# 8 Support Vector Machine - Hard Margin Classification

**Exercise 8.1**
Consider the following training data :

| Class | $x_1$ | $x_2$ |
|---|---|---|
| +1 | 1 | 1 |
| +1 | 2 | 2 |
| +1 | 2 | 0 |
| -1 | 0 | 0 |
| -1 | 1 | 0 |
| -1 | 0 | 1 |

1. Plot these six training points. Are the classes $\{+1, -1\}$ linearly separable ?
2. Infer the separating line which maximizes the margin and identify the support vectors.
3. If you remove one of the support vectors, does the size of the optimal margin decrease, stay the same, or increase ?
4. Is your previous answer also true for any dataset ? Provide a counterexample or give a short proof.

**Exercise 8.2**
Construct a support vector machine that computes the XOR function denoted $\oplus$. Use values of $+1$ and $-1$ (instead of $1$ and $0$) for both inputs and outputs, so that an example looks like the couple $([x_1, x_2], x_1 \oplus x_2)$ with $x_1, x_2 \in \{-1, 1\}$.

1. Map the input $[x_1, x_2]$ into a space consisting of $x_1$ and $x_1 x_2$. Draw the four input points in this space, and the maximal margin separator.
2. What is the margin ?
3. Draw the separating line back in the original Euclidian input space.

**Exercise 8.3**
Let us recall that the equation of a circle in the 2-dimensional plane is

$$(x_1 - a)^2 + (x_2 - b)^2 - r^2 = 0.$$

Show that every circular region is linearly separable from the rest of the 2-dimensional plane when we consider the feature space $(x_1, x_2, x_1^2, x_2^2)$.