Some Class
Random Examples

Your Name

Contents

Chapter 1

Naam van hoofdstuk

1.1 Random Examples

[colbacktitle=red!75!black]Limit of Sequence in $\mathbb{R}$Let $\{s_n\}$ be a sequence in $\mathbb{R}$. We say

$$\lim_{n\to\infty} s_n = s$$

where $s \in \mathbb{R}$ if $\forall$ real numbers $\epsilon > 0$ $\exists$ natural number N such that for $n > N$

$$s - \epsilon < s_n < s + \epsilon \text{ i.e. } |s - s_n| < \epsilon$$

Voorbeeldvraag 1

Is the set $x-$axis$\setminus\{$Origin$\}$ a closed set

Solution: We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls We will do topology in Normed Linear Space (Mainly $\mathbb{R}^n$ and occasionally $\mathbb{C}^n$)using the language of Metric Space

Herinnering 1.1.1 Topology

Topology is cool

Voorbeeld 1.1.1 (Open Set and Close Set)

Open Set:
- ϕ
- $\bigcup\limits_{x\in X} B_r(x)$ (Any $r > 0$ will do)
- $B_r(x)$ is open

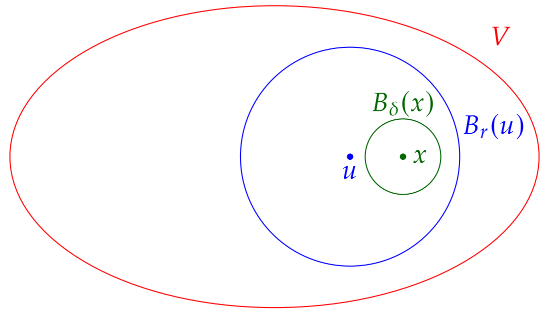
Closed Set:
- $X,\ \phi$
- $\overline{B_r(x)}$

$x-$axis $\cup$ $y-$axis

Belangrijk 1.1.1

If $x \in$ open set V then $\exists\ \delta > 0$ such that $B_\delta(x) \subset V$

Proof: By openness of V, $x \in B_r(u) \subset V$



Given $x \in B_r(u) \subset V$, we want $\delta > 0$ such that $x \in B_\delta(x) \subset B_r(u) \subset V$. Let $d = d(u,x)$. Choose δ such that $d + \delta < r$ (e.g. $\delta < \frac{r-d}{2}$)

If $y \in B_\delta(x)$ we will be done by showing that $d(u,y) < r$ but

$$d(u,y) \leqslant d(u,x) + d(x,y) < d + \delta < r$$

☺

Verduidelijking 1.1.1 hallo

By the result of the proof, we can then show...

Suppose $\vec{v}_1, \ldots, \vec{v}_n \in \mathbb{R}^n$ is subspace of $\mathbb{R}^n$.

Proposition 1.1.1

$1 + 1 = 2$.

1.2 Random

[colbacktitle=red!75!black]Normed Linear Space and Norm $\|\cdot\|$Let V be a vector space over $\mathbb{R}$ (or $\mathbb{C}$). A norm on V is function $\|\cdot\|\ V \longrightarrow \mathbb{R}_{\geqslant 0}$ satisfying

1. $\|x\| = 0 \iff x = 0\ \forall\ x \in V$
2. $\|\lambda x\| = |\lambda|\|x\|\ \forall\ \lambda \in \mathbb{R}(\text{or } \mathbb{C}),\ x \in V$
3. $\|x + y\| \leqslant \|x\| + \|y\|\ \forall\ x, y \in V$ (Triangle Inequality/Subadditivity)

And V is called a normed linear space.

• Same definition works with V a vector space over $\mathbb{C}$ (again $\|\cdot\| \to \mathbb{R}_{\geqslant 0}$) where 2 becomes $\|\lambda x\| = |\lambda|\|x\|$ $\forall\ \lambda \in \mathbb{C}$, $x \in V$, where for $\lambda = a + ib$, $|\lambda| = \sqrt{a^2 + b^2}$

Voorbeeld 1.2.1 ($p-$Norm)

$V = \mathbb{R}^m$, $p \in \mathbb{R}_{\geqslant 0}$. Define for $x = (x_1, x_2, \cdots, x_m) \in \mathbb{R}^m$

$$\|x\|_p = \left(|x_1|^p + |x_2|^p + \cdots + |x_m|^p\right)^{\frac{1}{p}}$$

(In school $p = 2$)

Special Case $p = 1$: $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_m|$ is clearly a norm by usual triangle inequality.

Special Case $p \to \infty$ ($\mathbb{R}^m$ with $\|\cdot\|_\infty$): $\|x\|_\infty = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$

For $m = 1$ these $p-$norms are nothing but $|x|$. Now exercise

Voorbeeldvraag 2

Prove that triangle inequality is true if $p \geqslant 1$ for $p-$norms. (What goes wrong for $p < 1$?)

Solution: **For Property 3 for norm-2**

When field is $\mathbb{R}$:

We have to show

$$
\begin{aligned}
&\sum_i (x_i + y_i)^2 \leqslant \left(\sqrt{\sum_i x_i^2} + \sqrt{\sum_i y_i^2}\right)^2 \\
\implies &\sum_i (x_i^2 + 2x_i y_i + y_i^2) \leqslant \sum_i x_i^2 + 2\sqrt{\left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]} + \sum_i y_i^2 \\
\implies &\left[\sum_i x_i y_i\right]^2 \leqslant \left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]
\end{aligned}
$$

So in other words prove $\langle x, y\rangle^2 \leqslant \langle x, x\rangle\langle y, y\rangle$ where

$$\langle x, y\rangle = \sum_i x_i y_i$$

Dit is mijn besluit na het bekijken van dit hoofdstuk.

1.3 Algorithms

Algorithm 1: what

Input: This is some input
Output: This is some output
/* This is a comment */

some code here;
$x \leftarrow 0$;
$y \leftarrow 0$;
if $x > 5$ **then**
  x is greater than 5 ; // This is also a comment
else
  x is less than or equal to 5;
end
foreach *y in 0..5* **do**
  $y \leftarrow y + 1$;
end
for y *in* 0..5 **do**
  $y \leftarrow y - 1$;
end
while $x > 5$ **do**
  $x \leftarrow x - 1$;
end
return Return something here;