TD 4
Unconstrained optimization

Exercice 1 :

$f(x,y) = x^2 + y^2$

$\nabla f(x,y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$ & point critique $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$H(f(x,y)) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} > 0$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ minimum global de f

Exercice 2 :

$f(x,y) = x^3 - y^2 + 6xy$

$\nabla f = \begin{bmatrix} 3x^2 + 6y \\ -3y^2 + 6x \end{bmatrix}$ $\quad 3x^2 + 6y = 0 \Rightarrow y = -x^2/2$

$\quad -3y^2 + 6x = 0 \Rightarrow x(x^3 - 8) = 0$

Points critiques : $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right\}$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $H(f(x,y)) = \begin{bmatrix} 0 & 6 \\ 6 & 0 \end{bmatrix}$ $\quad \lambda_1 = 6, \lambda_2 = -6 \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ point selle

$\begin{bmatrix} 2 \\ -2 \end{bmatrix}$ $H(f(x,y)) = \begin{bmatrix} 12 & 6 \\ 6 & 12 \end{bmatrix}$ $\quad (12-\lambda)^2 - 6^2 = 0 \quad , \lambda_1 = 6, \lambda_2 = 18$

$\Rightarrow \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ minimum global

Exercice 3 :

$f(x,y) = x^2 + y^2 + xy + x - 2y + 3$

Sous la forme $v = \begin{pmatrix} x \\ y \end{pmatrix}$

$f(v) = \frac{1}{2} [x, y] \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + [b_1, b_2] \begin{bmatrix} x \\ y \end{bmatrix} + C$

$= \frac{1}{2} d_{11} x^2 + d_{21} xy + \frac{1}{2} d_{22} y^2 + b_1 x + b_2 y + C$

$H(f) = \begin{bmatrix} d_{11} & d_{21} \\ d_{21} & d_{22} \end{bmatrix}$ $\quad A \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad \lambda_1 = 3, \lambda_2 = 1$

$A \succ 0$ donc f est strictement convexe