Supplementary material - Scalarizing Functions in Expensive Multi- and Many-objective Optimization

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I. IGD and Hypervolume values of WFG problems

We present the best, statistically similar to the best and the worst IGD and hypervolume values of WFG problems in Tables 1 and 2 respectively. The results are similar to our observations and analyis in the main manuscript.

II. Hypervolume on DTLZ problems

The hypervolume values of DTLZ problems are given in Table 3.

III. Approximated Pareto fronts

In this section, we plot the approximated Pareto fronts on DTLZ problems of the run with the best IGD value with different number of objectives. For two and three objectives, we used scatter plots and for five and 10 objectives, we used parallel coordinates plots. As can be seen in the plots that approximated Pareto fronts in using some scalarizing functions are closer to the actual Pareto fronts than some other functions. These plots resembles with our analysis of IGD and hypervolume values provided in main manuscript.

A. Two objectives DTLZ problems

The scatter plots of the approximated Pareto fronts of two objectives DTLZ problems are shown in Figures 1-4.

B. Three objectives DTLZ problems

The scatter plots of the approximated Pareto fronts of three objectives DTLZ problems are shown in Figures 5-8.

C. Five objectives DTLZ problems

The arallel coordinate plots of the approximated Pareto fronts of five objectives DTLZ problems are shown in Figures 9-12.

D. Ten objectives DTLZ problems

The parallel coordinate plots of the approximated Pareto fronts of ten objectives DTLZ problems are shown in Figures 13-16.

IV. Fitness landscapes

In this section, we show the scalarizing function values (notated by g) on different problems. Note that for a given number of objective value, we used the same training and testing data sets and also used the same weight vector when building surrogates. These results clearly showed the sensitivity of different functions to the number of objectives, which is also explained in details in the main manuscript.

A. Two Objectives DTLZ problems

The fitness landscapes for two objecives problems are shown in Figures 17-23.

B. Three Objectives DTLZ problems

The fitness landscapes of three objectives problems are shown in Figures 24-30.

C. Five objectives DTLZ problems

The fitness landscapes of five objectives problems are shown in Figures 31-37.

D. Ten Objectives DTLZ problems

The fitness landscapes of five objectives problems are shown in Figures 38-44.

Problem	k	WS	EWC	WPO	WN	WPR	TCH	ATCH	MTCH
WFG1	2	**1.31** (0.00)	1.68 (0.30)	**1.30** (0.01)	**1.31** (0.01)	**1.31** (0.01)	**1.31** (0.00)	**1.30** (0.01)	**1.31** (0.00)
	3	**1.55** (0.01)	1.82 (0.18)	**1.55** (0.01)	**1.55** (0.01)	**1.56** (0.01)	**1.55** (0.01)	**1.55** (0.01)	**1.55** (0.01)
	5	**2.05** (0.01)	2.36 (0.09)	**2.04** (0.02)	**2.05** (0.01)	**2.07** (0.01)	**2.04** (0.02)	**2.05** (0.01)	**2.05** (0.01)
	10	**3.12** (0.03)	3.58 (0.22)	**3.13** (0.04)	**3.10** (0.03)	**3.12** (0.04)	**3.11** (0.04)	**3.10** (0.04)	**3.11** (0.03)
WFG2	2	**0.38** (0.10)	0.46 (0.07)	**0.37** (0.07)	**0.29** (0.07)	**0.34** (0.06)	**0.37** (0.07)	**0.39** (0.06)	**0.38** (0.06)
	3	0.71 (0.09)	0.75 (0.07)	0.67 (0.05)	0.68 (0.08)	0.74 (0.06)	0.72 (0.08)	0.70 (0.08)	0.71 (0.08)
	5	**1.26** (0.20)	1.74 (0.25)	**1.30** (0.17)	**1.25** (0.22)	**1.46** (0.09)	**1.28** (0.23)	**1.26** (0.13)	**1.22** (0.19)
	10	**3.67** (0.34)	4.23 (0.61)	**3.86** (0.44)	**3.34** (0.64)	**3.81** (0.41)	**4.04** (0.50)	**3.85** (0.46)	**3.96** (0.32)
WFG3	2	0.31 (0.05)	0.49 (0.06)	0.26 (0.03)	0.25 (0.02)	0.37 (0.05)	0.32 (0.02)	0.31 (0.02)	0.32 (0.03)
	3	0.49 (0.04)	0.54 (0.06)	0.38 (0.04)	0.48 (0.02)	0.55 (0.03)	0.52 (0.03)	0.52 (0.03)	0.52 (0.04)
	5	0.70 (0.04)	0.76 (0.05)	0.64 (0.05)	0.68 (0.05)	0.71 (0.05)	0.71 (0.04)	0.70 (0.04)	0.71 (0.05)
	10	1.04 (0.08)	1.15 (0.08)	**0.87** (0.09)	1.03 (0.11)	1.06 (0.12)	1.07 (0.10)	1.06 (0.11)	1.06 (0.09)
WFG4	2	**0.21** (0.02)	0.40 (0.09)	**0.26** (0.05)	**0.21** (0.03)	**0.21** (0.03)	**0.26** (0.04)	**0.25** (0.06)	**0.24** (0.04)
	3	**0.56** (0.05)	0.81 (0.16)	**0.62** (0.05)	**0.48** (0.04)	**0.63** (0.06)	**0.58** (0.08)	**0.62** (0.06)	**0.61** (0.04)
	5	**1.64** (0.22)	2.45 (0.27)	**1.69** (0.13)	**1.51** (0.14)	**1.69** (0.07)	**1.78** (0.13)	**1.81** (0.10)	**1.82** (0.24)
	10	7.32 (0.50)	10.43 (0.96)	7.60 (0.29)	**6.50** (0.29)	**7.08** (0.45)	7.82 (0.47)	7.68 (0.40)	7.97 (0.38)
WFG5	2	**0.28** (0.04)	0.64 (0.05)	0.38 (0.04)	**0.29** (0.05)	0.37 (0.04)	0.40 (0.05)	0.39 (0.05)	0.40 (0.06)
	3	**0.72** (0.09)	0.93 (0.06)	0.78 (0.11)	**0.71** (0.09)	0.80 (0.09)	0.76 (0.07)	0.76 (0.05)	0.75 (0.06)
	5	2.33 (0.12)	2.39 (0.14)	2.25 (0.11)	2.36 (0.13)	2.48 (0.18)	2.29 (0.19)	2.32 (0.20)	2.34 (0.16)
	10	9.27 (0.29)	9.13 (0.39)	9.07 (0.33)	9.26 (0.27)	9.22 (0.23)	9.15 (0.32)	9.10 (0.36)	9.21 (0.28)
WFG6	2	0.40 (0.06)	0.57 (0.08)	0.39 (0.05)	0.35 (0.06)	0.41 (0.04)	0.42 (0.03)	0.42 (0.04)	0.44 (0.04)
	3	0.69 (0.02)	0.88 (0.08)	0.69 (0.03)	0.71 (0.03)	0.73 (0.04)	0.69 (0.02)	0.68 (0.02)	0.69 (0.02)
	5	1.65 (0.09)	2.07 (0.09)	1.63 (0.04)	1.77 (0.08)	1.67 (0.07)	**1.58** (0.04)	**1.57** (0.05)	**1.57** (0.04)
	10	8.14 (0.50)	8.71 (0.31)	8.52 (0.24)	8.62 (0.31)	**7.62** (0.39)	**7.43** (0.48)	**7.58** (0.43)	**7.61** (0.40)
WFG7	2	0.42 (0.06)	0.50 (0.06)	0.27 (0.03)	0.37 (0.06)	0.43 (0.03)	0.38 (0.01)	0.38 (0.02)	0.37 (0.02)
	3	0.79 (0.03)	0.90 (0.09)	0.73 (0.04)	0.77 (0.04)	0.78 (0.02)	0.79 (0.04)	0.78 (0.05)	0.77 (0.03)
	5	2.18 (0.16)	2.58 (0.16)	2.20 (0.11)	2.31 (0.24)	2.25 (0.10)	2.20 (0.12)	2.15 (0.07)	2.21 (0.12)
	10	10.09 (0.35)	10.17 (0.30)	**9.75** (0.28)	10.16 (0.42)	10.10 (0.43)	**9.57** (0.39)	**9.59** (0.34)	**9.83** (0.49)
WFG8	2	**0.68** (0.05)	0.84 (0.10)	**0.71** (0.05)	**0.72** (0.06)	**0.65** (0.04)	**0.70** (0.03)	**0.67** (0.03)	**0.69** (0.03)
	3	**0.99** (0.02)	1.13 (0.10)	**0.92** (0.03)	**0.99** (0.03)	**0.97** (0.02)	**0.97** (0.02)	**0.97** (0.02)	**0.96** (0.02)
	5	2.35 (0.13)	2.78 (0.19)	2.24 (0.12)	2.51 (0.16)	2.23 (0.09)	2.27 (0.10)	2.25 (0.11)	2.28 (0.15)
	10	9.80 (0.37)	10.07 (0.25)	9.49 (0.52)	9.95 (0.32)	9.75 (0.49)	9.58 (0.39)	9.51 (0.43)	9.55 (0.52)
WFG9	2	**0.26** (0.07)	0.49 (0.08)	0.35 (0.07)	0.29 (0.11)	0.33 (0.12)	0.34 (0.10)	0.35 (0.07)	0.36 (0.07)
	3	**0.62** (0.10)	0.91 (0.06)	0.70 (0.07)	0.70 (0.08)	0.79 (0.06)	0.68 (0.11)	0.67 (0.07)	0.66 (0.07)
	5	2.35 (0.12)	2.35 (0.15)	2.33 (0.16)	2.48 (0.23)	2.42 (0.14)	2.24 (0.17)	2.33 (0.17)	2.29 (0.12)
	10	**9.26** (0.25)	**9.13** (0.34)	**9.20** (0.22)	**9.22** (0.19)	**9.22** (0.30)	**9.19** (0.26)	**9.22** (0.21)	**9.23** (0.25)

Problem	k	PBI	IPBI	HypI	DomRank	MSD
WFG1	2	**1.29** (0.03)	**1.31** (0.01)	**1.39** (0.32)	**1.29** (0.02)	**1.33** (0.0
	3	**1.57** (0.03)	**1.67** (0.10)	2.10 (0.40)	**1.55** (0.02)	**1.55** (0.0
	5	**2.19** (0.14)	2.28 (0.10)	2.72 (0.30)	**2.20** (0.07)	**2.14** (0.2
	10	3.42 (0.09)	3.38 (0.05)	4.01 (0.10)	3.46 (0.27)	3.56 (0.2
WFG2	2	**0.29** (0.08)	0.43 (0.06)	**0.30** (0.08)	**0.36** (0.06)	**0.30** (0.0
	3	**0.62** (0.08)	0.78 (0.07)	0.82 (0.06)	0.67 (0.08)	**0.54** (0.6
	5	**1.34** (0.45)	1.71 (0.21)	1.80 (0.24)	**1.40** (0.27)	**1.39** (0.4
	10	**3.36** (0.86)	4.29 (0.47)	4.34 (0.44)	**4.19** (0.27)	4.25 (0.5
WFG3	2	0.35 (0.03)	0.44 (0.03)	**0.15** (0.05)	0.37 (0.04)	0.23 (0.0
	3	0.40 (0.04)	0.56 (0.03)	**0.27** (0.09)	0.46 (0.05)	0.39 (0.0
	5	**0.35** (0.06)	0.66 (0.06)	0.52 (0.18)	0.63 (0.04)	**0.42** (0.0
	10	**0.79** (0.09)	1.05 (0.08)	1.10 (0.11)	1.05 (0.13)	0.97 (0.1
WFG4	2	**0.19** (0.03)	0.32 (0.09)	**0.26** (0.13)	0.31 (0.02)	**0.28** (0.0
	3	**0.46** (0.06)	0.67 (0.10)	0.68 (0.25)	0.68 (0.08)	0.70 (0.1
	5	**1.52** (0.16)	2.39 (0.19)	3.00 (0.30)	2.50 (0.37)	2.72 (0.4
	10	9.84 (0.64)	10.37 (0.27)	11.89 (0.34)	11.67 (0.54)	11.84 (0.1
WFG5	2	0.32 (0.03)	0.48 (0.05)	**0.20** (0.06)	**0.26** (0.06)	0.34 (0.0
	3	**0.59** (0.03)	0.81 (0.06)	**0.64** (0.15)	**0.72** (0.12)	**0.61** (0.0
	5	**1.98** (0.23)	2.30 (0.14)	2.53 (0.15)	2.07 (0.10)	**2.03** (0.1
	10	9.36 (0.26)	9.12 (0.27)	9.43 (0.27)	9.39 (0.28)	**8.37** (0.4
WFG6	2	**0.29** (0.05)	0.42 (0.04)	**0.25** (0.06)	0.54 (0.04)	**0.31** (0.0
	3	**0.57** (0.05)	0.74 (0.05)	0.68 (0.23)	0.89 (0.08)	**0.63** (0.0
	5	**1.41** (0.06)	2.15 (0.10)	2.26 (0.18)	2.24 (0.15)	2.16 (0.1
	10	8.74 (0.29)	8.88 (0.19)	8.92 (0.17)	8.93 (0.18)	8.79 (0.2
WFG7	2	0.30 (0.03)	0.43 (0.05)	**0.15** (0.05)	0.34 (0.03)	0.24 (0.0
	3	0.63 (0.04)	0.77 (0.05)	**0.48** (0.08)	0.62 (0.02)	0.76 (0.1
	5	**1.85** (0.18)	2.36 (0.16)	2.13 (0.21)	2.16 (0.11)	2.58 (0.1
	10	10.17 (0.26)	**9.34** (0.43)	10.17 (0.42)	10.14 (0.35)	10.12 (0.1
WFG8	2	**0.67** (0.03)	**0.65** (0.03)	**0.66** (0.08)	**0.66** (0.04)	**0.69** (0.0
	3	**0.91** (0.03)	**0.98** (0.02)	1.14 (0.21)	**0.93** (0.11)	1.10 (0.1
	5	**2.07** (0.15)	**2.18** (0.08)	2.92 (0.15)	2.66 (0.30)	2.88 (0.1
	10	10.07 (0.23)	**8.32** (0.69)	10.12 (0.24)	10.01 (0.32)	10.06 (0.1
WFG9	2	**0.23** (0.07)	0.49 (0.06)	**0.12** (0.06)	0.48 (0.07)	0.38 (0.1
	3	**0.44** (0.10)	0.92 (0.07)	0.76 (0.24)	0.71 (0.11)	**0.60** (0.1
	5	**1.81** (0.14)	2.45 (0.15)	2.40 (0.28)	2.28 (0.11)	**2.04** (0.1
	10	**9.18** (0.24)	**9.25** (0.24)	9.31 (0.22)	**9.21** (0.24)	**9.26** (0.2

TABLE I: , Mean IGD values and standard deviation (in parentheses) for WFG problems. The values statistically similar to the best one are in bold, the best value is encircled the worst value is underlined

Problem	k	WS	EWC	WPO	WN	WPR	TCH	ATCH	MTCH
WFG1	2	**0.22** (0.00)	0.09 (0.06)	**0.22** (0.00)	**0.22** (0.00)	**0.22** (0.00)	**0.22** (0.00)	**0.22** (0.00)	**0.22** (0.00)
	3	**0.34** (0.00)	0.20 (0.06)	**0.34** (0.00)	**0.34** (0.00)	**0.33** (0.01)	**0.34** (0.00)	**0.34** (0.00)	**0.34** (0.00)
	5	**0.31** (0.00)	0.23 (0.03)	**0.31** (0.00)	**0.31** (0.00)	**0.31** (0.00)	**0.31** (0.00)	**0.31** (0.00)	**0.31** (0.00)
	10	**0.24** (0.00)	0.16 (0.03)	**0.23** (0.00)	**0.24** (0.00)	**0.23** (0.00)	**0.24** (0.00)	**0.24** (0.00)	**0.23** (0.00)
WFG2	2	**0.49** (0.05)	0.43 (0.03)	**0.49** (0.03)	**0.52** (0.05)	**0.50** (0.04)	**0.49** (0.03)	**0.48** (0.03)	**0.49** (0.03)
	3	**0.66** (0.05)	0.56 (0.05)	**0.70** (0.04)	**0.70** (0.04)	0.63 (0.02)	0.64 (0.04)	0.64 (0.03)	0.64 (0.03)
	5	**0.77** (0.06)	0.60 (0.06)	**0.72** (0.05)	**0.79** (0.05)	0.68 (0.04)	**0.73** (0.05)	**0.72** (0.03)	**0.72** (0.04)
	10	**0.76** (0.07)	0.58 (0.06)	**0.70** (0.06)	**0.80** (0.05)	0.67 (0.06)	0.68 (0.09)	0.68 (0.08)	**0.70** (0.06)
WFG3	2	0.49 (0.02)	0.41 (0.02)	0.51 (0.02)	0.52 (0.01)	0.46 (0.03)	0.49 (0.01)	0.49 (0.01)	0.49 (0.01)
	3	0.29 (0.02)	0.26 (0.03)	0.31 (0.01)	0.29 (0.01)	0.26 (0.01)	0.27 (0.01)	0.27 (0.01)	0.27 (0.01)
	5	0.13 (0.02)	0.11 (0.02)	0.14 (0.02)	0.13 (0.02)	0.13 (0.02)	0.12 (0.02)	0.12 (0.02)	0.11 (0.02)
	10	0.03 (0.01)	0.02 (0.01)	**0.04** (0.01)	**0.04** (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)
WFG4	2	**0.35** (0.01)	0.27 (0.02)	**0.33** (0.01)	**0.35** (0.02)	**0.35** (0.01)	**0.33** (0.01)	**0.33** (0.01)	**0.32** (0.02)
	3	**0.46** (0.01)	0.33 (0.03)	0.40 (0.03)	**0.48** (0.01)	0.40 (0.01)	0.41 (0.02)	0.42 (0.02)	0.42 (0.02)
	5	**0.55** (0.04)	0.33 (0.02)	0.44 (0.03)	**0.59** (0.03)	0.42 (0.02)	0.44 (0.03)	0.45 (0.04)	0.45 (0.03)
	10	0.40 (0.04)	0.28 (0.02)	0.37 (0.04)	**0.52** (0.02)	0.33 (0.04)	0.35 (0.04)	0.36 (0.04)	0.33 (0.03)
WFG5	2	0.29 (0.02)	0.20 (0.01)	0.27 (0.01)	0.29 (0.02)	0.28 (0.01)	0.26 (0.01)	0.26 (0.01)	0.27 (0.01)
	3	**0.34** (0.03)	0.26 (0.01)	0.33 (0.03)	**0.36** (0.03)	**0.34** (0.02)	0.33 (0.02)	0.32 (0.02)	0.33 (0.02)
	5	0.30 (0.02)	0.26 (0.01)	0.31 (0.02)	0.31 (0.02)	0.28 (0.02)	0.31 (0.02)	0.31 (0.02)	0.31 (0.01)
	10	**0.28** (0.02)	0.24 (0.01)	0.25 (0.02)	**0.28** (0.01)	0.26 (0.02)	**0.28** (0.01)	**0.28** (0.02)	**0.27** (0.02)
WFG6	2	0.24 (0.03)	0.21 (0.02)	0.26 (0.02)	0.27 (0.03)	0.24 (0.03)	0.23 (0.02)	0.23 (0.02)	0.23 (0.02)
	3	0.30 (0.01)	0.27 (0.02)	0.33 (0.02)	0.32 (0.03)	0.30 (0.02)	0.30 (0.01)	0.31 (0.01)	0.31 (0.01)
	5	0.39 (0.02)	0.28 (0.01)	0.39 (0.01)	0.36 (0.02)	0.39 (0.02)	0.40 (0.01)	0.40 (0.01)	0.40 (0.01)
	10	0.29 (0.02)	0.25 (0.01)	0.28 (0.02)	0.27 (0.02)	**0.32** (0.02)	**0.32** (0.03)	**0.32** (0.02)	**0.32** (0.02)
WFG7	2	0.28 (0.03)	0.24 (0.01)	0.31 (0.01)	0.32 (0.02)	0.25 (0.02)	0.26 (0.01)	0.26 (0.01)	0.26 (0.01)
	3	0.32 (0.02)	0.30 (0.02)	**0.42** (0.02)	0.36 (0.04)	0.32 (0.03)	0.32 (0.02)	0.33 (0.01)	0.33 (0.01)
	5	0.30 (0.03)	0.29 (0.01)	0.29 (0.01)	0.31 (0.01)	0.29 (0.02)	0.28 (0.01)	0.28 (0.01)	0.28 (0.02)
	10	0.27 (0.01)	0.28 (0.01)	0.28 (0.01)	0.27 (0.02)	0.27 (0.01)	0.27 (0.01)	0.26 (0.01)	0.27 (0.01)
WFG8	2	0.17 (0.01)	0.16 (0.01)	0.17 (0.01)	0.16 (0.01)	0.17 (0.02)	0.16 (0.01)	0.16 (0.01)	0.16 (0.01)
	3	0.22 (0.02)	0.21 (0.02)	**0.25** (0.01)	0.23 (0.03)	**0.24** (0.02)	**0.23** (0.01)	**0.24** (0.01)	**0.25** (0.01)
	5	0.24 (0.01)	0.23 (0.02)	0.27 (0.01)	0.24 (0.02)	0.24 (0.01)	0.24 (0.01)	0.23 (0.01)	0.24 (0.01)
	10	0.23 (0.01)	0.24 (0.01)	0.24 (0.01)	0.23 (0.01)	0.24 (0.01)	**0.25** (0.01)	0.24 (0.01)	0.24 (0.01)
WFG9	2	0.33 (0.02)	0.24 (0.03)	0.30 (0.02)	0.30 (0.04)	0.29 (0.04)	0.30 (0.03)	0.29 (0.02)	0.29 (0.02)
	3	**0.38** (0.05)	0.27 (0.02)	0.36 (0.02)	0.35 (0.03)	0.33 (0.03)	0.36 (0.04)	0.36 (0.03)	0.37 (0.03)
	5	0.30 (0.03)	0.27 (0.02)	0.29 (0.02)	0.28 (0.04)	0.28 (0.02)	0.31 (0.03)	0.30 (0.02)	0.30 (0.02)
	10	0.28 (0.02)	0.27 (0.01)	0.27 (0.01)	0.29 (0.02)	0.26 (0.01)	0.28 (0.01)	0.28 (0.01)	0.27 (0.01)

Problem	k	PBI	IPBI	HypI	DomRank	MSD
WFG1	2	**0.23** (0.01)	**0.22** (0.00)	**0.19** (0.07)	**0.21** (0.01)	**0.20** (0.02)
	3	**0.31** (0.02)	0.28 (0.04)	0.15 (0.13)	**0.32** (0.01)	**0.34** (0.01)
	5	**0.27** (0.04)	0.26 (0.03)	0.14 (0.08)	**0.28** (0.01)	**0.30** (0.03)
	10	0.18 (0.03)	**0.21** (0.02)	0.11 (0.06)	**0.22** (0.02)	0.18 (0.03)
WFG2	2	**0.52** (0.04)	0.46 (0.03)	**0.50** (0.04)	**0.48** (0.03)	**0.50** (0.03)
	3	0.63 (0.05)	0.60 (0.04)	0.53 (0.03)	0.64 (0.05)	**0.73** (0.05)
	5	**0.73** (0.11)	**0.70** (0.06)	0.59 (0.04)	**0.71** (0.05)	**0.73** (0.10)
	10	**0.77** (0.09)	0.65 (0.06)	0.57 (0.05)	0.60 (0.03)	0.64 (0.06)
WFG3	2	0.47 (0.01)	0.42 (0.02)	**0.56** (0.02)	0.45 (0.02)	0.52 (0.01)
	3	0.32 (0.02)	0.26 (0.02)	**0.36** (0.03)	0.28 (0.01)	0.31 (0.02)
	5	**0.21** (0.02)	0.14 (0.02)	0.16 (0.04)	0.12 (0.02)	**0.19** (0.03)
	10	**0.06** (0.01)	0.03 (0.01)	0.02 (0.01)	0.02 (0.01)	0.02 (0.01)
WFG4	2	**0.34** (0.02)	0.29 (0.03)	**0.34** (0.03)	0.28 (0.01)	0.32 (0.03)
	3	**0.46** (0.03)	0.36 (0.03)	0.40 (0.07)	0.35 (0.03)	0.38 (0.04)
	5	**0.59** (0.03)	0.37 (0.04)	0.31 (0.04)	0.32 (0.02)	0.35 (0.03)
	10	0.39 (0.02)	0.35 (0.02)	0.25 (0.01)	0.25 (0.01)	0.26 (0.01)
WFG5	2	0.28 (0.01)	0.22 (0.02)	**0.35** (0.02)	**0.33** (0.02)	0.30 (0.01)
	3	**0.35** (0.02)	0.28 (0.02)	**0.36** (0.06)	0.34 (0.04)	**0.38** (0.03)
	5	**0.38** (0.02)	0.27 (0.01)	0.25 (0.02)	0.29 (0.02)	0.30 (0.02)
	10	**0.30** (0.02)	0.24 (0.01)	0.23 (0.01)	0.23 (0.01)	0.24 (0.01)
WFG6	2	**0.31** (0.01)	0.24 (0.02)	**0.33** (0.03)	0.22 (0.02)	**0.30** (0.03)
	3	**0.41** (0.02)	0.31 (0.02)	0.36 (0.08)	0.26 (0.02)	0.36 (0.02)
	5	**0.50** (0.04)	0.31 (0.01)	0.27 (0.04)	0.28 (0.02)	0.29 (0.02)
	10	**0.31** (0.03)	0.26 (0.01)	0.23 (0.01)	0.23 (0.01)	0.25 (0.01)
WFG7	2	0.29 (0.01)	0.24 (0.01)	**0.37** (0.02)	0.29 (0.02)	**0.35** (0.02)
	3	0.39 (0.02)	0.31 (0.02)	**0.45** (0.03)	0.37 (0.01)	0.35 (0.03)
	5	**0.44** (0.03)	0.32 (0.02)	0.37 (0.04)	0.34 (0.01)	0.32 (0.01)
	10	0.29 (0.02)	**0.33** (0.01)	0.29 (0.02)	0.30 (0.01)	0.30 (0.02)
WFG8	2	0.16 (0.01)	0.17 (0.01)	**0.21** (0.01)	0.18 (0.01)	0.17 (0.01)
	3	**0.26** (0.01)	**0.24** (0.02)	0.23 (0.04)	**0.23** (0.01)	**0.24** (0.02)
	5	**0.31** (0.03)	0.26 (0.02)	0.22 (0.03)	0.24 (0.02)	0.23 (0.01)
	10	0.24 (0.01)	**0.26** (0.02)	0.23 (0.01)	0.24 (0.01)	0.23 (0.01)
WFG9	2	**0.34** (0.02)	0.25 (0.02)	**0.38** (0.02)	0.25 (0.02)	0.29 (0.05)
	3	**0.43** (0.04)	0.26 (0.02)	0.34 (0.09)	0.35 (0.04)	**0.39** (0.04)
	5	**0.44** (0.01)	0.26 (0.01)	0.28 (0.06)	0.30 (0.02)	0.34 (0.03)
	10	**0.32** (0.02)	0.27 (0.02)	0.26 (0.01)	0.27 (0.01)	0.27 (0.01)

TABLE II: Mean hypervolume values and standard deviation (in parentheses) for WFG problems. The values statistically similar to the best one are in bold, the best value is encircled the worst value is underlined

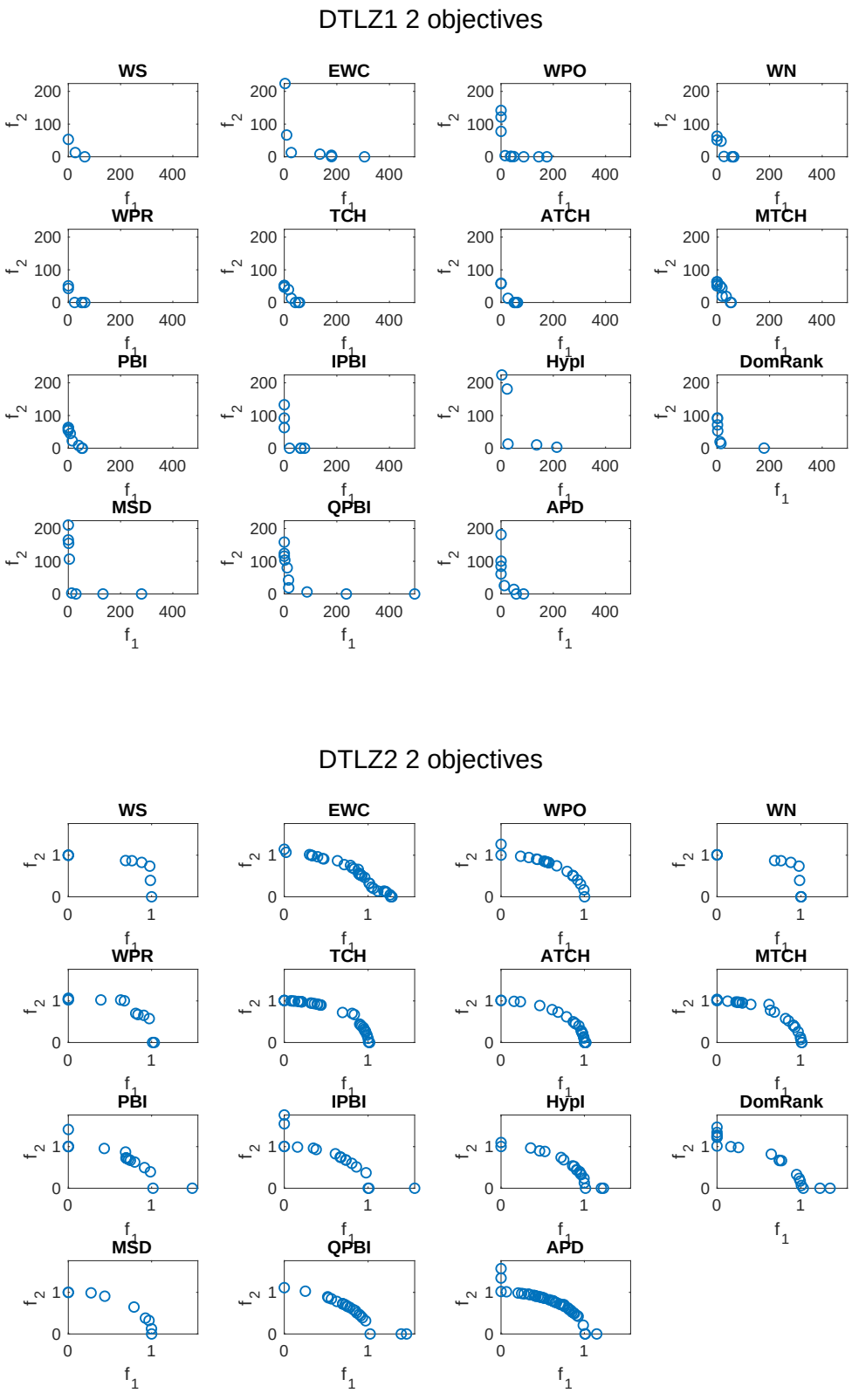
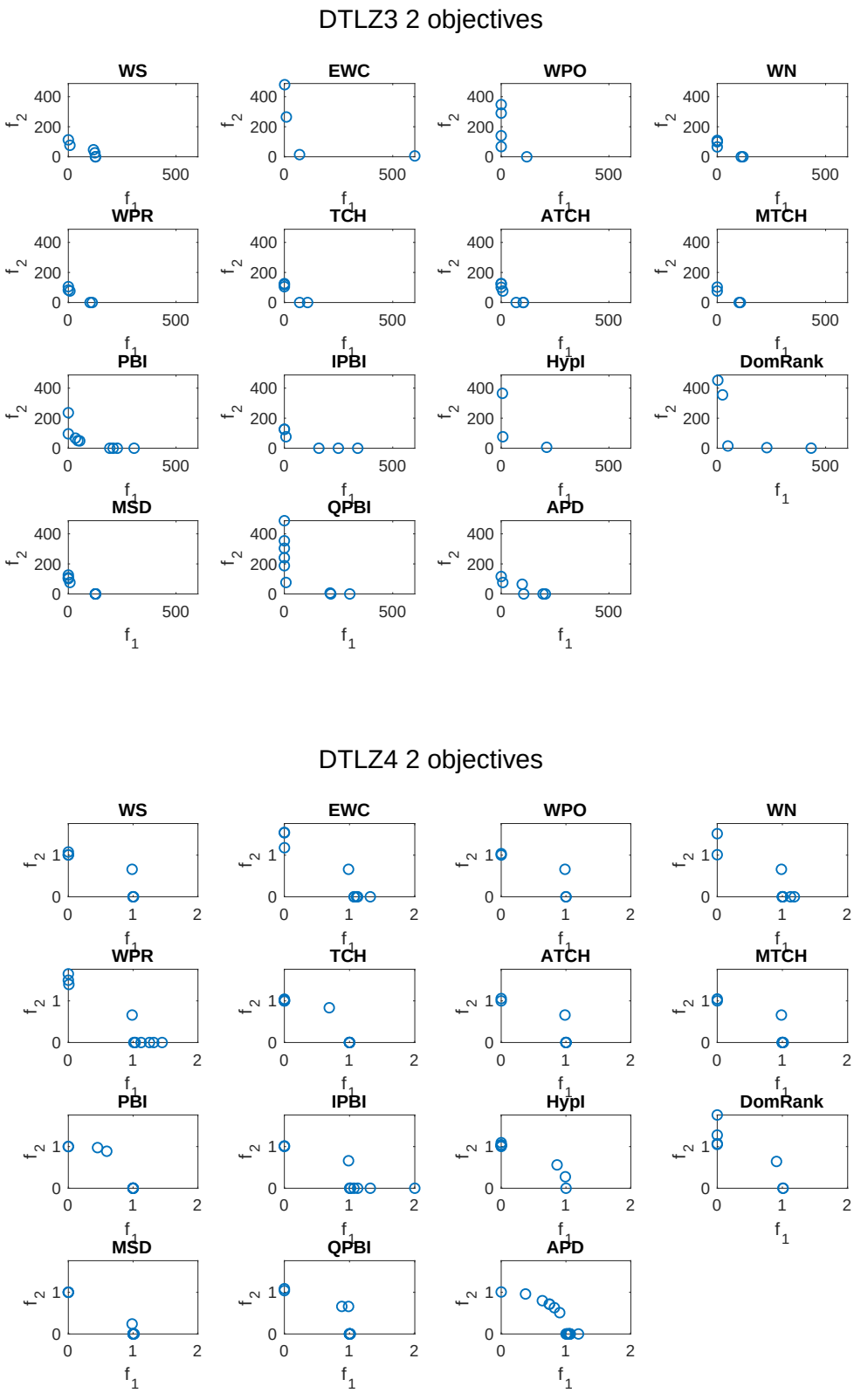

DTLZ1 2 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
f1
f2
DTLZ2 2 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
f1
f2

Fig. 1


DTLZ3 2 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
f1
f2
0
200
400
500

DTLZ4 2 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
f1
f2
0
1
2

Fig. 2

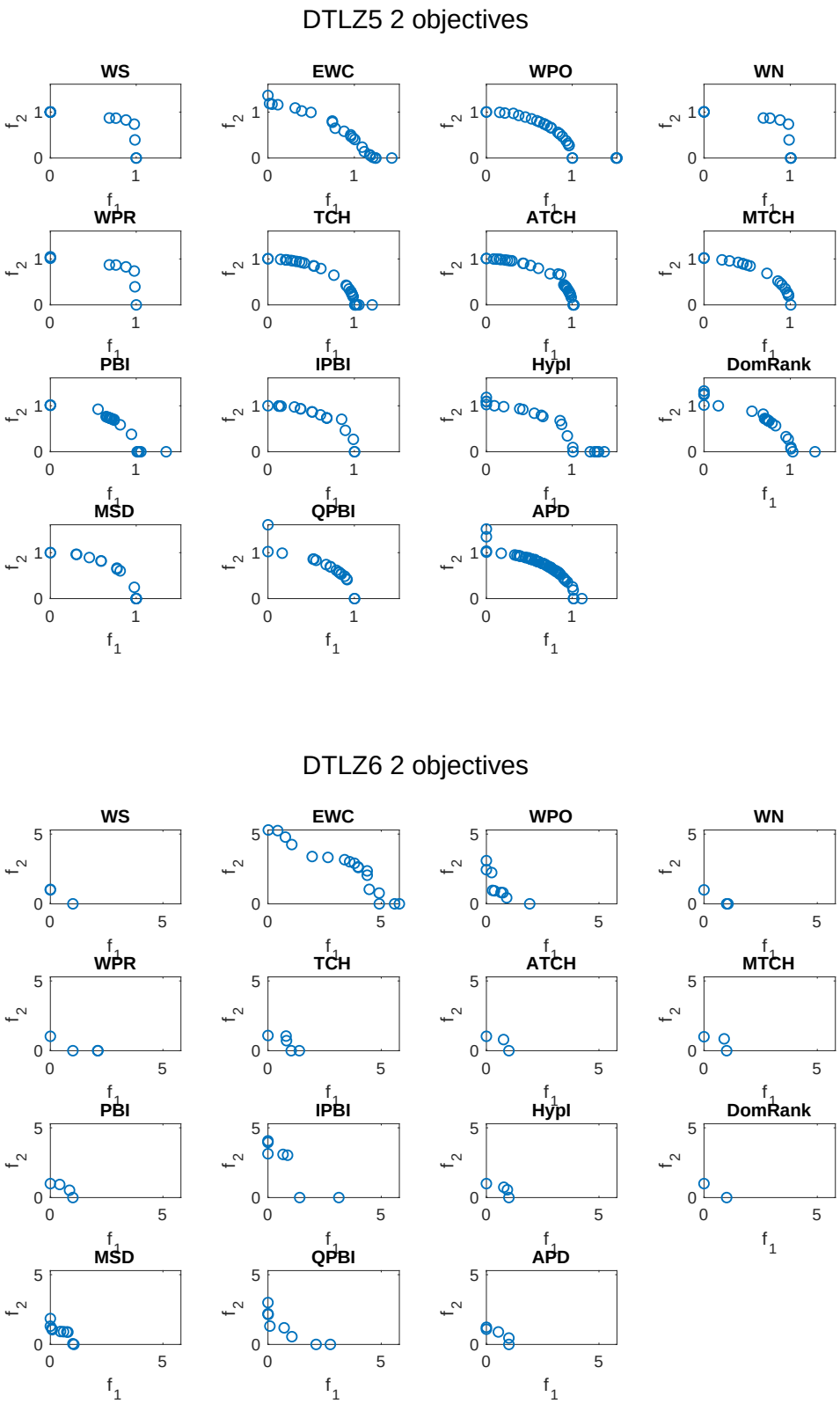

DTLZ5 2 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
f1
f2
DTLZ6 2 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
f1
f2

Fig. 3

Problem	k	WS	EWC	WPO	WN	WPR	TCH	ATCH	MTCH
DTLZ1	2	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	3	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	5	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	10	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
DTLZ2	2	0.58 (0.01)	0.53 (0.03)	**0.64** (0.00)	0.57 (0.01)	0.57 (0.01)	**0.63** (0.00)	**0.63** (0.00)	**0.63** (0.00)
	3	0.74 (0.01)	0.67 (0.03)	**0.78** (0.00)	0.73 (0.01)	0.74 (0.01)	**0.78** (0.00)	**0.78** (0.00)	**0.78** (0.00)
	5	0.88 (0.01)	0.74 (0.04)	0.89 (0.01)	0.88 (0.01)	0.89 (0.01)	0.87 (0.01)	0.88 (0.01)	0.88 (0.01)
	10	**0.84** (0.03)	0.73 (0.03)	**0.87** (0.02)	**0.85** (0.04)	0.82 (0.03)	**0.86** (0.03)	**0.85** (0.03)	**0.85** (0.02)
DTLZ3	2	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	3	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	5	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	10	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
DTLZ4	2	**0.54** (0.03)	0.40 (0.08)	**0.55** (0.01)	0.45 (0.09)	0.38 (0.07)	**0.56** (0.01)	**0.56** (0.00)	**0.56** (0.00)
	3	0.61 (0.07)	0.50 (0.08)	**0.70** (0.00)	0.48 (0.06)	0.51 (0.08)	**0.65** (0.06)	**0.65** (0.04)	**0.67** (0.05)
	5	0.71 (0.06)	0.63 (0.09)	**0.85** (0.01)	0.71 (0.06)	0.59 (0.09)	**0.73** (0.06)	0.69 (0.10)	0.70 (0.10)
	10	**0.89** (0.04)	0.82 (0.05)	**0.91** (0.03)	**0.88** (0.04)	0.78 (0.05)	**0.90** (0.05)	**0.90** (0.03)	**0.90** (0.04)
DTLZ5	2	0.58 (0.01)	0.53 (0.02)	**0.63** (0.00)	0.57 (0.01)	0.57 (0.01)	**0.63** (0.00)	**0.63** (0.00)	**0.63** (0.00)
	3	0.47 (0.00)	0.43 (0.03)	**0.52** (0.01)	0.47 (0.00)	0.46 (0.01)	**0.51** (0.00)	**0.51** (0.00)	**0.51** (0.01)
	5	0.40 (0.01)	0.34 (0.03)	**0.43** (0.00)	0.40 (0.01)	0.40 (0.01)	0.42 (0.01)	0.42 (0.01)	0.42 (0.00)
	10	0.36 (0.01)	0.31 (0.03)	**0.39** (0.00)	0.37 (0.00)	0.37 (0.01)	0.37 (0.00)	**0.37** (0.00)	**0.37** (0.01)
DTLZ6	2	**0.51** (0.06)	0.00 (0.00)	**0.48** (0.08)	**0.53** (0.04)	0.24 (0.19)	**0.53** (0.03)	**0.53** (0.07)	**0.52** (0.07)
	3	**0.32** (0.10)	0.00 (0.00)	0.15 (0.10)	0.25 (0.13)	0.09 (0.10)	**0.34** (0.10)	**0.30** (0.09)	**0.35** (0.09)
	5	0.05 (0.06)	0.00 (0.00)	0.01 (0.04)	0.08 (0.09)	0.01 (0.01)	0.05 (0.08)	0.05 (0.08)	0.01 (0.04)
	10	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.01)	0.00 (0.00)
DTLZ7	2	0.23 (0.01)	0.00 (0.00)	0.20 (0.03)	0.26 (0.02)	0.19 (0.03)	0.23 (0.01)	0.23 (0.02)	0.23 (0.01)
	3	0.21 (0.03)	0.00 (0.00)	0.18 (0.02)	0.17 (0.01)	0.17 (0.02)	0.22 (0.01)	0.21 (0.02)	0.22 (0.01)
	5	**0.15** (0.00)	0.00 (0.00)	0.04 (0.04)	**0.15** (0.00)	0.12 (0.02)	0.15 (0.01)	**0.15** (0.01)	0.15 (0.01)
	10	**0.05** (0.01)	0.00 (0.00)	0.00 (0.01)	**0.08** (0.01)	0.03 (0.01)	0.03 (0.01)	0.05 (0.01)	0.04 (0.01)

Problem	k	PBI	IPBI	HypI	DomRank	MSD	
DTLZ1	2	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
	3	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
	5	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
	10	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
DTLZ2	2	0.61 (0.01)	**0.62** (0.00)	0.61 (0.01)	0.61 (0.00)	0.60 (0.02)	0
	3	**0.78** (0.00)	0.68 (0.02)	**0.79** (0.01)	0.73 (0.02)	0.77 (0.01)	**0**
	5	**0.94** (0.00)	0.65 (0.02)	0.88 (0.02)	0.78 (0.02)	0.84 (0.07)	**0**
	10	**0.84** (0.03)	0.68 (0.01)	**0.86** (0.06)	0.77 (0.02)	**0.84** (0.02)	**0**
DTLZ3	2	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
	3	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
	5	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
	10	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
DTLZ4	2	**0.56** (0.01)	**0.54** (0.02)	**0.55** (0.02)	**0.53** (0.02)	**0.54** (0.04)	**0**
	3	**0.71** (0.01)	0.58 (0.11)	**0.62** (0.10)	0.58 (0.04)	**0.69** (0.06)	**0**
	5	**0.87** (0.02)	0.56 (0.13)	0.50 (0.13)	0.72 (0.09)	0.56 (0.17)	**0**
	10	**0.94** (0.03)	0.64 (0.05)	0.60 (0.06)	0.87 (0.04)	**0.89** (0.03)	**0**
DTLZ5	2	0.61 (0.00)	**0.62** (0.00)	0.62 (0.01)	0.61 (0.00)	0.60 (0.02)	0
	3	**0.52** (0.00)	0.42 (0.03)	**0.50** (0.01)	0.49 (0.01)	0.50 (0.01)	**0**
	5	**0.44** (0.01)	0.31 (0.03)	0.41 (0.01)	0.41 (0.01)	0.43 (0.01)	**0**
	10	**0.38** (0.01)	0.29 (0.03)	0.31 (0.03)	0.33 (0.02)	**0.38** (0.01)	**0**
DTLZ6	2	**0.56** (0.01)	0.01 (0.02)	0.34 (0.22)	0.34 (0.27)	**0.49** (0.06)	0
	3	**0.42** (0.06)	0.01 (0.01)	0.02 (0.06)	0.00 (0.00)	0.22 (0.18)	0
	5	**0.33** (0.04)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
	10	**0.17** (0.05)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0
DTLZ7	2	0.23 (0.01)	0.17 (0.00)	**0.31** (0.01)	0.18 (0.04)	0.24 (0.06)	0
	3	0.17 (0.02)	0.14 (0.02)	**0.31** (0.04)	0.14 (0.02)	0.20 (0.05)	0
	5	0.12 (0.03)	0.00 (0.00)	**0.19** (0.03)	0.03 (0.03)	0.11 (0.05)	0
	10	**0.07** (0.02)	0.00 (0.00)	0.03 (0.04)	0.01 (0.02)	0.00 (0.00)	0

TABLE III: Mean hypervolume values and standard deviation (in parentheses) for DTLZ problems. The values statistically similar to the best one are in bold, the best value is encircled the worst value is underlined

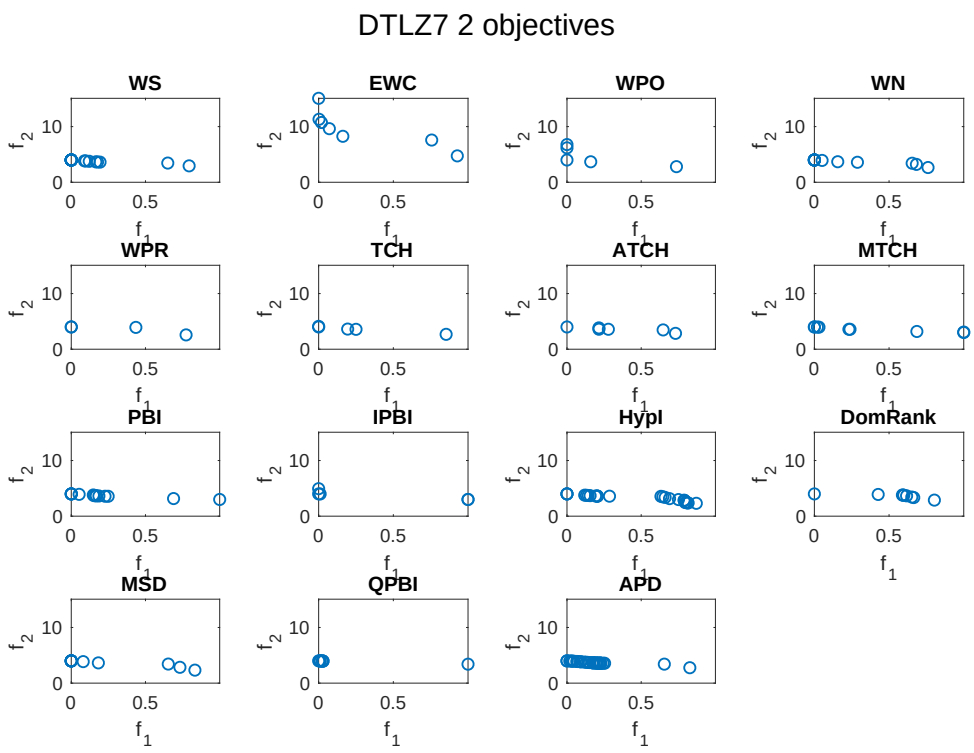


Fig. 4

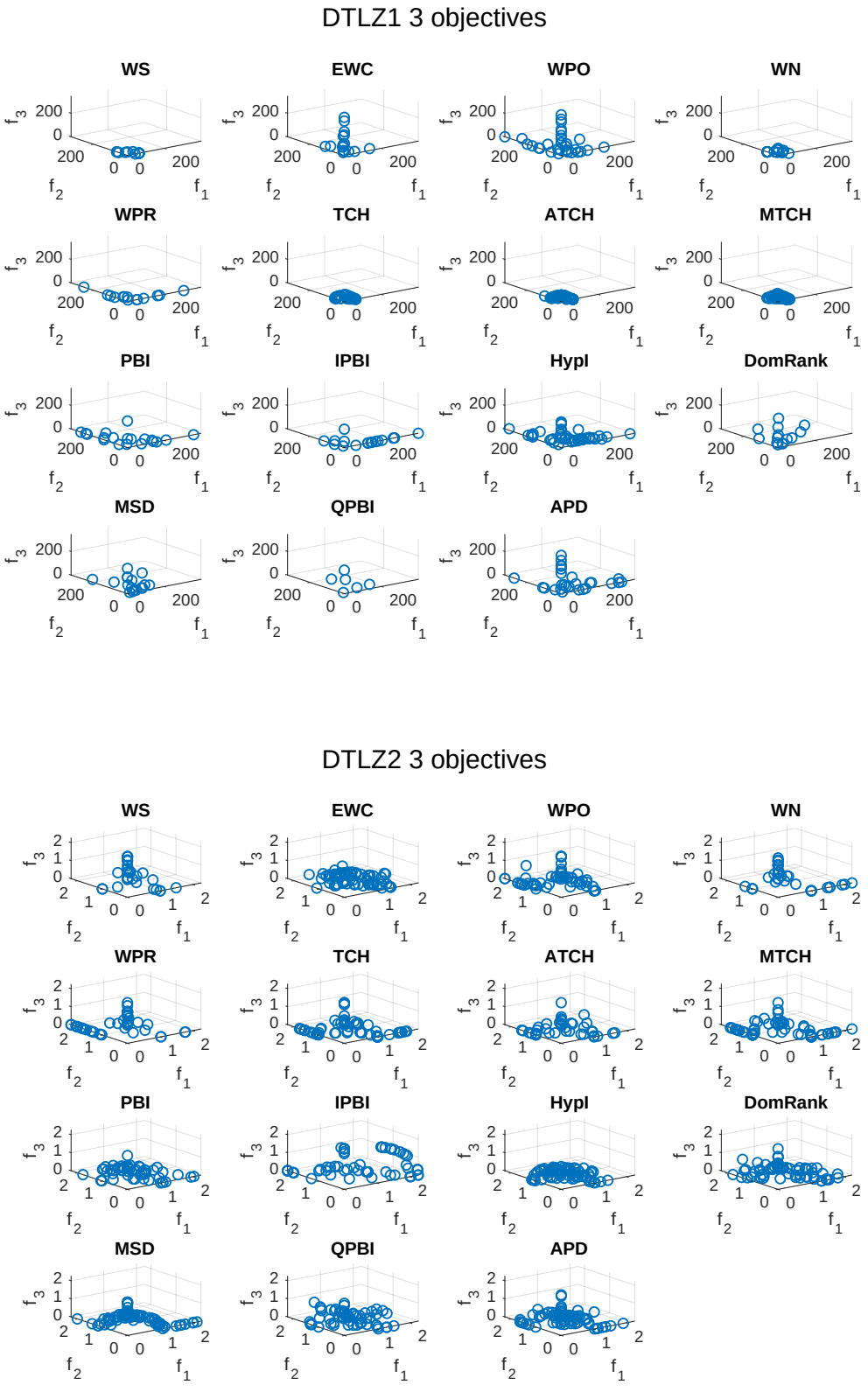

DTLZ1 3 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
DTLZ2 3 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD

Fig. 5

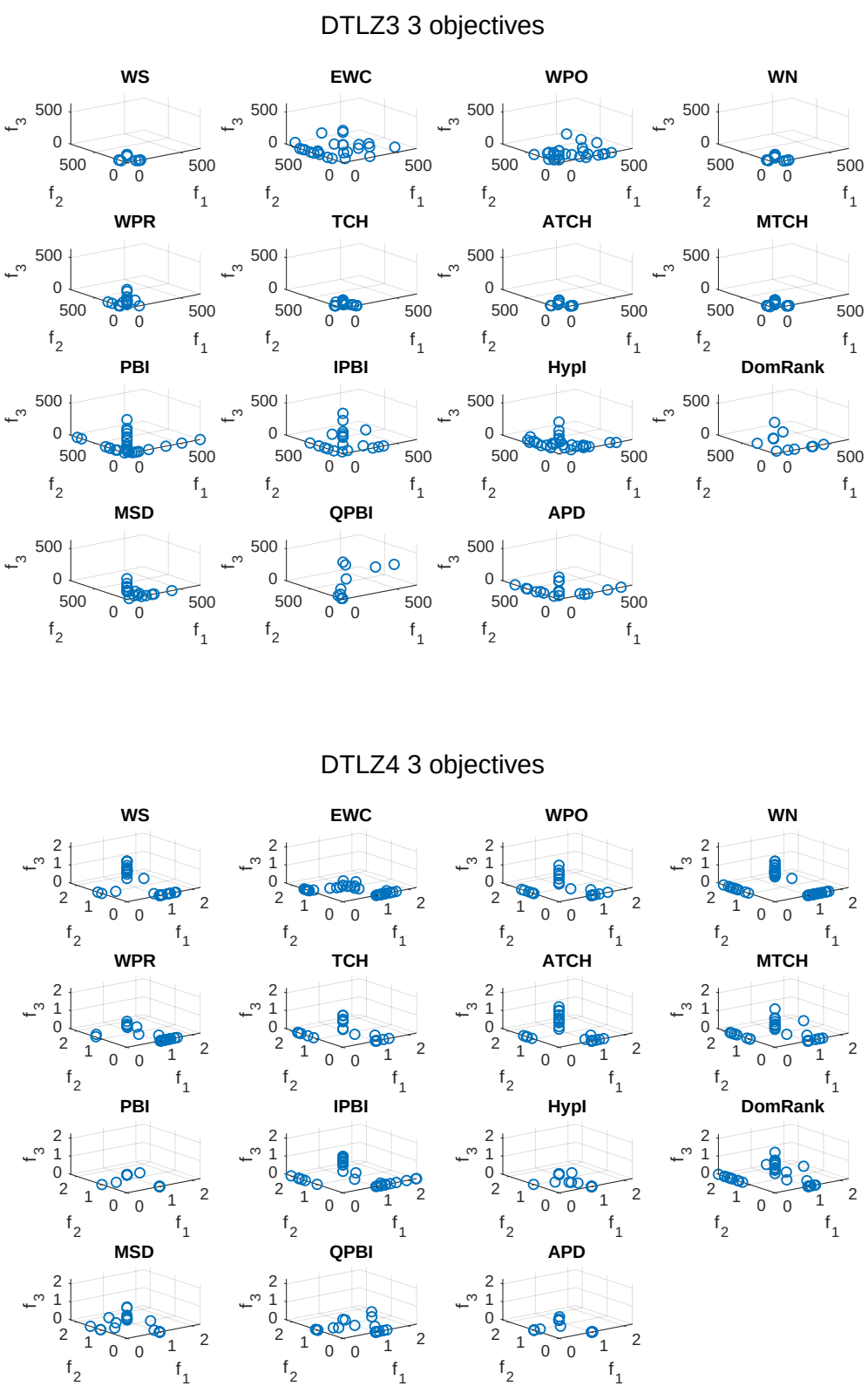

DTLZ3 3 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
DTLZ4 3 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD

Fig. 6

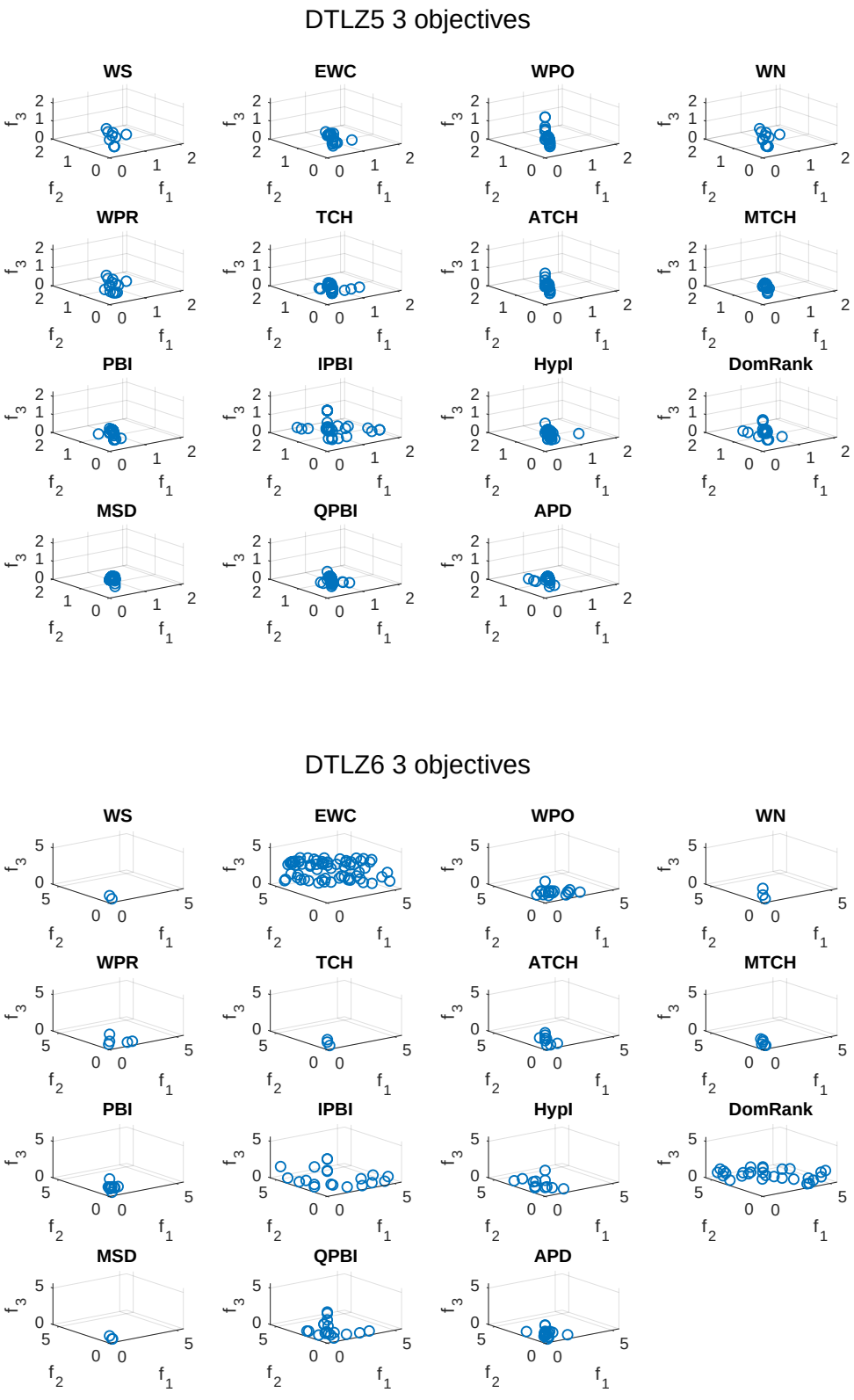

DTLZ5 3 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
DTLZ6 3 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD

Fig. 7

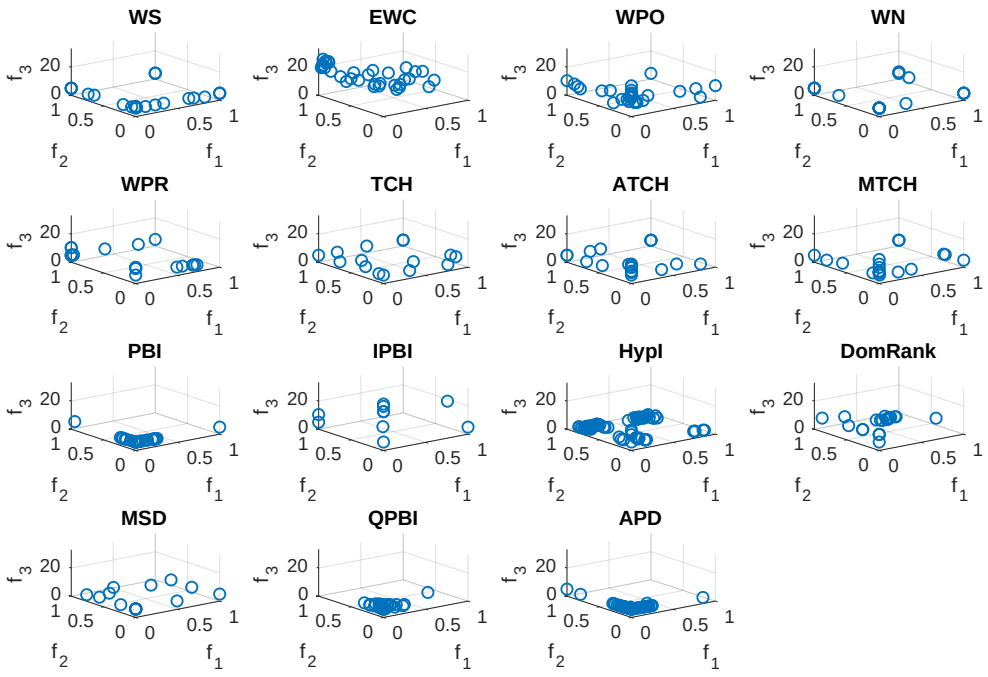
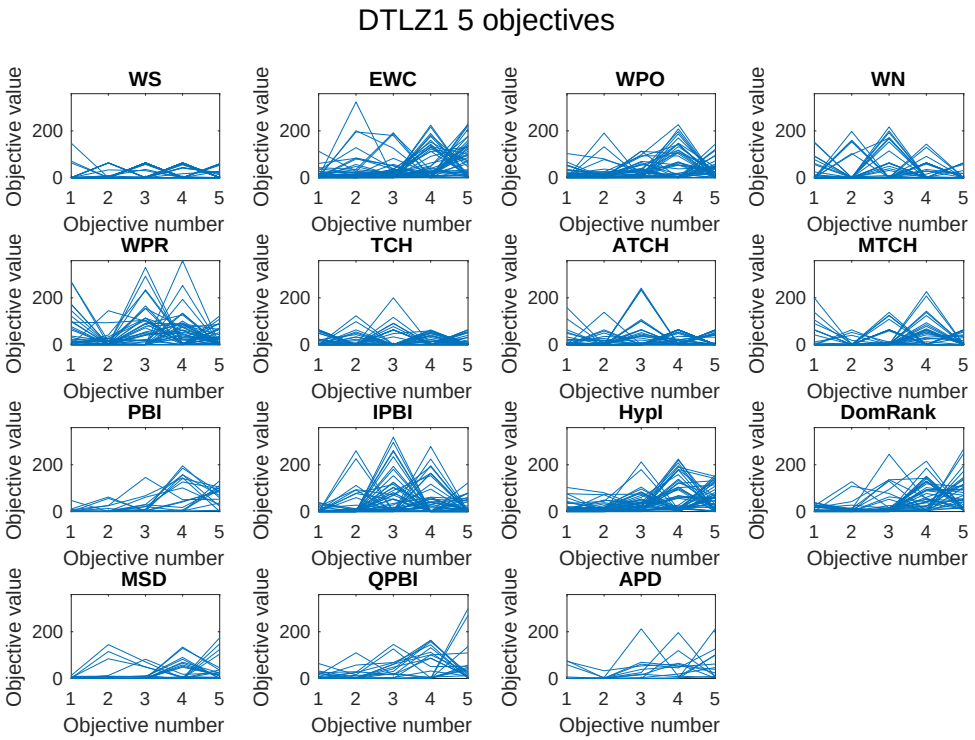

DTLZ7 3 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
f3
f2
f1

Fig. 8


DTLZ1 5 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

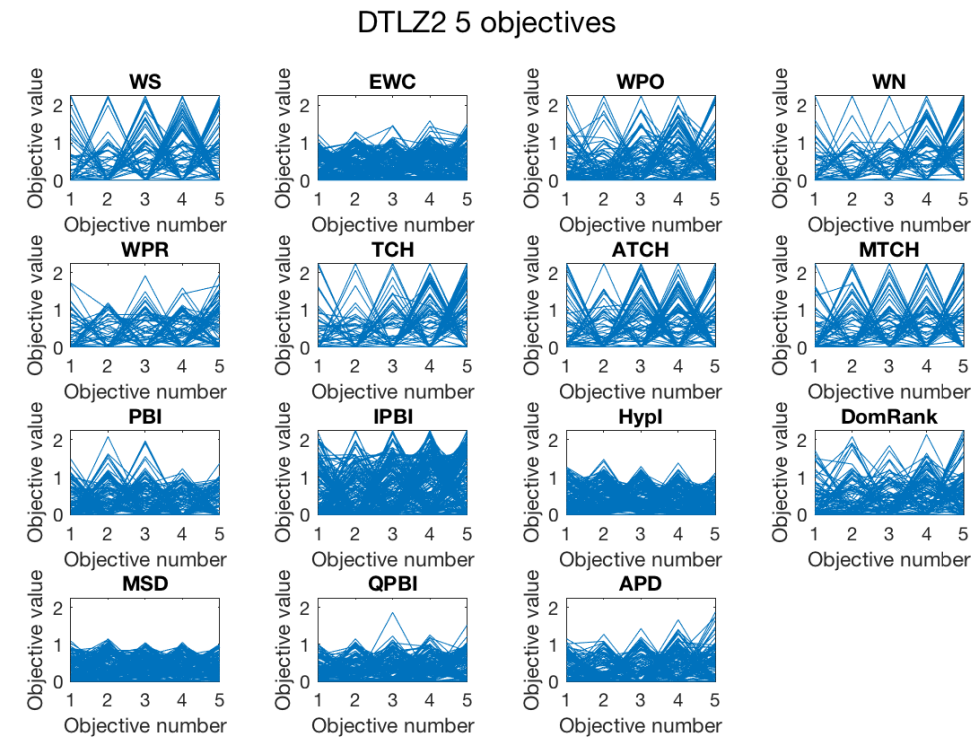
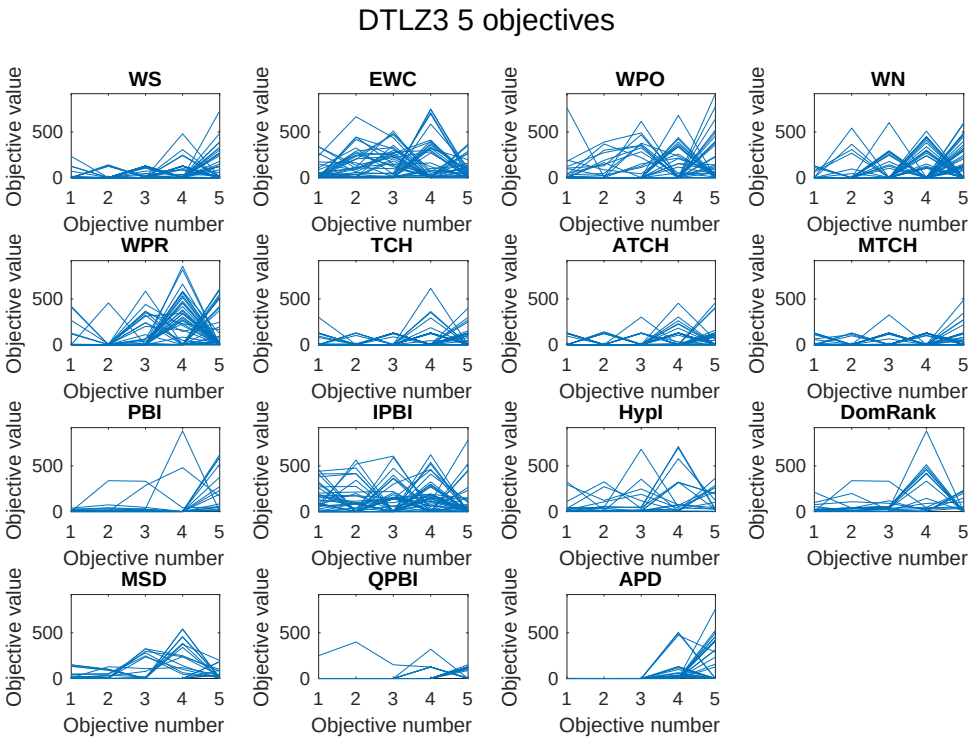

DTLZ2 5 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

Fig. 9


DTLZ3 5 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

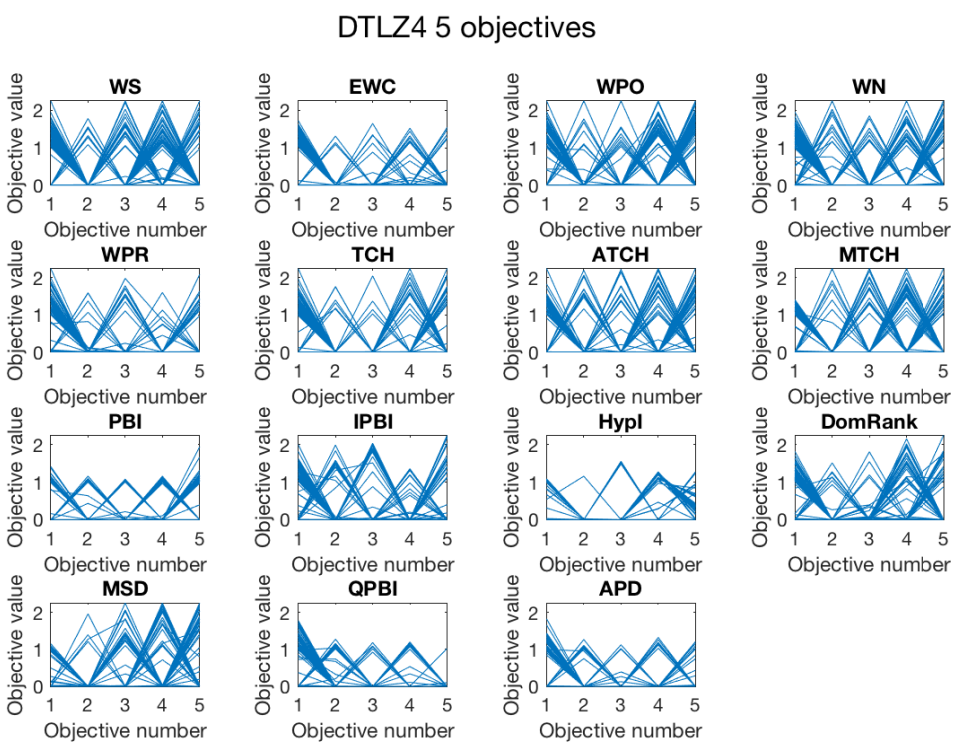
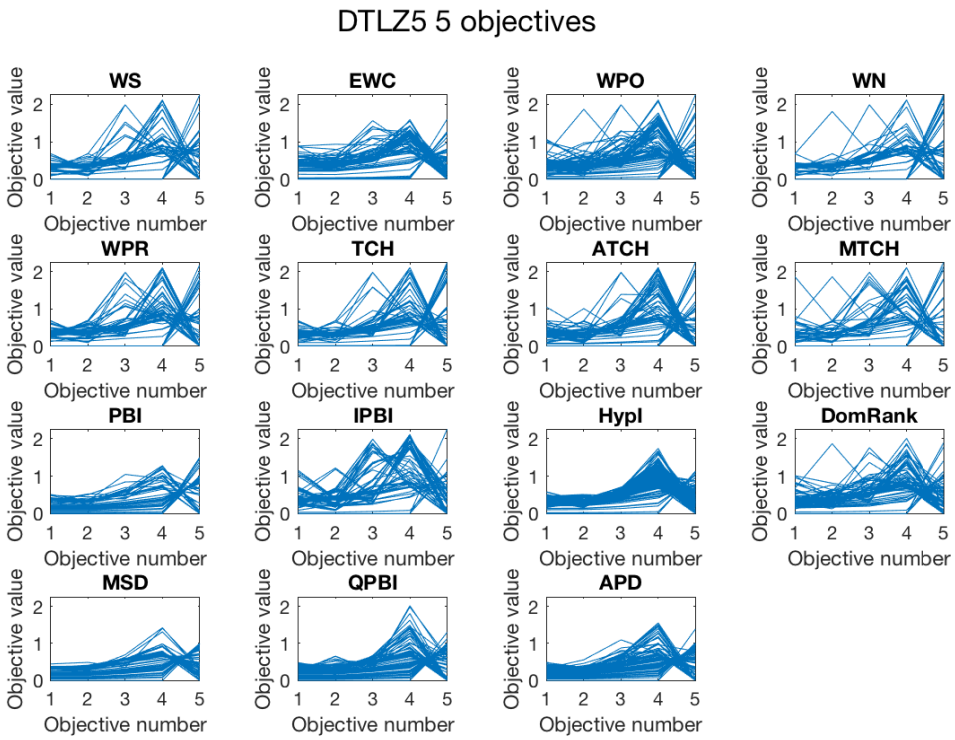

DTLZ4 5 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

Fig. 10


DTLZ5 5 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

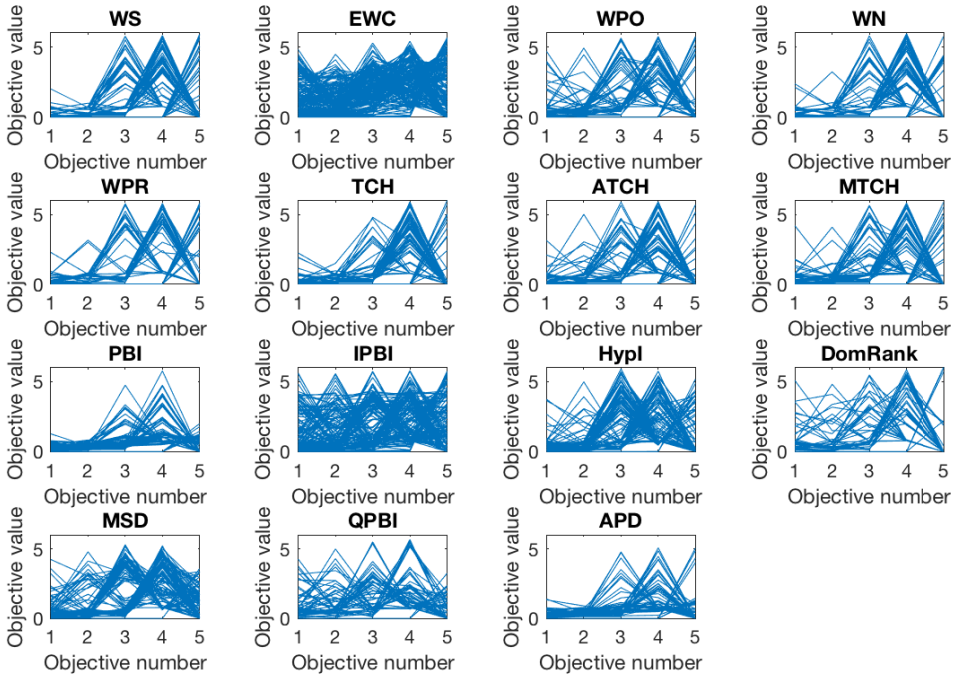

DTLZ6 5 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

Fig. 11

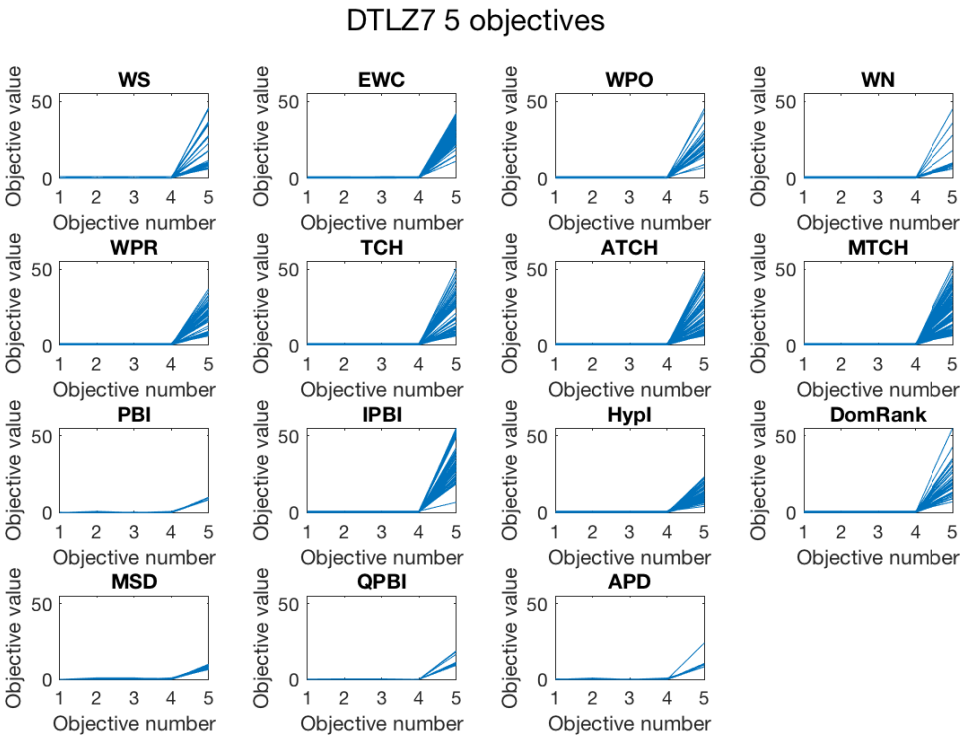
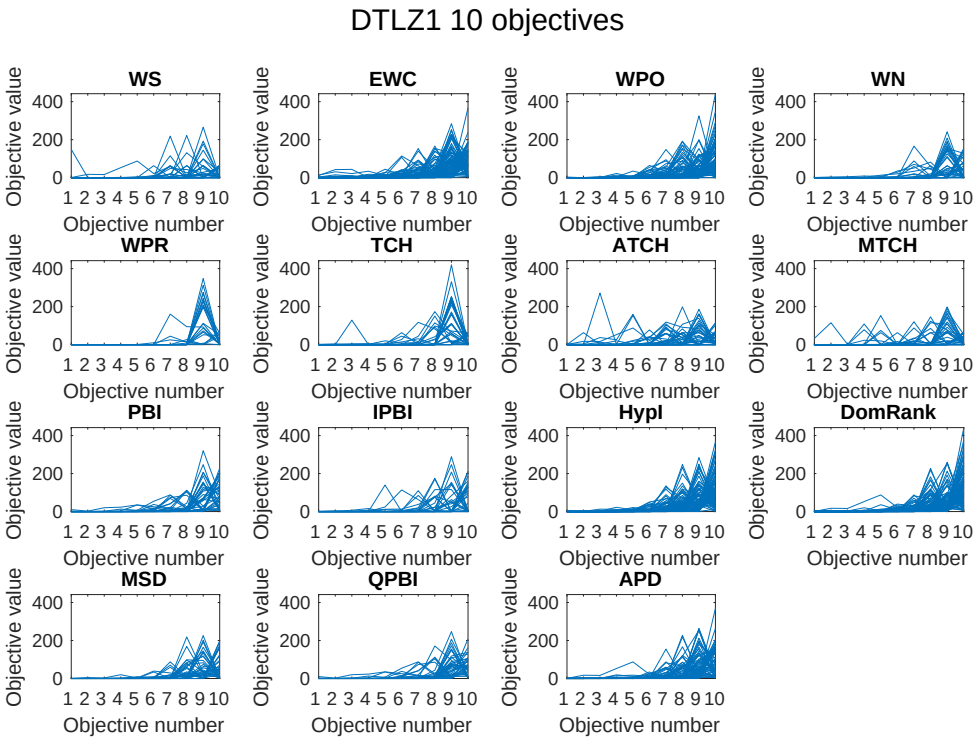

DTLZ7 5 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number
0
50
1
2
3
4
5

Fig. 12


DTLZ1 10 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

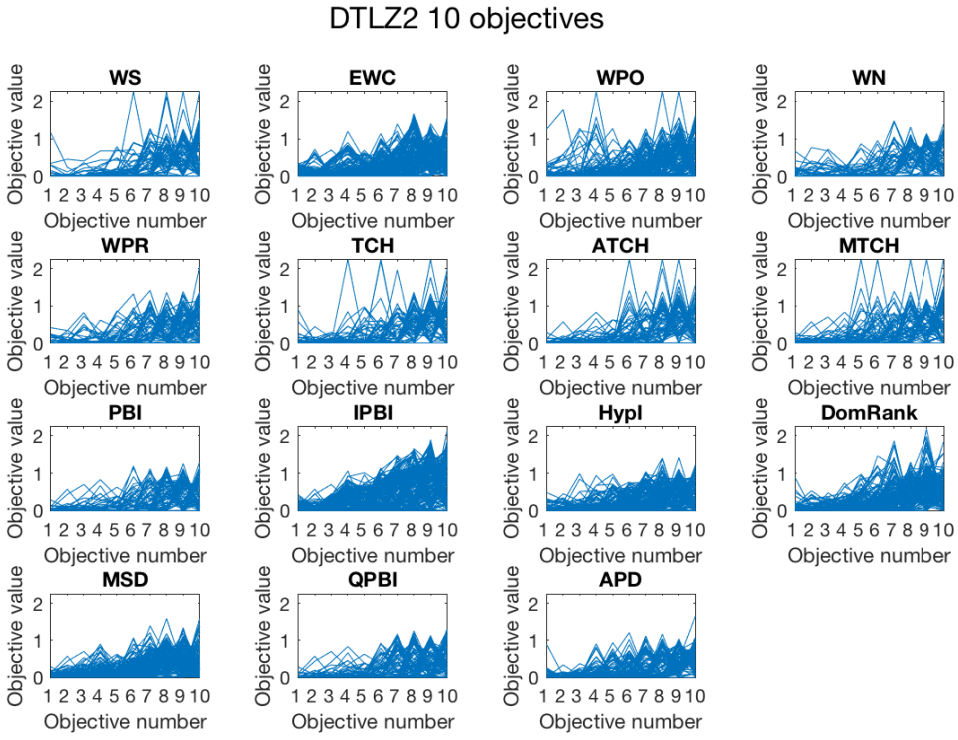
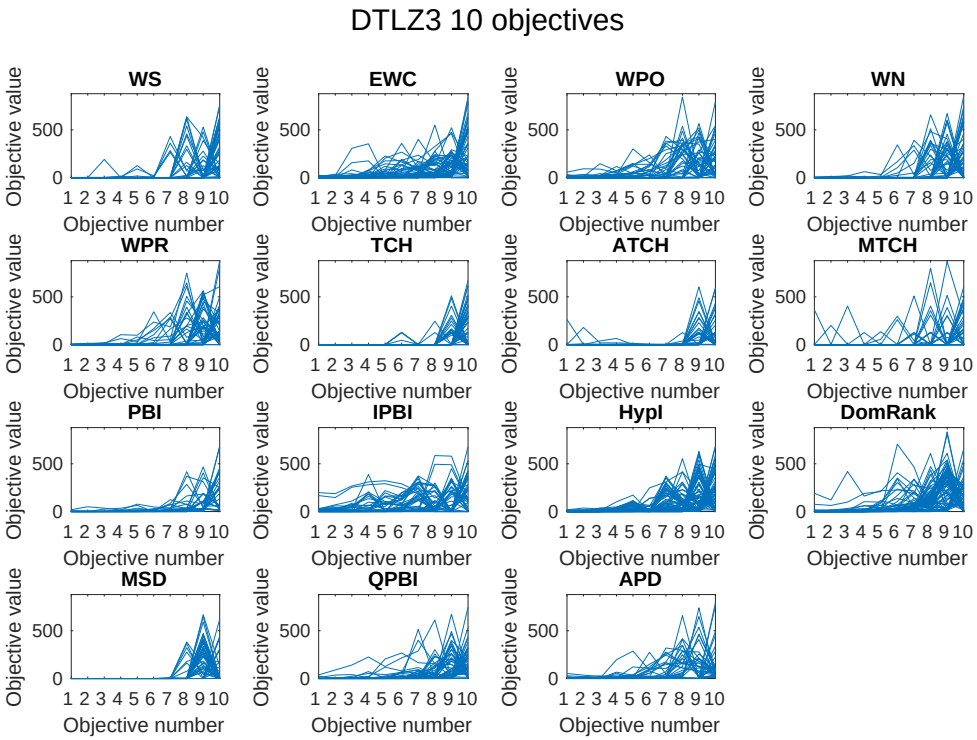

DTLZ2 10 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

Fig. 13


DTLZ3 10 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number
0
500
1 2 3 4 5 6 7 8 9 10

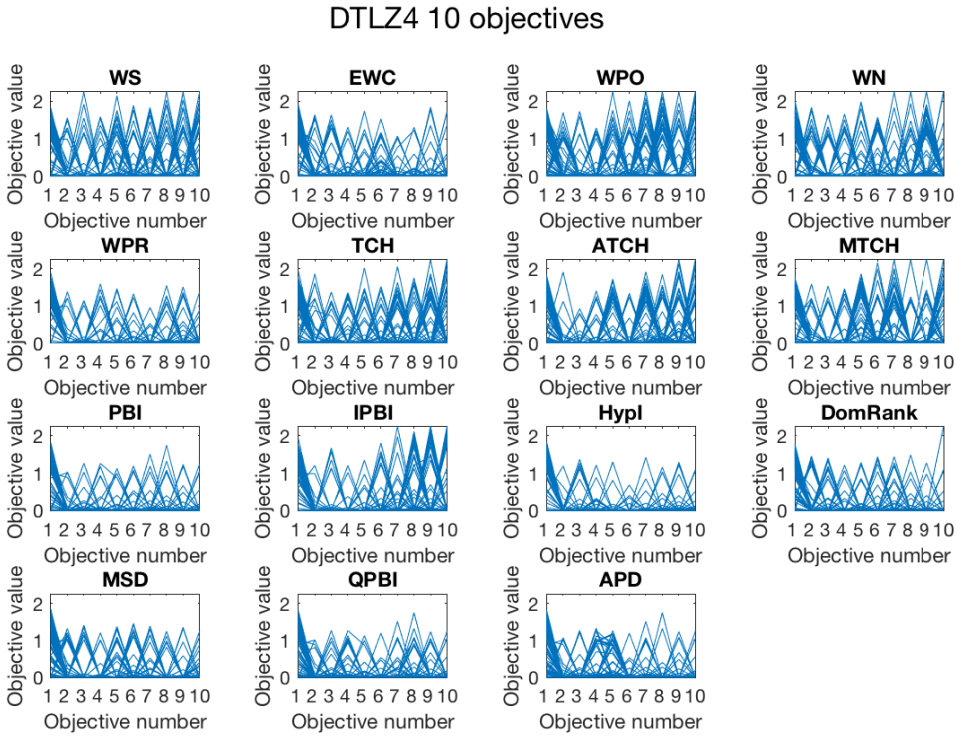
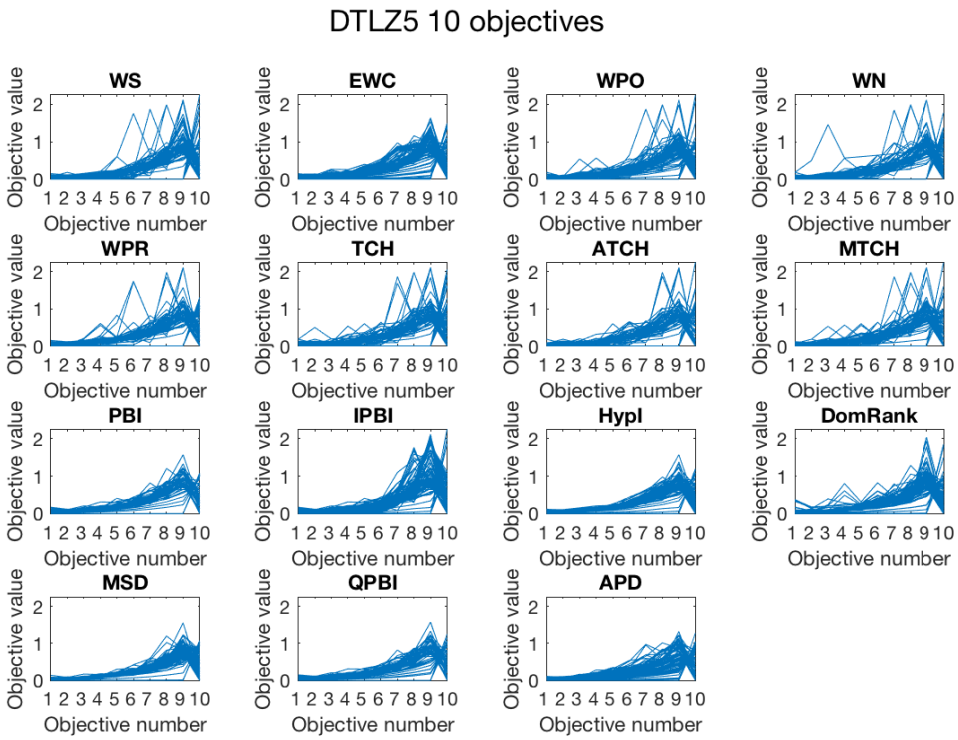

DTLZ4 10 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number
0
1
2
1 2 3 4 5 6 7 8 9 10

Fig. 14


DTLZ5 10 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

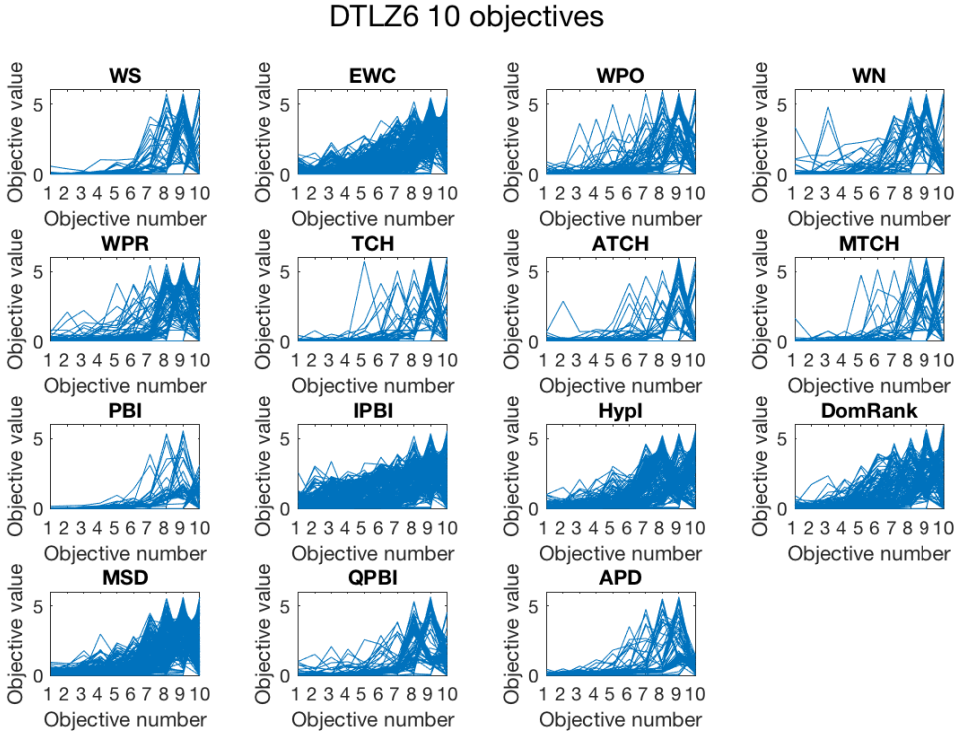

DTLZ6 10 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number

Fig. 15

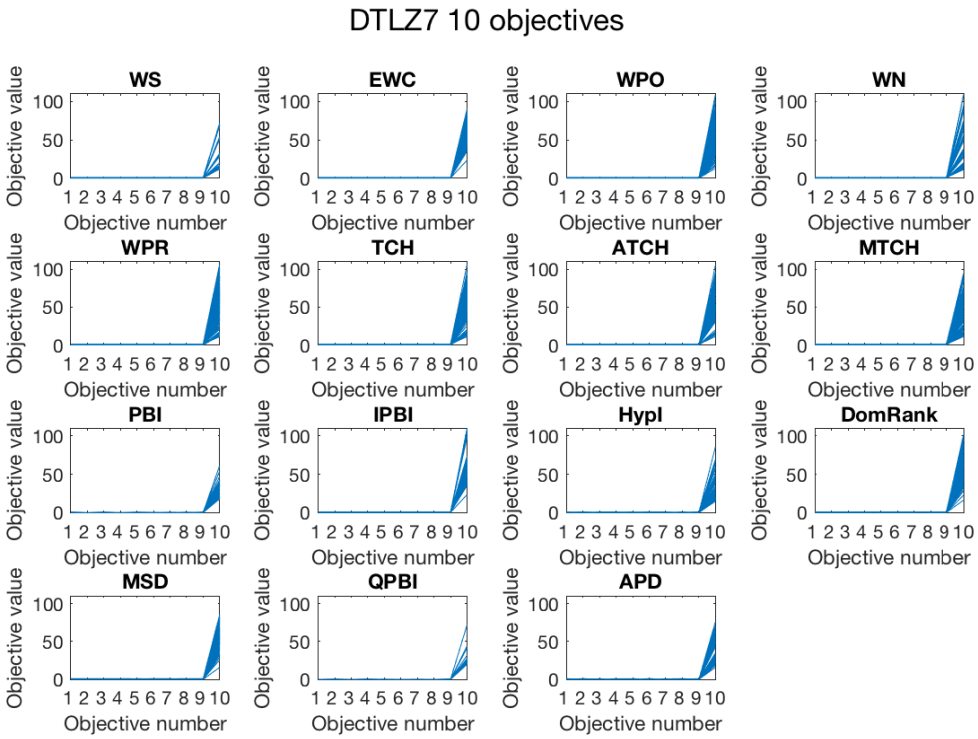

DTLZ7 10 objectives
WS
EWC
WPO
WN
WPR
TCH
ATCH
MTCH
PBI
IPBI
HypI
DomRank
MSD
QPBI
APD
Objective value
Objective number
100
50
0
1 2 3 4 5 6 7 8 9 10

Fig. 16

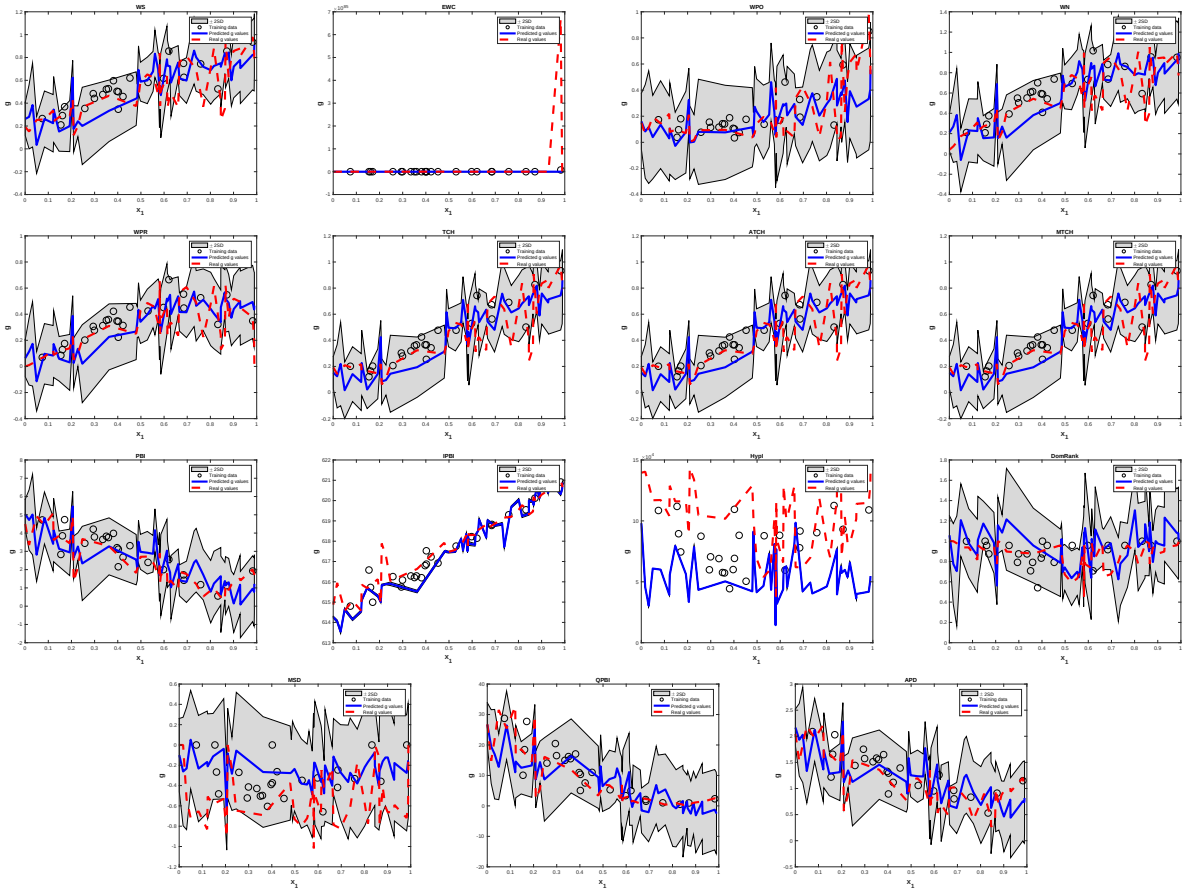


Fig. 17: Scalarizing function values (notated by g) with one decision variable value for DTLZ1 two objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

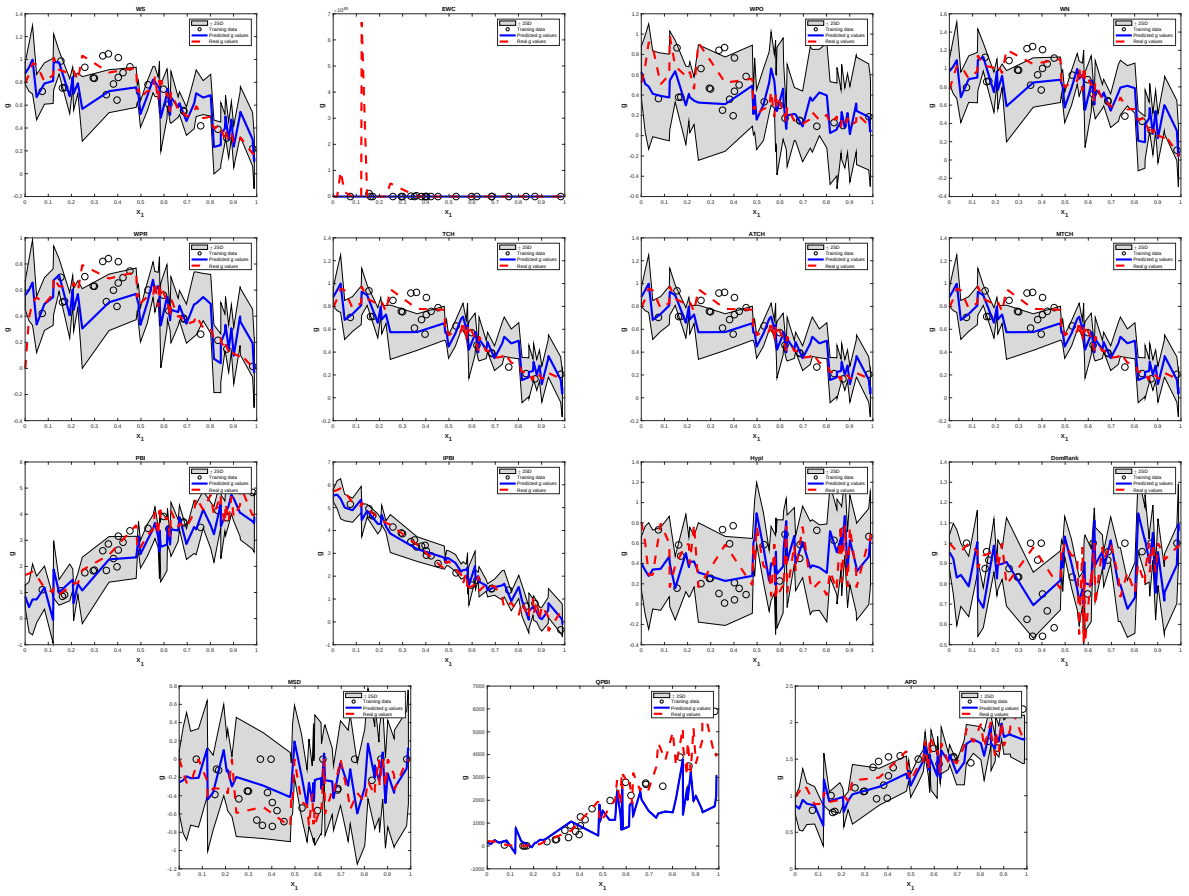


Fig. 18: Scalarizing function values (notated by g) with one decision variable value for DTLZ2 two objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

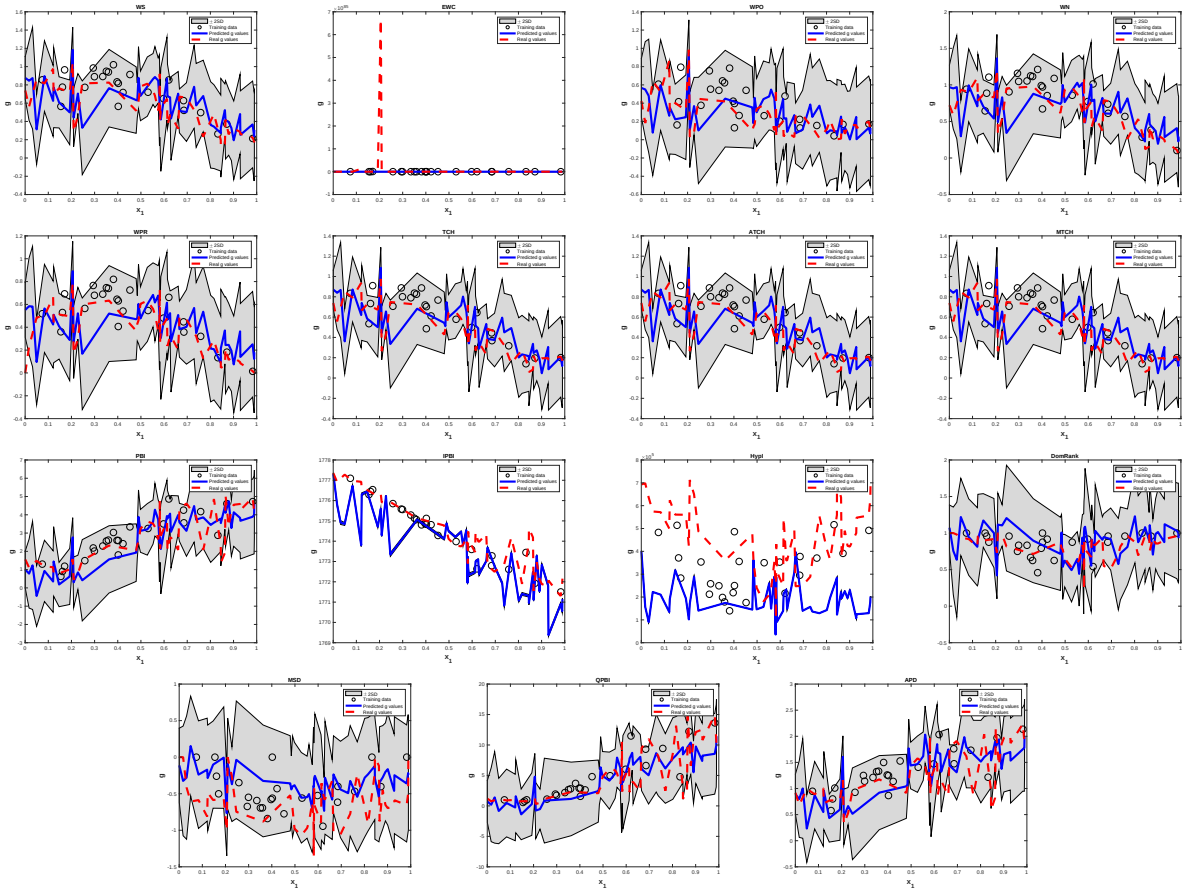


Fig. 19: Scalarizing function values (notated by g) with one decision variable value for DTLZ3 two objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

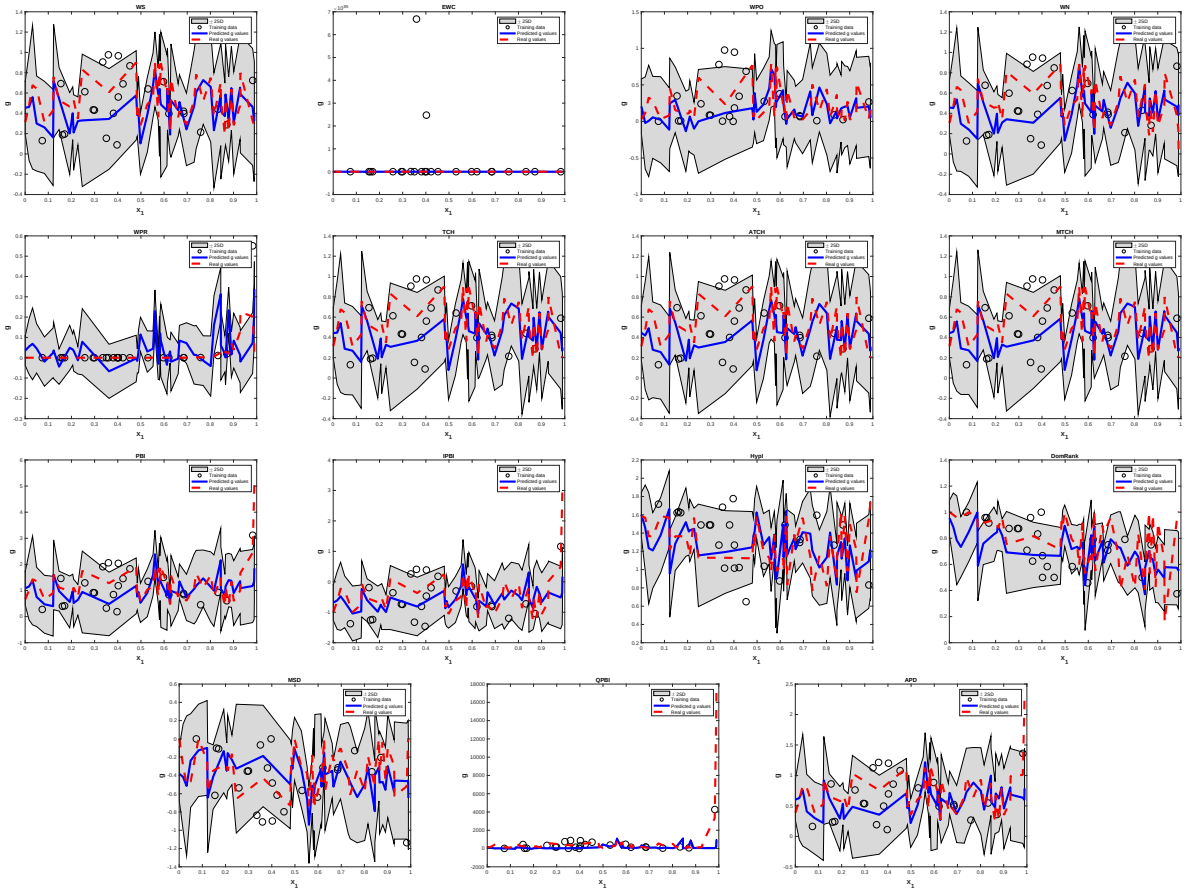


Fig. 20: Scalarizing function values (notated by g) with one decision variable value for DTLZ4 two objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

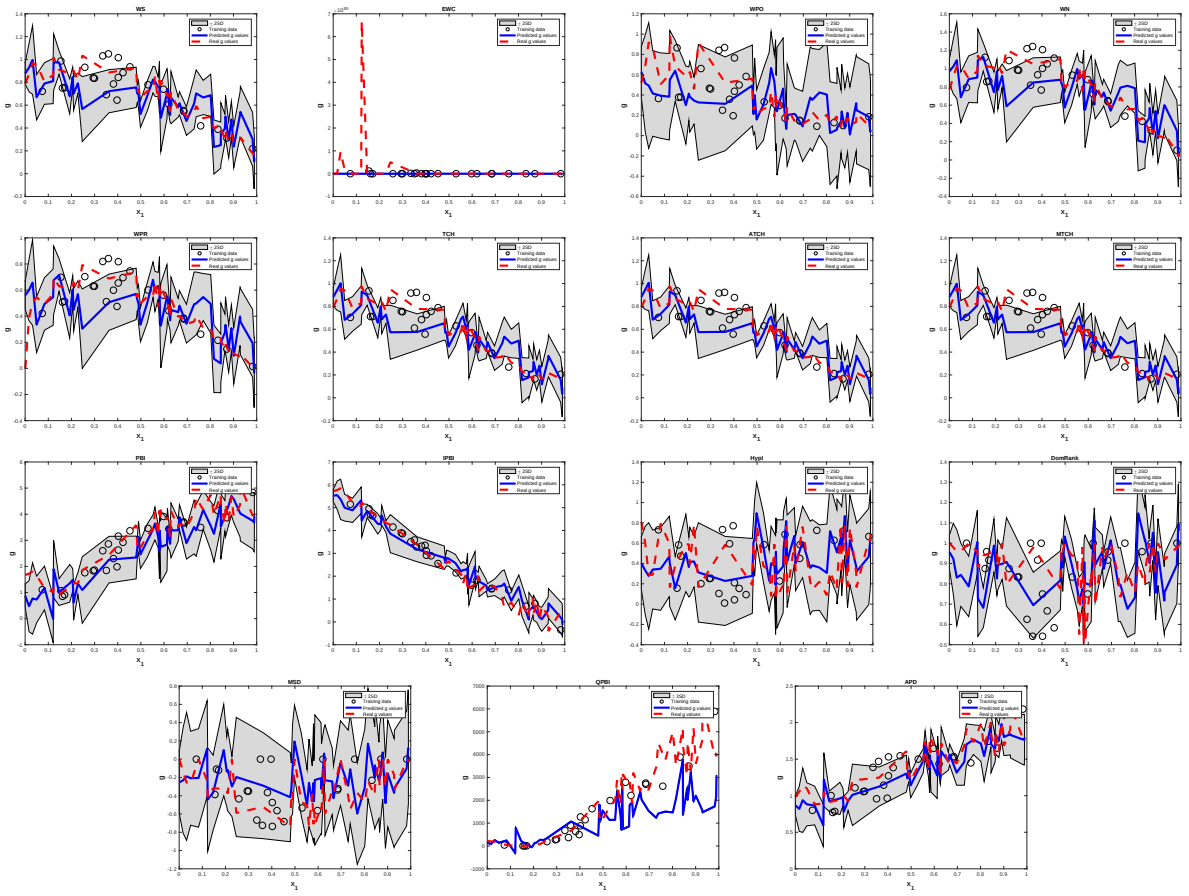


Fig. 21: Scalarizing function values (notated by g) with one decision variable value for DTLZ5 two objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

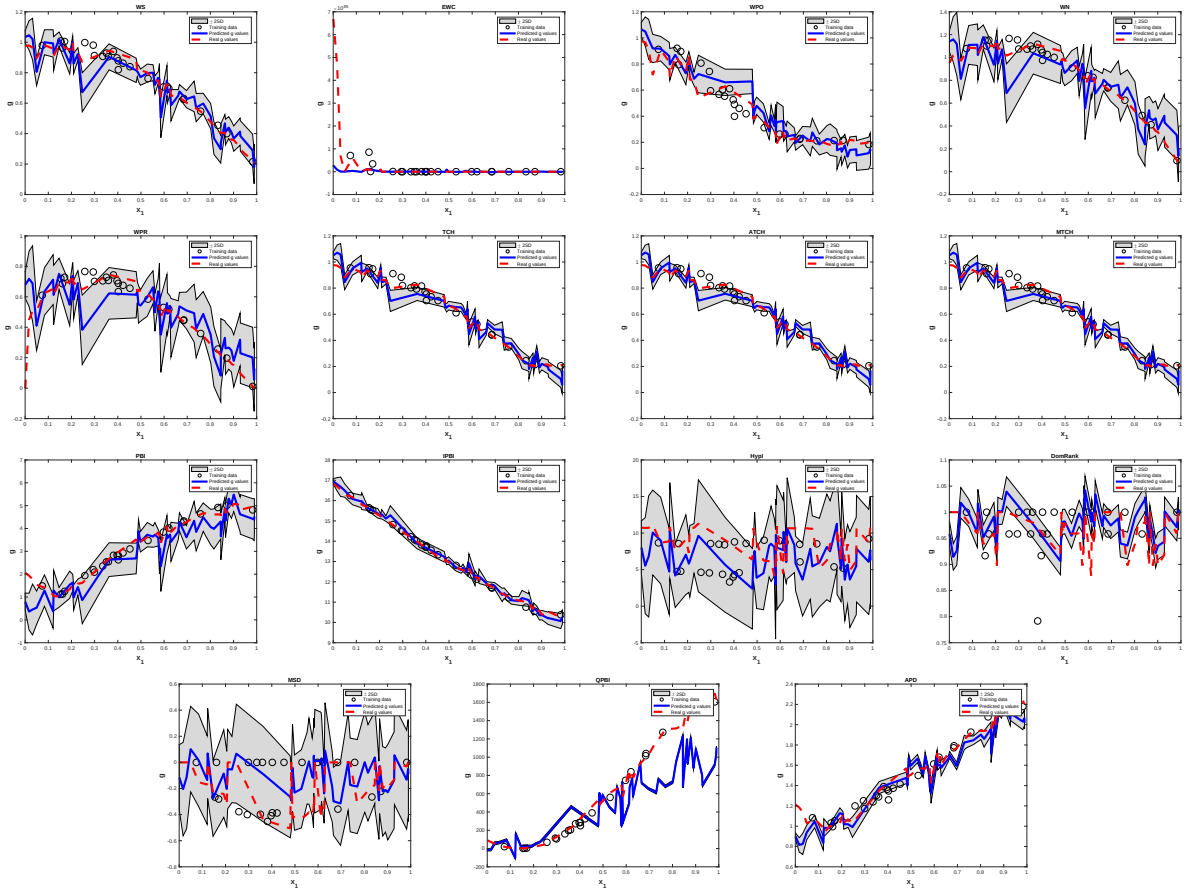


Fig. 22: Scalarizing function values (notated by g) with one decision variable value for DTLZ6 two objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

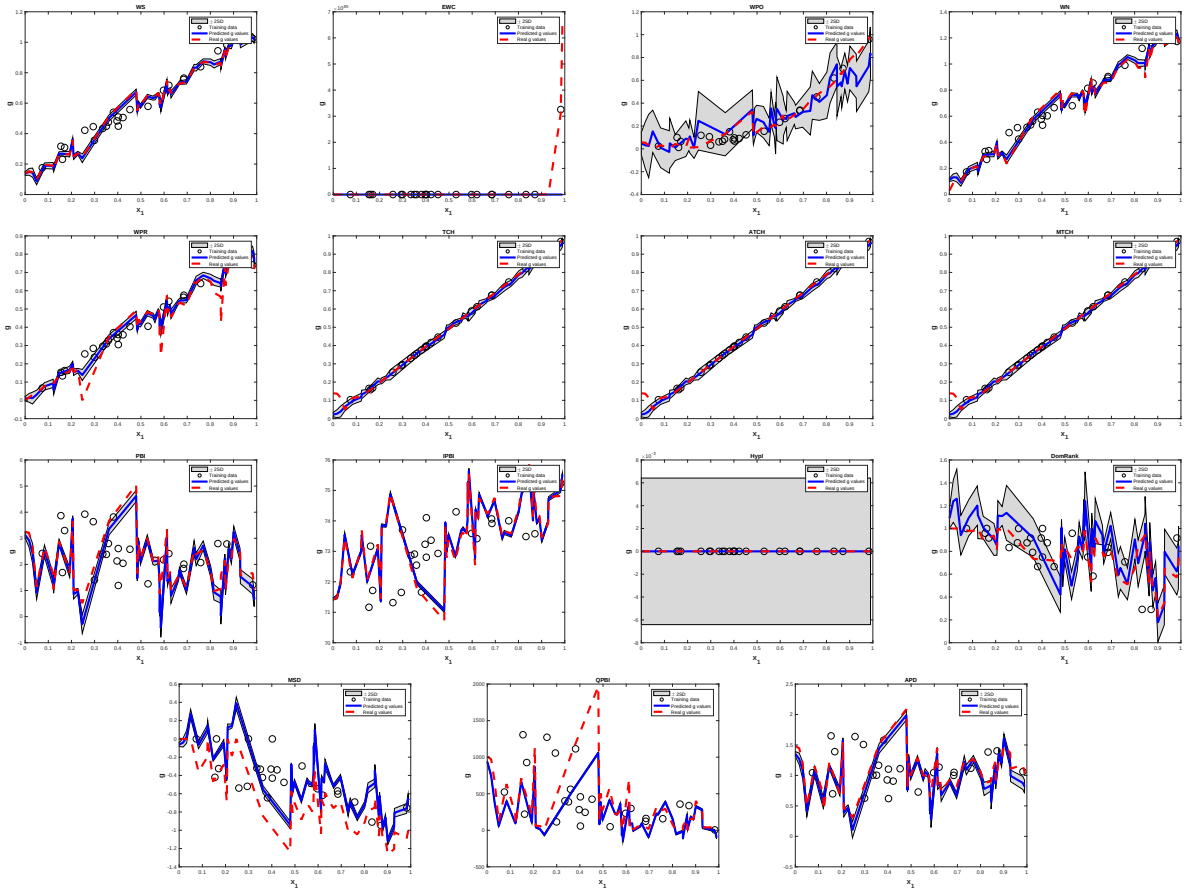


Fig. 23: Scalarizing function values (notated by g) with one decision variable value for DTLZ7 two objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

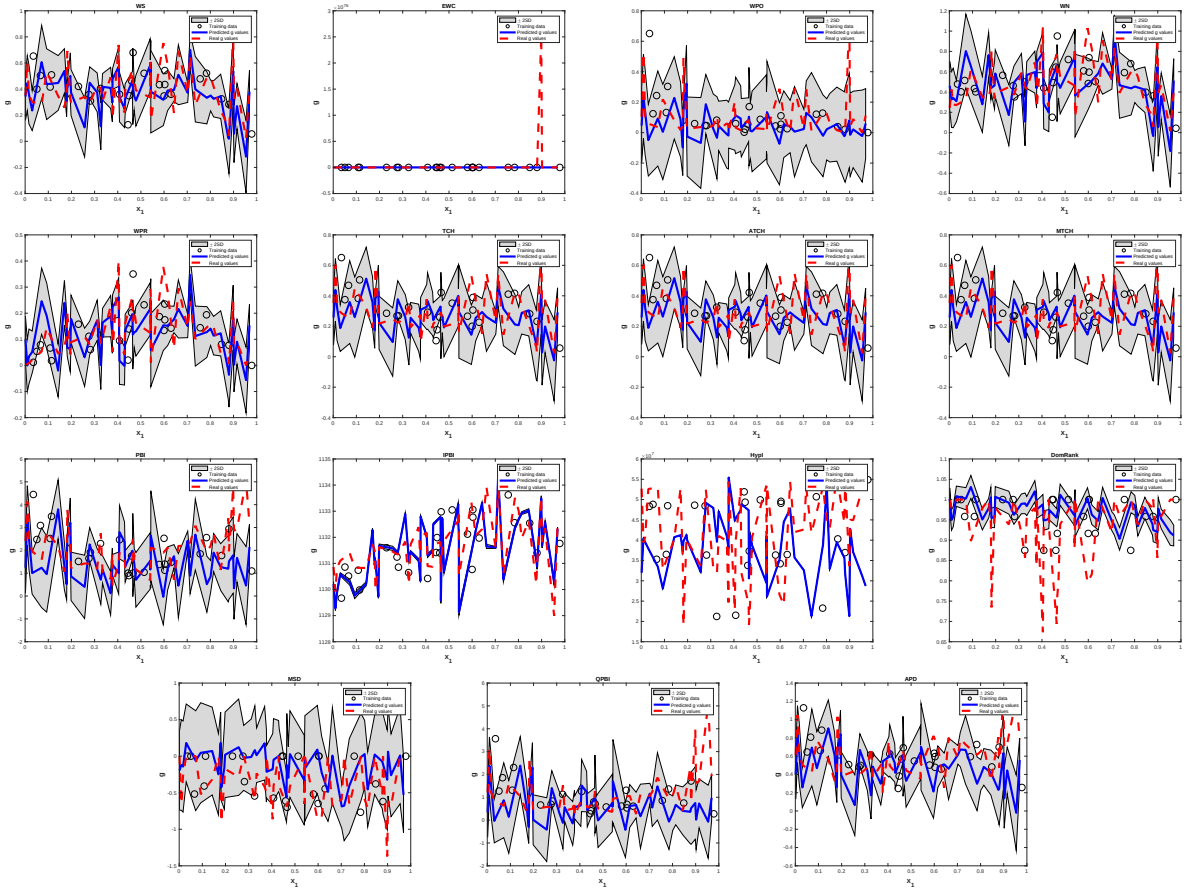


Fig. 24: Scalarizing function values (notated by g) with one decision variable value for DTLZ1 three objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

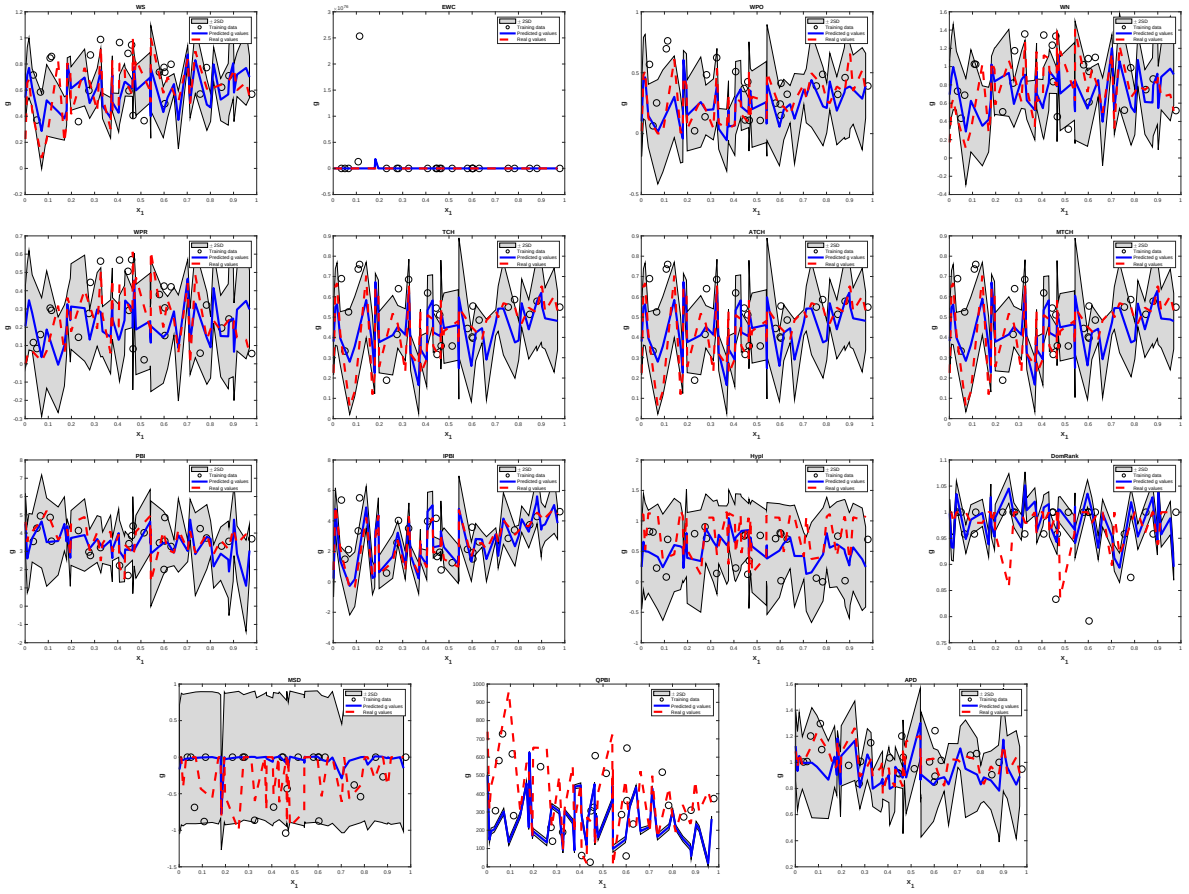


Fig. 25: Scalarizing function values (notated by g) with one decision variable value for DTLZ2 three objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

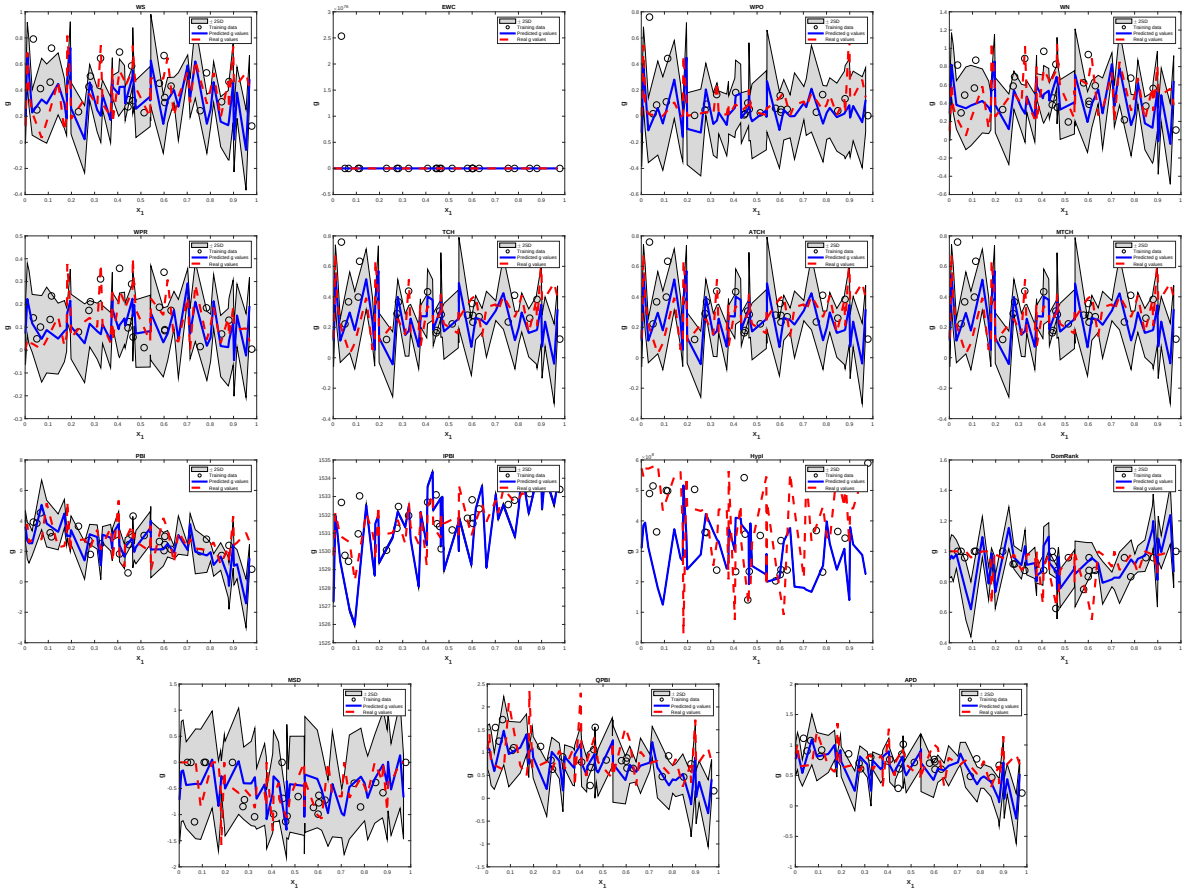


Fig. 26: Scalarizing function values (notated by g) with one decision variable value for DTLZ3 three objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

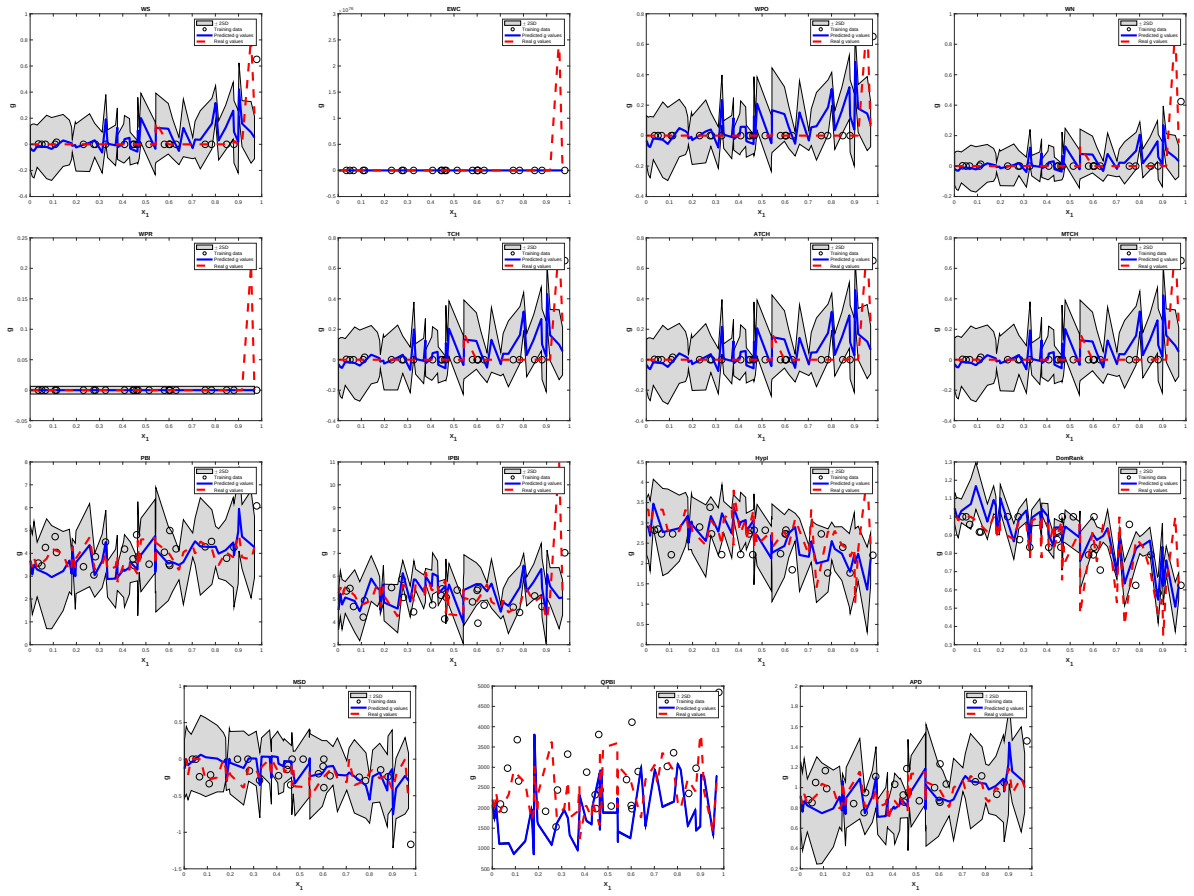


Fig. 27: Scalarizing function values (notated by g) with one decision variable value for DTLZ4 three objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

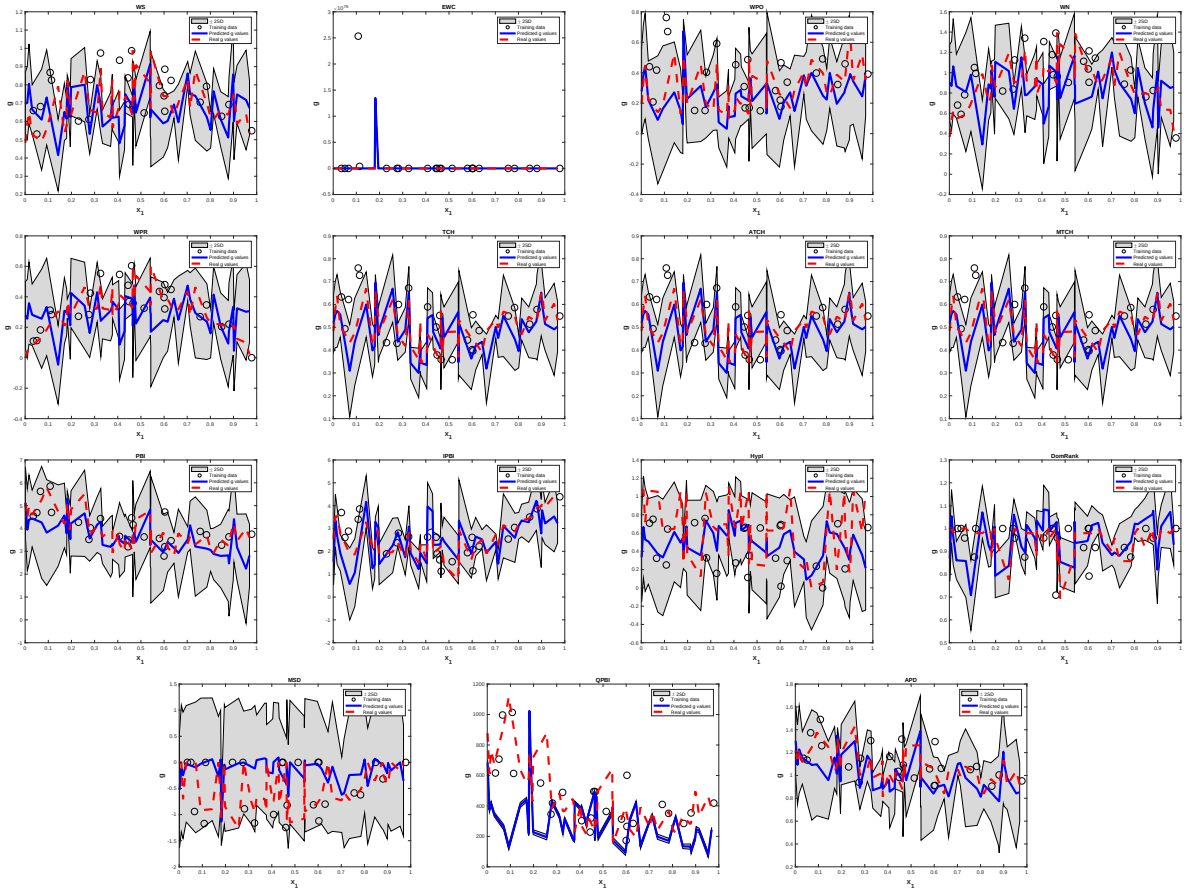


Fig. 28: Scalarizing function values (notated by g) with one decision variable value for DTLZ5 three objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

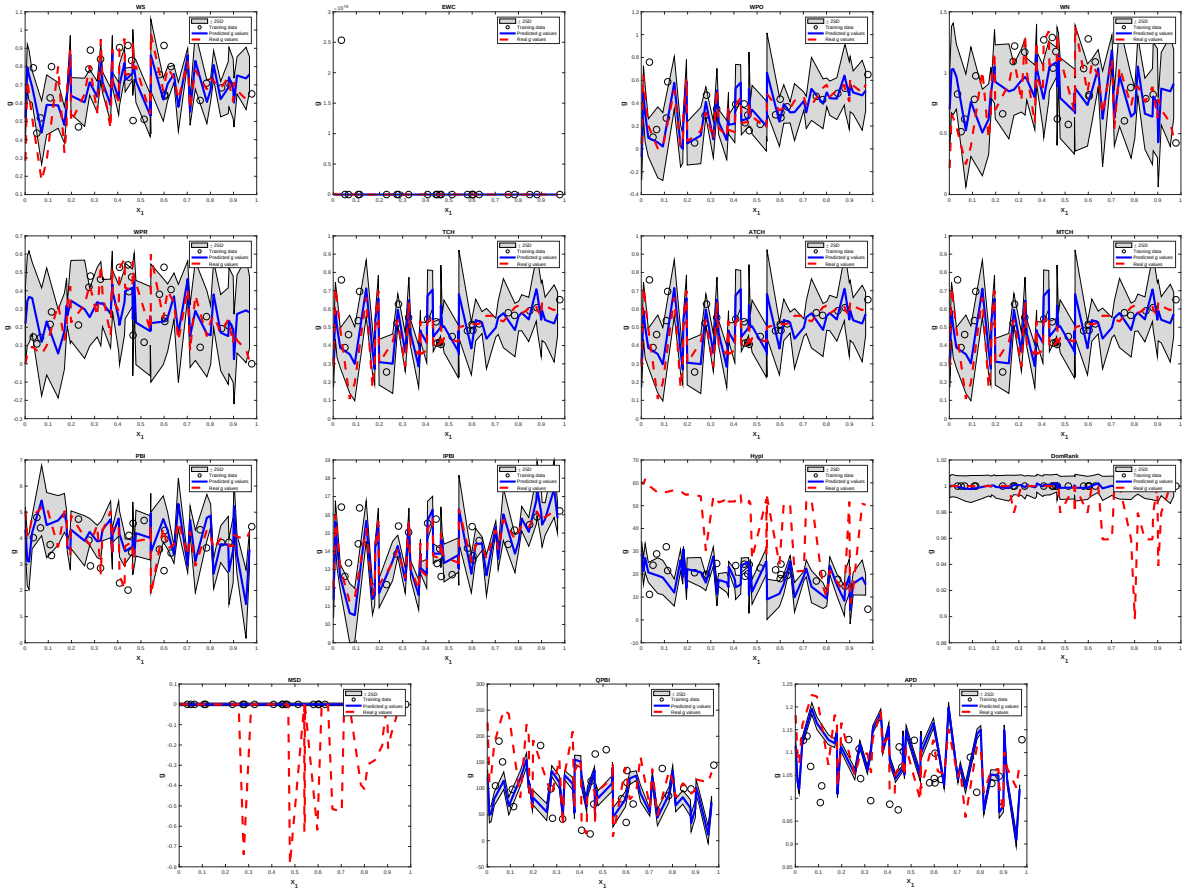


Fig. 29: Scalarizing function values (notated by g) with one decision variable value for DTLZ6 three objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

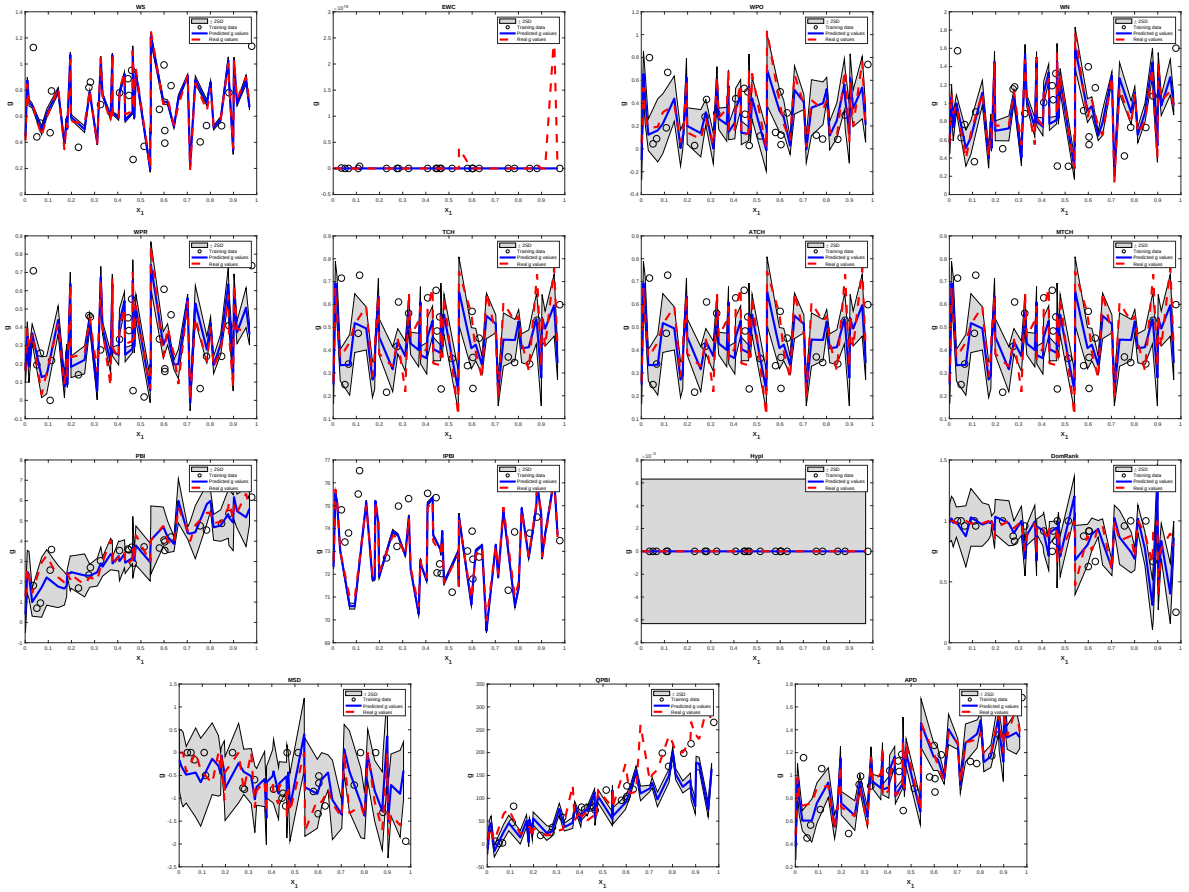


Fig. 30: Scalarizing function values (notated by g) with one decision variable value for DTLZ7 three objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

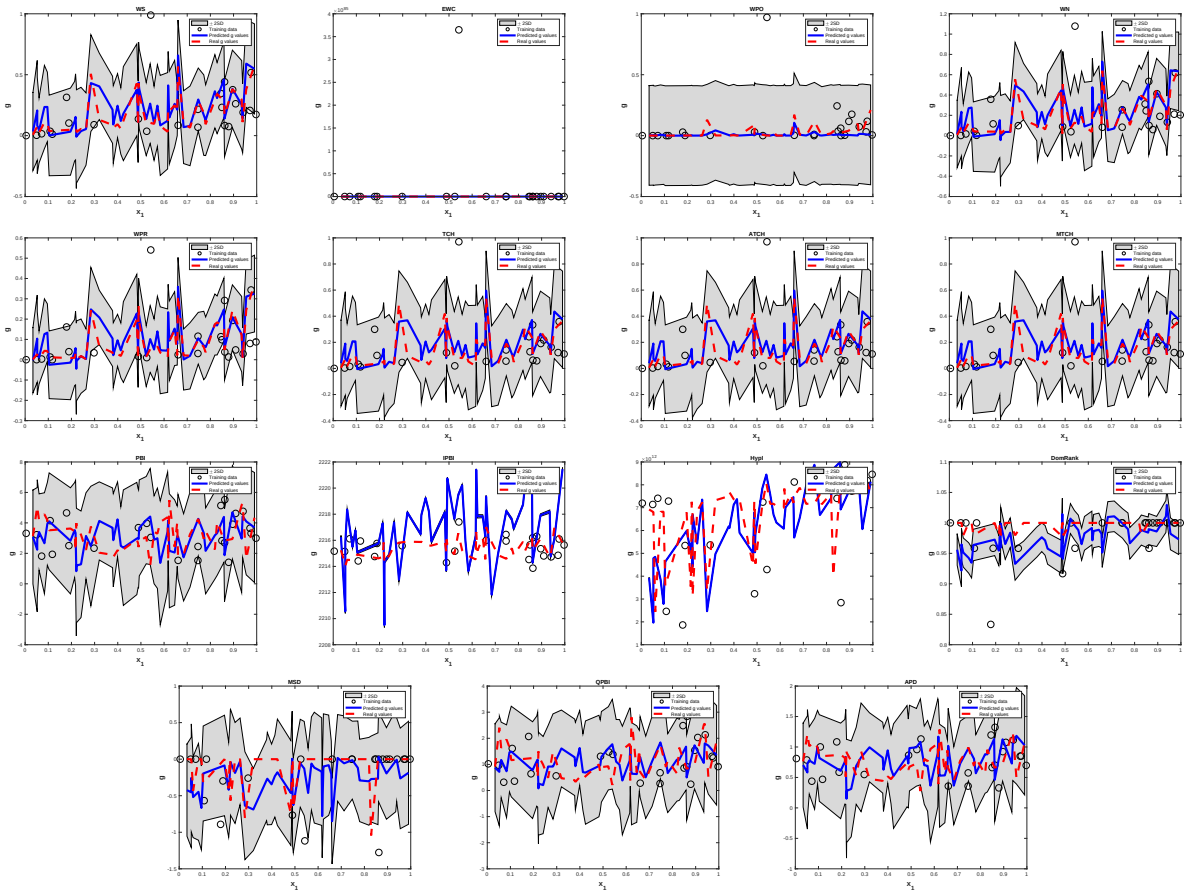


Fig. 31: Scalarizing function values (notated by g) with one decision variable value for DTLZ1 five objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

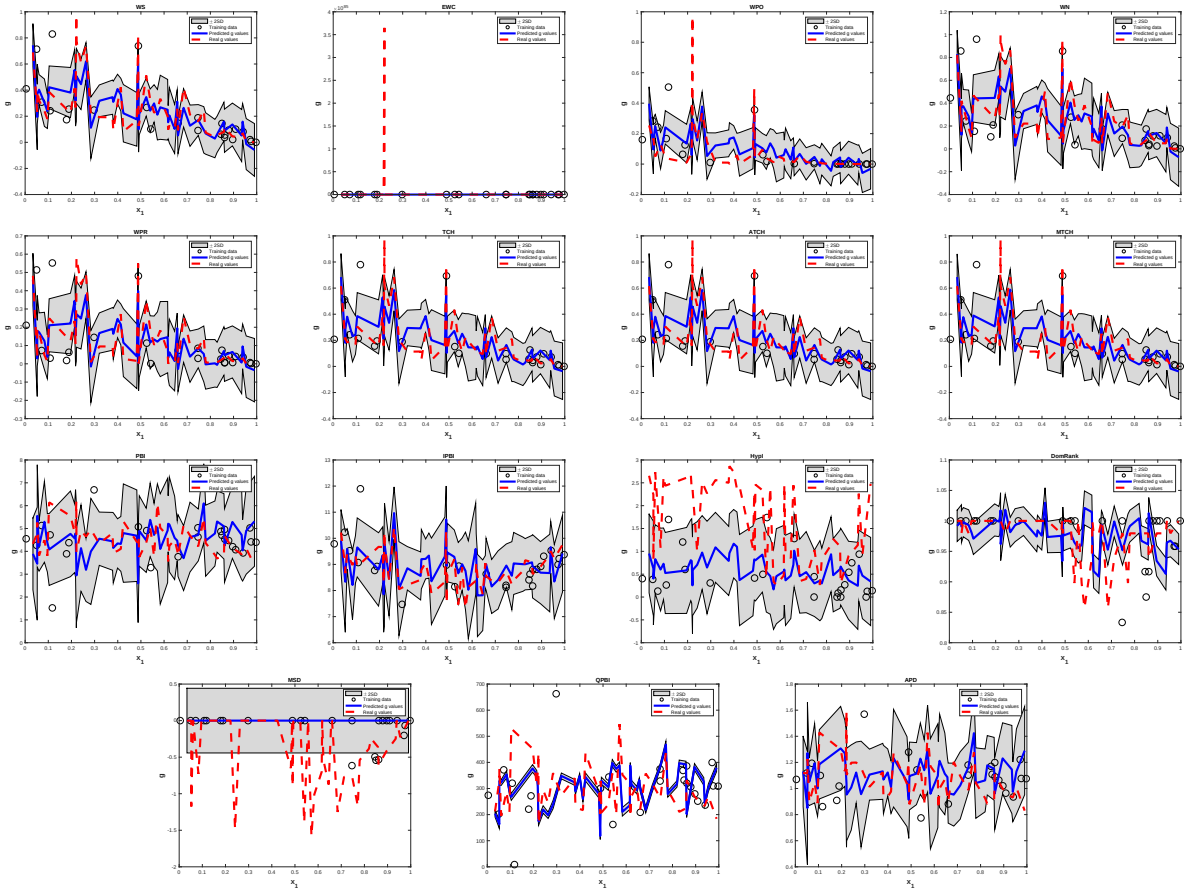


Fig. 32: Scalarizing function values (notated by g) with one decision variable value for DTLZ2 five objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

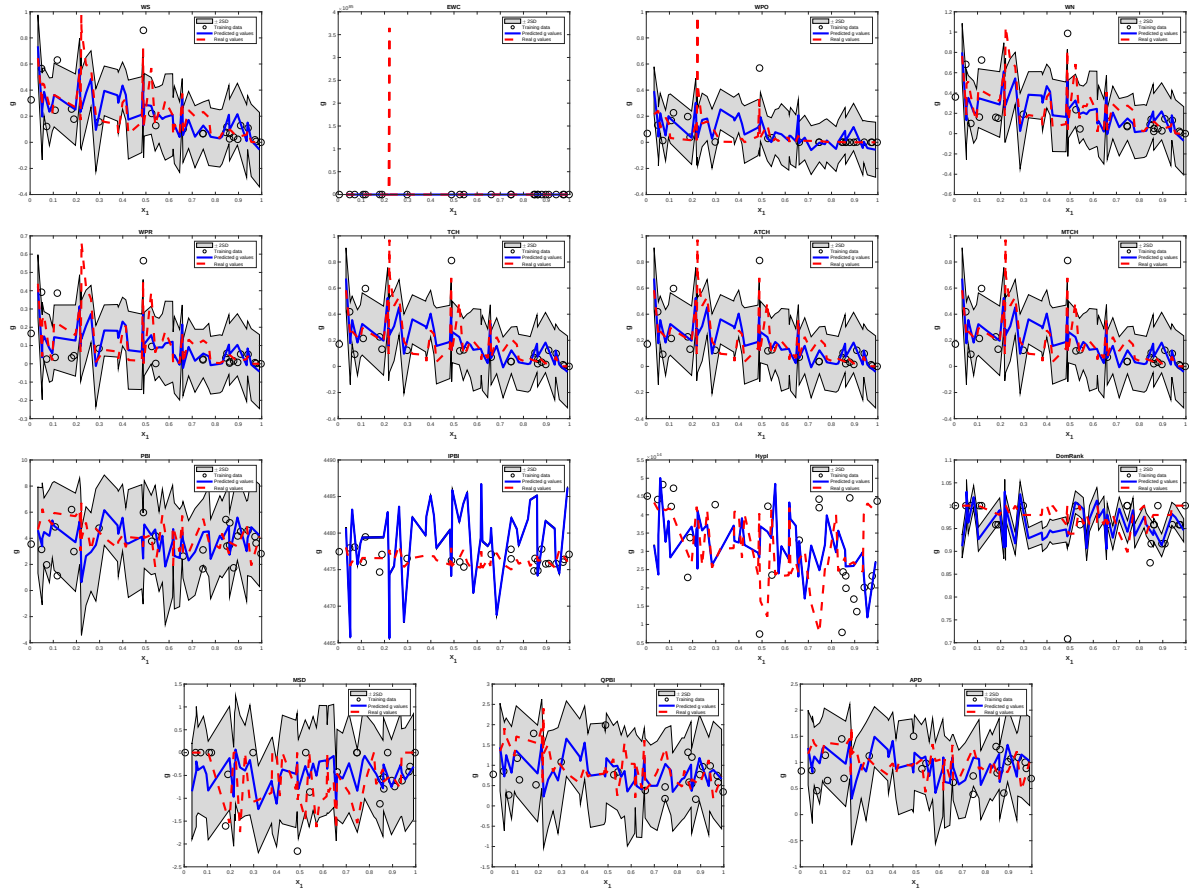


Fig. 33: Scalarizing function values (notated by g) with one decision variable value for DTLZ3 five objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

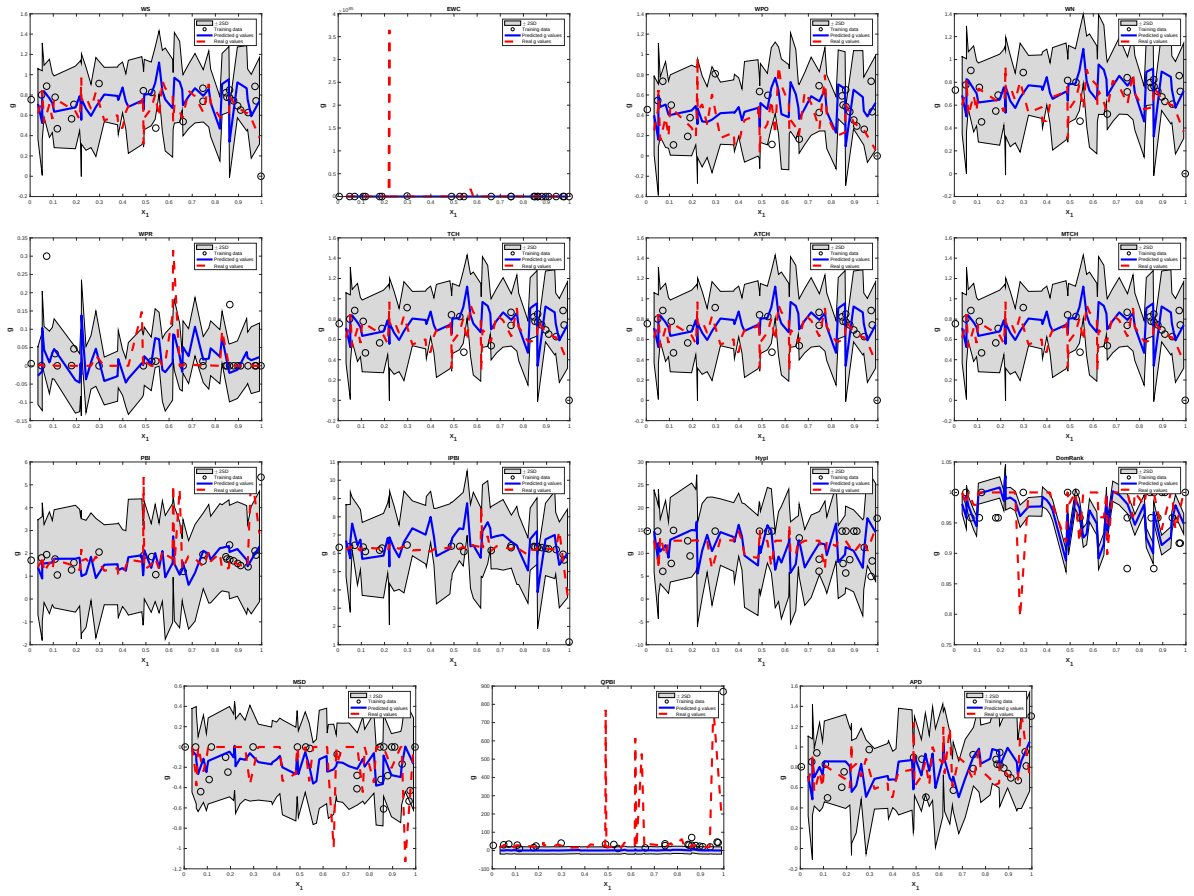


Fig. 34: Scalarizing function values (notated by g) with one decision variable value for DTLZ4 five objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

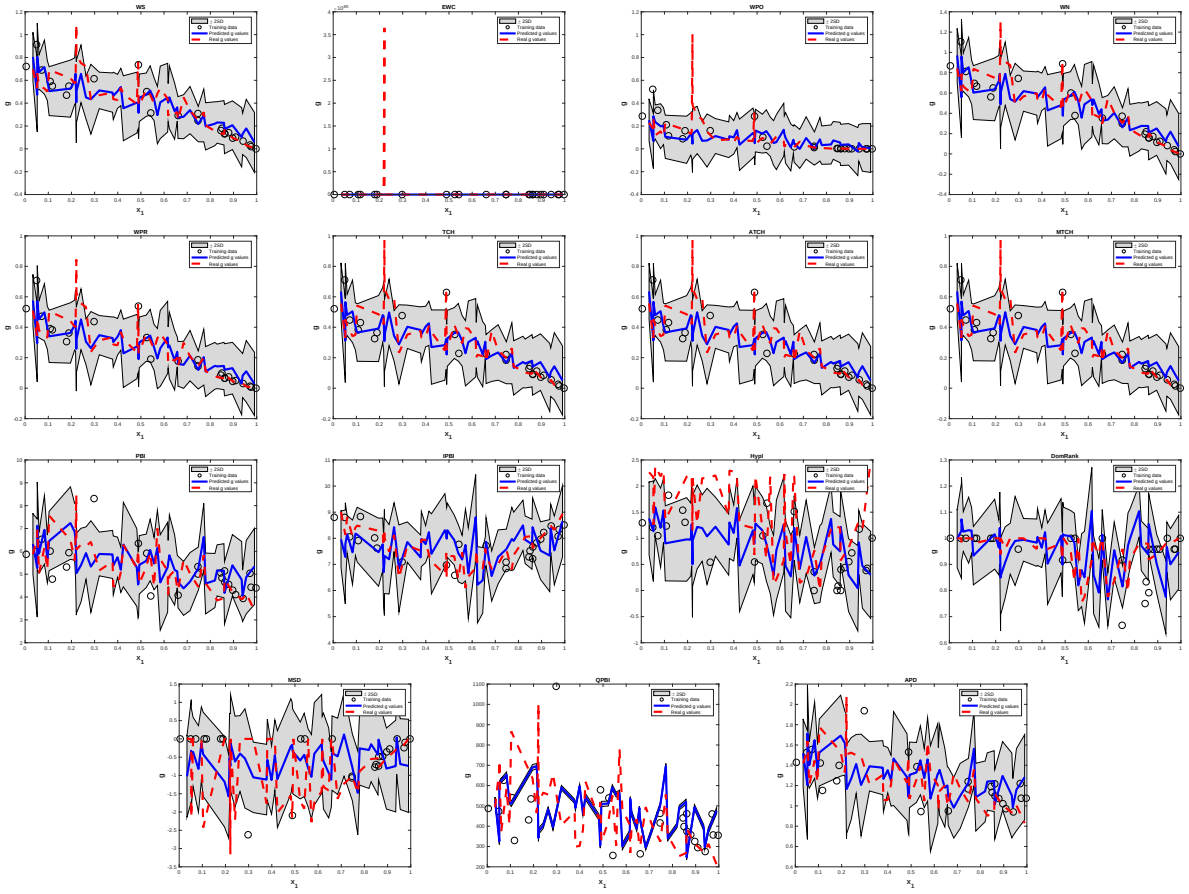


Fig. 35: Scalarizing function values (notated by g) with one decision variable value for DTLZ5 five objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

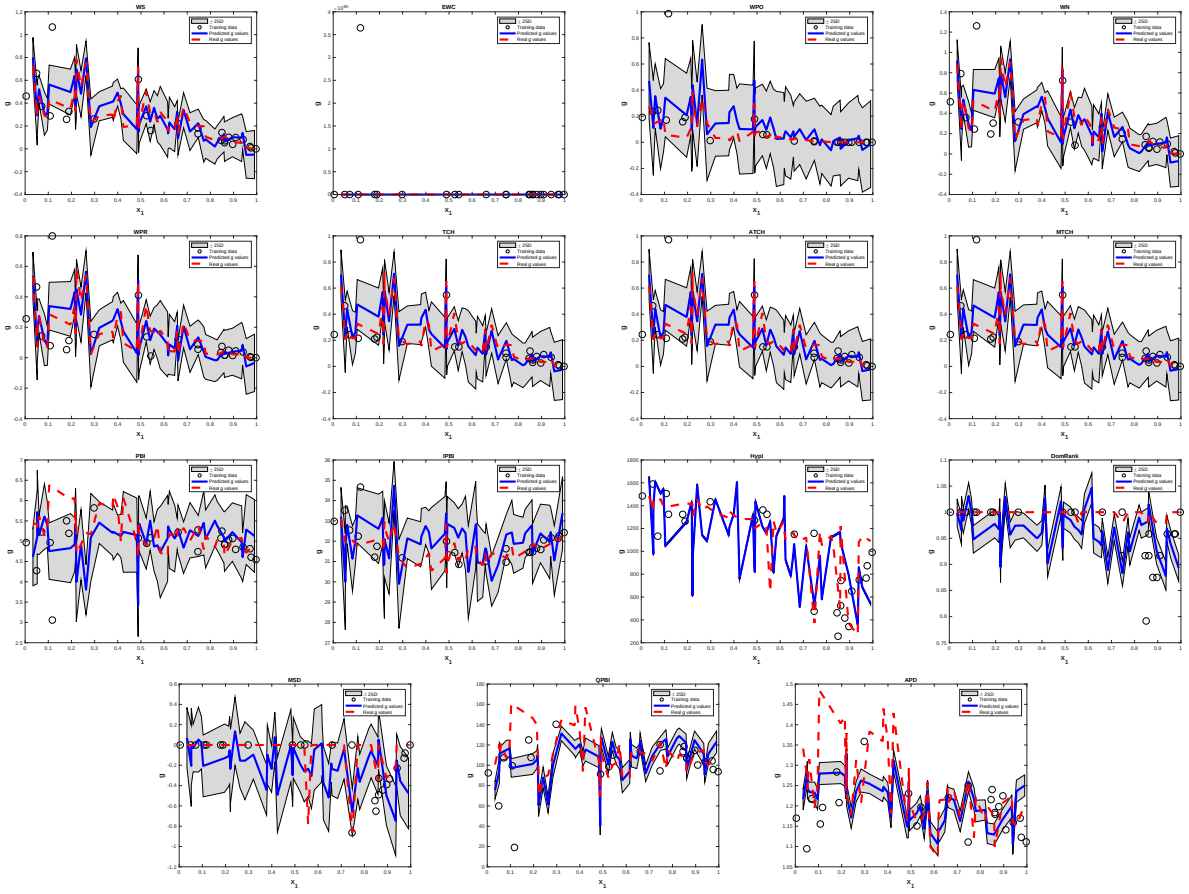


Fig. 36: Scalarizing function values (notated by g) with one decision variable value for DTLZ6 five objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

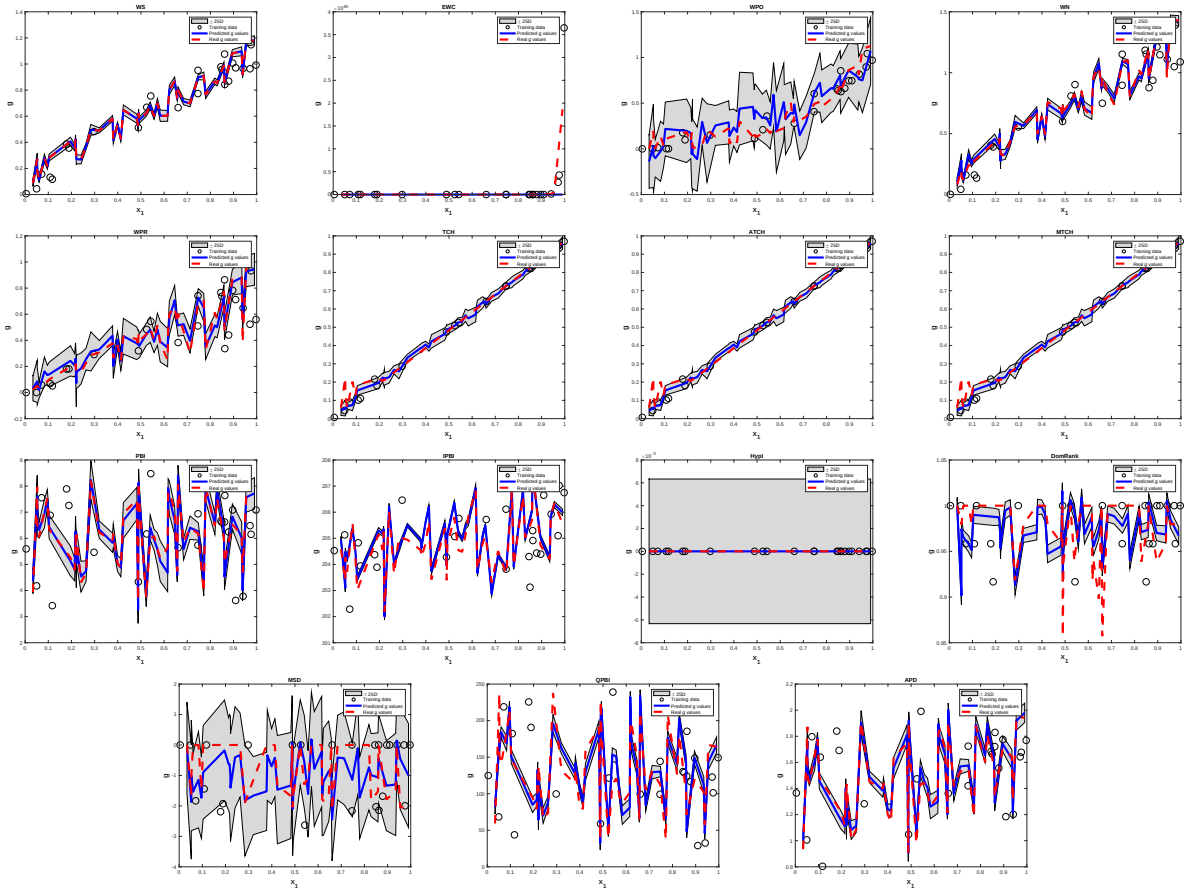


Fig. 37: Scalarizing function values (notated by g) with one decision variable value for DTLZ7 five objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

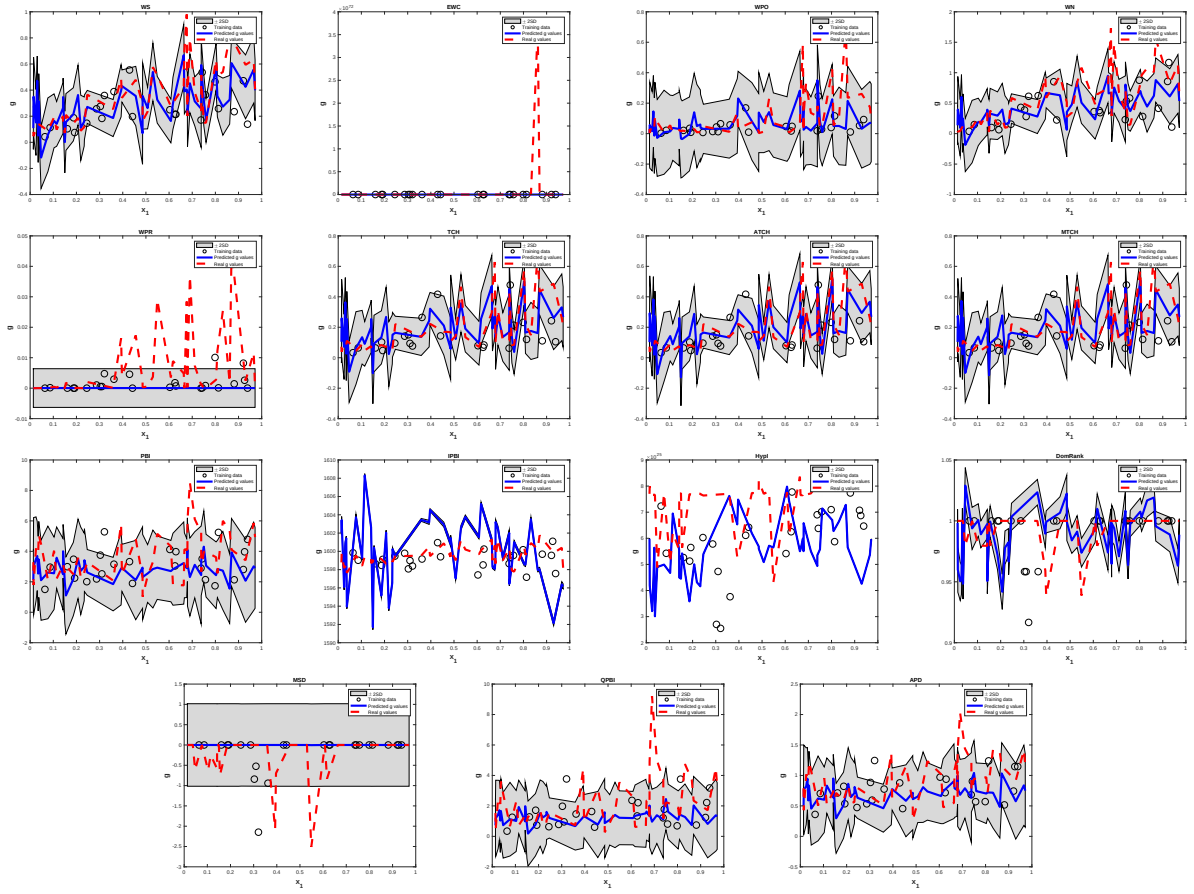


Fig. 38: Scalarizing function values (notated by g) with one decision variable value for DTLZ1 10 objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

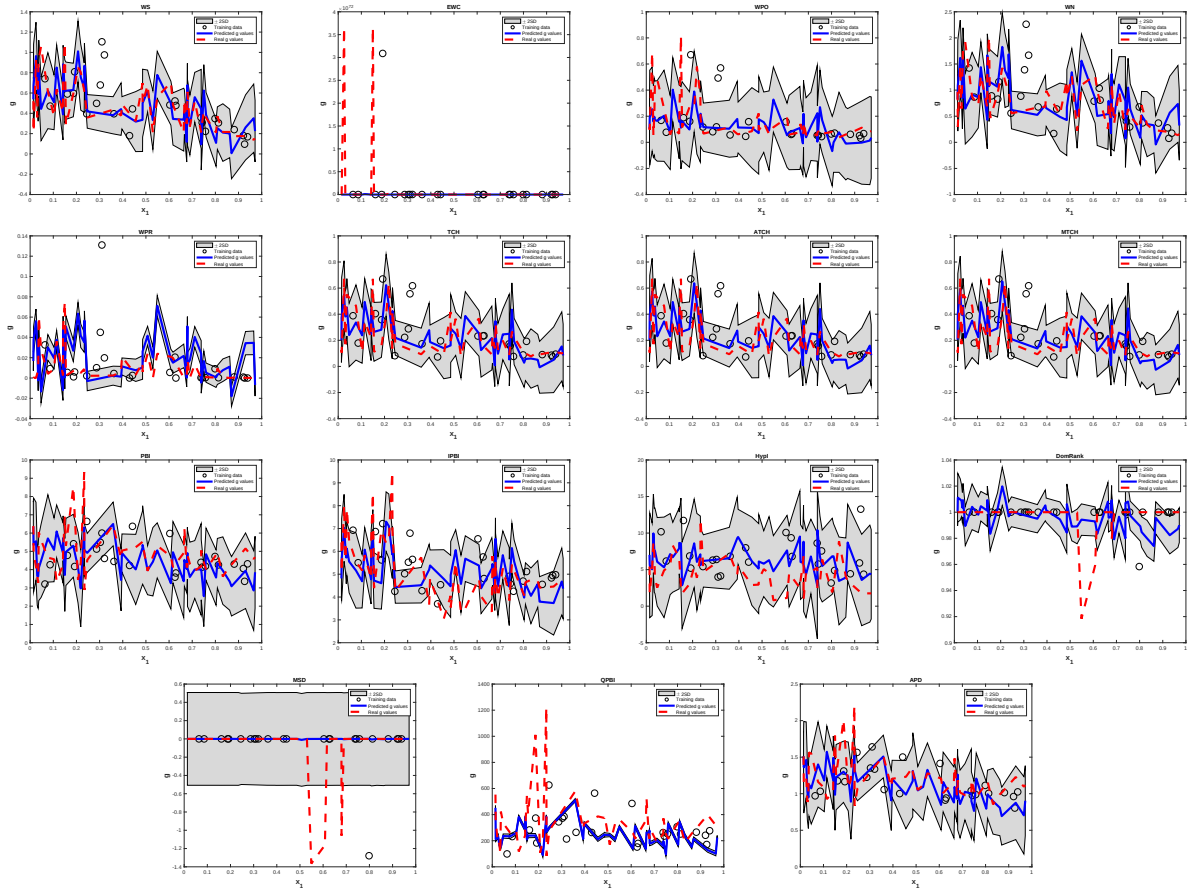


Fig. 39: Scalarizing function values (notated by g) with one decision variable value for DTLZ2 10 objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

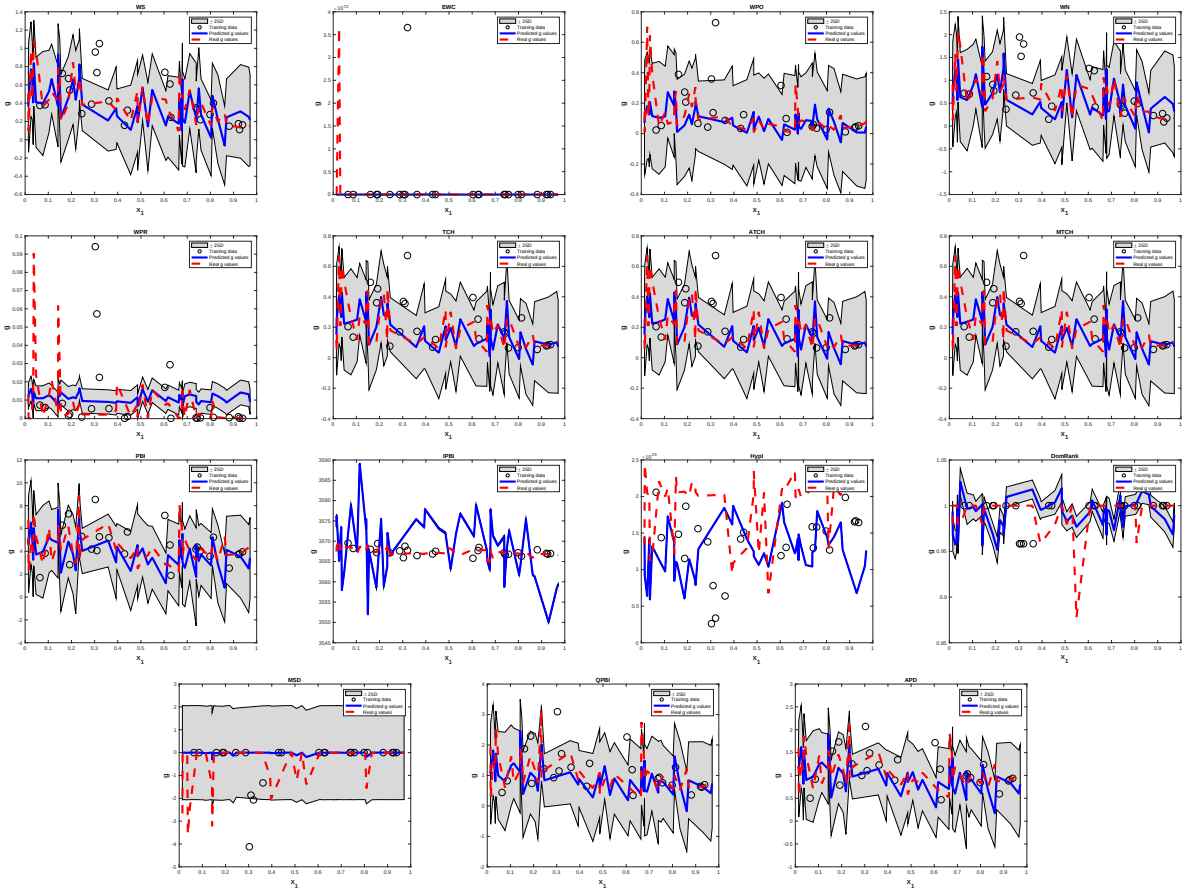


Fig. 40: Scalarizing function values (notated by g) with one decision variable value for DTLZ3 10 objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values

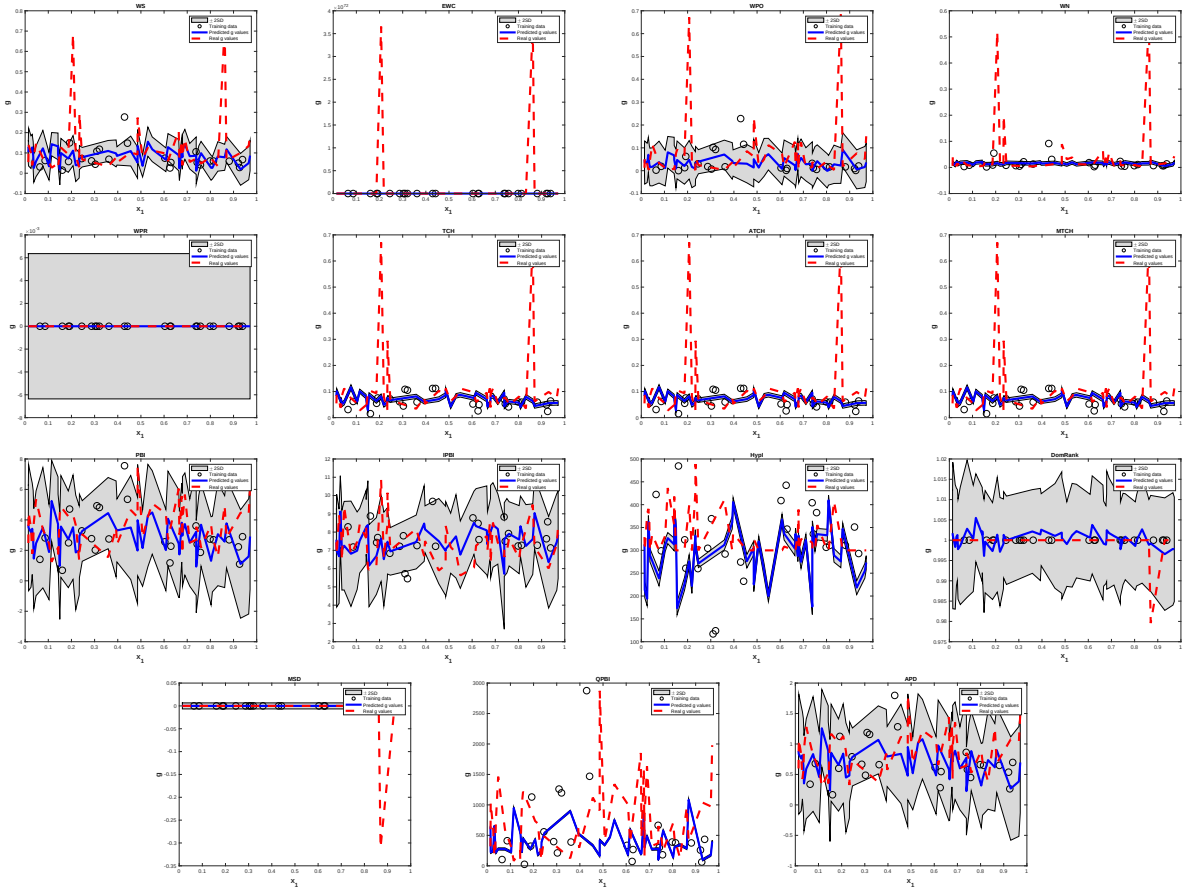


Fig. 41: Scalarizing function values (notated by g) with one decision variable value for DTLZ4 10 objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

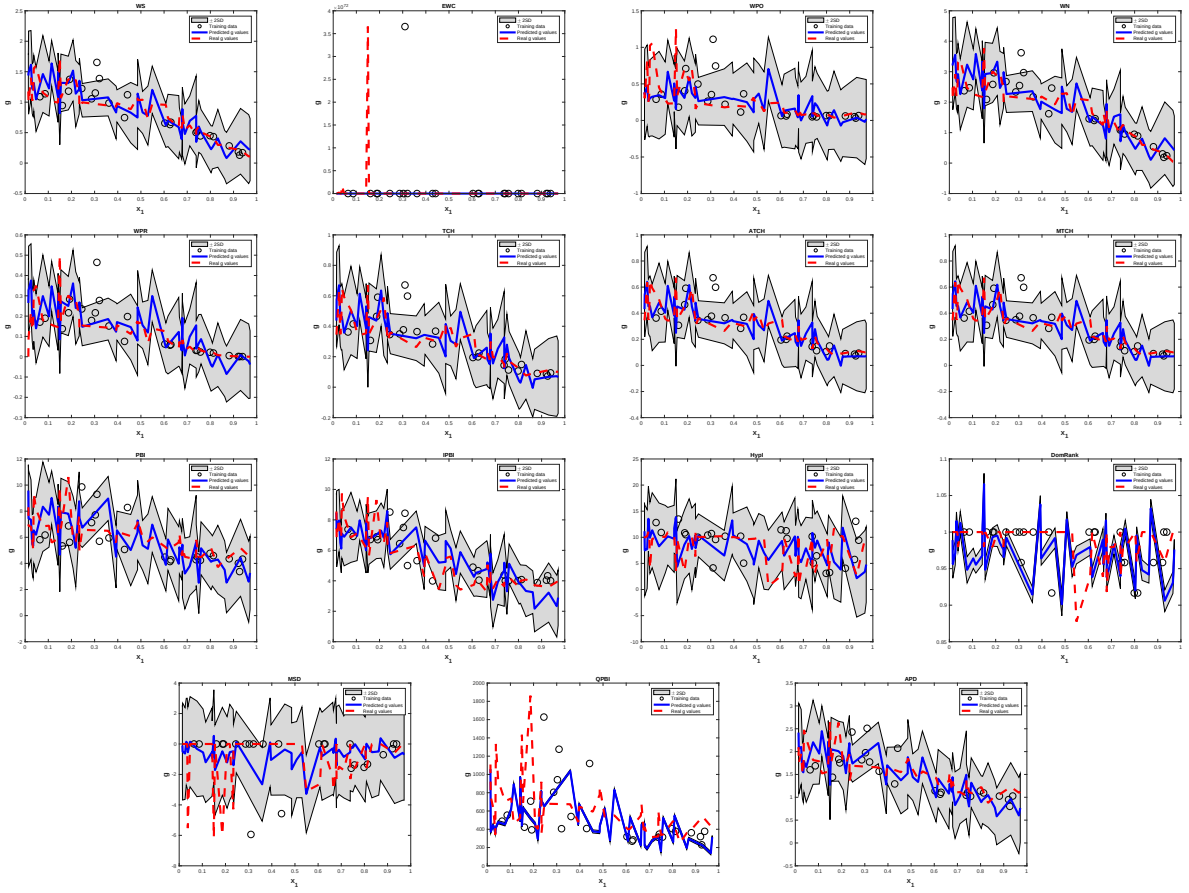


Fig. 42: Scalarizing function values (notated by g) with one decision variable value for DTLZ5 10 objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

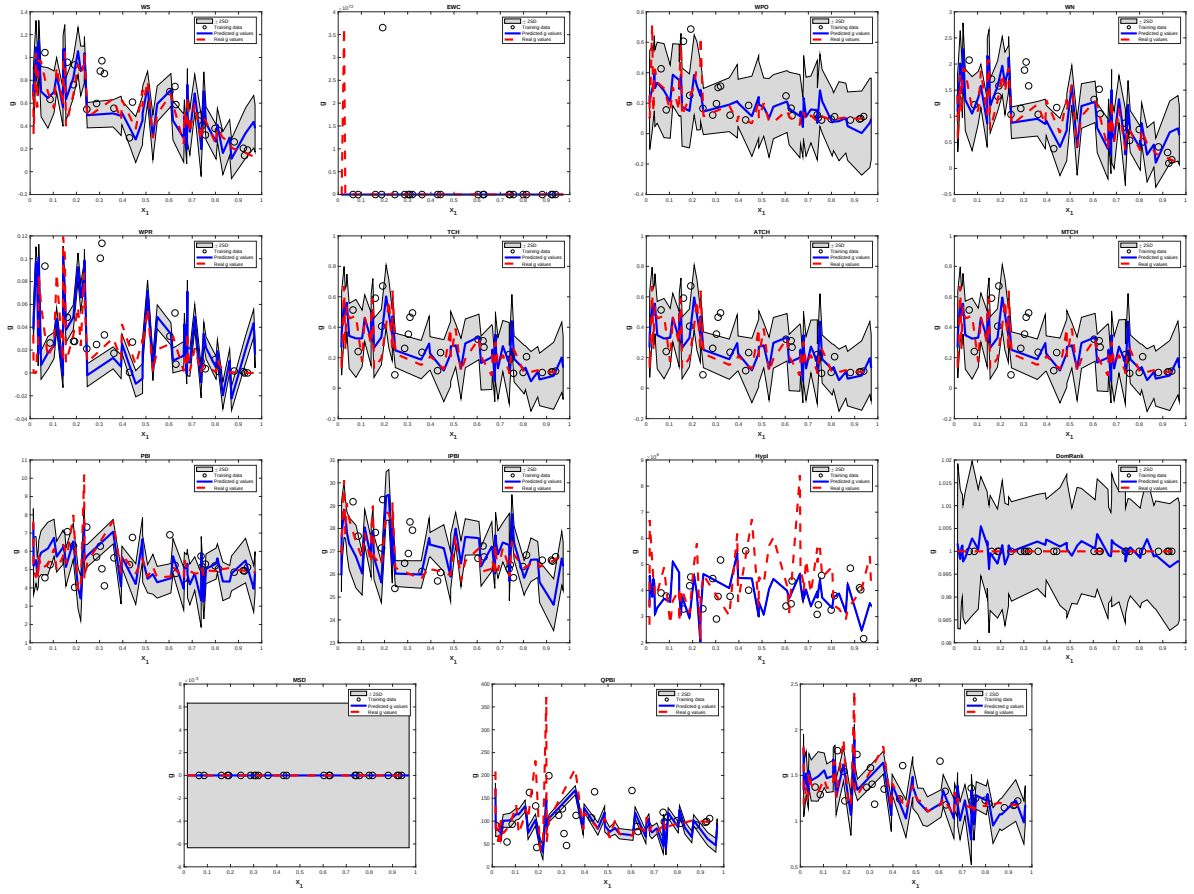


Fig. 43: Scalarizing function values (notated by g) with one decision variable value for DTLZ6 10 objectives, ±2SD represents the predicted g values with ± 2 standard deviations or uncertainty of the predicted values

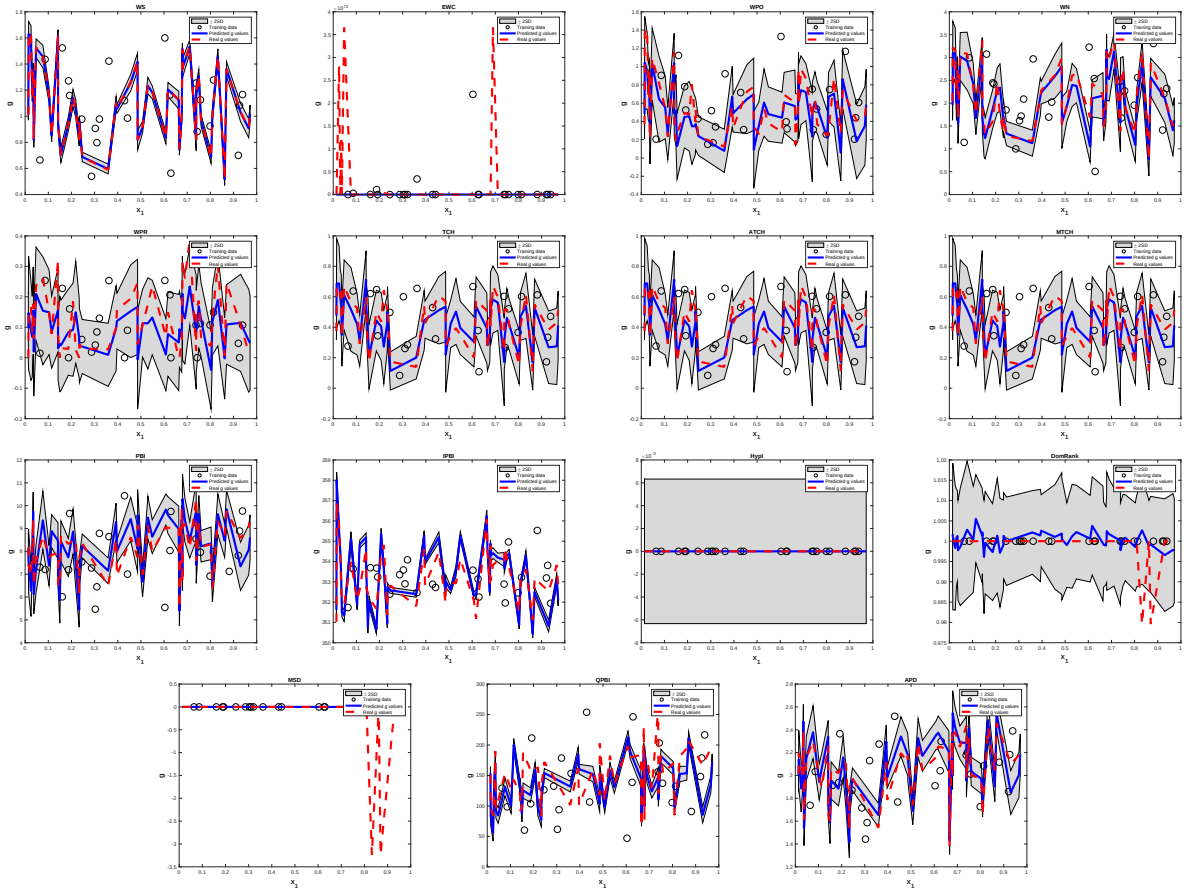


Fig. 44: Scalarizing function values (notated by g) with one decision variable value for DTLZ7 10 objectives, $\pm$2SD represents the predicted g values with $\pm$ 2 standard deviations or uncertainty of the predicted values