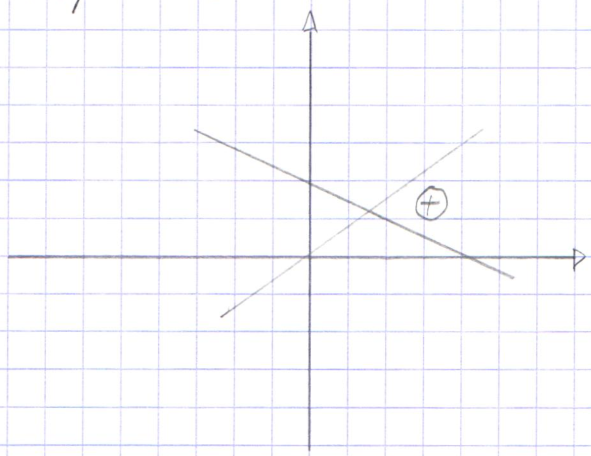
TD7: Constrained optimization

III/ $(x-2)^2+2(y-1)^2=C$



$\mu_1 C_1(X)=0$

① $\nabla L_0=\vec{0}$

$$L_0(X,Y,\mu_1,\mu_2)=(X-2)^2+2(Y-1)^2-\mu_1(-X-4Y+3)-\mu_2(X-Y)$$

$$\nabla L_0=\begin{pmatrix}2X-2+\mu_1-\mu_2\\4Y-1+4\mu_1+\mu_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\qquad \begin{aligned}X^*&=\mu_1/2+\mu_2/2+2\\Y^*&=-\mu_1-\mu_2/4+1\end{aligned}$$

Appliquer X^* et Y^* dans la condition KKT

$\mu_1(\mu_1/2-\mu_2/2-2+4\mu_1+\mu_2-4+3)=0$

$\mu_1(9\mu_1/2+\mu_2/2-3)=0$

$\mu_2(\mu_1/2+3\mu_2/4+1)=0$

Si $\mu_2^*=0$ $\mu_1^*=2/3$ Appliquer μ_1^* et μ_2^* en X^* et Y^*

$X^*=5/3$ $Y^*=2/3$

II/

$2x^2 + y^2 = C$

$L(X,Y,\lambda) = 2x^2 + y^2 - \lambda(2X + Y - 1)$

$$\nabla L_0 = \begin{pmatrix} 4X - \lambda \\ 2Y - \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{matrix} X^* = \lambda/4 \\ Y^* = \lambda/2 \end{matrix}$$

$$\begin{aligned} g(\lambda) &= 2\lambda^2/4^2 + (\lambda^2/4)(3\lambda/4 - 1) \\ &= 2\lambda^2/4^2 + \lambda^2/4 - 3\lambda^2/4 + \lambda \\ &= (\lambda^2 + 2\lambda - 6\lambda^2 + 2\lambda)/8 = -3\lambda^2/8 + \lambda \end{aligned}$$

$d(g(\lambda))/d\lambda = 0 \Rightarrow \lambda^* = 4/3$

On calcule X^* et Y^* avec λ^*

On obtiens donc $X^* = 1/5$ $Y^* = 2/3$

I/

Pour résoudre ce problème:

$x^2 = 16 \Leftrightarrow x = 4$ ou $x = -4$

Or $x = -4$ impossible puisque $x \geq 0$

donc $X^* = 4$