7 Constrained optimization

Exercise 7.1
Solve the optimization problem

$$\begin{array}{rll} \min\limits_{x\in\mathbb{R}} & -x & \\ \text{s.t.} & x^2 & = \quad 16 \\ & x & \geq \quad 0 \end{array}$$

with the dual Lagrangian.
Compare to the solution (still based on the dual Lagrangian) of the optimization problem

$$\begin{array}{rll} \min\limits_{x\in\mathbb{R}} & -x & \\ \text{s.t.} & \sqrt{x} & = \quad 2 \\ & x & \geq \quad 0. \end{array}$$

Exercise 7.2
Solve the optimization problem

$$\begin{array}{rll} \min\limits_{(x,y)\in\mathbb{R}^2} & 2x^2+y^2 & \\ \text{s.t.} & x+y & = \quad 1 \end{array}$$

1. with the KKT conditions
2. with the dual Lagrangian.

Exercise 7.3
Solve the optimization problem

$$\begin{array}{rll} \min\limits_{(x,y)\in\mathbb{R}^2} & (x-2)^2+2(y-1)^2 & \\ \text{s.t.} & x+4y & \leq \quad 3 \\ & x & \geq \quad y \end{array}$$

1. with the KKT conditions
2. with the dual Lagrangian.

Exercise 7.4
Solve the optimization problem

$$\begin{array}{rll} \min\limits_{x\in\mathbb{R}^n} & x^Tx & \\ \text{s.t.} & Ax & = \quad b \end{array}$$

with the dual Lagrangian. The matrix A has the size $m \times n$ with $m < n$ and rank$(A) = m$. The vector b belongs to $\mathbb{R}^m$.

Exercise 7.5
We want to use the Jupyter notebook to solve numerically a constrained optimization problem.

1. Run the Python notebook "TD7.ipynb". Explain carefully each command of this file. Some hints are given in the file "MLBD Optimization (scipy.optimize) example.pdf".
2. Modify this notebook to solve numerically exercise 7.3.