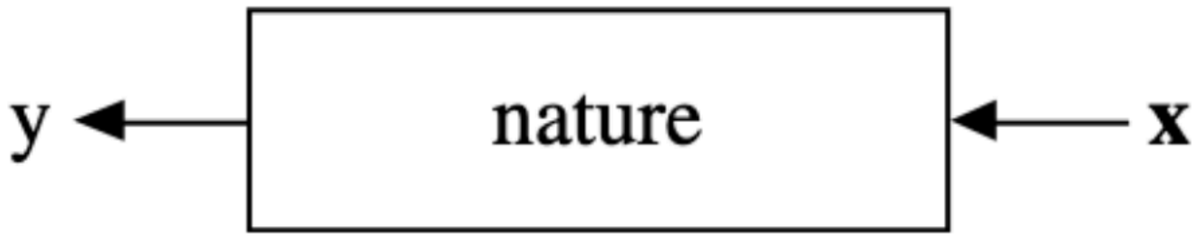
# Statistical Modeling: The Two Cultures

Sophia Lu

# Two Fundamentals of Statistical Order

- Data is generated by a black box:
  - $X$ (predictor) goes into the box
  - Generating process outputs $y$



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- **Prediction**: Predict what the responses are going to be for future input variables. **(Algorithmic modeling culture)**
- **Information**: To extract some information about how nature is associating the response variables to the input variables. **(Data modeling culture)**

# Data Modeling Culture

- Assumes a stochastic data generating process:
    - Example: $y_i \overset{iid}{\sim} f(X_i, \epsilon, \theta)$ where $X_i \in \mathbb{R}^p$, $y_i \in \mathbb{R}$, $\epsilon$ is random noise, and $\theta$ are parameters.

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  - Validation: Validate whether model is valid using hypothesis testing and residual analysis.

# Algorithmic Modeling Culture

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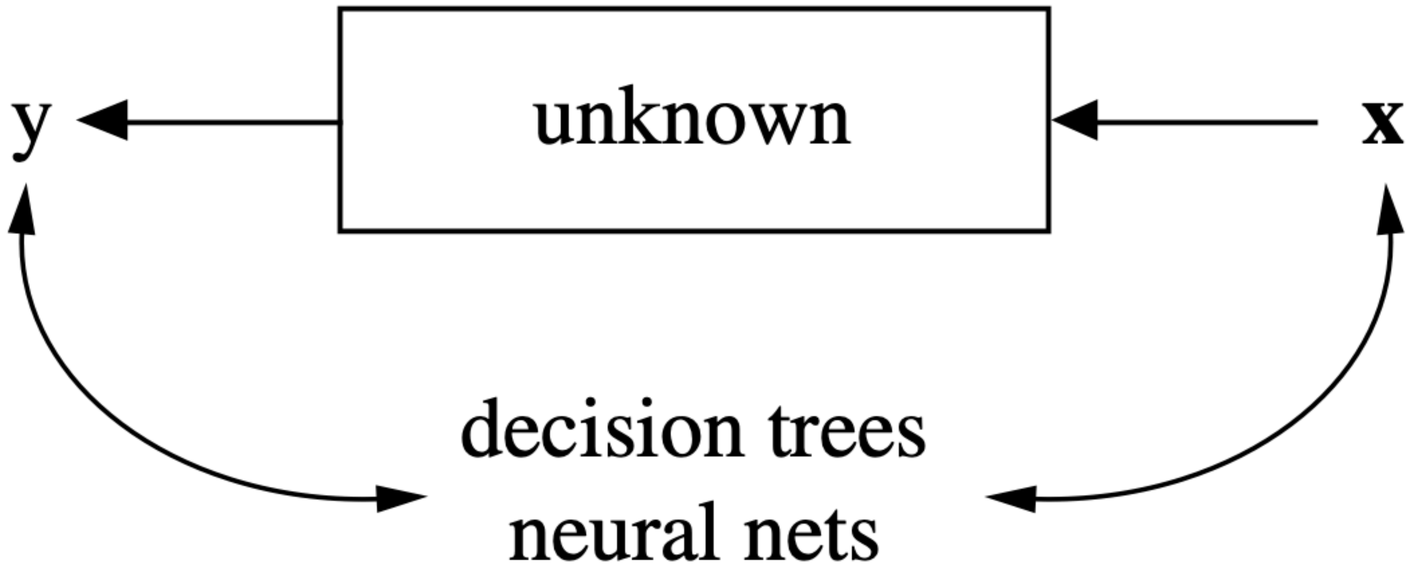
- Goal: find $f(x)$ that predicts well without making distributional assumptions.



# Algorithmic Modeling Culture

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- Validation: select model via predictive accuracy / quantifiable measure of performance.

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    - “Led to **irrelevant theory** and **questionable scientific conclusions**”.
    - “Kept statisticians from using more suitable algorithmic models”.
    - **“**Prevented statisticians from working on exciting new problems”.

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- Still, there have been some gems that have combined nice theory and significant applications. ”

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- I am deeply troubled by the current and past use of data models in applications, where quantitative conclusions are drawn and perhaps policy decisions made.”

# Breiman’s Criticism

- When data is gathered from complex systems, modeling the data generating process by a **simple** parametric model results in questionable conclusions.

# Breiman’s Three Lessons

- Rashomon: multiplicity of good models.
- Occam: conflict between simplicity and accuracy.
- Bellman: dimensionality – curse or blessing.

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- One might ask:
    - Are there redundant features?
    - What are the most effective / representative features to characterize a specific disease?

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- Feature selection is crucial:
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- **Challenge**: Unlike linear models (LASSO), neural networks do not inherently support global feature selection.

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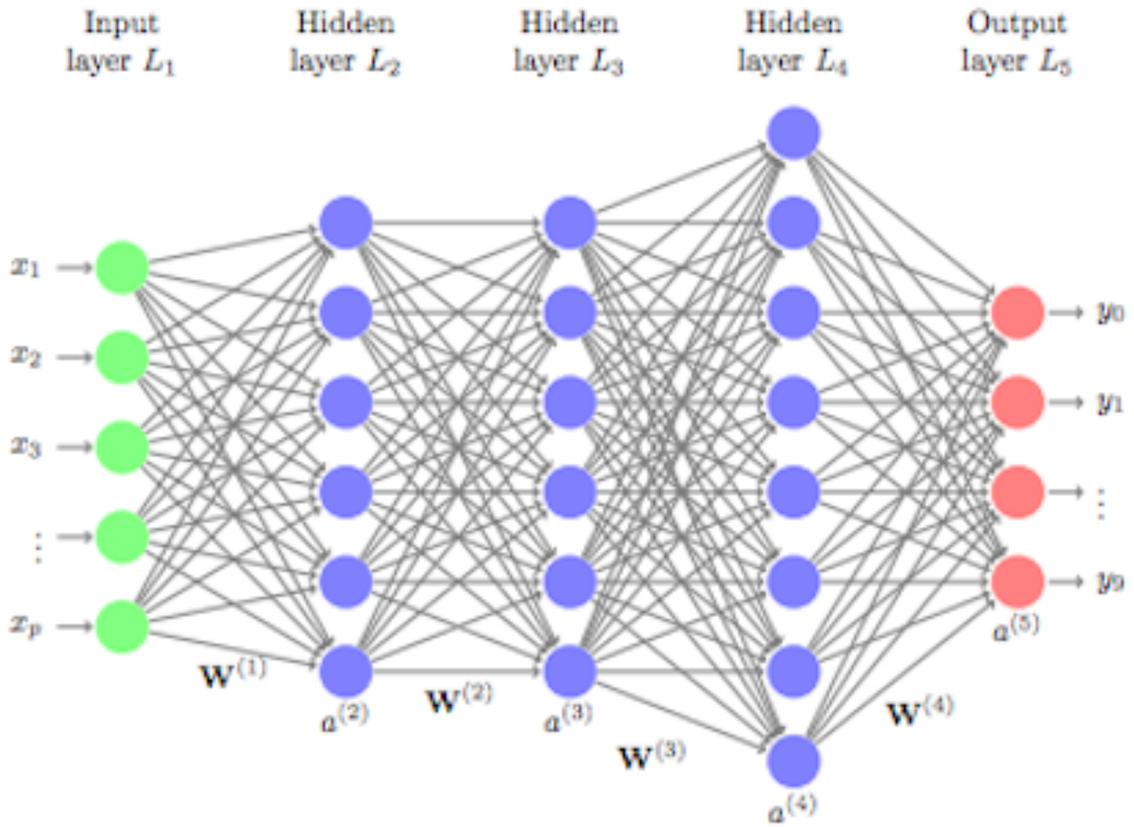

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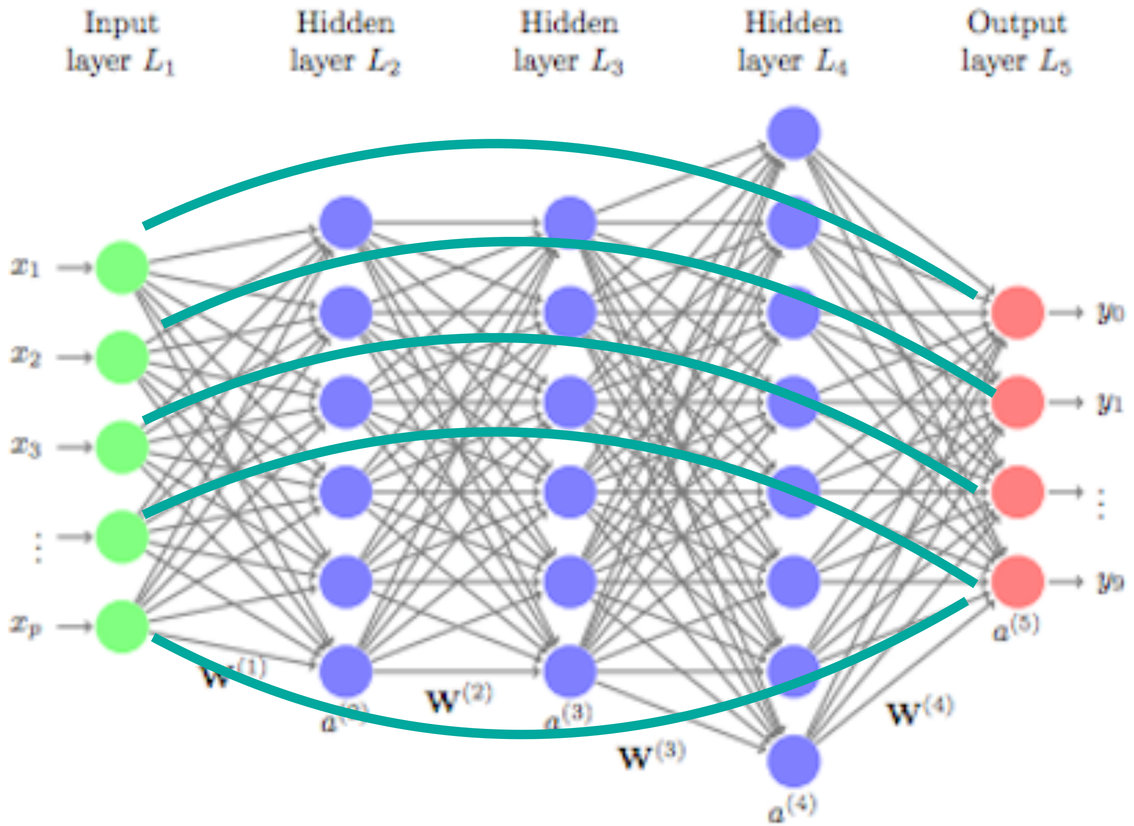


**skip connections**

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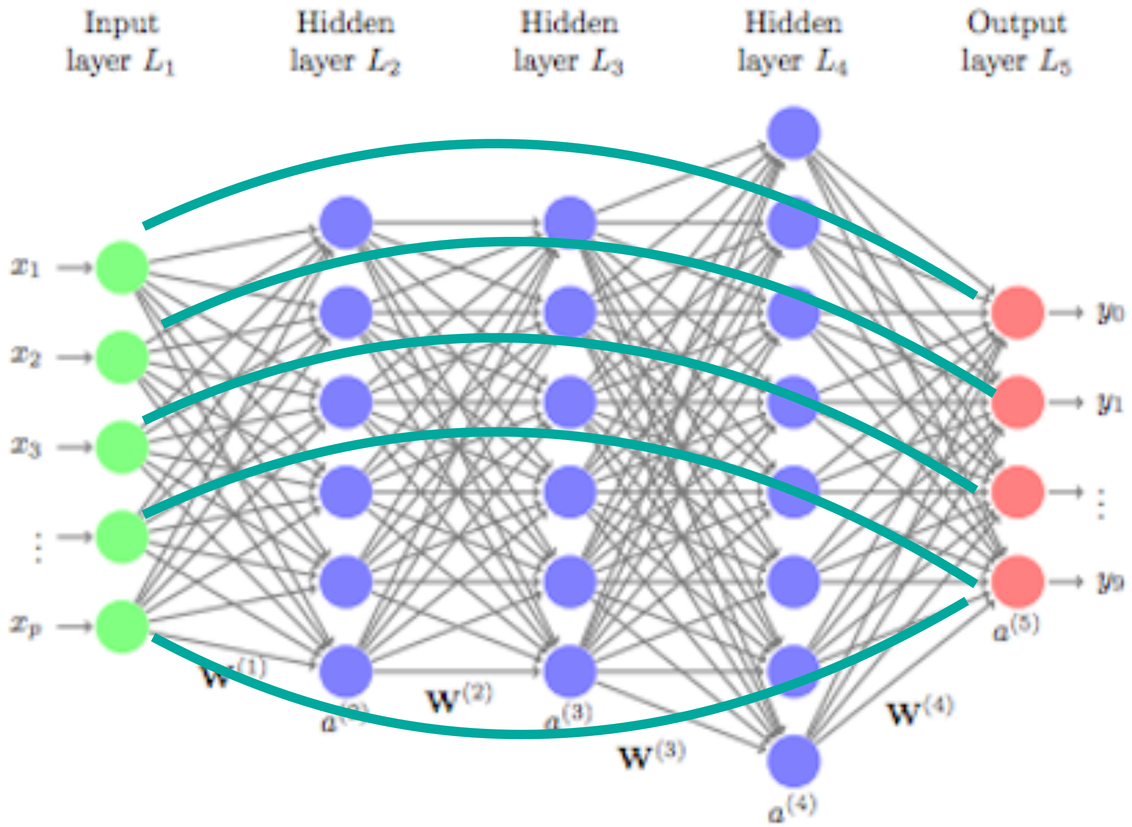


**Key constraint:**
A feature can only influence hidden layers if its corresponding skip connection (with linear coefficients) is active

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- Formally, objective function is as follows:

$$\min_{\theta, W} \quad L(\theta, W) + \lambda \|\theta\|_1$$

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Controls total amount of non-linearity involving feature $j$ according to relative effect importance of $X_j$.

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If $M = 0$, all hidden units are inactive and only skip connection remains.

**Exact Lasso.**

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If $M = \infty$, recovers standard unregularized feed-forward neural network.

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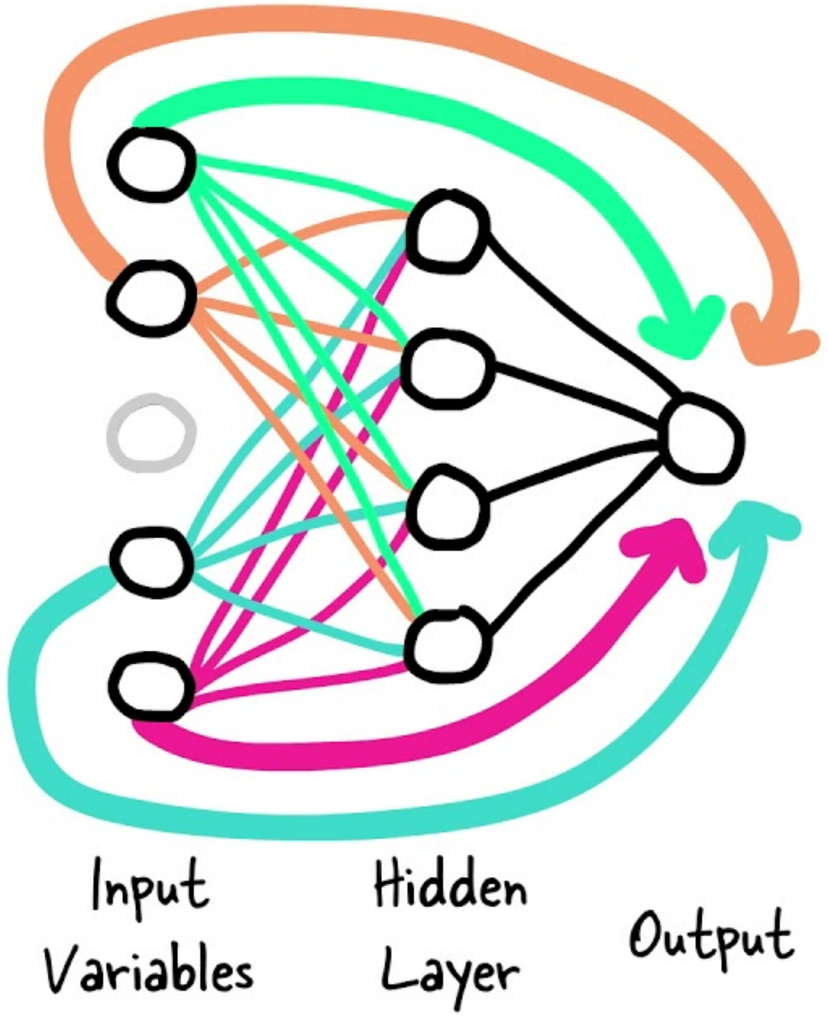


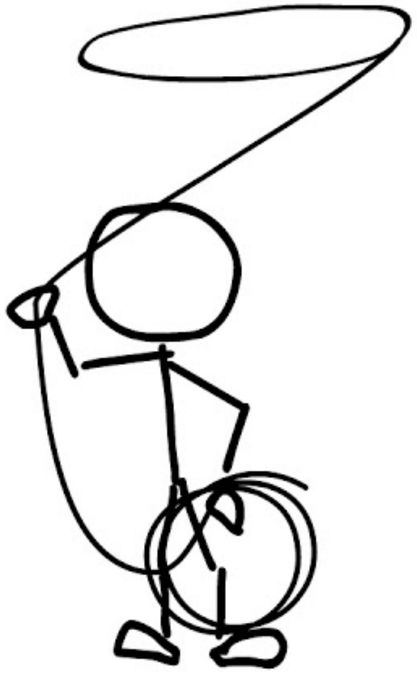

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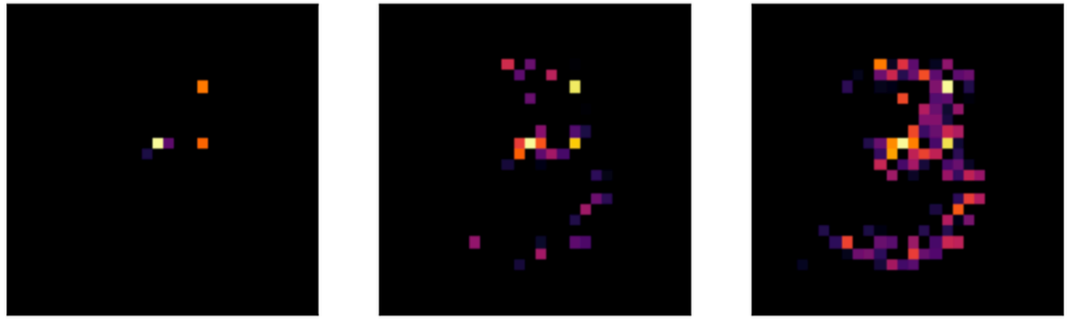

*Figure 7.* Results for LassoNet in choosing the most informative pixels of images of the digit 3 in the MNIST dataset, for three different penalty levels ($\lambda = 5, \lambda = 1, \lambda = 0.1$).

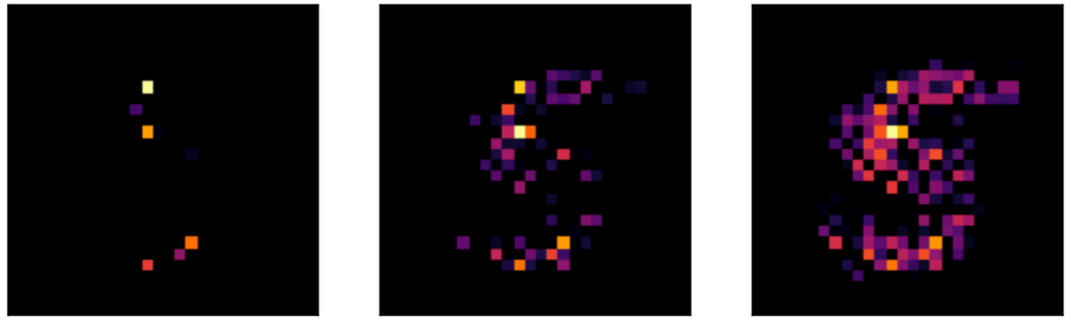

*Figure 8.* Results for LassoNet in choosing the most informative pixels of images of the digit 5 in the MNIST dataset, for the three penalty levels.

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# Breiman's Three Lessons

- When a model is fit to data to draw quantitate conclusions:
  - "The conclusions are about the model's mechanism, and not about nature's mechanism."
- Thus,
  - "If the model is a poor emulation of nature, the conclusions may be wrong."

# Efficient Generative Modeling via Penalized Optimal Transport Network

# Synthetic Data Generation

- **Why are synthetic data important?**

# Synthetic Data Generation

- Generation of **high-quality** synthetic data is important across many disciplines.
- Researchers and practitioners may want to use synthetic data for
  - Model evaluation
  - Data augmentation
  - Model selection

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- Generation of **high-quality** synthetic data is important across many disciplines.
- Researchers and practitioners may use synthetic data for
  - **Model evaluation:** When the true parameter of interest is unobserved (e.g., causal effects), researchers typically rely on Monte Carlo studies with discretionary assumptions to evaluate proposed methods. 
  - Data augmentation
  - Model selection

Athey, Susan, et al. "Using wasserstein generative adversarial networks for the design of monte carlo simulations." *Journal of Econometrics*

# Synthetic Data Generation

- Generation of **high-quality** synthetic data is important across many disciplines.
- Researchers and practitioners may use synthetic data for
  - **Model evaluation:** Faithful synthetic data allows for systematic evaluation of methods under conditions that closely reflect real-world scenarios.
  - Data augmentation
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# Synthetic Data Generation

- Generation of **high-quality** synthetic data is important across many disciplines.
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  - Model evaluation
  - **Data augmentation:** In likelihood-free inference, use synthetic data to directly approximate posterior and bypass likelihood evaluation. In machine learning, address class imbalance issues by adding synthetic data of minor groups.
  - Model selection

Frid-Adar, Maayan, et al. "Synthetic data augmentation using GAN for improved liver lesion classification." *2018 IEEE 15th international symposium on biomedical imaging.*

# Synthetic Data Generation

- Generation of **high-quality** synthetic data is important across many disciplines.
- Researchers and practitioners may use synthetic data for
    - Model evaluation
    - Data augmentation
    - **Model selection:** Evaluate empirical performance of estimators on synthetic datasets, and use these performance metrics to guide model selection process.

Ganganwar, Vaishali, and Ratnavel Rajalakshmi. "Employing synthetic data for addressing the class imbalance in aspect-based sentiment classification." *Journal of Information and Telecommunication*

# Synthetic Data Generation

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- Researchers and practitioners may use synthetic data for
    - Model evaluation
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- Goal: generating **high-quality** synthetic data whose distributions closely mirror the true data-generating mechanism.

# Generative Modeling

- **Generative models** aim to approximate the underlying probability distribution of observed data and produce new samples from the approximated distribution.

- Wasserstein Generative Adversarial Networks (WGANs) are a popular and powerful tool among generative models for synthetic data generation.

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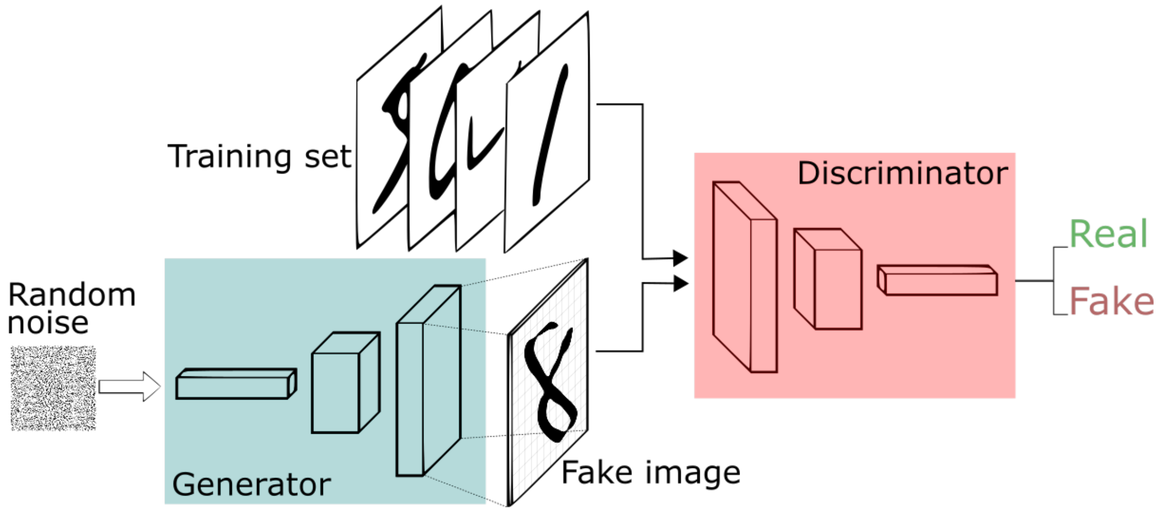

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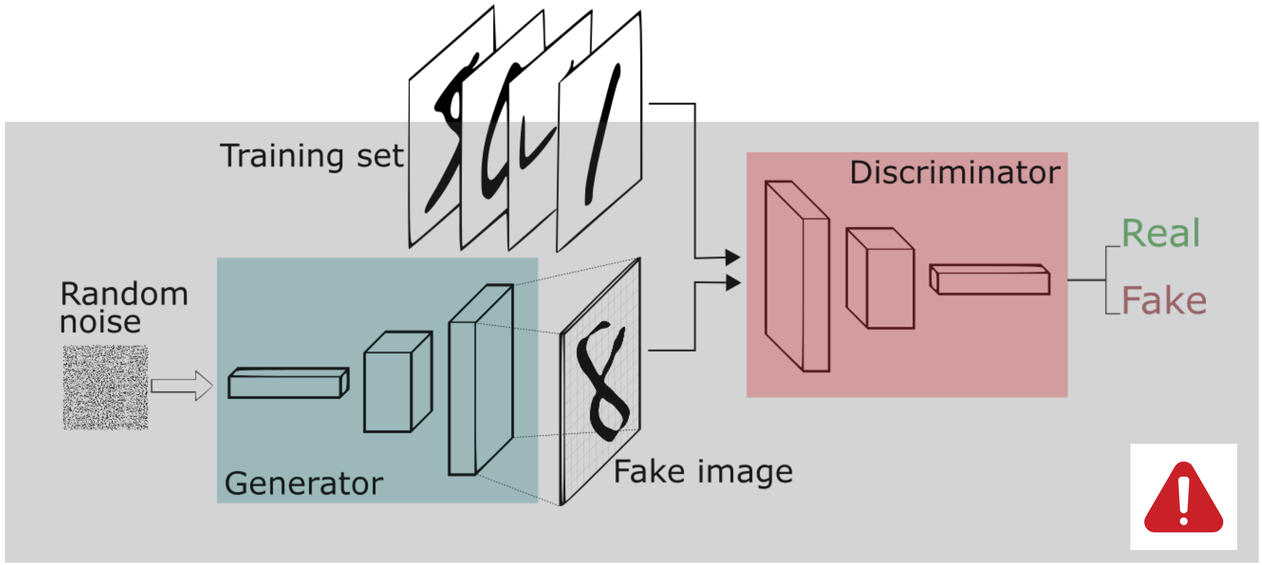


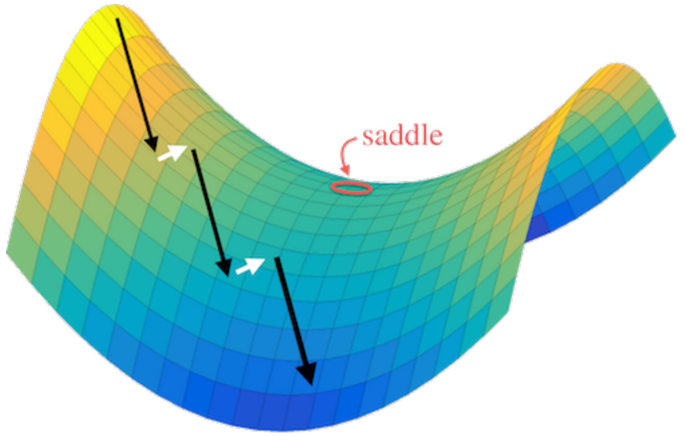

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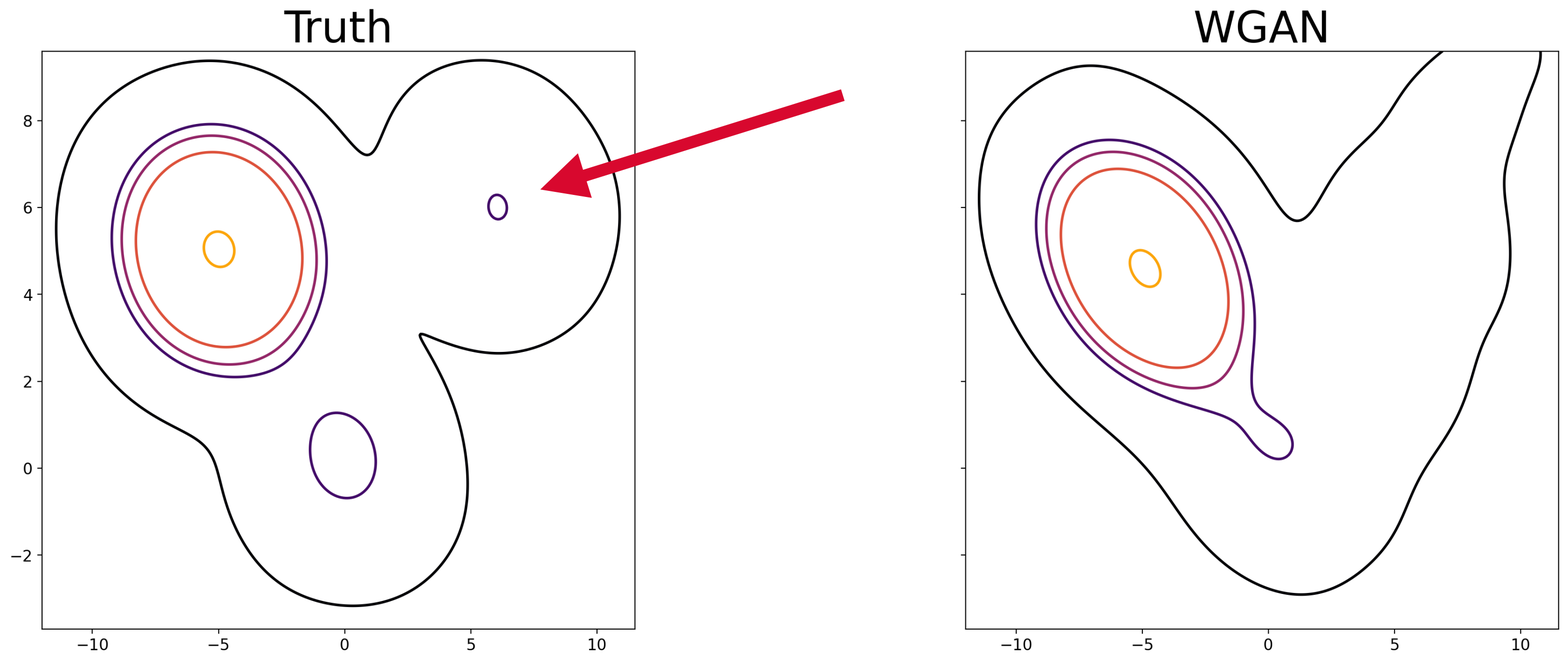

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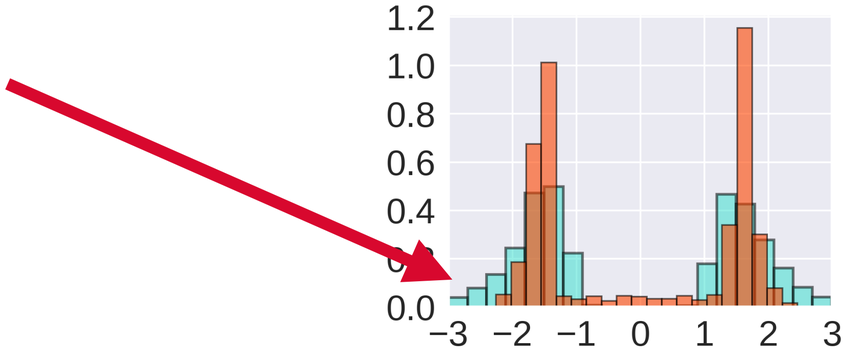

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        - Practical implications: omission of important extremal information.

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- **Causes of mode collapse:**
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# The POTNet Framework

## **P**enalized **O**ptimal **T**ransport **Net**work

- Since marginal distributions converge rapidly (no dependence on dimension), leverage marginal penalty as a form of regularization to guide alignment of joint distributions.
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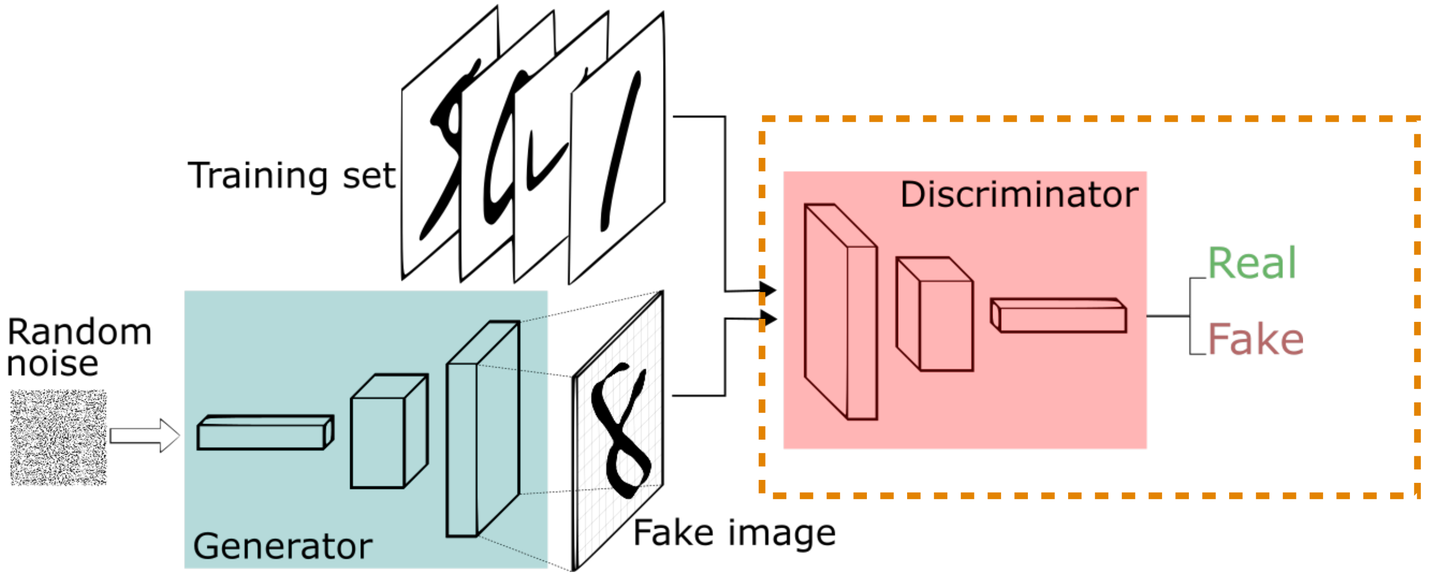

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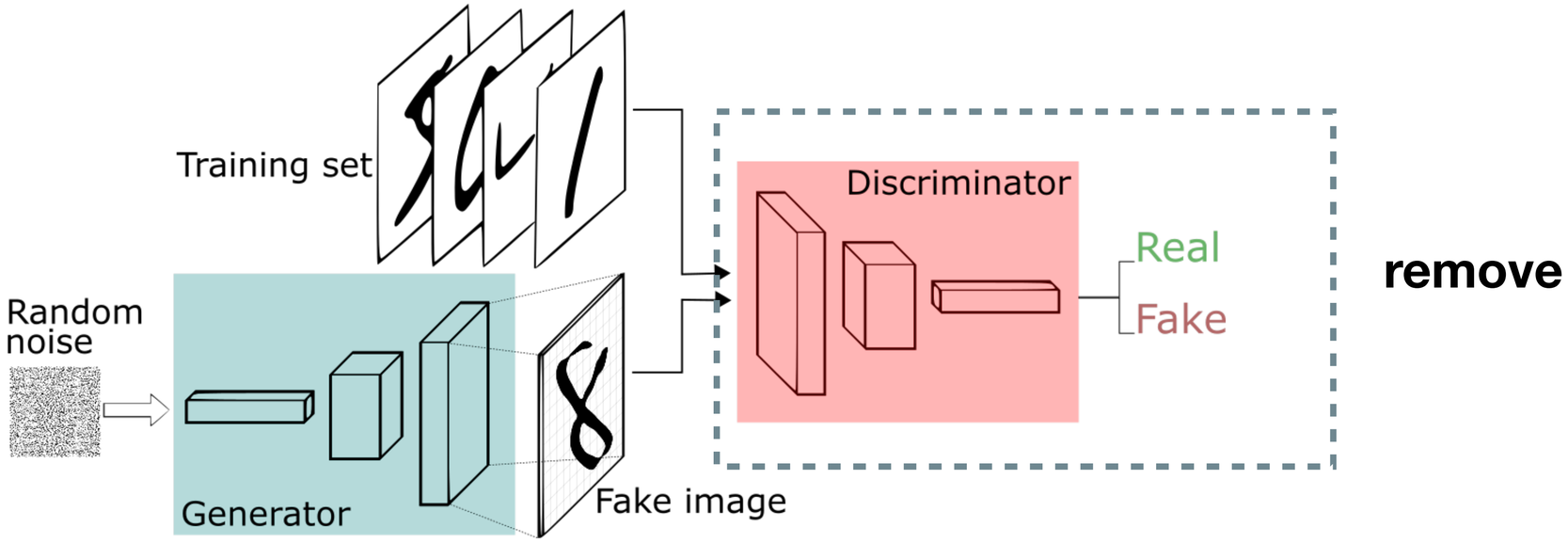


**remove**

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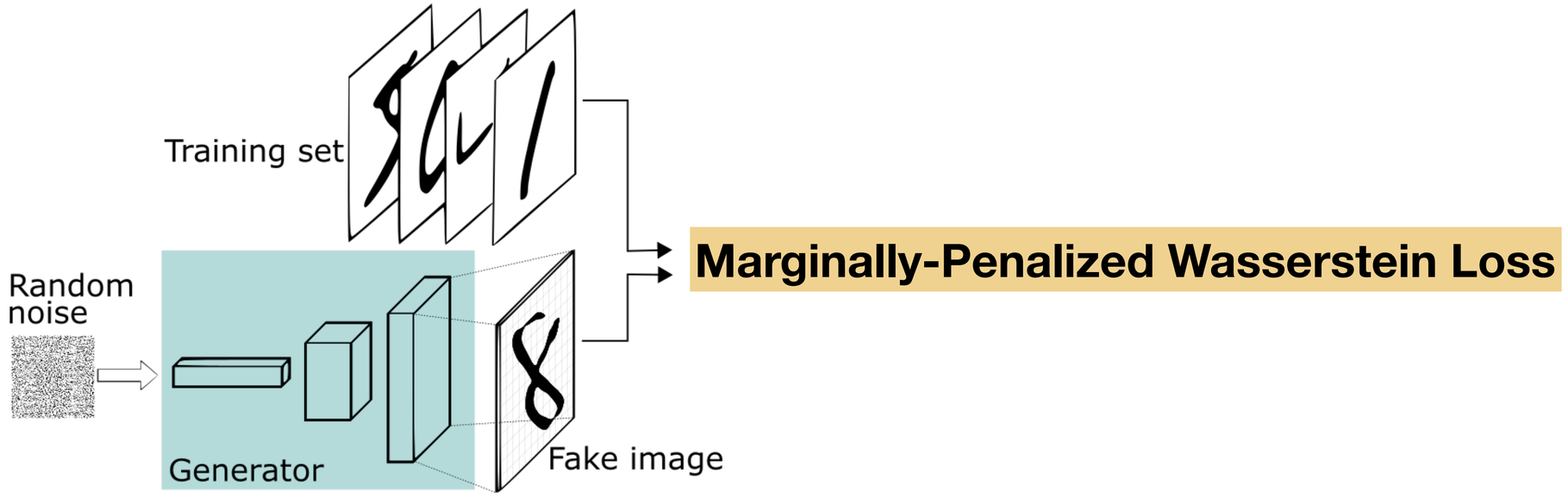

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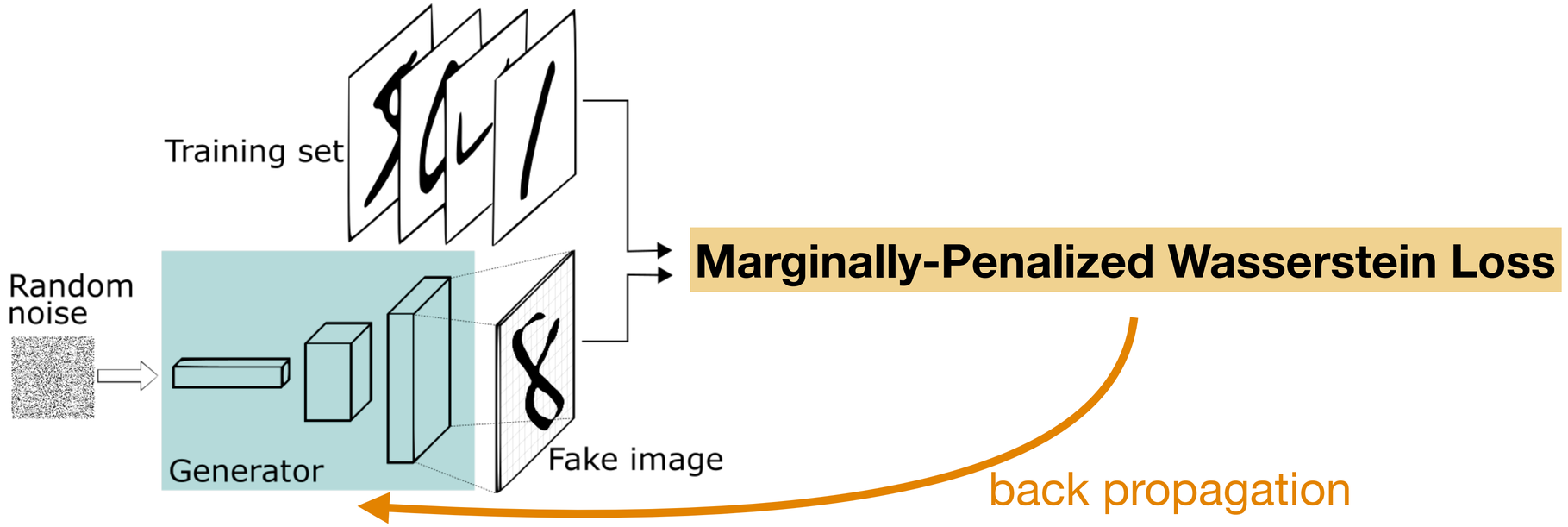

# The POTNet Framework

## Marginally-Penalized Wasserstein Distance

**Marginally-Penalized Wasserstein (MPW) Distance**

$$\mathcal{D}(\mu,\nu) := \underbrace{W_1(\mu,\nu)}_{joint\ distance} + \sum_{j=1}^{d} \underbrace{\lambda_j W_1((p_j)_*\mu, (p_j)_*\nu)}_{marginal\ penalty}$$

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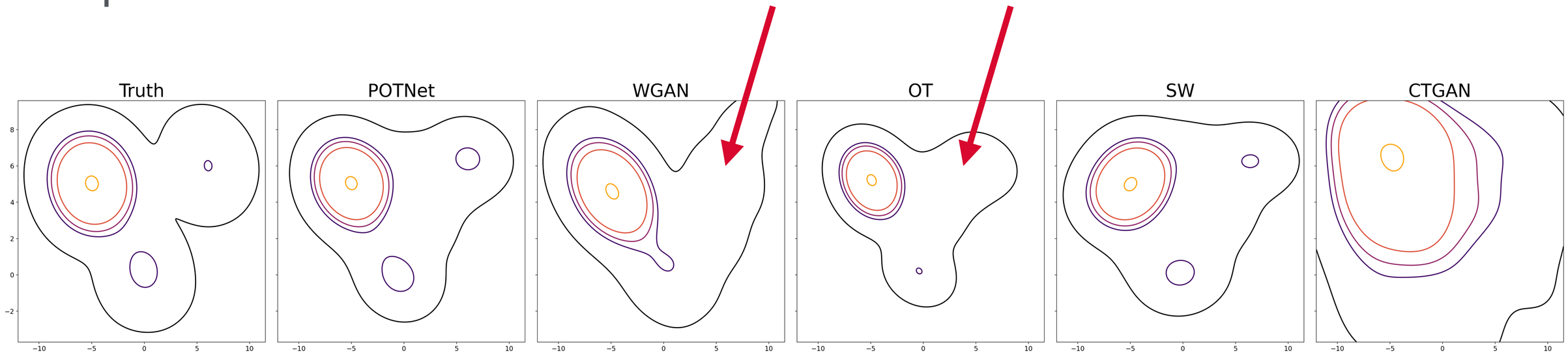
regularization factor on the $j$th marginal

- Use the marginal distribution to **constrain** possible mass allocations and use the joint loss to **identify** the correct mass allocation.

# The POTNet Framework

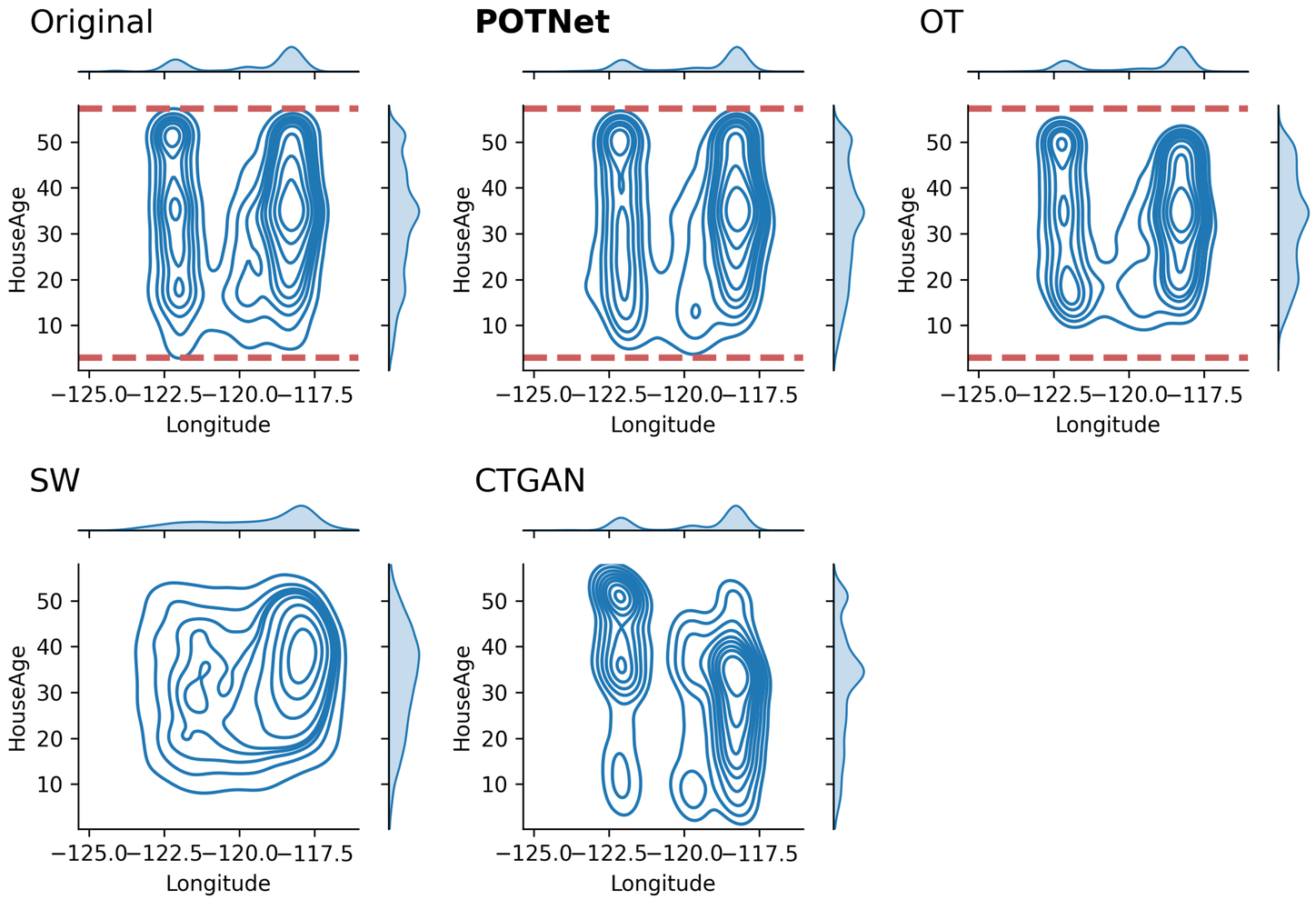
## Empirical Evaluations




20 dimensional mixture of Gaussian example

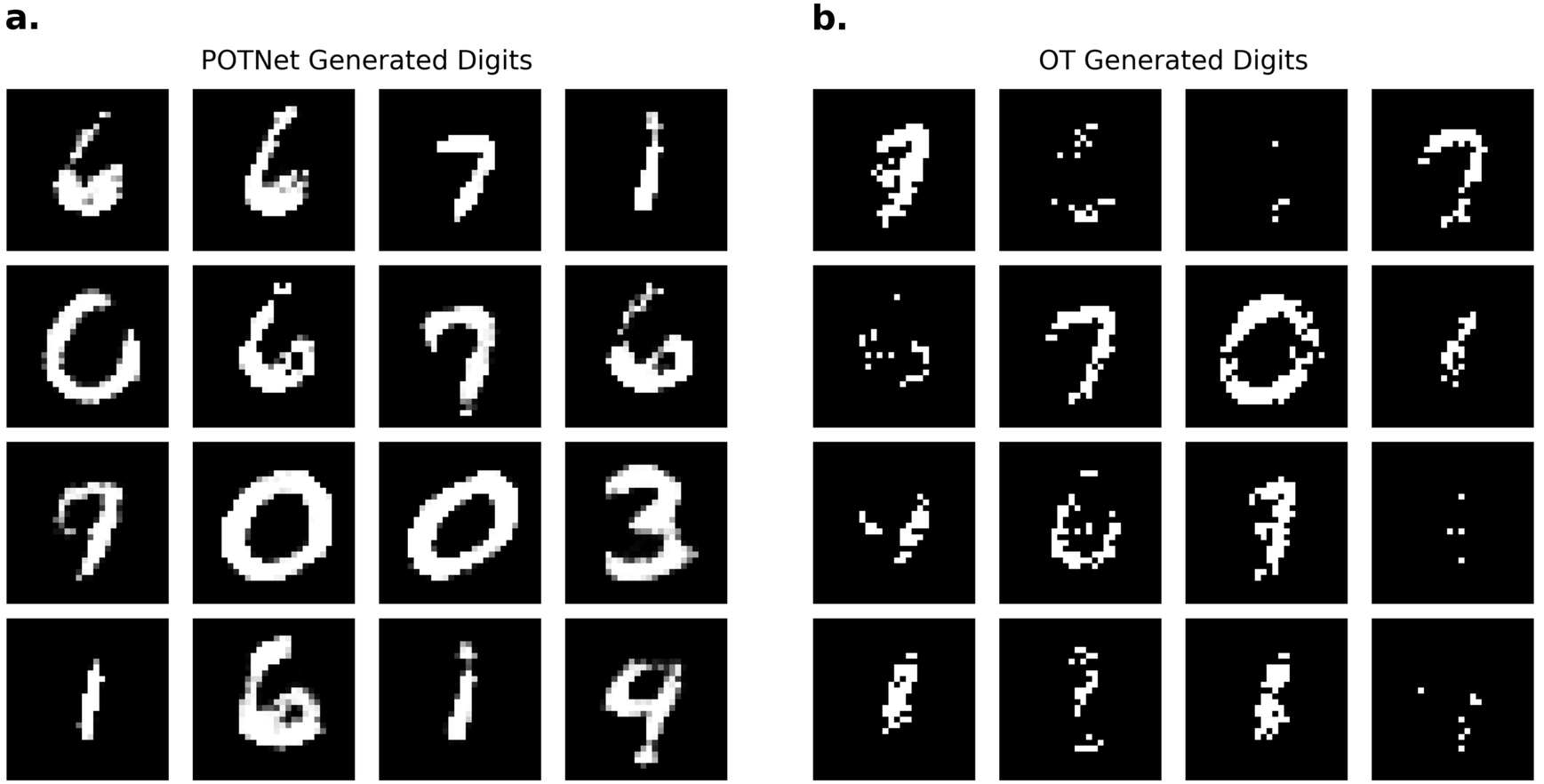
# The POTNet Framework

## Real Data Examples: California Housing Data



# The POTNet Framework

## Real Data Examples: MNIST Digit Generation




Using feedforward neural network architecture

# Potential connections to LassoNet…?

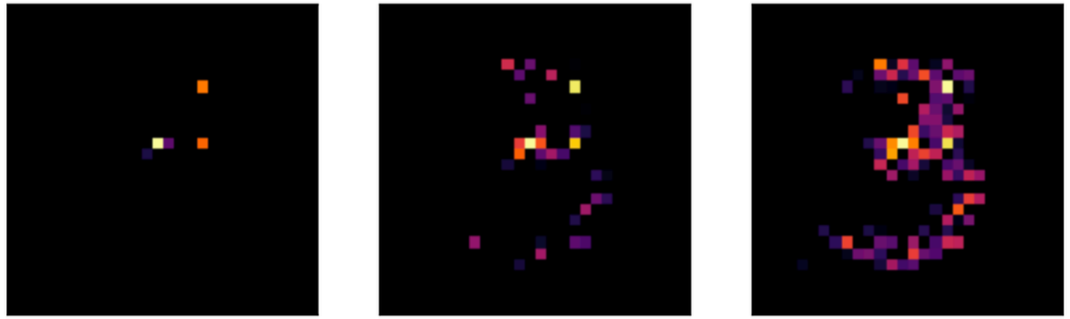


*Figure 7.* Results for LassoNet in choosing the most informative pixels of images of the digit 3 in the MNIST dataset, for three different penalty levels ($\lambda = 5, \lambda = 1, \lambda = 0.1$).

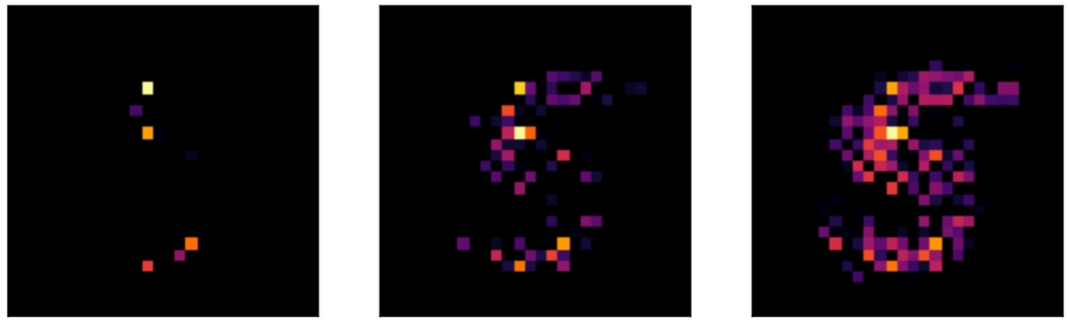


*Figure 8.* Results for LassoNet in choosing the most informative pixels of images of the digit 5 in the MNIST dataset, for the three penalty levels.