

Analysis on MoreLoRA

$$W = W_0 + \Delta$$

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1 LoRA

$$\Delta = AB, A \in \mathbf{R}^{m \times r}, \text{rank} \Delta \leq r, B \in \mathbf{R}^{r \times n}$$

$$\begin{aligned} \frac{\partial L}{\partial A} &= \frac{\partial L}{\partial W} B^T \\ \frac{\partial L}{\partial B} &= A^T \frac{\partial L}{\partial W} \end{aligned}$$

2 AddLoRA

$$\Delta = AI_{r(1 \times n/r)} + I_{r(m/r, 1)}B, \text{rank} \Delta \leq 2r, A \in \mathbf{R}^{m \times r}, B \in \mathbf{R}^{r \times n}$$

$$\begin{aligned} \frac{\partial L}{\partial A} &= \frac{\partial L}{\partial W} I_{r(1 \times n/r)}^T \\ \frac{\partial L}{\partial B} &= I_{r(m/r \times 1)}^T \frac{\partial L}{\partial W} \end{aligned}$$

3 HadamardLoRA

$$\Delta = A_1 B_1 \odot A_2 B_2, \text{rank} \leq \left(\frac{r}{2}\right)^2$$

$$A_1, A_2 \in \mathbf{R}^{m \times r/2}, B_1, B_2 \in \mathbf{R}^{r/2 \times n}$$

$$\begin{aligned} \frac{\partial L}{\partial A_1} &= \left(\frac{\partial L}{\partial W} \odot (A_2 B_2) \right) B_1^T \\ \frac{\partial L}{\partial B_1} &= A_1^T \left(\frac{\partial L}{\partial W} \odot (A_2 B_2) \right) \end{aligned}$$

$$\Delta = (A_1 I_{r(1 \times n/r)} + I_{r(m/r, 1)} B_1) \odot (A_2 I_{r(1 \times n/r)} + I_{r(m/r, 1)} B_2), \text{rank} \leq \left(\frac{2r}{2}\right)^2 = r^2$$

$$A_1, A_2 \in \mathbf{R}^{m \times r/2}, B_1, B_2 \in \mathbf{R}^{r/2 \times n}$$

$$\begin{aligned} \frac{\partial L}{\partial A_1} &= \left(\frac{\partial L}{\partial W} \odot (A_2 I_{r(1 \times n/r)} + I_{r(m/r, 1)} B_2) \right) I_{r(1 \times n/r)}^T \\ \frac{\partial L}{\partial B_1} &= I_{r(m/r \times 1)}^T \left(\frac{\partial L}{\partial W} \odot (A_2 I_{r(1 \times n/r)} + I_{r(m/r, 1)} B_2) \right) \end{aligned}$$