

#4.1. Számítsuk ki a megadott görbék ívhosszát az adott intervallumon!

a) $\mathbf{r}(t) = (t)\hat{\mathbf{i}} + (\sqrt{6}t^2/2)\hat{\mathbf{j}} + (t^3)\hat{\mathbf{k}}, \quad t \in [0; 2]$

b) $\mathbf{r}(t) = (t \cos t)\hat{\mathbf{i}} + (t \sin t)\hat{\mathbf{j}}, \quad t \in [0; 1]$

c) $\mathbf{r}(t) = (e^t \cos t)\hat{\mathbf{i}} + (e^t \sin t)\hat{\mathbf{j}} + (e^t)\hat{\mathbf{k}}, \quad t \in [0; 2\pi]$

d) $\mathbf{r}(t) = (t - \sin t)\hat{\mathbf{i}} + (1 - \cos t)\hat{\mathbf{j}}, \quad t \in [0; 2\pi]$

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$$\begin{aligned} L &= \int_0^2 \|\dot{\mathbf{r}}(t)\| \, dt = \int_0^2 \sqrt{1^2 + (\sqrt{6}t)^2 + (3t)^2} \, dt = \int_0^2 \sqrt{1 + 6t^2 + 9t^4} \, dt \\ &= \int_0^2 1 + 3t^2 \, dt = [t + t^3]_0^2 = 10 \end{aligned}$$