroll the second die

Chance Node 3: 9, the expected value to get if choosing to reroll the third die

ii) Following a similar tree filling pattern from part i

Initial roll (1,2,4)



I would choose to reroll the first die since the expected value is maximized with that course of action.

b) i) The First tree is the accurate representation since after the first chance nodes there's a terminal node or another chance node which accounts for Albertbot's interference

ii) $R_H = \frac{A+B+C+D}{4}$ As each action is equally probable

or $\frac{X}{4} + \frac{Y_p}{4}$ where $X = A+B+C$ $Y_p = D$

$R_{AH} = \frac{(Dp + (1-p)A) + (Dp + (1-p)B) + (Dp + (1-p)C) + D}{4}$

$= \frac{3Dp + D}{4} + \frac{(1-p)}{4}(A+B+C) = D\left(\frac{3p+1}{4}\right) + (A+B+C)\frac{(1-p)}{4}$

Choose $X = A+B+C$ $Y_p = D$ then $R_{AH} = \frac{1-p}{4}X + \frac{3p+1}{4}Y_p$

iii) We want $R_{AH} > R_A \Rightarrow \frac{1-p}{4}X + \frac{3p+1}{4}Y_p > \frac{X}{4} + \frac{Y_p}{4}$

$(1-p)X + (3p+1)Y_p > X + Y_p$

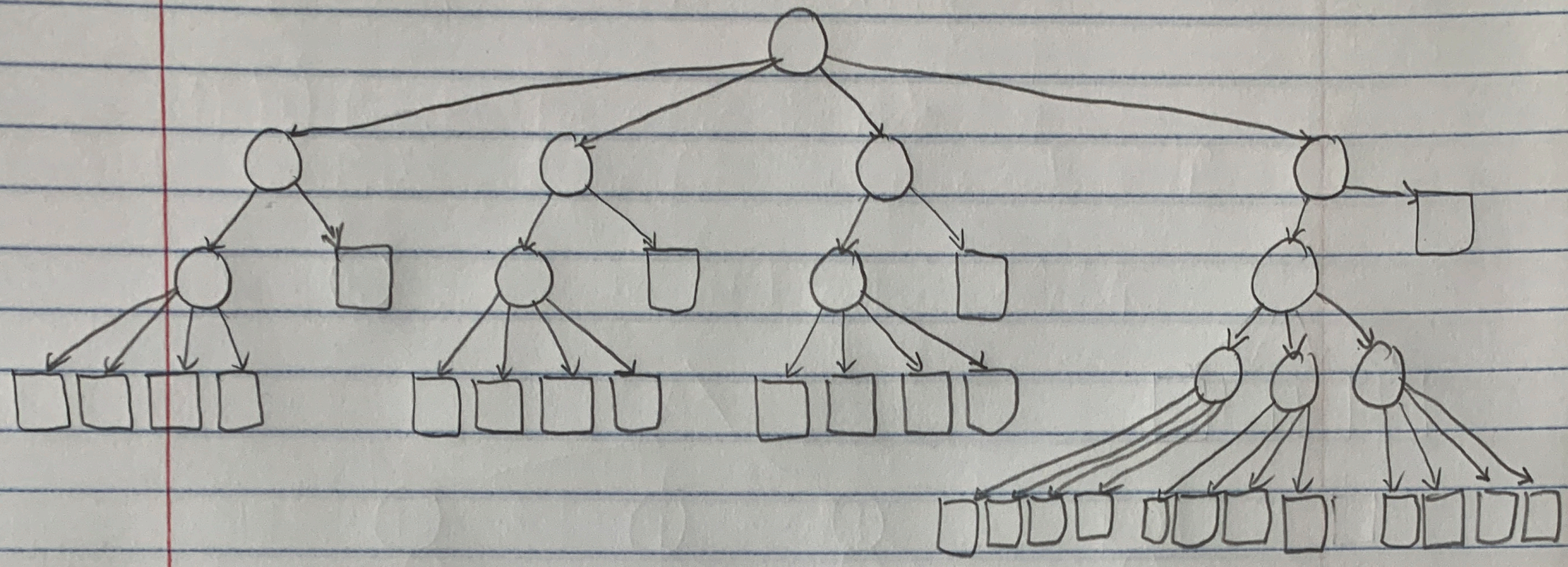
$-pX + 3pY_p > 0$ $\quad -X + 3Y_p > 0$ Plug in X and Y_p

$-(A+B+C) + 3D > 0$ $\quad 3D - A - B - C > 0$

or $3D > A+B+C$ or $D > \frac{A+B+C}{3}$

iv) $D > \frac{A+B+C}{3}$ is true when not rerolling is on average better than choosing to randomly reroll one of the three dice

c) i) The new expectimax tree would look like this:



ii) $R_{DH} = \Big((Dp + (1-p)A) + (Dp + (1-p)B) + (Dp + (1-p)C)$

$+ \big((1-p)D + p\big(\frac{A+B+C}{3}\big)\big)\Big)/4$

$= \big(3Dp + (1-p)(A+B+C) + D - Dp + \frac{p}{3}(A+B+C)\big)/4$

Let $X = A+B+C$ and $Y_p = D$ then

$R_{DH} = (2Y_p \cdot p + Y_p + (1-p)X + \frac{p}{3}X)/4$

$= \big((1-\frac{2p}{3})X + (2p+1)Y_p\big)/4$

iii) $R_{DH} > R_{AH} \Rightarrow \frac{(1-\frac{2p}{3})X + (2p+1)Y_p}{4} > \left(\frac{1-p}{4}\right)X + \frac{(3p+1)Y_p}{4}$

$= \frac{X}{4} - \frac{2Xp}{12} + \frac{2pY_p}{4} + \frac{Y_p}{4} > \frac{X}{4} - \frac{Xp}{4} + \frac{3pY_p}{4} + \frac{Y_p}{4}$

$= -\frac{2Xp}{3} + \frac{pY_p}{2} > -Xp + 3pY_p \quad -\frac{2X}{3} + \frac{Y_p}{2} > -X + 3Y_p$

$\frac{X}{3} > \frac{5Y_p}{2} \quad X > \frac{15Y_p}{2} \Rightarrow A+B+C > \frac{15D}{2}$